1. If the point \( P \left( -\frac{5}{2}, 3 \right) \) lies on ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) whose eccentricity is \( \frac{1}{4} \), then the value of \( a^2 + b^2 \) is equal to

- (1) 30
- (2) 31
- (3) 29
- (4) 41

Ans. (2)
We know that, eccentricity of ellipse

\[ e = \sqrt{1 - \frac{b^2}{a^2}} \]

\[ e = \frac{1}{4} = \sqrt{1 - \frac{b^2}{a^2}} \]

\[ \Rightarrow \frac{1}{16} = 1 - \frac{b^2}{a^2} \]

\[ b^2 = \frac{15a^2}{16} \]

The point \( P \left( -\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}} \right) \) lie on ellipse

so put \( \frac{x^2}{a^2}, \frac{y^2}{b^2} = 1 \)

\[ \frac{x^2}{a^2} = 1 - \frac{2}{5} \]

\[ \frac{9}{b^2} = 1 \]

\[ \frac{32}{5a^2} + \frac{9}{b^2} = 1 \]

\[ b^2 = \frac{15a^2}{16} \]

\[ \Rightarrow \frac{32}{5a^2} + \frac{12}{5a^2} = 1 \]

\[ 16 \left[ \frac{3 \times 2 + 9}{a^2} \right] = 1 \]

then the value of \( a^2 + b^2 = 31 \)

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2. The value of \( \cos \left( \frac{2\pi}{7} \right) + \cos \left( \frac{4\pi}{7} \right) + \cos \left( \frac{6\pi}{7} \right) \) is

(1) \(-1\)  (2) \(-\frac{1}{2}\)  (3) \(-\frac{1}{3}\)  (4) \(-\frac{1}{4}\)

Ans. (2)

Sol.
\[ \cos \left( \frac{2\pi}{7} \right) + \cos \left( \frac{4\pi}{7} \right) + \cos \left( \frac{6\pi}{7} \right) \]

\[ \Rightarrow \cos \left( \frac{2\pi}{7} \right) + \cos \left( \frac{2\pi}{7} \right) + \sin \left( \frac{3 \times 2\pi}{7} \right) \]

\[ \Rightarrow -\cos \left( \frac{2\pi}{7} \right) \sin \left( \frac{2\pi}{7} \right) \]

\[ \Rightarrow \sin \left( \frac{2\pi}{7} \right) \]

\[ \Rightarrow -2 \sin \left( \frac{2\pi}{7} \right) \cos \left( \frac{3\pi}{7} \right) \]

\[ \Rightarrow \frac{1}{2} \sin \left( \frac{6\pi}{7} \right) \]

\[ \Rightarrow \frac{6\pi}{7} \]

\( \Rightarrow \)
3. The value of \( \sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{5\pi}{4}\right) \) is equal to

\[
(1) \frac{11\pi}{12} \quad (2) \frac{\pi}{4} \quad (3) \frac{5\pi}{3} \quad (4) \frac{7\pi}{6}
\]

**Ans.** (1)

**Sol.**
\[
\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{5\pi}{4}\right)
\]
\[
= \sin^{-1}\left(\sin\frac{\pi}{3}\right) + \cos^{-1}\left(-\cos\frac{\pi}{6}\right) + \tan^{-1}\left(-\tan\frac{\pi}{4}\right)
\]
\[
= \frac{\pi}{3} + \frac{\pi}{6} - \frac{\pi}{4}
\]
\[
= \frac{11\pi}{12}
\]

---

4. Five numbers \(x_1, x_2, x_3, x_4, x_5\) are selected randomly from set \(\{1, 2, 3, \ldots, 18\}\) such that \(x_1 < x_2 < x_3 < x_4 < x_5\) then the probability that \(x_2 = 7\) and \(x_4 = 11\) is.

\[
(1) \frac{1}{34} \quad (2) \frac{1}{68} \quad (3) \frac{1}{36} \quad (4) \frac{1}{17}
\]

**Ans.** (2)

**Sol.**
Total ways = \(18C_5\)
Since \(x_1 \in [1, 6], x_3 \in [8, 10], x_5 \in [12, 18]\)
Favourable ways = \(6C_1 \times 3C_1 \times 1C_1\)
Probability = \(\frac{6C_1 \times 3C_1 \times 1C_1}{18C_5}\)

\[
= \frac{\frac{6 \times 3 \times 7}{3 \times 2 \times 1}}{\frac{18 \times 17 \times 16 \times 15 \times 14}{5 \times 4 \times 3 \times 2 \times 1}} = \frac{1}{68}
\]

