Chapter 4: DETERMINANTS

Exercise - 4.1

Q1. Evaluate the determinants in Exercises 1 and 2.

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$$

A.1. $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 2\times(-1) - 4\times(-5) = -2+20 = 18$

Q2. **(i)** $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ **(ii)** $\begin{vmatrix} x^2 - x + 1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

A.2. **(i)** $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

$= \cos\theta \times \cos\theta - (-\sin\theta)\times\sin\theta$

$= \cos^2\theta + \sin^2\theta$

$=1$

(ii) $\begin{vmatrix} x^2 - x + 1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

$= (x^2 - x + 1)(x+1) - (x-1)(x+1)$

$= x^3 + x^2 - x^2 - x + x + 1 - x^2 + 1$

$= x^3 - x^2 + 2$

Q3. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that | 2A | = 4 | A |

A.3. Given, $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

So, $2A = 2\begin{bmatrix} 1 & 4 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$.

L.H.S. $= |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 2\times4 - 4\times8 = 8-32 = -24$

R.H.S. $= 4|A| = 4\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 4(1\times2 - 2\times4)$

$= 4[2 - 8]$

$= 4[-6] = -24.$

L·H·S = R·H·S

Q4. **If A=** $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ **,then show that | 3 A | = 27 | A |**

A.4. Given, A= $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

Then, |A|= $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

$= 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$

$= 4\times1 - 2\times0$

$= 4$

And 3A= $3\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$

L·H·S·= |3A|= $\begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$

$= 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$

$=3[36 - 6\times0]$

$= 108$

R·H·S = 27. |A| = 27× 4 = 108

∴ L·H·S· = R·H·S·

Q5. Evaluate the determinants

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ **(ii)** $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

(ii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ **(iv)** $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

A.5. **(i)** $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

$= 3\begin{vmatrix} 0 & -1 \\ -5 & 0 \end{vmatrix} -(-1)\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} + (-2)\begin{vmatrix} 0 & 0 \\ 3 & -5 \end{vmatrix}$

$= 3\left[0(-5)(-1)\right] + 1(0-3(-1) - 2[0\times(-5) - 0\times 3]$

$= -15 + 3$

$= -12$

(ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

$= 3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} - (-4)\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$

$=3[1\times1-(-2)\times3]+4[1\times1-(-2)\times2]+5[3\times1-2\times1]$

$= 3[1+6]+4[1+4]+5(3-2)$

$= 3\times7+4\times5+5\times1$

$= 21+20+5$

$= 46$

(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

$= \begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} - 1\begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2\begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix}$

$= 0 - 1\ [-1\times0 - (-2)\times(-3)] + 2[-1\times3 - (-2)\times0]$

$= (-1)\times(-6)+2(-3)$

$= 6 - 6$

$= 0$

(iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

$= 2\begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - 0\begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} + 3\begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix}$

$= 2[2\times0-(-5)\times(-1)]+3[(-1)\times(-1)-(-2)\times2]$

$= 2(-5)+3\ (1+4)$

$= 2(-5) + 3\times(5)$

$= -10+15$

$= 5$

Q6. **If A=** $\begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$ **,find | A |**

A.6. A= $\begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$

So,|A|= $\begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$

$= 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$

$= 1[1\times(-9) - 4\times(-3)] - 1[2\times(-9) - 5\times(-3)] - 2[2\times4 - 5\times1]$

$= 1[-9 + 12] - 1[-18+15] - 2[8 - 5]$

$= 3 + 3 - 6$

$= 0$

Q7. Find values of x, if

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ **(ii)** $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

A.7. **(i)** $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2 \times 1 - 4 \times 5 = 2x \times x - 6 \times 4$

$\Rightarrow 2 - 20 = 2x^2 - 24$

$\Rightarrow 24 - 18 = 2x^2$

$\Rightarrow x^2 = \frac{6}{2}$

$\Rightarrow x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow 2 \times 5 - 4 \times 3 = 5 \times x - 2x \times 3$

$\Rightarrow 10 - 12 = 5x - 6x.$

$\Rightarrow -2 = -x$

$\Rightarrow x = 2.$

Q8. **If** $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$**, then x is equal to**

(A) 6 **(B) ± 6** **(C) – 6** **(D) 0**

A.8. $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

=>x x x– 1 8 × 2 = 6 × 6– 18 × 2

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm\sqrt{36}$

$\Rightarrow x = \pm 6$

∴ Option B is correct

Exercise - 4.2

Using the property of determinants and without expanding in Exercises 1 to 7, prove that:

Q1. $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$

A.1. L·H·S = $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$

Applying $C_1 \to C_1 + C_2$

= $\begin{vmatrix} x+a & a & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix}$ (∴ C_1 and C_2 are same or proportional)

= 0 = R·H·S·

Q2. $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

A.2. L·H·S = $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

Applying = $C_1 \to C_1 + C_2 + C_3$

= $\begin{vmatrix} a-b+b-c+c-a & b-c & c-a \\ b-c+c-a+a-b & c-a & a-b \\ c-a+a-b+b-c & a-b & b-c \end{vmatrix}$

= $\begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}$ (C_1=0 ∴ $|A|$=0))

$= 0 =$ R·H·S·

Q3. $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$

A.3. L·H·S·$= \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

$= \begin{vmatrix} 2 & 7 & 65-2 \\ 3 & 8 & 75-3 \\ 5 & 9 & 86-5 \end{vmatrix}$ applying $C_3 \to C_3 - C_1$

$= \begin{vmatrix} 2 & 7 & 63 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix}$

$= 9 \begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix}$ taking 9 common from C_3.

$= 9 \times 0$ $(\because C_2 = C_3)$

$= 0$

Q4. $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

A.4. L·H·S$= \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$

$= \begin{vmatrix} 1 & bc & ab+ac \\ 1 & ca & bc+ab \\ 1 & ab & ac+bc \end{vmatrix}$

$= \begin{vmatrix} 1 & bc & ab+bc+ac \\ 1 & ca & ab+bc+ac \\ 1 & ab & ab+bc+ac \end{vmatrix}$ applying $C_3 \to C_3 + C_2$.

$= (ab+bc+ac) \begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix}$ {taking $(ab+bc+ac)$ from C_3}

$= (ab+bc+ac) \times 0 (\because C_1 = C_3)$

$= 0 =$ R.H.S

Q5. $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

A.5. L.H.S = $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z-1x \\ a+b & p+q & x+y \end{vmatrix}$

$= \begin{vmatrix} b+c+c+a+a+b & q+r+r+p+p+q & y+z+z+x+x+y \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$ applying $R_1 \to R_1+R_2+R_3$

$= \begin{vmatrix} 2(a+b+c) & 2(p+q+r) & 2(x+y+z) \\ +c & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$

$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$ Taking2 common from R_1

$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ c+a-(a+b+c) & (r+p)-(p+q+r) & z-x-(x+y+z) \\ a+b-(a+b+c) & (p+q)-(p+q+r) & x+y-(x+y+z) \end{vmatrix}$ $R_2 \to R_2 - R_1$, and$R_3 \to R_3 - R_1$

$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$

$= 2\begin{vmatrix} a+b+c-b-c & p+q+r-q-r & x+y+z-y-E \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$ = $R_1 \to R_2+R_3R_1$

$= 2\begin{vmatrix} a & p & x \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$

$= 2\times(-1)\times(-1)\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

$= 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

Q6. $\begin{vmatrix} a & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$

A.6. L·H·S· = $\begin{vmatrix} a & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = \Delta$ Say

$= \begin{vmatrix} 0 & (-1)(-a) & (-1)b \\ (-1)a & 0 & (-1)(c) \\ (-1)(-b) & (-1)(-c) & 0 \end{vmatrix}$ Take (−1) common from each row

$= (-1)\times(-1)\times(-1) \begin{vmatrix} 0 & -a & b \\ a & 0 & c \\ -b & -c & 0 \end{vmatrix}$

$= -\begin{vmatrix} 0 & -a & b \\ a & 0 & c \\ -b & -c & 0 \end{vmatrix}$

$= -\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$ $\because$ det (A) = det (A`)

$= -\Delta$

$\Rightarrow \Delta + \Delta = 0$

$\Rightarrow 2\Delta = 0$

$\Rightarrow \Delta = 0$ = R·H·S·

Q7. $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$

A.7. L·H·S = $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$

$= abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$ taking a, b& c common from R_1, R_2 and R_3 respectively

$= a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$ Taking a, b,&c, common from c_1, c_2,&c_3 respectively

$= a^2b^2c^2 \begin{vmatrix} 0 & 2 & 0 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$ $R_1 \Rightarrow R_1+R_3$

$= a^2b^2c^2 \times(-2) \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$ expand along R_1

$= a^2b^2c^2 \times(-2)\left(1\times(-1) - 1\times 1\right)$

$= a^2b^2c^2 \times (-2) \times (-2)$

$= 4a^2b^2c^2$

= R·H·S·

By using properties of determinants, in Exercises 8 to 14, show that:

Q8. **(i)** $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

A.8. **(i)** L·H·S = $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

$= \begin{vmatrix} 1 & a & a^2 \\ 1-1 & b-a & b^2-a^2 \\ 1-1 & c-a & c^2-a^2 \end{vmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$= \begin{vmatrix} 1 & a & a^2 \\ 0 & (b-a) & b^2-a^2 \\ 0 & (c-a) & c^2-a^2 \end{vmatrix}$

Expand along C_1 = $1\begin{vmatrix} (b-a) & b^2-a^2 \\ (c-a) & c^2-a^2 \end{vmatrix} - 0\begin{vmatrix} a & a^2 \\ c-a & c^2-a^2 \end{vmatrix} + 0\begin{vmatrix} a & a^2 \\ b-a & b^2-a^2 \end{vmatrix}$

$= \begin{vmatrix} (b-a) & (b-a)(b+a) \\ (c-a) & (c-a)(c+a) \end{vmatrix}$

$= (b-a)(c-a)\begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix}$ Take $(b-a)$ & $(c-a)$ common from R_1, & R_2

$=(b-a)(c-a)[(c+a)-(b+a)]$

$=(b-a)(c-a)[(c+a-b-a)]$

$=(b-a)(c-a)(c-b)$

$=(-1)(a-b)\times(-1)(b-c)(c-a)$

$=(a-b)(b-c)(c-a)$.

= R·H·S·

(ii) L·H·S $= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

$$=\begin{vmatrix} 1 & 1-1 & 1-1 \\ a & b-a & c-a \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix} \quad \begin{matrix} c_2 \to c_2 - c_1 \\ c_3 \to c_3 - c_1 \end{matrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & (b-a)\left(b^2+a^2+ab\right) & (c-a)\left(c^2+a^2+ab\right) \end{vmatrix} \begin{cases} \because x^3-y^3 \\ =(x-y)\left(x^2+y^2+xy\right) \end{cases}$$

Expanding along R_1

$$= \begin{vmatrix} b-a & c-a \\ \left(b-a(b^2+a^2+ab\right) & (c-a)\left(c^2+a^2+ab\right) \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & 1 \\ b^2+a^2+ab & c^2+a^2+ab \end{vmatrix} \begin{cases} \text{Taking}\,(b-a)\ \&(c-a) \\ \text{common from}\, c_1\ \& c_3 \end{cases}$$

$= (b-a)(c-a)\ \{(c^2 + a^2 + ab)- (b^2 + a^2 + ab)\}$

$= (b-a)\ (c-a)\ \{c^2 + a^2 + ab-b^2-a^2-ab\}$

$= (b-a)\ (c-a)(c^2-b^2 + ac-ab)$

$= (b-a)(c-a)(c-b)(c + b) + (c-b)\ a\}$

$= (b-a)(c-a)(c-b)(a + c + b).$

$= (-1)(a-b)(c-a)(-1)(b-c)(a + c + b)$

$= (a-b)(b-c)(c-a)(a + c + b)$

= R·H·S·

Q9. $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

A.9. L·H·S· $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$

$$= \frac{1}{xyz}\begin{vmatrix} x\cdot x & x\cdot x^2 & x\cdot yz \\ y\cdot y & y\cdot y^2 & y\cdot zx \\ z\cdot z & z\cdot z^2 & z\cdot xy \end{vmatrix}$$ Multiplying R_1, R_2& R_3 by *x*, *y*&*z*.

$$= \frac{1}{xyz}\begin{vmatrix} x^2 & x^3 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix}.$$

$$= \frac{xyz}{xyz}\begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$$ Taking *xyz* common from c_3.

$$= \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 - x^2 & y^3 - x^3 & 1-1 \\ z^2 - x^2 & 2^3 - x^3 & 1-1 \end{vmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1. \end{matrix}$$

$$= \begin{vmatrix} x^2 & x^3 & 1 \\ (y-x)(y+x) & (y-x)\left(y^2 + x^2 + xy\right) & 0 \\ (z-x)(z+x) & (2-x)\left(z^2 + x^2 + 2x\right) & 0 \end{vmatrix}$$

Expanding along c_3 we get,

$$= 1 \begin{vmatrix} (y-x)(y+x) & (y-x)\left(y^2 + x^2 + xy\right) \\ (z-x)(z+x) & (z-x)\left(z^2 + x^2 + zx\right) \end{vmatrix}$$

$= (y-x)(z-x). \begin{vmatrix} y+x & y^2 + x^2 + xy \\ z+x & z^2 + x^2 + zx \end{vmatrix}$ Taking $(y-x)$ & $(z-x)$ common from R_1 and R_2.

