General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question paper contains 38 questions. **All** questions are **compulsory**.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.*

(iii) *In Section **A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In Section **B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In Section **C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In Section **D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In Section **E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.*

(ix) *Use of calculator is **NOT** allowed.*

SECTION – A

This section comprises of **20** Multiple Choice Questions (MCQs) of **1** mark each. **$20 \times 1 = 20$**

1. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is

(A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. If vector $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, then which of the following is correct ?

(A) $\vec{a} \parallel \vec{b}$

(B) $\vec{a} \perp \vec{b}$

(C) $|\vec{b}| > |\vec{a}|$

(D) $|\vec{a}| = |\vec{b}|$

3. $\int_{-1}^{1} \frac{|x|}{x}\, dx$, $x \neq 0$ is equal to

(A) -1

(B) 0

(C) 1

(D) 2

4. Which of the following is not a homogeneous function of x and y ?

(A) $y^2 - xy$

(B) $x - 3y$

(C) $\sin^2 \frac{y}{x} + \frac{y}{x}$

(D) $\tan x - \sec y$

5. If $f(x) = |x| + |x-1|$, then which of the following is correct ?

(A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$.

(B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$.

(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.

(D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.

6. If A is a square matrix of order 2 such that det (A) = 4, then det (4 adj A) is equal to :

(A) 16

(B) 64

(C) 256

(D) 512

7. If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E/\overline{F})$ is equal to :

(A) $\frac{1}{6}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{7}{9}$

8. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in [0, 2] is :

(A) 0
(B) 2
(C) 4
(D) 5

9. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$, $C = [9\ 8\ 7]$, which of the following is defined ?

(A) Only AB
(B) Only AC
(C) Only BA
(D) All AB, AC and BA

10. If $\int \frac{2^{\frac{1}{x}}}{x^2}\, dx = k \cdot 2^{\frac{1}{x}} + C$, then k is equal to

(A) $\frac{-1}{\log 2}$
(B) $-\log 2$
(C) -1
(D) $\frac{1}{2}$

11. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is

(A) $\frac{\pi}{6}$
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{2}$

12. The integrating factor of differential equation $(x + 2y^3) \frac{dy}{dx} = 2y$ is

(A) $e^{\frac{y^2}{2}}$
(B) $\frac{1}{\sqrt{y}}$
(C) $\frac{1}{y^2}$
(D) $e^{-\frac{1}{y^2}}$

13. If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x is equal to

(A) 0
(B) 1
(C) 7
(D) ± 7

14. The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If $Z = 18x + 9y$ be the objective function, then

(A) Z is maximum at (2, 72), minimum at (15, 20)
(B) Z is maximum at (15, 20) minimum at (40, 15)
(C) Z is maximum at (40, 15), minimum at (15, 20)
(D) Z is maximum at (40, 15), minimum at (2, 72)

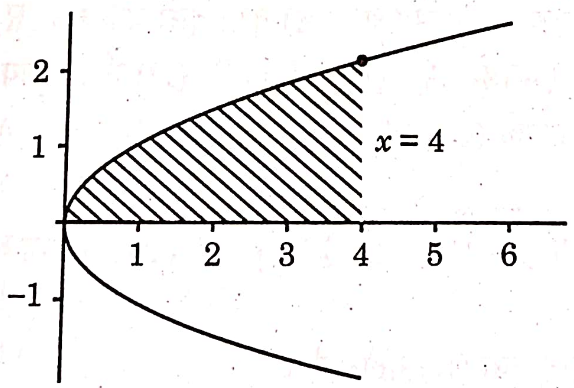
15. If A and B are invertible matrices, then which of the following is not correct ?

(A) $(A + B)^{-1} = B^{-1} + A^{-1}$
(B) $(AB)^{-1} = B^{-1}A^{-1}$
(C) $\text{adj}(A) = |A| A^{-1}$
(D) $|A|^{-1} = |A^{-1}|$

16. If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct ?

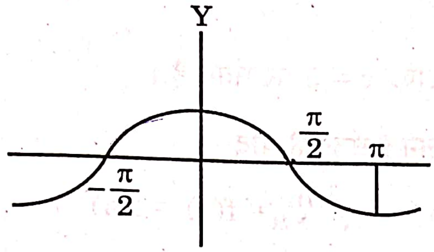
(A) It will only have a maximum value.
(B) It will only have a minimum value.
(C) It will have both maximum and minimum values.
(D) It will have neither maximum nor minimum value.