5. Number of real roots of equation \(x^4 - 4x + 1 = 0\) is

\[
(1) 4 \quad (2) 2 \quad (3) 1 \quad (4) 0
\]

**Ans.** (2)

**Sol.**
Let \(f(x) = x^4 - 4x + 1\)
\(f'(x) = 4x^3 - 4\)
f'(0) = 0 \Rightarrow x = 1\)
f(1) = 2 and \(f(0) = 1\)
Two real roots

6. There are 16 identical cubes in which 5 are red and 11 are blue, the number of ways in which these cubes can be arranged in a row such that between any two red cubes there are at least two blue cubes, is

**Ans.** (56)

**Sol.**
\(x_1R_1x_2R_2x_3R_3x_4R_4x_5\)
x_1 + x_2 + x_3 + x_4 + x_5 = 11
\geq 0 \geq 2 \geq 2 \geq 2 \geq 0\)
where \(x_1, x_2 \geq 0\) and \(x_2, x_3, x_4, x_5 \geq 2\)

\[
x_1 + x_2 + x_3 + x_4 + x_5 = 11\]

\[
\text{Number of ways} = \binom{11 + 5 - 1}{5 - 1} = \binom{15}{4} = 56
\]
7. If \( f(x) = \frac{2e^{-2x}}{e^{2x} - e} \) then the value of \( \frac{1}{100} \cdot f\left(\frac{1}{100}\right) + \frac{2}{100} \cdot f\left(\frac{2}{100}\right) + \ldots + f\left(\frac{99}{100}\right) \) is

(1) 98  (2) 99  (3) 100  (4) 97

Ans. (2)

Sol.
\[
f(1-x) + f(x) = \frac{2e^{2x-1}(x-1)}{e^{2x-1} + e} + \frac{2e^{2x-1}}{e^{2x-1} + e} = \frac{2e^{2x}}{e^{2x} + e} + \frac{2e^{2x}}{e^{2x} + e} = \frac{2e}{e^{2x} + e} + \frac{2e}{e^{2x} + e}
\]

\[
f(1-x) + f(x) = 2
\]

Now
\[
\frac{f\left(\frac{1}{100}\right) + f\left(\frac{99}{100}\right) + 2 \cdot f\left(\frac{2}{100}\right) + \ldots + f\left(\frac{100}{100}\right)}{2}
\]

\[
= \frac{2 \cdot 1 + 2 \cdot 2 + \ldots + 2 \cdot 100}{49}
\]

\[
= 98 + 1 = 99
\]

8. In a binomial distribution \( n = 7 \) and \( P(x = 3) = 5P(x = 4) \), then the value of sum of mean and variance is

(1) 14  (2) 27  (3) 31  (4) 35

Ans. (2)

Sol.
Given \( P(x = 3) = 5P(x = 4) \)

\( C_3 P = 5(C_4 P) \)

\( q = 5P \)

\( p + q = 1 \) \( \ldots (1) \)

\( 6p - 1 = 0 \) \( \ldots (2) \)

\( p = \frac{1}{6} \)

\( q = \frac{5}{6} \)

\( \text{mean} = np = 7 \cdot \frac{1}{6} = \frac{7}{6} \)

\( \text{variance} = npq = \frac{7}{6} \cdot \frac{5}{6} \cdot \frac{35}{36} \)

\( \text{sum of mean and variance} = \frac{7}{6} + \frac{35}{36} = \frac{42}{36} + \frac{35}{36} = \frac{77}{36} \)
9. A rectangle whose two adjacent vertices are \((1, 2)\) and \((3, 6)\) is inscribed in a circle. If the equation of a diameter of circle is \(2x - y + 4 = 0\), then the area of the rectangle is

\[
\begin{align*}
\text{(1) } & 6 \\
\text{(2) } & 8 \\
\text{(3) } & 16 \\
\text{(4) } & 20
\end{align*}
\]

**Ans.** \(\text{(3)}\)

**Sol.**

\[
\text{Length of AB} = \sqrt{2^2 + 4^2} = 2\sqrt{5}
\]

Slope of AB = \(\frac{6-2}{3-1} = 2\)

\[
\Rightarrow AB \parallel \text{diameter } 2x - y + 4 = 0
\]

\[
AD = 2(\text{AE}) = 2 \times \frac{8}{\sqrt{2^2 + 1^2}} = \frac{8}{\sqrt{5}}
\]

Area of rectangle = \(\frac{8}{\sqrt{5}} \times 2\sqrt{5} = 16\)

**10.** If the coefficient of \(x^{10}\) in the expansion of \(\left(\frac{x^{1/2} + 5^{1/2}}{x^{1/2}}\right)^{80}\), where \(i\) is co-prime with 5 then value of \(k\) is