$= (y-x)(z-x)[(y + x)(z^2 + x^2 + zx) - (z + x)(y^2 + x^2 + xy)]$

$= (y-x)(z-x)[yz^2 + yx^2 + xyz + xz^2 + x^3 + x^2z - zy^2 - zx^2 - xyz - xy^2 - x^3 - x^2y]$

$= y-x)(z-x)[yz^2 - zy^2 + xz^2 - xy^2]$

$= (y-x)(z-x)[yz (z-y) + x(z^2-y^2)]$

$= y-x)(z-x)(z-y)[yz + x (z + y)]$ $\left\{ \begin{matrix} \because z^2 - y^2 = \\ (z-y)(z+y) \end{matrix} \right\}$

$= (-1)(x-y)(z-x)(-1)(y-z)[yz + 1\, xz + xy]$

$= (x-y)(y-z)(z-x)(xy + yz + xz).$ = R·H·S·

Q10. (i) $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

(ii) $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$.

A.10. (i) L·H·S· = $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

$$= \begin{vmatrix} x+4+2x+2x & 2x+x+4+2x & 2x+2x+x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} \quad R_1 \to R_1 + R_2 + R_3$$

$$= \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Taking (5x + 4) common from R_1.

$$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}.$$

$$= (5x + 4) \begin{vmatrix} 1-1 & 1-1 & 1 \\ 2x-2x & x+4-2x & 2x \\ 2x-(x+4) & 2x-(x+4) & x+4 \end{vmatrix} \begin{matrix} C_1 \to C_1 - C_3 \\ C_2 \to C_2 - C_3 \end{matrix}$$

$$= (5x + 4) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 4-x & 2x \\ x-4 & x-4 & x+4 \end{vmatrix}$$

Expanding along R_1, we get,

$$= (5x + 4) \times 1 \begin{vmatrix} 0 & 4-x \\ x-4 & x-4 \end{vmatrix}.$$

$= (5x + 4)[0 – (4 – x)(x – 4)]$

$= (5x + 4)(4 – x)(4 – x)$

$= (5x + 4)(4 – x)^2$ = R·H·S

(ii) L·H·S = $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}.$

$$= \begin{vmatrix} y+k+y+y & y+y+k+y & y+y+y+k \\ y & y+k & y \\ y & y & y+k \end{vmatrix} R_1 \to R_1 + R_2 + R_3$$

$$= \begin{vmatrix} 3y+k & 3y+k & 3y+k \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

$$= (3y + k) \begin{vmatrix} 1 & 1 & 1 \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

$$= (3y + k) \begin{vmatrix} 1 & 1-1 & 1-1 \\ y & y+k-y & y-y \\ y & y-y & y+k-y \end{vmatrix} \begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{matrix}$$

$$= (3y + k) \begin{vmatrix} 1 & 0 & 0 \\ y & k & 0 \\ y & 0 & k \end{vmatrix}.$$

$= (3y + k) \times 1. \begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix}$ Expand along R_1

$= k^2 (3y + k)$ = R·H·S·

Q11. (i) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

(ii) $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$.

A.11.(i) L·H·S = $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$= \begin{vmatrix} a-b-c+2b+2c & 2a+b-c-a+2c & 2a+2b+c-ab \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ $R_1 \to R_1 + R_2 + R_3$

$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ Taking $(a+b+c)$ common from R_1

$= (a+b+c)\begin{vmatrix} 1 & 1-1 & 1-1 \\ 2b & b-c-a-2b & 2b-2b \\ 2c & 2c-2c & c-a-b-2c \end{vmatrix}$ $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix}$.

$= (a+b+c) \times 1. \begin{vmatrix} -(a+b+c) & 0 \\ 0 & -(a+b+c) \end{vmatrix}$ Expand along R_1

$= (a+b+c)\{(a+b+c)^2 - 0\}$

$= (a+b+c)^3$ = R·H·S

(ii) L·H·S· = $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$.

$= \begin{vmatrix} x+y+2z+x+y & x & y \\ z+y+z+2x+y & y+z+2x & y \\ z+x+z+x+2y & x & z+x+2y \end{vmatrix}$ $C_1 \to C_1 + C_2 + C_3$.

$= \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$.

$= 2\,(k + y + z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$ Taking 2(k + y + z) common from C_1.

$= 2(k + y + z)\begin{vmatrix} 1 & x & y \\ 1-1 & y+z+2x-x & y-y \\ 1-1 & x-x & z+x+2y-y \end{vmatrix}$ $R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_1$

$= 2(k + y + z)\begin{vmatrix} 1 & x & y \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{vmatrix}$

Expand along C_1,

$= 2(k + y + z) \times 1\begin{vmatrix} x+y+z & 0 \\ 0 & x+y+z \end{vmatrix}$

$= 2(k + y + z) \times \{(k + y + z)^2 - 0\}$

$= 2(k + y + z)^3$ = R·H·S·

Q12. $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \left(1-x^3\right)^2$

A.12. L·H·S = $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$

$= \begin{vmatrix} 1+x^2+x & x+1+x^2 & x^2+x+1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$ $R_1 \to R_1 + R_2 + R_3$.

$= (1 + x^2 + x)\begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$ Taking (1 + x^2 + x) common from R_1.

$= (1 + x^2 + x)\begin{vmatrix} 1 & 1-1 & 1-1 \\ x^2 & 1-x^2 & x-x^2 \\ x & x^2-x & 1-x \end{vmatrix}$

$= \left(1+x^2+x\right)\begin{vmatrix} 1 & 0 & 0 & 0 \\ x^2 & (1-x)(1+x) & x(1-x) & \\ x & x(x-1) & (1-x) & \end{vmatrix}$

$= (1 + x^2 + x) \times 1\begin{vmatrix} -(1-x)(1+x) & x(1-x) \\ (-1)x(1-x) & (1-x) \end{vmatrix}$ Expand along R_1.

$= (1 + x^2 + x)(1 - x)^2\begin{vmatrix} 1+x & x \\ -x & 1 \end{vmatrix}$

$= (1 + x^2 + x)(1 - x)^2 [(1 + x) \times 1 - (-x) x].$

$= (1 + x^2 + x)(1 - x)^2 (1 + x + x^2).$

$= \{(1 + x^2 + x)(1 - x)\}^2$

$= \{1 - x + x^2 - x^3 + x - x^2\}^2$

$= (1 - x^3)^2 =$ R·H·S·

Q13. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$

A.13. L·H·S· = $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$.

$$= \begin{vmatrix} 1+a^2-b^2-b(-2b) & 2ab+a(-2b) & -2b \\ 2ab-b(2a) & 1-a^2+b^2+a(2a) & 2a \\ 2b-b\left(1-a^2-b^2\right) & -2a+a\left(1-a^2-b^2\right) & 1-a^2-b^2 \end{vmatrix} \quad \begin{matrix} C_1 \to C_1 - bC_3 \\ C_2 \to C_2 - aC_3. \end{matrix}$$

$$= \begin{vmatrix} 1+a^2-b^2+2b^2 & 2ab-2ab & -2b \\ 2ab-2ab & 1-a^2+b^2+2a^2 & 2a \\ 2b-b+a^2b+b^3 & -2a+a-a^3-ab^2 & 1-a^2-b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b\left(1+a^2+b^2\right) & -a\left(1+a^2+b^2\right) & 1-a^2-b^2 \end{vmatrix}$$

$= (1 + a^2 + b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2+b^2 \end{vmatrix}$ Taking $(1 + a^2 + b^2)$ common from C_1 and C_2.

$= (1 + a^2 + b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b-b(1) & -a-b(0) & 1-a^2-b^2-b(-2b) \end{vmatrix}$ $R_3 \to R_3 - b R_1$

$$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2+b^2 \end{vmatrix}$$

$= (1 + a^2 + b^2)^2 \times 1 \begin{vmatrix} 1 & 2a \\ -a & 1-a^2+b^2 \end{vmatrix}$ Expand along C_1

$= (1 + a^2 + b^2)^2 [(1 - a^2 + b^2) - 2a (-a)]$

$= (1 + a^2 + b^2)^2 (1 - a^2 + b^2 + 2a^2)$

$= (1 + a^2 + b^2)^2 (1 + a^2 + b^2)$

$= (1 + a^2 + b^2)^3 =$ R·H·S·

Q14. $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

A.14. L·H·S = $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$

$= \frac{1}{abc}\begin{vmatrix} a(a^2+1) & ab^2 & ac^2 \\ a^2b & b(b^2+1) & bc^2 \\ a^2c & b^2c & c(c^2+1) \end{vmatrix} \begin{matrix} C_1 \to aC_1 \\ C_2 \to bC_3 \\ C_3 \to cC_3. \end{matrix}$

$= \frac{abc}{abc}\begin{bmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2. \\ a^2 & b^2 & c^2+1 \end{bmatrix}$ Taking a, b&c common from R_1, R_2&R_3

$= \begin{bmatrix} 1+a^2+b^2+c^2 & b^2 & c^2 \\ a^2+b^2+1+c^2 & b^2+1 & c^2 \\ a^2+b^2+c^2+1 & b^2 & c^2+1 \end{bmatrix}$ $C_1 \to C_2 + C_3$.

$= (1 + a^2 + b^2 + c^2)\begin{bmatrix} 1 & b^2 & c^2 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{bmatrix}$ Taking $(1 + a^2 + b^2c^2)$ common from C_1.

$= (1 + a^2 + b^2 + c^2)\begin{bmatrix} 1 & b^2 & c^2 \\ 1-1 & b^2+1-b^2 & c^2-c^2 \\ 1-1 & b^2-b^2 & c^2+1-c^2 \end{vmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$= (1 + a^2 + b^2 + c^2\begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$= (1 + a^2 + b^2 + c^2) \times 1 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ Expand along C_1

$= (1 + a^2 + b^2 + c^2) (|\times| - 0 \times 0)$

$= 1 + a^2 + b^2 + c^2$

= R·H·S·

Choose the correct answer in Exercises 15 and 16.

Q.15 Let A be a square matrix of order 3 × 3, then | kA| is equal to

(A) k| A| (B) k^2 | A | (C) k^3 | A | (D) $3k$ | A |

A.15. Let A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. and |A| = $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Then, KA = $k\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$= \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{4} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{bmatrix}$$

$$\therefore |\mathrm{KA}| = \begin{vmatrix} \mathrm{k}_{a11} & \mathrm{k}a_{12} & \mathrm{k}_{13} \\ \mathrm{k}_{a_{21}} & \mathrm{k}_{22} & ka_{23} \\ \mathrm{k}_{31} & \mathrm{k}_{a_{32}} & k_{a_{33}} \end{vmatrix}$$

$$= \mathrm{k}^3 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{22} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= k^3 |\mathrm{A}|$$

So, option c is correct.

Q16. Which of the following is correct

(A) Determinant is a square matrix.
(B) Determinant is a number associated to a matrix.
(C) Determinant is a number associated to a square matrix.
(D) None of these

A.16. Option 'C' is correct as determinant is a number associated to a square matrix.

Exercise - 4.3

Q1. Find area of the triangle with vertices at the point given in each of the following:

(i) (1, 0), (6, 0), (4, 3)
(ii) (2, 7), (1, 1), (10, 8)
(iii) (–2, –3), (3, 2), (–1, –8)

A.1.(i) Area of triangle is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left| [1\times(0\times1-3\times1)-0\times(6\times1-4\times1)+1(6\times3-4\times0)] \right|$$

$$= \frac{1}{2}\left|[-3+18]\right| = \frac{15}{2} = 7.5 \text{ sq. units.}$$

(ii) Area of the triangle is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left| [2(1 \times 1) - 8 \times 1) - 7 (1 \text{ k } 1 - 10 \times 1) + 1(8 \times 1 - 10 \times 1)] \right|$$

$$= \frac{1}{2}\left|[2(1-8)-7(1-10)+1(8-10)]\right|$$

$$= \frac{1}{2}\left|(2\times(-7)-7\times(-9)+21\times(-2)]\right|$$

$$= \frac{1}{2}\left|[-14+63-2]\right| = \frac{1}{2}|47|$$

$$= \frac{47}{2} = 23.5 \; sq. \text{ units}$$

(iii) Area of triangle is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}.$$

$$= \frac{1}{2}\left| [-2 (2 \times 1 - (-8) \times 1) - (-3) (3 \times 1(-1) \times 1) \times 1 (3 \times (-8) - (2) \times (-1))] \right|$$

$$= \frac{1}{2}\left|[-2\times10+3(4)+(-24+2)]\right|$$

$$= \frac{1}{2}\left|[-20+12-22]\right|$$

$$= \frac{1}{2}|-30| = \frac{30}{2} = 15 \text{ sq. units.}$$

Q2. Show that points
A (a, $b + c$), B (b, $c + a$), C (c, $a + b$) are collinear.

