JEE (Main) 2020

COMPUTER BASED TEST (CBT)
Memory Based Questions & Solutions

Date: 06 September, 2020 (SHIFT-1) | TIME: (9.00 a.m. to 12.00 p.m)
Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS
1. If \( \sum_{i=1}^{n} (x_i - a) = n \) and \( \sum_{i=1}^{n} (x_i - a)^2 = na \) then the standard deviation of variate \( x_i \):

(1) \( \sqrt{a^2 - 1} \)  \hspace{1cm} (2) \( \sqrt{a - 1} \)  \hspace{1cm} (3) \( \sqrt{n^2 a - 1} \)  \hspace{1cm} (4) \( \sqrt{a^2 n^2 - n} \)

Ans. (2)

Sol. S.D. = \[ \sqrt{\frac{\sum (x_i - a)^2}{n} - \left( \frac{\sum (x_i - a)}{n} \right)^2} \]

= \[ \sqrt{\frac{na}{n} - \left( \frac{n}{n} \right)^2} = \sqrt{a - 1} \]

2. Negation of \( p \lor (q \land \sim p) \) is:

(1) \( p \land q \)  \hspace{1cm} (2) \( \sim p \lor \sim q \)  \hspace{1cm} (3) \( \sim p \lor q \)  \hspace{1cm} (4) \( \sim p \land \sim q \)

Ans. (4)

Sol. Given statement is \( p \lor (q \land \sim p) \)

\[ \therefore \text{Negation is } \sim (p \lor (q \land \sim p)) \]

= \( \sim p \land \sim q \lor \sim p \land p \)

= \( \sim p \land \sim q \lor \sim p \land q \)

= \( \sim p \land \sim q \)

3. There are three families in which 2 families has 3 members each and third family has 4 members. They are arranged in a line, then probability that members of same family are together, is:

(1) \( \frac{1}{700} \)  \hspace{1cm} (2) \( \frac{3}{700} \)  \hspace{1cm} (3) \( \frac{3}{720} \)  \hspace{1cm} (4) \( \frac{9}{730} \)

Ans. (1)

Sol. \( P(A) = \frac{3 \times 3! \times 3 \times 4!}{10!} = \frac{6 \times 6 \times 6}{10 \times 9 \times 8 \times 7 \times 6 \times 5} = \frac{1}{700} \)

4. If roots of quadratic equation \( x^2 - 64x + 256 = 0 \) are \( \alpha \) & \( \beta \) then \( \left( \frac{\alpha^3}{\beta^2} \right)^\frac{1}{6} + \left( \frac{\beta^3}{\alpha^2} \right)^\frac{1}{6} = \)

(1) 2  \hspace{1cm} (2) 6  \hspace{1cm} (3) -2  \hspace{1cm} (4) 5

Ans. (1)

Sol. \( \alpha + \beta = 64, \alpha \beta = 256 \)

\[ \text{Now } = \left( \frac{\alpha^3}{\beta^2} \right)^\frac{1}{6} + \left( \frac{\beta^3}{\alpha^2} \right)^\frac{1}{6} \]
\[
\frac{3^\frac{3}{8} \cdot 5^\frac{3}{8}}{(\alpha \beta)^{1/6}} = \frac{\alpha + \beta}{(\alpha \beta)^{1/3}} = \frac{64}{(256)^{1/8}} = \frac{64}{32} = 2
\]

5. \(\lim_{x \to 1} \frac{(x-1)^2 \cos \theta}{(x-1)\sin(x-1)}\) is equal to

\begin{align*}
(1) & \quad 2 & (2) & \quad 0 & (3) & \quad 1 & (4) & \text{Does not exist} \\
\end{align*}

Ans. (2)

Sol.
\[
\lim_{x \to 1} \frac{2(x-1) \cdot (x-1)^2 \cos(x-1)^2}{2(x-1)} = 0
\]

6. Let \(I_1 = \int_0^1 (1-x^{50})^{100} \, dx\) and \(I_2 = \int_0^1 (1-x^{50})^{101} \, dx\)

and \(I_1 = \lambda \cdot I_2\), then \(\lambda\) is

\begin{align*}
(1) & \quad \frac{5051}{5050} & (2) & \quad \frac{5050}{5051} & (3) & \quad 1 & (4) & \frac{5049}{5050} \\
\end{align*}

Ans. (1)

Sol.
\[
\lambda = \frac{\frac{1}{0} (1-x^{50})^{100} \, dx}{\frac{1}{0} (1-x^{50})^{101} \, dx} = \frac{I_1}{I_2}
\]

\[
I_2 = \int_0^1 (1-x^{50})^{100} \, dx
\]

\[
I_2 = I_1 \int_0^1 (1-x^{50})^{100} \, dx
\]

\[
I_2 = I_1 - \left[ \frac{-x(1-x^{50})^{100}}{5050} \right]_0^1 = \frac{1}{5050} \int_0^1 (1-x^{50})^{100} \, dx
\]

\[
I_2 = I_1 - \frac{I_2}{5050}
\]

\[
\Rightarrow \lambda = \frac{I_1}{I_2} = \frac{5051}{5050}
\]
7. If \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) & \( \mathbf{d} \) are position vectors of point A, B, C and D respectively in 3-D space no three of A, B, C, D are collinear and satisfy the relation \( 3\mathbf{a} - 2\mathbf{b} + \mathbf{c} - 2\mathbf{d} = 0 \) then

(1) A, B, C, and D coplanar
(2) the line joining points B and D divides the line joining points A and C in the ratio 2 : 1
(3) the line joining points A and C divides the line joining points B and D in the ratio 1 : 2
(4) the four vectors \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) & \( \mathbf{d} \) are linearly independent

**Ans.** (1)

**Sol.**

\[
3\mathbf{a} + \mathbf{c} = 2\mathbf{b} + \mathbf{d}
\]

\[
\frac{3\mathbf{a} + \mathbf{c}}{3 + 1} = \frac{\mathbf{b} + \mathbf{d}}{2}
\]

Point P divides the line joining A and C in ratio 1 : 3 and bisects the line joining B and D.