**Ans.** \(\text{(05,00)}\)

**Sol.**

\[
T_{11} = 60C_i = \left(\frac{x^{1/4}}{x^{1/2}}\right)^{60-i} \left(\frac{5^{1/2}}{x^{1/2}}\right)^i
\]

\[
\frac{60 - i}{2} = \frac{i}{3} = 10
\]

\[
180 - 3r = 2r = 60
\]

\[5r = 120, r = 24
\]

\[
T_{24+1} = 60C_{24} \left(\frac{1}{5^{1/2}}\right)^{36} \left(\frac{5^{1/2}}{x^{1/2}}\right)^{24} = 60C_{24} \times 5^3
\]

Exponent of 5 in \(60!\) = \(\text{Int}\left(\frac{60}{5}\right) + \text{Int}\left(\frac{60}{25}\right) = 12 + 2 = 14\)

Exponent of 5 in \(24!\) = \(\text{Int}\left(\frac{24}{5}\right) = 4\)

Exponent of 5 in \(36!\) = \(\text{Int}\left(\frac{36}{5}\right) + \text{Int}\left(\frac{36}{25}\right) = 7 + 1 = 8\)

Exponent of 5 in \(60C_{24}\) = \(14 - 4 - 8 = 2\)

\[k = 2 + 3 = 5\]

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**11.** If the non-real roots of \(z^2 = i\) the vertices of a polygon then the area of the polygon is

\[
\begin{align*}
\text{(1) } & \frac{\sqrt{3}}{4} \\
\text{(2) } & \frac{\sqrt{3}}{2} \\
\text{(3) } & \frac{\sqrt{5}}{2} \\
\text{(4) } & \frac{\sqrt{5}}{4}
\end{align*}
\]

**Ans.** \(\text{(2)}\)

**Sol.**

\[
z^2 = i
\]

\[
|z|^2 = |i|
\]

\[
|z| = 1
\]

\[
z^3 = 1
\]
12. If \( \frac{dy}{dx} \left( \frac{2^x - 1}{2^x - 1} \right) = 0 \) and \( y(1) = 1 \) then the value of \( y(2) \) is

\[ \begin{align*}
(1) \log_2 \left( \frac{3}{4} \right) \\
(2) \log_2 \left( \frac{3}{4} \right) \\
(3) \log_3 \left( \frac{4}{3} \right) \\
(4) \log_3 \left( \frac{4}{3} \right)
\end{align*} \]

Ans. (3)

Sol.

\( \frac{2^y}{2^y - 1} \frac{dy}{dx} = \frac{2^x}{2^x - 1} \frac{dx}{dx} = 0 \)

\( \ln(2^y - 1) + \ln(2^x - 1) = \text{Inc} \)

\( (2^y - 1)(2^x - 1) = c \)

given \( y(1) = 1 \)

\( x = 1, y = 1 \)

then \( c = 1 \)

now \( x = 2 \)

\( (2^y - 1)(3) = 1 \)

\( 2^y - 1 = \frac{1}{3} \)

\( 2^y = \frac{4}{3} \)

\( y = \log_2 \left( \frac{4}{3} \right) \)

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13. If \( \int e^{x^2 - 1} \frac{dx}{(x + 1)^2} \) then the value of \( \frac{d^3}{dx^3} \left( f(x) \right) \) at \( x = 1 \) is

\[ \begin{align*}
(1) \frac{1}{4} \\
(2) \frac{3}{4} \\
(3) \frac{5}{4} \\
(4) 1
\end{align*} \]

Ans. (2)

Sol.

\begin{align*}
\int e^{x^2 - 1} \frac{dx}{(x + 1)^2} \\
\int e^{x - 1} \frac{2}{x + 1} \frac{dx}{(x + 1)^2}
\end{align*}

\[ f(x) = \frac{e^{x - 1}}{x + 1} + C \]

\( f(x) = \frac{x - 1}{x + 1} \)

\( f'(x) = \frac{2}{(x + 1)^2} \)

\( f''(x) = \frac{4}{(x + 1)^3} \)

\( f'''(x) = \frac{12}{(x + 1)^4} \)

\( f''''(x) = \frac{24}{(x + 1)^5} \)
14. Evaluate the integral $\int_2^6 \frac{x^3 + 1}{e^{4x} + 1} dx$.

Ans. \(3\)

Sol. Let $F(x) = \frac{x^3 + 1}{e^{4x} + 1}$.

So $F(-x) = \frac{x^3 + 1}{e^{-4x} + 1}$.