A.2. The area of triangle from by the given points is area (Δ ABC) = $\frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} a+b+c+1 & b+c & 1 \\ b+c+a+1 & c+a & 1 \\ c+a+b+1 & a+b & 1 \end{vmatrix} \quad C_1 \to C_1 + C_2 + C_3$$

$$= \frac{1}{2}(a+b+c+1)\begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix} \quad \text{Taking } (a + b + c + 1) \text{ common from } C_1$$

$= \frac{(a+b+c+1)}{2} \times 0 \quad \{\because c = c_3\}$

= 0

Hence the points A, B C are collinear.

Q3. Find values of k if area of triangle is 4 sq. units and vertices are
(i) (k, 0), (4, 0), (0, 2) (ii) (–2, 0), (0, 4), (0, k)

A.3(i) Area of the triangle = 4 sq. units (given)

$$\Rightarrow \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 4$$

$$\Rightarrow \frac{1}{2}\left| [k(0-2) - 0(4-0) + 1(8-0)] \right| = 4$$

$$\Rightarrow \frac{1}{2}\left| [-2k + 8] \right| = 4$$

$$\Rightarrow \frac{2}{2}\left| [-k + 4] \right| = 4$$

$\Rightarrow -k + 4 = \pm 4$ $\left\{\begin{matrix} r\,|x| = a \\ \Rightarrow x = \pm a \end{matrix}\right\}$

When $\Rightarrow k + 4 = 4$ and when $-k + 4 = -4$

$\Rightarrow -k = 4 - 4$ $\Rightarrow -k = -4 - 4$

$\Rightarrow \boxed{k = 0}$ $\Rightarrow \boxed{k = 8}$

(ii) Area of the triangle = 4 sq units

$$\Rightarrow \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} = 4.$$

$$\Rightarrow \frac{1}{2}\left| [-2(4 \times 1 - k \times 1) - 0(0 \times 1 - k \times 1) + 0(0 \times 1 - 4 \times 1)] \right| = 4$$

$$\Rightarrow \frac{1}{2}\left| [-2(4 - k) \right| = 4$$

$$\Rightarrow \frac{-2}{2}\left| -4 + k \right| = 4$$

$\Rightarrow -4 + k = \pm 4$ $\left\{\begin{matrix} \because |x| = a \\ \Rightarrow x = \pm a. \end{matrix}\right\}$

When $-4 + k = 4$ and when $-4 + k = -4$

$\Rightarrow k = 4 + 4$ $\Rightarrow k = 4 - 4$

$\Rightarrow \boxed{k = 8}$ $\Rightarrow \boxed{k = 0}$

Q4. (i) Find equation of line joining (1, 2) and (3, 6) using determinants.
(ii) Find equation of line joining (3, 1) and (9, 3) using determinants

A.4. (i) Let P (x, y) be any point on line joining A (1, 2) & B(3, 6)

Then, area of triangle (ABP) = 0 { ∵ the point are collinear

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$\Rightarrow 1\,(6\times 1 - y\times 1) - 2\,(3\times 1 - x\times 1) + 1\,(3\times y - 6\times k) = 0$

$\Rightarrow 6 - y - 6 + 2x + 3y - 6x = 0$

$\Rightarrow 2y - 4x = 0$

$\Rightarrow 2y = 4x$

$\Rightarrow y = 2x$. Is the required equation of line

(ii) Let P(k, y) be any point on line-joining A (3, 1) and B (9, 3)

Then area of triangle A B P = 0

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$\Rightarrow 3(3\times 1 - y\times 1) - 1(9\times 1 - x\times 1) + 1(9\times y - 3\times x) = 0$

$\Rightarrow 9 - 3y - 9 + x + 9y - 3x = 0$

$\Rightarrow 6y - 2x = 0$

$\Rightarrow 2x = 6y$

$\Rightarrow x = 3y\cdot$ is the required equation of line

Q5. If area of triangle is 35 sq units with vertices (2, – 6), (5, 4) and (k, 4). Then k is
(A) 12 (B) –2 (C) –12, –2 (D) 12, –2

A.5. Given,

Area of triangle = 35 sq. Units

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = 35.$$

$\Rightarrow \frac{1}{2}\left| [2\,(4\times 1 - 4\times 1) - (-6)(5\times 1 - k\times 1) + 1\,(5\times 4 - k\times 4] \right| = 35.$

$\Rightarrow \frac{1}{2}\left| [2\,(4-4) + 6(5-k) + 1\,(20-4k)] \right| = 35.$

$\Rightarrow \frac{1}{2}\left| [50 - 10k] \right| = 35$

$\Rightarrow \frac{2}{2}\left| [25 - 5k] \right| = 35.$

$\Rightarrow 25 - 5k = \pm 35$ $\left\{\begin{matrix} \because |x| = a \\ \Rightarrow x = \pm a \end{matrix}\right\}$

When $25 - 5k = 35$.

$\Rightarrow -5k = 35 - 25$ and $25 - 5k = -35$

$\Rightarrow -5k = 10 \Rightarrow -5k = -35 - 25.$

$\Rightarrow k = \frac{10}{-5} \Rightarrow -5k = -60$

$\Rightarrow \boxed{k = -2} \Rightarrow k = \frac{-60}{-5}$

$\Rightarrow \boxed{k = 12}$

$\therefore$ Option D is correct.

Exercise – 4.4

Write Minors and Cofactors of the elements of following determinants:

Q1. **(i)** $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ **(ii)** $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

A.1.(i) We know that,

Minor of element a_{ij} is m_{ij} and its co-factor is $A_{ij} = (-1)^{i+j} M_{ij}$

So,

$M_{11} = 3$ and $A_{11} = (-1)^{1+1} M_{11} = 1 \times 3 = 3$

$M_{12} = 0$ and $A_{12} = (-1)^{1+2} M_{12} = -1 \times 0 = 0$

$M_{21} = -4$ and $A_{21} = (-1)^{2+1} M_{21} = (-1) \times (-4) = 4$

$M_{22} = 2$ and $A_{22} = (-1)^{2+2} M_{22} = 1 \times 2 = 2$

(ii) Given A = $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

So,

$M_{11} = d$ and $A_{11} = (-1)^{1+1} M_{11} = 1 \times d = d$

$M_{12} = b$ and $A_{12} = (-1)^{1+2} M_{12} = (-1) \times b = -b$

$M_{21} = c$ and $A_{21} = (-1)^{2+1} M_{21} = (-1) \times c = -c$

$M_{22} = a$ and $A_{22} = (-1)^{2+2} M_{22} = 1 \times a = a$

Q2. **(i)** $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ **(ii)** $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

A.2.(i) Given, A = $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

So,

$M_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ and $A_{11} = (-1)^{1+1} M_{11} = 1 \times 1 = 1$

$M_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 \times 1 - 0 \times 0 = 0$ and $A_{12} = (-1)^{1+2} M_{12} = -1 \times 0 = 0$

$M_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0 \times 0 - 0 \times 1 = 0$ and $A_{13} = (-1)^{1+3} M_{13} = 1 \times 0 = 0$

$M_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 \times 1 - 0 \times 0 = 0$ and $A_{21} = (-1)^{2+1} M_{21} = -1 \times 0 = 0.$

$M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \times 1 - 0 \times 0 = 1$ and $A_{22} = (-1)^{2+2} M_{22} = 1 \times 1 = 1.$

$M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 1 \times 0 - 0 \times 0 = 0$ and $A_{23} = (-1)^{2+3} M_{23} = -1 \times 0 = 0$

$M_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 \times 0 - 0 \times 1 = 0$ and $A_{31} = (-1)^{3+1} M_{31} = 1 \times 0 = 0$

$M_{32} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 \times 0 - 0 \times 1 = 0$ and $A_{32} = (-1)^{3+2} M_{32} = -1 \times 0 = 0$

$M_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \times 1 - 0 \times 0 = 1$ and $A_{33} = (-1)^{3+3} M_{33} = 1 \times 1 = 1$

(ii) Given, $\Delta = \begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

$M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 5 \times 2 - (-1) \times 1 = 10 + 1 = 11$ and $A_{11} = (-1)^{1+1} M_{11} = 1 \times 11 = 11$

$M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 3 \times 2 - (-1) \times 0 = 6$ and $A_{12} = (-1)^{1+2} M_{12} = -1 \times 6 = -6.$

$M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 \times 1 - 5 \times 0 = 3$ and $A_{13} = (-1)^{1+3} M_{13} = 1 \times 3 = 3$

$M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 \times 2 - 4 \times 1 = -4$ and $A_{21} = (-1)^{2+1} M_{21} = -1 \times (-4) = 4$

$M_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 1 \times 2 - 4 \times 0 = 2$ and $A_{23} = (-1)^{2+2} M_{23} = 1 \times 2 = 2$

$M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \times 1 - 0 \times 0 = 1$ and $A_{23} = (-1)^{2+3} M_{23} = -1 \times 1 = -1.$

$M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 \times (-1) - 4 \times 5 = -20$ and $A_{31} = (-1)^{3+1} M_{31} = 1 \times (-20) = -20.$

$M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = 1 \times (-1) - 4 \times 3 = -1 - 12 = -13$ and $A_{32} = (-1)^{3+2} M_{32} = -1 \times (-13) = 13$

$M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 1 \times 5 - 3 \times 0 = 5$ and $A_{33} = (-1)^{3+3} M_{33} = 1 \times 5 = 5.$

Q3. Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

A.3. Given, $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Co-factors of elements of second row,

$A_{21} = (-1)^{2+1} \times \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = (-1) \times (3 \times 3 - 8 \times 2) = (-1)(9 - 16) = (-1) \times (-7) = 7.$

$A_{22} = (-1)^{2+2} \times \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 1 \times (5 \times 3 - 8 \times 1) = 15 - 8 = 7.$

$A_{23} = (-1)^{2+3} \times \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = (-1) \times (5 \times 2 - 3 \times 1) = (-1)(10 - 3) = (-1)(7) = -7.$

$\therefore \Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$

$= 2 \times 7 + 0 \times 7 + 1 \times (-7) = 14 + 0 - 7 = 7.$

Q4. Using Cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

A.4. Given, $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

Co-factor of elements of third column

$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & y \\ 1 & 2 \end{vmatrix} = (-1)^4 (1 \times z - y \times 1) = (z - y)$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = (-1)^5 (1 \times z - 1 \times x) = (-1) \times (z - x) = -(z - x)$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = (-1)^6 (1 \times y - 1 \times x) = (y - x)$

$\therefore \Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$

$= yz\,(z-y) + zx\,[-(z-x)] + xy\,(y-x)$

$= y\,z^2 - y^2 z - z^2 x + zx^2 + xy^2 - x^2 y.$

$= yz^2 - y^2 z + (xy^2 - xz^2) + (zx^2 - x^2 y)$

$= yz\,(z-y) + x\,(y^2 - z^2) - x^2\,(y-z)$

$= -yz\,(y-z) + x\,(y+z)\,(y-z) - x^2\,(y-z)$

$= (y-z)\,[-yz + x\,(y+z) - x^2]$

$= (y-z)\,[-yz + xy + xz - x^2]$

$= (y-z)\,[-y\,(z-x) + x\,(z-x)]$

$= (y-z)\,(z-x)\,(x-y)$

Q5. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is Cofactors of a_{ij} then value of Δ is given by

(A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ **(B)** $a_{11}\,A_{11} + a_{12}\,A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$ **(D)** $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

A.5. Option D is correct.

Exercise – 4.5

Find adjoint of each of the matrices in Exercise 1 and 2.

Q1. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

A.1. We have,

$A_{11} = (-1)^{1+1} \times 4 = 4$

$A_{12} = (-1)^{1+2} \times 3 = -3$

$A_{21} = (-1)^{2+1} \times 2 = -2$

$A_{22} = (-1)^{2+2} \times 1 = 1.$

$$\text{adj}\,A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}' = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

Q2. $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

A.2. We have,

$$A_{11} = (-1)^{1+1} \times \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = (3\times 1 - 5\times 0) = 3$$

$$A_{12} = (-1)^{1+2} \times \begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = (-1)(2\times 1 - 5\times(-2)) = -1(2+10) = -12$$

$$A_{13} = (-1)^{1+3} \times \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = [2\times 0 - (-2)\times 3] = 6$$

$$A_{21} = (-1)^{2+1} \times \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = (-1)(-1 \times 1 - 2 \times 0) = 1$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = (1 \times 1 - (2) \times (-2)) = (1+4) = 5$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = (-1)(1 \times 0 - (-1) \times (-2)) = (-1)(0-2) = 2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = (-1 \times 5 - 3 \times 2) = (-5-6) = -11 \; A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = (-1)(1 \times 5 - 2 \times 2) = (-1)(5-4) = -1.$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = (1 \times 3 - (-1) \times 2) = (3+2) = 5$$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}.$$

Verify A (adj A) = (adj A)A = |A|I in Exercise 3 and 4

Q3. $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

A.3. Let $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$. Then, $|A| = \left(2 \times (-6) - 3 \times (-4)\right) = -12 + 12 = 0$

So, $A_{11} = (-1)^{1+1} \times (-6) = -6$

$A_{12} = (-1)^{1+2} \times (-4) = 4$

$A_{21} = (-1)^{2+1} \times (3) = -3$

$A_{22} = (-1)^{2+2} \times 2 = 2$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}.$$

So, $A.(\text{adj } A) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 2 \times (-6) + 3 \times 4 & 2 \times (-3) + 3 \times 2 \\ (-4) \times (-6) + (-6) \times 4 & (-4) \times (-3) + (-6) \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} -12+12 & -6+6 \\ 24-24 & 12-12 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$(\text{adj } A).A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

$$=\begin{bmatrix}-6\times 2+(-3)\times(-4) & -6\times 3+(-3)\times(-6)\\ 4\times 2+2\times(-4) & 4\times 3+2\times(-6)\end{bmatrix}$$

$$=\begin{bmatrix}-12+12 & -18+18\\ 8-8 & 12-12\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$|A|\text{I}=0\times\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\therefore A(\text{adj } A)=(\text{adj } A)A=|A|\text{I}$$

Q4. $\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{bmatrix}$

A.4. Let A = $\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{bmatrix}$ Then, $|A|=\begin{vmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{vmatrix}$

$$|A|=1\times\begin{vmatrix}0 & -2\\ 0 & 3\end{vmatrix}-(-1)\begin{vmatrix}3 & -2\\ 1 & 3\end{vmatrix}+2\begin{vmatrix}3 & 0\\ 1 & 0\end{vmatrix}$$

$$=1\times(0\times 3-0\times(-2))+1(3\times 3-(-2)\times 1)+2(3\times 0-0\times 1)=9+2=11$$

$$A_{11}=(-1)^{1+1}\begin{vmatrix}0 & -2\\ 0 & 3\end{vmatrix}=(0\times 3-(-2)\times 0)=0$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix}3 & -2\\ 1 & 3\end{vmatrix}=(-1)\times(3\times 3-(-2)\times 1)=(-1)(9+2)=-11$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}3 & 0\\ 1 & 0\end{vmatrix}=(3\times 0-0\times 1)=0$$

$$A_{21}=(-1)^{2+1}\begin{vmatrix}-1 & 2\\ 0 & 3\end{vmatrix}=(-1)[-1\times 3-2\times 0]=3$$

$$A_{22}=(-1)^{2+2}\begin{vmatrix}1 & 2\\ 1 & 3\end{vmatrix}=(1\times 3-2\times 1)=1$$

$$A_{23}=(-1)^{2+3}\begin{vmatrix}1 & -1\\ 1 & 0\end{vmatrix}=(-1)(1\times 0-(-1)\times 1)=-1$$

$$A_{31}=(-1)^{3+1}\begin{vmatrix}-1 & 2\\ 0 & -2\end{vmatrix}=(-1\times(-2)-2\times 0)=2$$

$$A_{32}=(-1)^{3+2}\begin{vmatrix}1 & 2\\ 3 & -2\end{vmatrix}=(-1)[1\times(-2)-2\times 3]=(-1)(-2-6)=8$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix}1 & -1\\ 3 & 0\end{vmatrix} = \left(1\times 0-(-1)\times 3\right) = 3$$

$$\text{adj } A = \begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix} = \begin{bmatrix}0 & 3 & 2\\ -11 & 1 & 8\\ 0 & -1 & 3\end{bmatrix}$$

$$\text{So, } A.(\text{adj } A) = \begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{bmatrix}\begin{bmatrix}0 & 3 & 2\\ -11 & 1 & 8\\ 0 & -1 & 3\end{bmatrix}$$

$$= \begin{bmatrix}1\times 0+(-1)(-11)+2\times 0 & 1\times 3+(-1)\times 1+2\times(-1) & 1\times 2+(-1)\times 8+2\times 3\\ 3\times 0+0\times(-11)+(-2)\times 0 & 3\times 3+0\times 1+(-2)(-1) & 3\times 2+0\times 8+(-2)\times 3\\ 1\times 0+0\times(-11)+3\times 0 & 1\times 3+0\times 1+3\times(-1) & 1\times 2+0\times 8+3\times 3\end{bmatrix}$$

$$= \begin{bmatrix}11 & 3-1-2 & 2-8+6\\ 0 & 9+2 & 6-6\\ 0 & 3-3 & 2+9\end{bmatrix} = \begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$$

$$(\text{adj } A)A = \begin{bmatrix}0 & 3 & 2\\ -11 & 1 & 8\\ 0 & -1 & 3\end{bmatrix}\begin{bmatrix}1 & -1 & 2\\ 3 & 0 & -2\\ 1 & 0 & 3\end{bmatrix}$$

$$= \begin{bmatrix}0\times 1+3\times 3+2\times 1 & 0\times(-1)+3\times 0+2\times 0 & 0\times 2+3\times(-2)+3\times 2\\ -11\times 1+1\times 3+8\times 1 & -11\times(-1)+1\times 0+8\times 0 & -11\times 2+1\times(-2)+8\times 3\\ 0\times 1+(-1)\times 3+3\times 1 & 0\times(-1)+(-1)\times 0+3\times 0 & 0\times 2+(-1)\times(-2)+3\times 3\end{bmatrix}$$

$$= \begin{bmatrix}9+2 & 0 & -6+6\\ -11+3+8 & 11 & -22-2+24\\ -3+3 & 0 & 2+9\end{bmatrix} = \begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$$

$$\text{And } |A|I = 11\times\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$$

$$\therefore A.(\text{adj } A) = (\text{adj } A)A = |A|I$$

Find the inverse of each of the matrices (if it exists) given in Exercise 5 to 11.

Q5. $\begin{bmatrix}2 & -2\\ 4 & 3\end{bmatrix}$

A.5. Let $A = \begin{bmatrix}2 & -2\\ 4 & 3\end{bmatrix}$. Then $|A| = 2\times 3-(-2)\times 4 = 6+8 = 14 \neq 0$

So, A^{-1} exist

$$\therefore \text{adj}A = \begin{bmatrix}3 & 2\\ -4 & 2\end{bmatrix}\left\{\because \text{Adj of } \begin{bmatrix}a & B\\ c & d\end{bmatrix} = \begin{bmatrix}d & -b\\ -c & a\end{bmatrix}\right\}$$

Now, $A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

Q6. $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

A.6. Let $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$. Then $|A| = \left(-1\times 2 - 5A(-3)\right) = -2+15 = 13 \neq 0$

So, A^{-1} exist

And $\text{adj } A = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

Q7. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

A.7. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$. Then $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{vmatrix}$

$\Rightarrow |A| = 1\times\begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} - 0\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} + 0\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix}$

$= 1\times(2\times 5 - 4\times 0) = 10 \neq 0$

So, A^{-1} exist

$A_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} = (10-0) = 0$

$A_{12} = (-1)^{1+2}\begin{vmatrix} 0 & 4 \\ 0 & 2 \end{vmatrix} = (-1)(0-0) = 0.$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = (0-0) = 0.$

$A_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = (-1)(10-0) = -10.$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = (5-0) = 5.$

$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = (-1)(0-0) = 0$

$A_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = (8-6) = 2$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = (-1)(4-0) = -4.$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = (2-0) = 2.$$

$$\text{So, adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

Q8. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

A.8. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$. Then $|A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix}$

$$\Rightarrow |A| = 1\begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} - 0\begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} + 0\begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix}$$

$$= (-3-0) = -3 \neq 0$$

So, A^{-1} exist

Now, $A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = -3-0 = -3$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = (-1)(-3-0) = 3$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix} = (6-15) = -9$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} = (-1)(0-0) = 0$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 0 \\ 5 & -1 \end{vmatrix} = (-1-0) = -1$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 5 & 2 \end{vmatrix} = (-1)(2-0) = -2$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} = (0-0) = 0$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = (-1)(0-0) = 0$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix}1 & 0\\ 3 & 3\end{vmatrix} = 3-0 = 3$$

$$\text{adj } A = \begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix} = \begin{bmatrix}-3 & 0 & 0\\ 3 & -1 & 0\\ -9 & -2 & 3\end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{-3}\begin{bmatrix}-3 & 0 & 0\\ 3 & -1 & 0\\ -9 & -2 & 3\end{bmatrix}$$

Q9. $\begin{bmatrix}2 & 1 & 3\\ 4 & -1 & 0\\ -7 & 2 & 1\end{bmatrix}$

A.9. Let $A = \begin{bmatrix}2 & 1 & 3\\ 4 & -1 & 0\\ -7 & 2 & 1\end{bmatrix}$. Then , $|A| = \begin{vmatrix}2 & 1 & 3\\ 4 & -1 & 0\\ -7 & 2 & 1\end{vmatrix}$

$$|A| = 2\begin{vmatrix}-1 & 0\\ 2 & 1\end{vmatrix} - 1\begin{vmatrix}4 & 0\\ -7 & 1\end{vmatrix} + 3\begin{vmatrix}4 & -1\\ -7 & 2\end{vmatrix}$$

$= 2(-1-0) - 1(4-0) + 3(8-7)$

$= -2 - 4 + 3 = -3 \neq 0.$

So, A^{-1} exist

Now, $A_{11} = (-1)^{1+1}\begin{vmatrix}-1 & 0\\ 2 & 1\end{vmatrix} = -1 - 0 = -1$

$A_{12} = (-1)^{1+2}\begin{vmatrix}4 & 0\\ -7 & 1\end{vmatrix} = (-1)(4-0) = -4$

$A_{13}\ (-1)^{1+3}\begin{vmatrix}4 & -1\\ -7 & 2\end{vmatrix} = (8-7) = 1$

$A_{21} = (-1)^{2+1}\begin{vmatrix}1 & 3\\ 2 & 1\end{vmatrix} = (-1)(-6) = 5.$

$A_{22} = (-1)^{2+2}\begin{vmatrix}2 & 3\\ -7 & 1\end{vmatrix} = (2-(21)) = 2 + 21 = 23$

$A_{23} = (-1)^{2+3}\begin{vmatrix}2 & 1\\ -7 & 2\end{vmatrix} = (-1)(4-(-7)) = (-1)(4+7) = -11$

$A_{31} = (-1)^{3+1}\begin{vmatrix}1 & 3\\ -1 & 0\end{vmatrix} = (0-(-3)) = 3$

$A_{32} = (-1)^{3+2}\begin{vmatrix}2 & 3\\ 4 & 0\end{vmatrix} = (-1)(0-12) = 12$

$A_{33} = (-1)^{3+3}\begin{vmatrix}2 & 1\\ 4 & -1\end{vmatrix} = (-2-4) = -6.$

So, $A = \begin{bmatrix}A_{11} & A_{41} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix} = \begin{bmatrix}-1 & 5 & 3\\ -4 & 23 & 12\\ 1 & -11 & -6\end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}\text{ adj } A = \frac{1}{-3}\begin{bmatrix}-1 & 5 & 3\\ -4 & 23 & 12\\ 1 & -11 & -6\end{bmatrix}$

Q10. $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$

A.10. Let $A = \begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$ Then $|A| = \begin{vmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{vmatrix}$

$\Rightarrow |A| = 1\times\begin{vmatrix}2 & -3\\ -2 & 4\end{vmatrix} - (-1)\begin{vmatrix}0 & -3\\ 3 & 4\end{vmatrix} + 2\begin{vmatrix}0 & 2\\ 3 & -2\end{vmatrix}$

$= (8-6) + (0-(-9)) + 2(0-6)$

$= 2 + 9 - 12 = -1 \neq 0$

So, A^{-1} exist

$A_{11} = (-1)^{1+1}\begin{vmatrix}2 & -3\\ -2 & 4\end{vmatrix} = (8-6) = 2$

$A_{12} = (-1)^{1+2}\begin{vmatrix}0 & -3\\ 3 & 4\end{vmatrix} = (-1)(0-(-9)) = -9$

$A_{13} = (-1)^{1+3}\begin{vmatrix}0 & 2\\ 3 & -2\end{vmatrix} = (0-6) = -6.$

$A_{21} = (-1)^{2+1}\begin{vmatrix}-1 & 2\\ -2 & 4\end{vmatrix} = (-1)(-4-(-4)) = 0$