Therefore, the integral becomes:

$$\int_2^6 F(x)dx = \int_0^2 F(x) + F(-x) dx = \int_0^2 x^3 dx$$

$$= \left[ \frac{x^4}{4} \right]_0^2 = \left( \frac{16}{4} - 0 \right) = 6$$

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15. The sides of triangles are $10 + x$, $10 + x$, $20 - 2x$, if the area of the triangle is maximum at $x = k$ then the value of $3k^2$ is

(1) 3  (2) 6  (3) 7  (4) 10

Ans. (4)

Sol. 

$$A = \frac{1}{2} (20 - 2x^2) \sqrt{(10 + x)^2 - \frac{(20 - 2x^2)}{2}}$$

$$= \sqrt{(10 - x^2)(10 + x^2) - (10 - x^2)^2}$$

$$= \sqrt{100 - x^4 + x^2}$$

Differentiating,$$
\frac{dA}{dx} = \frac{-2x}{\sqrt{100 - x^4 + x^2}}$$

Maxima $x = \sqrt{10}$ is

$3k^2 = 3 \cdot \frac{10}{3} = 10$

16. If the sum of series $\frac{1}{5}$, $\frac{2}{65}$, $\frac{3}{325}$, $\frac{4}{1025}$, $\frac{5}{2061}$, $\ldots$ upto 10 terms is $\frac{m}{n}$, where $m$ & $n$ are co-prime, then the value of $m + n$ is

Ans. 16.00

Sol. 

$$S = \frac{10}{4^2 + 1} + \frac{10}{(2^2 + 1)(2^2 - 2) + 1}$$

$$= \frac{10}{2^2 + 1} - \frac{1}{2^2 - 2} + 1$$

$$= \frac{1}{1} \left[ \frac{1}{5} \right] + \frac{1}{13} \left[ \frac{1}{13} \right] \frac{1}{25} + \ldots$$

$$= \frac{1}{1} \left[ \frac{1}{5} \right] \frac{1}{221}$$

$$= \frac{220}{221} \cdot \frac{1}{4} \cdot \frac{55}{221} \cdot \frac{5}{11} \cdot \frac{n}{m}$$

so, the value of $m + n = 5 + 11 = 16$

17. In an isosceles triangle ABC, with $AB = AC$, vertex $A(6, 1)$, base $BC$ is $2x + y = 4$ and point $B$ lies on $x + 3y = 7$, if centroid of triangle is $(\alpha, \beta)$ find the value of $15(\alpha + \beta)$ is
(1) 43  (2) b1  (3) 4y  (4) 52

Ans. (2)

Sol. Let C be \((x, y)\)

B is point of intersection of \(2x + y = 4\) and \(x + 3y = 7\)

\[ \Rightarrow B(1, 2) \]

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\[ A(6, 1) \]

\[ B \]

\[ C(x, y) \]

\[ G(\alpha, \beta) \]

\[ \therefore \text{ABC is isosceles AG \parallel BC} \]

\(\text{(Slope of AG)}(\text{Slope of BC}) = -1\)

\[ \Rightarrow \frac{\beta - 1}{\alpha - 6} = -1 \Rightarrow \alpha - 2\beta = 4 \quad \ldots \ldots \ldots (1) \]

Now centroid \(G(\alpha, \beta) = \left(\frac{x_1 + 7, y_1 + 3}{3}, \frac{3}{3}\right)\)

\[ \Rightarrow C(x, y) = (3\alpha - 7, 3\beta - 3) \text{ lies on } 2x + y = 4 \]

\[ \Rightarrow 2(3\alpha - 7) + 3\beta - 3 = 4 \]

\[ \Rightarrow 2\alpha + \beta = 7 \quad \ldots \ldots \ldots (2) \]

from (1) and (2)

\[ \alpha = \frac{18}{5}; \beta = \frac{-1}{5} \]

\[ \alpha + \beta = \frac{17}{5} \Rightarrow 15(\alpha + \beta) = 51 \]

18. Which of the following is logically equivalent to \((2(p \land q) \land q) \land q\)

\[ p \rightarrow q \quad (2) p \rightarrow (p \land q) \quad (3) p \rightarrow (p \lor q) \quad (4) p \rightarrow (p \lor q) \]

Ans. (3)

Sol.

\[ \sim (p \land q) \land q = \sim (p \land \sim q) \land q \]

\[ = \sim p \lor (\sim q \land q) \]

\[ = \sim p \lor \perp \perp \perp \perp \perp \perp \]

Option (C) \(p \rightarrow (p \land q) \land q \rightarrow (p \land q) \land q \land q = \perp \perp \perp \perp \perp \perp \perp \perp \perp \perp \)
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