$A_{22} = (-1)^{2+2}\begin{vmatrix}1 & 2\\ 3 & 4\end{vmatrix} = (4-6) = -2$

$A_{23} = (-1)^{2+3}\begin{vmatrix}1 & -1\\ 3 & -2\end{vmatrix} = (-1)(-2(-3)) = -1$

$A_{31} = (-1)^{3+1}\begin{vmatrix}-1 & 2\\ 2 & -3\end{vmatrix} = (3-4) = -1$

$A_{32} = (-1)^{3+2}\begin{vmatrix}1 & 2\\ 0 & -3\end{vmatrix} = (-1)(-3-0) = 3$

$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = (2 - 0) = 2$

So, adj A = $\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$

$\because A^{-1} = \frac{1}{|A|}$ adj A $= \frac{1}{-1}\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

Q11. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\propto & \sin\propto \\ 0 & \sin\propto & -\cos\propto \end{bmatrix}$

A.11. Let A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\propto & \sin\propto \\ 0 & \sin\propto & -\cos\propto \end{bmatrix}$. Then |A| = $\begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\propto & \sin\propto \\ 0 & \sin\propto & -\cos\propto \end{vmatrix}$

$\Rightarrow |A| = 1\begin{vmatrix} \cos\propto & \sin\propto \\ \sin\propto & -\cot\propto \end{vmatrix} - 0\begin{vmatrix} 0 & \sin\propto \\ 0 & -\cos\propto \end{vmatrix} + 0\begin{vmatrix} 0 & \cos\propto \\ 0 & \sin\propto \end{vmatrix}$

$= -\cos^2\propto - \sin^2\propto = -\left(\cos^2\propto + \sin^2\propto\right) = -1 \neq 0$

So, A^{-1} exist

Now $A_{11} = (-1)^{1+1}\begin{vmatrix} \cos\propto & \sin\propto \\ \sin\propto & -\cos\propto \end{vmatrix} = -\cos^2\alpha - \sin^2\alpha = -\left(\cos^2\propto + \sin^2\propto\right) = -1$

$A_{12} = (-1)^{1+2}\begin{vmatrix} 0 & \sin\propto \\ 0 & -\cot\propto \end{vmatrix} = (-1)(0 - 0) = 0$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 0 & \cos\propto \\ 0 & \sin\propto \end{vmatrix} = (0 - 0) = 0.$

$A_{21} = (-1)^{2+1}\begin{vmatrix} 0 & 0 \\ \sin\propto & -\cos\propto \end{vmatrix} = (-1)(0 - 0) - 0.$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 0 \\ 0 & -\cos\propto \end{vmatrix} = -\cos\propto - 0 = -\cos\propto$

$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 0 & \sin\propto \end{vmatrix} = (-1)(\sin\propto) = -\sin\propto.$

$A_{31} = (-1)^{3+1}\begin{vmatrix} 0 & 0 \\ \cos\propto & \sin\propto \end{vmatrix} = 0 - 0 = 0$

$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 0 \\ 0 & \sin\propto \end{vmatrix} = (-1)(\sin\propto - 0) = -\sin\propto.$

$A_{33}=(-1)^{3+3}\begin{vmatrix}1 & 0\\ 0 & \cos\propto\end{vmatrix}=\cos\propto.-0=\cos\propto.$

So, $\text{adj } A=\begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix}=\begin{bmatrix}-1 & 0 & 0\\ 0 & -\cos\propto & -\sin\propto\\ 0 & -\sin\propto & \cos\propto\end{bmatrix}$

$\therefore\ A^{-1}=\dfrac{1}{|A|}\text{ adj } A=\dfrac{1}{-1}\begin{bmatrix}-1 & 0 & 0\\ 0 & -\cos\propto & -\sin\propto\\ 0 & -\sin\propto & \cos\propto\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\propto & \sin\propto\\ 0 & \text{cosin}\propto & -\cos\propto\end{bmatrix}$

Q12. Let $A=\begin{bmatrix}3 & 7\\ 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}6 & 8\\ 7 & 9\end{bmatrix}$. Verify that $(AB)^{-1}=B^{-1}A^{-1}$.

A.12. Given, $A=\begin{bmatrix}3 & 7\\ 2 & 5\end{bmatrix}$. Then $|A|=\begin{vmatrix}3 & 7\\ 2 & 5\end{vmatrix}=(15-14)=1\neq 0$

So, A^{-1} exist.

$\text{Adj}A=\begin{bmatrix}5 & -7\\ -2 & 3\end{bmatrix}\left\{\therefore\ \text{adj of }\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}.\right\}$

$\therefore\ A^{-1}=\dfrac{1}{|A|}\text{ adj}A=\dfrac{1}{1}\begin{bmatrix}5 & -7\\ -2 & 3\end{bmatrix}=\begin{bmatrix}5 & -7\\ -2 & 3\end{bmatrix}$

And $B=\begin{bmatrix}6 & 8\\ 7 & 9\end{bmatrix}$. Then, $|B|=\begin{vmatrix}6 & 8\\ 7 & 9\end{vmatrix}=54-56=-2\neq 0$

So B^{-1} exist

$\text{adj } B=\begin{bmatrix}9 & -8\\ -7 & 6\end{bmatrix}$

$\therefore\ B^{-1}=\dfrac{1}{|B|}\text{ adj } B=\dfrac{1}{-2}\begin{bmatrix}9 & -8\\ -7 & 6\end{bmatrix}$

Now, $AB=\begin{bmatrix}3 & 7\\ 2 & 5\end{bmatrix}\begin{bmatrix}6 & 8\\ 7 & 9\end{bmatrix}=\begin{bmatrix}3\times6+7\times7 & 3\times8+7\times9\\ 2\times6+5\times7 & 2\times8+5\times9\end{bmatrix}=\begin{bmatrix}18+49 & 24+63\\ 12+35 & 16+45\end{bmatrix}=\begin{bmatrix}67 & 87\\ 47 & 61\end{bmatrix}$

$|AB|=\begin{vmatrix}67 & 87\\ 47 & 61\end{vmatrix}=67\times61-87\times47=4087-4089=-2\neq0$

So, $(AB)^{-1}$ exist

$\text{adj }(AB)=\begin{bmatrix}61 & -87\\ -47 & 67\end{bmatrix}$

$\therefore\ (AB)^{-1}=\dfrac{1}{|AB|}\text{ adj }(AB)=\dfrac{1}{-2}\begin{bmatrix}61 & -87\\ -47 & 67\end{bmatrix}$ = L.H.S.

And R.H.S $=B^{-1}A^{-1}=\dfrac{1}{-2}\begin{bmatrix}9 & -8\\ -7 & 6\end{bmatrix}\begin{bmatrix}5 & -7\\ -2 & 3\end{bmatrix}$

$$=\frac{1}{-2}\begin{bmatrix}9\times5+(-8)(-2) & 9\times(-7)+(-8)\times3\\ -7\times5+6\times(-2) & -7\times(-7)+6\times3\end{bmatrix}$$

$$=\frac{1}{-2}\begin{bmatrix}45+16 & -63-24\\ -35-12 & 49+18\end{bmatrix}$$

$$=\frac{1}{-2}\begin{bmatrix}61 & -87\\ -47 & 67\end{bmatrix}$$

$\therefore$ L.H.S. = R.H.S.

Q13. f $A = \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2 - 5A+7I=O$. Hence find A^{-1}

A.13. Given, $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

we have, $A^2 = A\cdot A = \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$$=\begin{bmatrix}3\times3+1\times(-1) & 3\times1+1\times2\\ (-1)\times3+(-1)\times2 & (-1)\times1+2\times2\end{bmatrix}$$

$$=\begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$$

Hence, $A^2 - 5A+71=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$$=\begin{bmatrix}8-5\times3+7\times1 & 5-5\times1+7\times0\\ -5-5\times(1)+7\times0 & 3-5\times2+7\times1\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=0$$

Now, $A^2 - 5A+7I=0$.

$A\cdot A - 5A= -7I$.

$\Rightarrow A\,A(A^{-1})-5(AA^{-1})= -7I(A^{-1})$ (Multiplying by A^{-1} on both sides)

$\Rightarrow AI - 5I= -7A^{-1}$

$\Rightarrow 7A^{-1}= -(AI - 5I)$

$\Rightarrow A^{-1}=\frac{1}{7}[-A+5I]$

$$=\frac{1}{7}\left\{(-1)\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+5\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right\}$$

$$=\frac{1}{7}\left\{\begin{bmatrix}(-1)\times3+5\times1 & -1+5\times0\\ (-1)\times(-1)+5\times0 & (-1)\times2+5\times1\end{bmatrix}\right\}$$

$$A^{-1}=\frac{1}{7}\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}$$

Q14. For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = O$.

A.14. Given, $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

We have, $A^2 = A \cdot A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3\times 3+2\times 1 & 3\times 2+2\times 1 \\ 1\times 3+|\times| & 1\times 2+1\times 1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

As $A^2 + a\,A + bI = 0$.

$$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 11+3a+b & 8+2a+0 \\ 4+a+0 & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Equating the (Corresponding) elements of the matrices,

$11+3a+b=0$ --------(i)

$8+2a=0$ ------(ii)

$4+a=0$ ------(iii)

$3+a+b=0$ ------(iv)

Foreq^n(iii),

$4+a=0$

$\Rightarrow$ $\boxed{a = -4}$

Put a value of a in equation (iv) we gets,

$3+(-4)+b=0$

$\Rightarrow -1+b=0$

$\Rightarrow \boxed{b = 1}$

Q15. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$.

A.15. Given, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$.

$$15A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A^2 = A\,.\,A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = A^2 . A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

Then , $A^3 - 6A^2 + 5A + 11I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6 \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8-24+5+11 & 7-12+5+0 & 1-6+5+0 \\ -23+18+5+0 & 27-48+10+11 & -69+84-15 \\ 32-42+10+0 & -13+18-5+0 & 58-84+15+11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0.$$

$\therefore\ A^{-1} \times (A^3 - 6A^2 + 5A + 11I) = 0$

$\Rightarrow A^2 - 6A + 5I + 11A^{-1} = 0$

$\Rightarrow 11A^{-1} = 6A - 5I - A^2$

$$\Rightarrow 11A^{-1} = 6 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 6-5-4 & 6-2 & 6-1 \\ 6+3 & 12-5-8 & -18+14 \\ 12-7 & -6+3 & 18-5-14 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

Q16. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}.

A.16. We have, $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Then, $A^2 = A.A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 1+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$A^3 = A^2 . A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36+18-4 & -21+30-9-0 & 21-30+9-0 \\ -21+30-9-0 & 22-36+18-4 & -21+30-9-0 \\ 21-30+9-0 & -21+30-9-0 & 22-36+18-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0.$$

Now, $A^{-1}\left[A^3 - 6A^2 + 9A - 4I\right] = 0$

$\Rightarrow A^2 - 6A + 9I - 4A^{-1} = 0.$

$\Rightarrow 4A^{-1} = A^2 - 6A + 9I$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Q17. Let A be a nonsingular square matrix of order 3×3 Then $|adj\ A|$ is equal to

(A) $|A|$ (B) $|A|^2$ (C) $|A|^3$ (D) $3|A|$

A.17. As A is a non-singular matrik of or 3×3.

$|\text{adj } A| = |A|^{3-1} = |A|^2$

$\therefore$ Option (B) is correct.

Q18. s an invertible matrix of order 2, then det (A^{-1}) is equal to

(A) $\det(A)$ (B) $\frac{1}{\det(A)}$ (C) 1 (D) 0

A.18. Given, A is an invertible matrix of order 2.

$AA^{-1} = I.$

$\Rightarrow |AA^{-1}| = |I|.$

$\Rightarrow |A||A^{-1}| = 1$

$\Rightarrow |A^{-1}| = \frac{1}{|A|}$

$\Rightarrow \det(A^{-1}) = \frac{1}{\det(A)}$

$\therefore$ Option (B) is correct.

Exercise 4.6

Examine the consistency of the system of equations in Exercise 1 to 6.

Q1. $x + 2y = 2$
$2x + 3y = 3$

A.1. The given system of equation can be written in the form of AK = B

Where A = $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and K = $\begin{bmatrix} x \\ y \end{bmatrix}$

Now, $|A| = 3 \times 1 - 2 \times 2 = 3 - 4 = -1 \neq 0$.

$\therefore$ The system has unique sol^n and hence equation are consistent

Q2. $2x - y = 5$
$x + y = 4$

A.2. The given system of eq^n can be written in the form AK = B where

A = $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$ and K = $\begin{bmatrix} x \\ y \end{bmatrix}$.

Now, $|A| = 2 - (-1) = 2 + 1 = 3 \neq 0$.

$\therefore$ The system of eq^{n}is consistent.

Q3. $2x + 3y = 5$
$2x + 6y = 8$

A.3. The given system of equation can be written in the form AK = B

Where A = $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$, B = $\begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and K = $\begin{bmatrix} x \\ y \end{bmatrix}$

Now, $|A| = 6 - 6 = 0$

adj A = $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

$\therefore$ (adj A) B = $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30-24 \\ -10+8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq 0$.

Hence the given system of eq^n are inconsistent.

Q4. $x + y + z = 1$
$2x + 3y + 2z = 2$
$ax + ay = 2az = 4$

A.4. The given system of eq^n using matrik method can be ekpressed as

AK = B

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

Now, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{vmatrix} = 1\begin{vmatrix} 3 & 2 \\ a & 2a \end{vmatrix} - 1\begin{vmatrix} 2 & 2 \\ a & 2a \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ a & a \end{vmatrix}$

$= (6a - 2a) - (4a - 2a) + (2a - 3a)$

$= 4a - 2a - a$

$= a \neq 0.$

Hence, the given system of eq^n is consistent

Q5.
$$3x - y - 2z = 2$$
$$2y - z = -1$$
$$3x - 5y = 3$$

A.5. The given system of eq^n using matrix form can be written as AK = B

$$\Rightarrow \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

Now, $|A| = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

$= 3\begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} + (-2)\begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix}$

$= 3(0 - 5) + 1(0 + 3) - 2(0 - 6)$

$= -15 + 3 + 12$

$= -15 + 15$

$= 0$

$A_{11} = (-1)^2 \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} = (0 - 5) = -5.$

$A_{12} = (-1)^3 \begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} = -(0 + 3) = -3$

$A_{13} = (-1)^4 \begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix} = (0 - 6) = -6.$

$A_{21} = (-1)^3 \begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} = -(0 - 10) = 10.$

$A_{22} = (-1)^4 \begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} = (0 + 6) = 6.$

$$A_{23} = (-1)^5 \begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix} = -(-15+3) = 12.$$

$$A_{31} = (-1)^4 \begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix} = (1+4) = 5.$$

$$A_{32} = (-1)^5 \begin{vmatrix} 3 & -2 \\ 0 & -1 \end{vmatrix} = -(-3-0) = 3.$$

$$A_{33} = (-1)^6 \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = (6-0) = 6$$

$$\text{adj A} = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$\therefore (\text{adj A}).B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -10-10+15 \\ -6-6+9 \\ -12-12+18 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq 0$$

$\therefore$ The given system of equation is inconsistent.

Q6.
$$5x - y + 4z = 5$$
$$2x + 3y + 5z = 2$$
$$5x - 2y + 6z = -1$$

A.6. The given system of eq^n in matrix form is AK = B

$$\Rightarrow \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$\text{Then } |A| = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix} = 5(18+10) - (-1)(12-25) + 4(-4-15)$$

$= 5 \times 28 - 13 + 4 \times (-9)$

$= 140 - 13 - 76$

$= 51 \neq 0$.

The given system of eq^n are consistent

Solve system of linear equations, using matrix method, in Exercise 7 to 14.

Q7.
$$5x + 2y = 4$$
$$7x + 3y = 5$$

A.7. The given system of eq^n in matrix form is AK = B.

i.e, $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Then, $|A| = 5 \times 3 - 7 \times 2 = 15 - 14 = 1 \neq 0$

$\therefore$ The system has a unique solution.

And $X = A^{-1}B = \dfrac{\text{adj}(A)}{|A|}.B$

$\rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1}\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 12-10 \\ -28+25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

$\therefore k = 2$ and $y = -3$

Q8. $2x - y = -2$

$3x + 4y = 3$

A.8. The given system of eq^n in matrix form is AK = B

i.e, $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 8 + 3 = 11 \neq 0$

$\therefore$ The system has a unique solution.

And $X = A^{-1}B = \dfrac{\text{adj}(A)}{|A|}.B$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}.\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$= \frac{1}{11}\begin{bmatrix} -8+3 \\ 6+6 \end{bmatrix} = \begin{bmatrix} -5/11 \\ 12/11 \end{bmatrix}$

$\therefore k = \dfrac{-5}{11}$ and $y = \dfrac{12}{11}$

Q9. $4x - 3y = 3$

$3x - 5y = 7$

A.9. The given system of eq^n can be written in matrix form as

AK = B

$\Rightarrow \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix} = -20 + 9 = -11 \neq 0$

So, the system has a unique solution

And $K = A^{-1}B = \dfrac{\text{adj}(A)}{|A|}.B = \dfrac{1}{-11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} -15+21 \\ -9+28 \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} 6 \\ 19 \end{bmatrix} = \begin{bmatrix} -6/11 \\ -19/11 \end{bmatrix}$$

$\therefore k = \frac{-6}{11}$ and $y = \frac{-19}{11}$

Q10. $5x+2y=3$
$3x+2y=5$

A.10. The given system of equation in matrix form is

AK = B

$$\Rightarrow \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Then $|A| = \begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix} = 10-6 = 4 \neq 0.$

Hence, the system has unique solution

i.e, $X = A^{-1}B = \frac{\text{adj}(A)}{|A|}.B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 6+0 \\ -9+25 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}.$$

$\therefore x = -1$ and $y = 4$

Q11. $2x+y+z=1$
$x-2y-z=\frac{3}{2}$
$3y-5z=9$

A.11. The given system of solution in matrix form is AK = B.

$$\Rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

Now, $|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix} = 2(10+3)-1(-5-0)+1(3-0) = 26+5+3 = 34 \neq 0.$

So, the system has a unique solution.

Now, $A_{11} = (-1)^2\begin{vmatrix} -2 & 1 \\ 3 & -5 \end{vmatrix} = 10+3 = 13$

$A_{12} = (-1)^3\begin{vmatrix} 1 & -1 \\ 0 & -5 \end{vmatrix} = -(-5-0) = 5$

$$A_{13} = (-1)^4 \begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} = (3-0) = 3$$

$$A_{21} = (-1)^3 \begin{vmatrix} 1 & 1 \\ 3 & -5 \end{vmatrix} = -(-5-3) = 8.$$

$$A_{22} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 0 & -5 \end{vmatrix} = (-10-0) = -10.$$

$$A_{23} = (-1)^5 \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = -(6-0) = -6.$$

$$A_{31} = (-1)^4 \begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix} = (-1+2) = 1.$$

$$A_{32} = (-1)^5 \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -(-2-1) = 3$$

$$A_{33} = (-1)^6 \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = (-4-1) = -5.$$

So, $A^{-1} = \dfrac{\text{adj}(A)}{|A|} = \dfrac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$

$$\therefore K = A^{-1}B = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13+12+9 \\ 5-15+27 \\ 3-9-45 \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ -3/2 \end{bmatrix}$$

$\therefore x = 1, y = ½$ and $z = -3/2$.

Q12.
$$\begin{aligned} x - y + z &= 4 \\ 2x + y - 3z &= 0 \\ x + y + z &= 2 \end{aligned}$$

A.12. The given system of eq^n can be written in matrix form as AK = B.

$$\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}.$$

Then, $|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1(1+3) - (-1)(2+3) + 1(2-1) = 4 + 5 + 1 = 10 \neq 0.$

$\therefore$ The sol^n of the system is unique.

Now, $A_{11} = (1)\begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} = 1+3 = 4$

$$A_{12} = (-1)\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -(2+3) = -5$$

$$A_{13} = (1)\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = (2-1) = 1$$

$$A_{21} = (-1)\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -(-1-1) = +2$$

$$A_{22} = (1)\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1-1 = 0$$

$$A_{23} = (-1)\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = -(1+1) = -2$$

$$A_{31} = (1)\begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = (3-1) = 2$$

$$A_{32} = (-1)\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -(-3-2) = 5$$

$$A_{33} = (1)\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1+2 = 3.$$

So, $A^{-1} = \dfrac{\text{adj}(A)}{|A|} = \dfrac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$\therefore K = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4-0+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore x = 2, y = -1$ and $z = 1$.

Q13.
$$2x+3y+3z=5$$
$$x-2y+z=-4$$
$$3x-y-2z=3$$

A.13. The given system of equation can be represented in matrix form as AK = B

$$\Rightarrow \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

Then $|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 2(4+1) - 3(-2-3) + 3(-1+6)$ = 10 + 15 + 15 = 40 ≠ 0.

Now, $A_{11} = \begin{vmatrix} -2 & 1 \\ -1 & -2 \end{vmatrix} = 4 + 1 = 5$

$A_{12} = (-1)\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = -(-2-3) = 5$

$A_{13} = \begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix} = -1 + 6 = 5$

$A_{21} = (-1)\begin{vmatrix} 3 & 3 \\ -1 & -2 \end{vmatrix} = -(-6+3) = 3$

$A_{22} = \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = -4 - 9 = -13$

$A_{23} = (-1)\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} = -(-2-9) = 11$

$A_{31} = \begin{vmatrix} 3 & 3 \\ -2 & 1 \end{vmatrix} = 3 + 6 = 9$

$A_{32} = (-1)\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -(2-3) = 1$

$A_{33} = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -4 - 3 = -7$

So, $A^{-1} = \dfrac{\text{adj}(A)}{|A|} = \dfrac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$

So, K = A^{-1}B

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \dfrac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

$= \dfrac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix} = \dfrac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

$\therefore x = 1$, $y = 2$ and $z = -1$.

Q14. $x - y + 2z = 7$
$3x + 4y - 5z = -5$
$2x - y + 3z = 12$

A.14. The given system of eq^n can be written in matrix form as

AK = B

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

Now, $|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix} = 1(12-5)-(-1)(9+10)+2(-3-8) = 7 + 19 - 22 = 4 \neq 0.$

$\therefore$ The system has a unique solution.

So, $A_{11} = \begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix} = 12 - 5 = 7$

$$A_{12} = (-1)\begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} = -(9+10) = -19$$

$$A_{13} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -\ 3 - 8 = -11$$

$$A_{21} = (-1)\begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} = -(-3+2) = 1$$

$$A_{22} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1$$

$$A_{23} = (-1)\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -(-1+2) = -1$$

$$A_{31} = \begin{vmatrix} -1 & 2 \\ 4 & -5 \end{vmatrix} = 5 - 8 = -3$$

$$A_{32} = (-1)\begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -(-5-6) = 11$$

$$A_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} = 4 + 3 = 7$$

ie, $A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$

So, $K = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\therefore x = 2, y = 1$ and $z = 3$.

Q15. If $A=\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -5 \\ 1 & 1 & -5 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$$2x-3y+5z=11$$
$$3x+2y-4z=-5$$
$$x+y-2z=-3$$

A.15. Given, $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

$= 2(-4+4)-(-3)(-6+4)+5(3-2)$

$= 0 + 3(-2) + 5(1)$

$= -6 + 5 = -1 \neq 0$

$\therefore A^{-1}$ exist and $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

Now, $A_{11} = \begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} = (-4+4) = 0$

$A_{12} = (-1)\begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} = -(-6+4) = 2$

$A_{13} = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3-2 = 1$

$A_{21} = (-1)\begin{vmatrix} -3 & 5 \\ 1 & -2 \end{vmatrix} = -(6-5) = -1$

$A_{22} = \begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix} = -4-5 = -9$

$A_{23} = (-1)\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -(2+3) = -5$

$A_{31} = \begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix} = 12-10 = 2.$

$A_{32} = (-1)\begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix} = -(-8-15) = 23$

$A_{33} = \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = 4+9 = 13.$

$$\therefore \; A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of eq^n in the matrix form is AX = B

$$\Rightarrow \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$\therefore$ The solution is unique we have,

$$X = A^{-1} B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore x = 1$, $y = 2$ and $z = 3$.

Q16. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is `60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is `90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is `70. Find cost of each item per kg by matrix method.

A.16. Let k, y and z be the cost per kg of onion, wheat and rice. Then, we form a system of equations as follows

$$4k + 3y + 2z = 60$$

$$2k + 4y + 6z = 90$$

$$6k + 2y + 3z = 70$$

In matrix form we can write,

$$AX = B$$

$$\Rightarrow \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

Then, $|A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix} = 4(12-12) - 3(6-36) + 2(4-24)$

$$= 0 - 3(-30) + 2(-20)$$

$$= 90 - 40$$

$$= 50 \neq 0.$$

∴ The system has a unique solution.

Now, $A_{11} = \begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix} = 12 - 12 = 0$

$$A_{12} = (-1)\begin{vmatrix} 2 & 6 \\ 6 & 3 \end{vmatrix} = -(6-36) = 30$$

$$A_{13} = \begin{vmatrix} 2 & 4 \\ 6 & 2 \end{vmatrix} = 4 - 24 = -20$$

$$A_{21} = (-1)\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = -(9-4) = -5$$

$$A_{22} = \begin{vmatrix} 4 & 2 \\ 6 & 3 \end{vmatrix} = 12 - 12 = 0$$

$$A_{23} = (-1)\begin{vmatrix} 4 & 3 \\ 6 & 2 \end{vmatrix} = -(8-18) = 10$$

$$A_{31} = \begin{vmatrix} 3 & 2 \\ 4 & 6 \end{vmatrix} = 18 - 8 = 10$$

$$A_{32} = (-1)\begin{vmatrix} 4 & 2 \\ 2 & 6 \end{vmatrix} = -(24-4) = -20$$

$$A_{33} = \begin{vmatrix} 4 & 3 \\ 2 & 4 \end{vmatrix} = 16 - 6 = 10$$

So, $A^{-1} = \dfrac{\text{adj}(A)}{|A|} = \dfrac{1}{50}\begin{bmatrix} 0 & -3 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$

Hence, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$= \frac{1}{850}\begin{bmatrix} -450+700 \\ 1800-1400 \\ -1200+900+700 \end{bmatrix}$$

$$= \frac{1}{850}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

∴ $x = 5, y = 8$ and $z = 8$

The cost of onion, wheat and rice per kg are ₹ 5, ₹ 8 and ₹ 8 respectively.

Miscellaneous Exercise

Q1. Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ **is independent of** θ.

A.1. $\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

$= x\left(-x^2-1\right)-\sin\theta\left(-x\sin\theta-\cos\theta\right)+\cos\theta\left(-\sin\theta+x\cos\theta\right)$

$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \cos\theta\sin\theta + x\cos^2\theta$

$= -x^3 - x + x\left(\sin^2\theta+\cos^2\theta\right)$

$= -x^3 - x + x$

$= -x^3$, which is independent of θ.

Q2. Without expanding the determinants, prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

Solution. L.H.S.= $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix}$

Multiplying R_1, by a, R_2 by b and R_3 by c

$$= \frac{1}{abc}\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix}$$

Taking abc common from $c_3 = \frac{abc}{abc}\begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$

Inter changing c_1 and $c_3 = -\begin{vmatrix} 1 & a^3 & a^2 \\ 1 & b^3 & b^2 \\ 1 & c^3 & c^2 \end{vmatrix}$

Inter changing c_2 and c_3 $= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = R.H.S$

Q3. **Evaluate** $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

A.3 $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

$$= -\sin\alpha\left(-\sin^2\beta\sin\alpha - \sin\alpha\cos^2\beta\right) - 0\left(\cos\alpha\cos\beta\sin\alpha\sin\beta - \cos\alpha\sin\beta\sin\alpha\cos\beta\right) + \cos\alpha\left(\cos\alpha\cos^2\beta + \cos\alpha\sin^2\beta\right)$$

$$= \sin^2\alpha\left(\sin^2\beta + \cos^2\beta\right) + \cos^2\alpha\left(\cos^2\beta + \sin^2\beta\right)$$

$$= \sin^2\alpha + \cos^2\alpha$$

$$= 1$$

Q4. If a, b and c are real numbers and

$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$ Show that either a+b+c=0 or a=b=c=0

Solution. $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Taking $2(a+b+c)$ common from R_1

$$\Rightarrow 2(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Either $2(a+b+c) = 0$ i.e. a+b+c=0 or

$$\begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

$c_2 \to c_2 - c_1$ and $c_3 \to c_3 - c_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & b-a \\ a+b & c-a & c-b \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow \begin{vmatrix} b-c & b-a \\ c-a & c-b \end{vmatrix} = 0$$

$$\Rightarrow (b-c)(c-b)-(b-a)(c-a)=0$$

$$\Rightarrow bc-b^2-c^2+cb-bc+ab+ac-a^2=0$$

$$\Rightarrow -a^2-b^2-c^2+ab+bc+ca=0$$

Multiplying by -2

$$\Rightarrow 2a^2+2b^2+2c^2-2ab-2bc-2ca=0$$

$$\Rightarrow a^2+a^2+b^2+b^2+c^2+c^2--2ab-2bc-2ca=0$$

$$\Rightarrow \left(a^2+b^2-2ab\right)+\left(a^2+c^2-2ac\right)+\left(b^2+c^2-2bc\right)=0$$

$$\Rightarrow (a-b)^2+(a-c)^2+(b-c)^2=0$$

$$\Rightarrow a-b=0, b-c=0, c-a=0$$

$$\left[\because x^2+y^2+z^2=0 \Rightarrow x=0, y=0, z=0\right]$$

$$\Rightarrow a=b, b=c, c=a$$

$$\Rightarrow a=b=c$$

$\therefore$ Either a+b+c=0 or a=b=c

Q5. Solve the equation $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, a \neq 0$

Solution.

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$R_1 \to R_1+R_2+R_3$$

$$\begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Taking 3x+a common from R_1

$$(3x+a)\begin{vmatrix}1 & 1 & 1\\ x & x+a & x\\ x & x & x+a\end{vmatrix}=0$$

Either $3x+a=0 \Rightarrow x=-a/3$

Or

$$\begin{vmatrix}1 & 1 & 1\\ x & x+a & x\\ x & x & x+a\end{vmatrix}=0$$

$c_2 \to c_2-c_1, c_3 \to c_3-c_1$

$$\begin{vmatrix}1 & 0 & 0\\ x & a & 0\\ x & 0 & a\end{vmatrix}=0$$

Expanding along $R_1,\ a^2=0$

$\Rightarrow a=0$

$\therefore\ x=-a/3$ is the only solution.

Q6. Prove that $\begin{vmatrix}a^2 & bc & ac+c^2\\ a^2+ab & b^2 & ac\\ ab & b^2+bc & c^2\end{vmatrix}=4a^2b^2c^2$

Solution. $L.H.S=\begin{vmatrix}a^2 & bc & ac+c^2\\ a^2+ab & b^2 & ac\\ ab & b^2+bc & c^2\end{vmatrix}$

$$=\begin{vmatrix}a^2 & bc & c(a+c)\\ a(a+b) & b^2 & ac\\ ab & b(b+c) & c^2\end{vmatrix}$$

Taking a, b, c common from c_1, c_2 and c_3 respectively

$$= abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$R_1 \to R_1 - R_2 - R_3$

$$= abc\begin{vmatrix} -2b & -2b & 0 \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$c_2 \to c_2 - c_1$

$$= abc\begin{vmatrix} -2b & 0 & 0 \\ a+b & -a & a \\ b & c & c \end{vmatrix}$$

Expanding along R_1

$$abc(-2b)(-2ac) = 4a^2 + b^2 + c^2 = R.H.S.$$

Q7. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ find $(AB)^{-1}$

A.7.

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$B = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$|B| = 1(3-0) - 2(-1-0) - 2(2-0) = 1 \neq 0$

$\Rightarrow B^{-1}$ exists.

$B_{11} = 3$ $B_{12} = 1$ $B_{13} = 2$

$B_{21} = 2$ $B_{22} = 1$ $B_{23} = 2$

$B_{31} = 6$ $B_{32} = 2$ $B_{33} = 5$

Hence, adj $B = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Thus, $B^{-1} = \dfrac{1}{|B|}\text{adj } B = \dfrac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

We know that : $(AB)^{-1} = B^{-1}A^{-1}$, then

$$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Q8. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$. Verify that

(i) $[\text{adj } A]^{-1} = \text{adj }(A^{-1})$ (ii) $(A^{-1})^{-1} = A$

A.8. **(i)** $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

SO, $|A| = 1(15-1) - 2(10-1) + 1(2-3)$

$= 14 - 18 - 1$

$= -5 \neq 0$

$\therefore A^{-1}$ exist.

$$A_{11} = \begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} = 15 - 1 = 14$$

$$A_{12} = (-1)\begin{vmatrix} 2 & 1 \\ 1 & 5 \end{vmatrix} = -(10-1) = -9$$

$$A_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 2 - 3 = -1$$

$$A_{21} = (-1)\begin{vmatrix} 2 & 1 \\ 1 & 5 \end{vmatrix} = -(10-1) = -9$$

$$A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} = 5 - 1 = 4$$

$$A_{23} = (-1)\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -(1-2) = 1$$

$$A_{31} = \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = 2 - 3 = -1$$

$$A_{32} = (-1)\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -(1-2) = 1$$

$$A_{33}=\begin{vmatrix}1 & 2\\ 2 & 3\end{vmatrix}=3-4=-1$$

$$\text{So, } \operatorname{adj}(A)=\begin{bmatrix}14 & -9 & -1\\ -9 & 4 & 1\\ -1 & 1 & -1\end{bmatrix}$$

$$\text{Let } B=\operatorname{adj}(A)=\begin{bmatrix}14 & -9 & -1\\ -9 & 4 & 1\\ -1 & 1 & -1\end{bmatrix}$$

$$\text{So, } |B|=14(-4-1)-(-9)(9+1)-1(-9+4)$$

$$=-70+90+5=25\neq 0$$

$\therefore$ B^{-1} exist

$$B_{11}=\begin{vmatrix}4 & 1\\ 1 & -1\end{vmatrix}=-4-1=-5$$

$$B_{12}=(-1)\begin{vmatrix}-9 & 1\\ -1 & -1\end{vmatrix}=-(9+1)=-10$$

$$B_{13}=\begin{vmatrix}-9 & 4\\ -1 & 1\end{vmatrix}=-9+4=-5$$

$$B_{21}=(-1)\begin{vmatrix}-9 & -1\\ 1 & -1\end{vmatrix}=-(9+1)=-10$$

$$B_{22}=\begin{vmatrix}14 & -1\\ -1 & -1\end{vmatrix}=(-14-1)=-15$$

$$B_{23}=(-1)\begin{vmatrix}14 & -9\\ -1 & 1\end{vmatrix}=-(14-9)=-5$$

$$B_{31}=\begin{vmatrix}-1 & -1\\ 4 & 1\end{vmatrix}=-9+4=-5$$

$$B_{32}=(-1)\begin{vmatrix}14 & -1\\ -9 & 1\end{vmatrix}=-(14-9)=-5$$

$$B_{33}=\begin{vmatrix}14 & -9\\ -9 & 4\end{vmatrix}=56-81=-25$$

$$\text{So, } B^{-1}=\frac{1}{|B|}\operatorname{adj} B$$

$$=\frac{1}{25}\begin{bmatrix}-5 & -10 & -5\\ -10 & -15 & -5\\ -5 & -5 & -25\end{bmatrix}$$

$$=\begin{bmatrix}-1/5 & -2/5 & -1/5\\ -2/5 & -3/5 & -1/5\\ -1/5 & -1/5 & -1\end{bmatrix}$$

Let C = $A^{-1}=\frac{1}{|A|}\text{adj A}=\frac{1}{-5}\begin{bmatrix}14 & -9 & -1\\ -9 & 4 & 1\\ -1 & 1 & -1\end{bmatrix}$

$$C=\begin{bmatrix}-14/5 & 9/5 & 1/5\\ 9/5 & -4/5 & -1/5\\ 1/5 & -1/5 & 1/5\end{bmatrix}$$

So,

$$C_{11}=\begin{vmatrix}-4/5 & -1/5\\ -1/5 & 1/5\end{vmatrix}=\frac{-4}{25}-\frac{1}{25}=\frac{-5}{25}=-\frac{1}{5}$$

$$C_{12}=(-1)\begin{vmatrix}9/5 & -1/5\\ 1/5 & 1/5\end{vmatrix}=-\left(\frac{9}{25}+\frac{1}{25}\right)=-\frac{10}{25}=\frac{-2}{5}$$

$$C_{13}=\begin{vmatrix}9/5 & -4/5\\ 1/5 & -1/5\end{vmatrix}=\frac{-9}{25}+\frac{4}{25}=\frac{-5}{25}=-\frac{1}{5}$$

$$C_{21}=(-1)\begin{vmatrix}9/5 & 1/5\\ -1/5 & 1/5\end{vmatrix}=(-1)\left(\frac{9}{25}+\frac{1}{25}\right)=-\frac{10}{25}=\frac{-2}{5}$$

$$C_{22}=\begin{vmatrix}-14/5 & 1/5\\ 1/5 & 1/5\end{vmatrix}=\frac{-14}{25}-\frac{1}{25}=\frac{-15}{25}=-\frac{3}{5}$$

$$C_{23}=(-1)\begin{vmatrix}-14/5 & 9/5\\ 1/5 & -1/5\end{vmatrix}=-\left(\frac{14}{25}-\frac{9}{25}\right)=-\frac{5}{25}=\frac{-1}{5}$$

$$C_{31}=\begin{vmatrix}9/5 & 1/5\\ -4/5 & -1/5\end{vmatrix}=\frac{-9}{25}+\frac{4}{25}=\frac{-5}{25}=-\frac{1}{5}$$

$$C_{32}=(-1)\begin{vmatrix}-14/5 & 1/5\\ 9/5 & -1/5\end{vmatrix}=-\left(\frac{14}{25}-\frac{9}{25}\right)=-\frac{5}{25}=\frac{-1}{5}$$

$$C_{33}=\begin{vmatrix}-14/5 & 9/5\\ 9/5 & -4/5\end{vmatrix}$$

$$=\frac{56}{25}-\frac{81}{25}=\frac{-25}{25}=-1.$$

So, adj C $=\begin{bmatrix}-1/5 & -2/5 & -1/5\\ -2/5 & -3/5 & -1/5\\ -1/5 & -1/5 & -1\end{bmatrix}$

Hence, B^{-1} = adj C

$$\Rightarrow (\text{adj A})^{-1}=\text{adj}(A^{-1}) .$$

(ii) Then, $C^{-1}=\frac{1}{|C|}\text{adj }(C)$

$$|c|=\frac{-14}{5}\left(\frac{-4}{25}-\frac{1}{25}\right)-\frac{9}{5}\left(\frac{9}{25}+\frac{1}{25}\right)+\frac{1}{5}\left(\frac{-9}{25}+\frac{4}{25}\right)$$

$$= \frac{-14}{5}\left(\frac{-5}{25}\right) - \frac{9}{5}\left(\frac{10}{25}\right) + \frac{1}{5}\left(\frac{-5}{25}\right)$$

$$= \frac{14}{25} - \frac{18}{25} - \frac{1}{25} = -\frac{5}{25} = -\frac{1}{5}$$

So, $C^{-1} = \frac{1}{(-1/5)} \begin{bmatrix} -1/5 & -2/5 & -1/5 \\ -2/5 & -3/5 & -1/5 \\ -1/5 & -1/5 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = A$$

Hence, $\left(C^{-1}\right) = A \Rightarrow \left(A^{-1}\right)^{-1} = A$

Q9. Evaluate $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

A.9. $\Delta = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(x+y) & y & x+y \\ 2(x+y) & x+y & x \\ 2(x+y) & x & y \end{vmatrix}$$

Taking 2(k+ y) as common from C_1

$$= 2(x+y) \begin{vmatrix} 1 & y & x+y \\ 1 & x+y & x \\ 1 & x & y \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$& $R_2 \to R_2 - R_3$

$$= 2(x+y) \begin{vmatrix} 0 & -x & y \\ 0 & y & x-y \\ 1 & y & y+k \end{vmatrix}$$

Expanding along C_1

$$= 2(x+y)\{(-x)(x-y) - y.y\}$$

$$= 2(x+y)(-x^2 + xy - y^2)$$

$$= -2(x+y)(x^2 - xy + y^2)$$

$$= -2\{x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3\}$$

$$= -2(x^3 + y^3)$$

Q10. Evaluate $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$

A.10 . $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 0 & -y & 0 \\ 0 & y & -x \\ 1 & x & x+y \end{vmatrix}$$

Expanding along C_1,

$$= \{(-y)(-x) - y.0\}$$

$= xy$

Using properties of determinants in Exercise 11 to 15, prove that:

Q11. $\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$

Solution

$$L.H.S. = \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

$c_3 \to c_3 + c_1$

$$= \begin{vmatrix} \alpha & \alpha^2 & \alpha+\beta+\gamma \\ \beta & \beta^2 & \alpha+\beta+\gamma \\ \gamma & \gamma^2 & \alpha+\beta+\gamma \end{vmatrix}$$

$$= (\alpha+\beta+\gamma) \begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta & \beta^2 & 1 \\ \gamma & \gamma^2 & 1 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= (\alpha+\beta+\gamma) \begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta-\alpha & \beta^2-\alpha^2 & 0 \\ \gamma-\alpha & \gamma^2-\alpha^2 & 0 \end{vmatrix}$$

Expanding along c_3

$$=(\alpha+\beta+\gamma)\begin{vmatrix}\beta-\alpha & \beta^2-\alpha^2\\ \gamma-\alpha & \gamma^2-\alpha^2\end{vmatrix}$$

$$=(\alpha+\beta+\gamma)\begin{vmatrix}\beta-\alpha & (\beta-\alpha)(\beta+\alpha)\\ \gamma-\alpha & (\gamma-\alpha)(\gamma+\alpha)\end{vmatrix}$$

$$\Rightarrow(\alpha+\beta+\gamma)(\beta-\alpha)(\gamma-\alpha)\begin{vmatrix}1 & \beta+\gamma\\ 1 & \gamma+\alpha\end{vmatrix}$$

$$\Rightarrow(\alpha+\beta+\gamma)(\beta-\alpha)(\gamma-\alpha)\{\gamma+2-(\beta-\gamma)\}$$

$$\Rightarrow(\alpha+\beta+\gamma)(\beta-\alpha)(\gamma-\alpha)(\gamma-\beta)$$

$$\Rightarrow(\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)=R.H.S.$$

Q12. $\begin{vmatrix}x & x^2 & 1+px^3\\ y & y^2 & 1+py^3\\ z & z^2 & 1+pz^3\end{vmatrix}=(1+pxyz)(x-y)(y-z)(z-x)$

Solution.

$$L.H.S.=\begin{vmatrix}x & x^2 & 1+px^3\\ y & y^2 & 1+py^3\\ z & z^2 & 1+pz^3\end{vmatrix}$$

$$=\begin{vmatrix}x & x^2 & 1\\ y & y^2 & 1\\ z & z^2 & 1\end{vmatrix}+\begin{vmatrix}x & x^2 & px^3\\ y & y^2 & py^3\\ z & z^2 & pz^3\end{vmatrix}$$

$$=\triangle_1+\triangle_2------(1)$$

$$Now\triangle_2=\begin{vmatrix}x & x^2 & px^3\\ y & y^2 & py^3\\ z & z^2 & pz^3\end{vmatrix}$$

$$=pxyz\begin{vmatrix}1 & x & x^2\\ 1 & y & y^2\\ 1 & z & z^2\end{vmatrix}$$

$c_1 \leftrightarrow c_3$

$$=-pxyz\begin{vmatrix}x^2 & x & 1\\ y^2 & y & 1\\ z^2 & z & 1\end{vmatrix}$$

$c_1 \leftrightarrow c_2$

$$=pxyz\begin{vmatrix}x & x^2 & 1\\ y & y^2 & 1\\ z & z^2 & 1\end{vmatrix}=pxyz\triangle_1$$

Putting value in (1)

$\Rightarrow \triangle_1 + pxyz\triangle_1$

$= (1+pxyz)\triangle_1 - - - - - (2)$

$$Now \triangle_1 = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} x & x^2 & 1 \\ y-x & y^2-x^2 & 0 \\ z-x & z^2-x^2 & 0 \end{vmatrix}$$

Expanding along c_3

$$\triangle_1 = \begin{vmatrix} (y-x) & (y-x)(y+x) \\ (z-x) & (z-x)(z+x) \end{vmatrix}$$

$$= (y-x)(z-x)\begin{vmatrix} 1 & y+x \\ 1 & z+x \end{vmatrix}$$

$$= (y-x)(z-x)(z+x-y-x)$$

$$= (x-y)(y-z)(z-x)$$

Putting $\triangle_1$ in (2)

$$L.H.S. = (1+pxyz)(x-y)(y-z)(z-x) = R.H.S$$

Q13. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

Solution.

$$L.H.S. = \begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$$

$c_1 \to c_1 + c_2 + c_3$

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix}$$

$$(a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$=\left(a+b+c\right)\begin{vmatrix}1 & -a+b & -a+c \\ 0 & 3b+a-b & -b+a \\ 0 & -a+b+a-b & 3c+a-c\end{vmatrix}$$

Expanding along c_1

$$=\left(a+b+c\right)\begin{vmatrix}2b+a & a-b \\ a-c & 2c+a\end{vmatrix}$$

$$=\left(a+b+c\right)\left[4bc+2ab+2ac+a^2+ac+ab-bc\right]$$

$$=\left(a+b+c\right)\left(3ab+3bc+3ac\right)$$

$$=3\left(a+b+c\right)\left(ab+bc+ac\right)$$

$$=R.H.S.$$

Q14. Using properties of determinants,

Prove that: $\begin{vmatrix}1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q\end{vmatrix}=1$

Solution.

$$L.H.S.=\begin{vmatrix}1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q\end{vmatrix}$$

$R_2 \to R_2-2R_1$

$R_3 \to R_3-3R_1$

$$=\begin{vmatrix}1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 3 & 7+3p\end{vmatrix}$$

$R_3 \to R_3-3R_2$

$$=\begin{vmatrix}1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 0 & 1\end{vmatrix}$$

Expanding along c_1

$$\Delta=\begin{vmatrix}1 & 2+p \\ 0 & 1\end{vmatrix}=1=R.H.S.$$

Q15. $\begin{vmatrix}\sin\alpha & \cos\alpha & \cos\left(\alpha+\delta\right) \\ \sin\beta & \cos\beta & \cos\left(\beta+\delta\right) \\ \sin\gamma & \cos\gamma & \cos\left(\gamma+\delta\right)\end{vmatrix}=0$

Solution.

$$L.H.S. = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix}$$

$$= \frac{1}{\sin\delta\cos\delta}\begin{vmatrix} \sin\alpha\sin\delta & \cos\alpha\cos\delta & \cos(\alpha+\delta) \\ \sin\beta\sin\delta & \cos\beta\cos\delta & \cos(\beta+\delta) \\ \sin\gamma\sin\delta & \cos\gamma\cos\delta & \cos(\gamma+\delta) \end{vmatrix}$$

Applying $\cos(A+B) = \cos A\cos B - \sin A\sin B$ in c_3

$c_1 \to c_1 + c_3$

$$\Delta = \frac{1}{\sin\delta\cos\delta}\begin{vmatrix} \cos\alpha\cos\delta & \cos\alpha\cos\delta & \cos\alpha\cos\delta - \sin\alpha\sin\delta \\ \cos\beta\cos\delta & \cos\beta\cos\delta & \cos\beta\cos\delta - \sin\beta\sin\delta \\ \cos\gamma\cos\delta & \cos\gamma\cos\delta & \cos\gamma\cos\delta - \sin\gamma\sin\delta \end{vmatrix}$$

$c_1 = c_2$

$\therefore \Delta = 0 = R.H.S.$

Q16. Solve the system of equations

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$

$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

A.16. Given equations are :-

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

This system of equation can be written, in matrix form, as AX= B, Where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, \quad X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$|A| = 2(120-45) - 3(-80-30) + 10(36+36)$

$= 150 + 330 + 720$

$= 1200 \neq 0$

i.e., A^{-1} exists.

Now, we find adj A.

$A_{11} = 75$ $\quad$ $A_{12} = 110$ $\quad$ $A_{13} = 72$

$A_{21} = 150$ $A_{22} = -100$ $A_{23} = 0$

$A_{31} = 75$ $A_{32} = 30$ $A_{33} = -24$

Since,

$$\text{adj A} = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Thus, $A^{-1} = \frac{1}{|A|}\text{adj A} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

$$X = A^{-1} B$$

$$\Rightarrow \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

$\therefore \Rightarrow \frac{1}{x} = \frac{1}{2}, \quad \frac{1}{y} = \frac{1}{3}, \quad \frac{1}{z} = \frac{1}{5}$

$\Rightarrow x = 2, y = 3, z = 5.$

Q17. Choose the correct answer

If a, b, c are in AP, then the determinant $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$

A. 0
B. 1
C. Z
D. 2x

Answer A $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$

Q18. If x, y, z are nonzero real numbers, then the inverse of matrix A = $\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is

(A) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$ (B) $xyz\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(C) $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ (D) $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

A.8. $A=\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

$|A| = x(yz-0)-0(0-0)+0(0-0)$

$= xyz \neq 0$

i.e., A^{-1} exists.

Therefore, $A_{11} = yz$ $A_{12} = 0$ $A_{13} = 0$

$A_{21} = 0$ $A_{22} = xz$ $A_{23} = 0$

$A_{31} = 0$ $A_{32} = 0$ $A_{33} = xy$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{xyz}\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$= \begin{bmatrix} 1/x & 0 & 0 \\ 0 & 1/y & 0 \\ 0 & 0 & 1/z \end{bmatrix}$$

$$= \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

Hence, option (A) is correct.

Q19. Let A = $\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \leq \theta \leq 2\pi$. Then

(A) Det (A) = 0 (B) Det (A) $\in (2, \infty)$

(C) Det (A) $\in (2, 4)$ (D) Det (A) $\in [2, 4]$

A.19. $A=\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$

Expanding along C_1

$$=1(1+\sin^2\theta)+\sin\theta(-\sin\theta+\sin\theta)+1(\sin^2\theta+1)$$

$= 2\left(1+\sin^2\theta\right)$

Given that : $0 \le \theta \le 2\pi$

$\Rightarrow 0 \le \sin\theta \le 1$

$\Rightarrow 0 \le \sin^2\theta \le 1$

$\Rightarrow 1 \le 1+\sin^2\theta \le 2$

$\Rightarrow 2 \le 2\left(1+\sin^2\theta\right) \le 4$

$\Rightarrow \det(A) \in [2,4]$

Therefore, the correct option is (D).