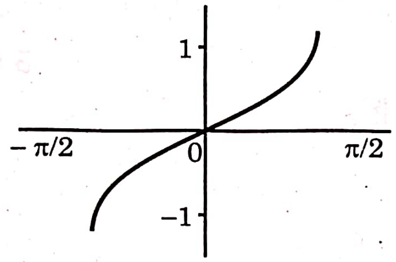
17. The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by



(A) $\int_0^4 x \, dx$
(B) $\int_0^2 y^2 \, dy$
(C) $2\int_0^4 \sqrt{x} \, dx$
(D) $\int_0^4 \sqrt{x} \, dx$

18. The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse ?



(A)


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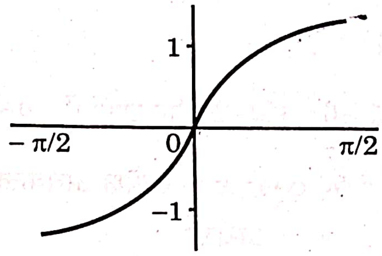

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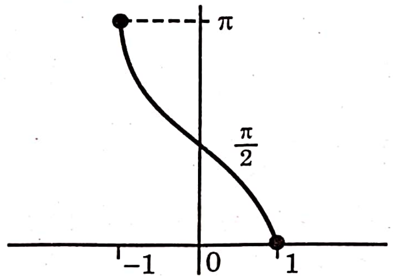

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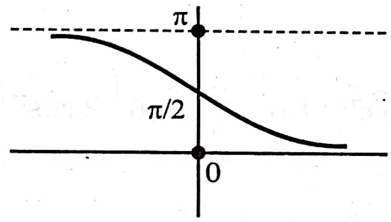


Assertion – Reason Based Questions

Direction : Question numbers **19** and **20** are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A)** : Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5$, $\forall x \in Z$ is a bijective.

Reason (R) : A function is a bijective if it is both surjective and injective.

20. **Assertion (A)** : $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k \;, & x > 5 \end{cases}$

is continuous at $x = 5$ for $k = \frac{5}{2}$.

Reason (R) : For a function f to be continuous at $x = a$,
$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.

SECTION – B

$5 \times 2 = 10$

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$.

OR

(b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.

22. Evaluate : $\tan^{-1}\left[2 \sin\left(2 \cos^{-1} \frac{\sqrt{3}}{2}\right)\right]$

23. The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.

24. Find the intervals in which function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$ is (i) increasing (ii) decreasing.

25. (a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings.

OR

(b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, –1, 0) respectively.

SECTION – C

$6 \times 3 = 18$

This section comprises of **6** Short Answer (SA) type questions of **3** marks each.

26. The side of an equilateral triangle is increasing at the rate of 3 cm/s. At what rate its area increasing when the side of the triangle is 15 cm ?

27. Solve the following linear programming problem graphically :

Maximise $Z = x + 2y$

Subject to the constraints :

$x - y \geq 0$

$x - 2y \geq -2$

$x \geq 0,\ y \geq 0$

28. (a) Find : $\int \frac{x + \sin x}{1 + \cos x}\, dx$

OR

(b) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$

29. (a) Verify that lines given by $\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ and $\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines.

OR

(b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.

30. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows :

X	0	2	4	5
P(X)	p	2p	3p	p

Where X is the number of students absent.

(i) Calculate p. 1

(ii) Calculate the mean of the number of absent students on Saturday. 2

OR

(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

31. Sketch the graph of $y = | x + 3 |$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.

SECTION – D $4 \times 5 = 20$

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

32. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(b) If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

33. Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$.

34. (a) Find the image A' of the point A(1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A'.

OR

(b) Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point Q(2, 4, –1) is 7 units. Also, find the equation of line joining P and Q.

35. A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions :

- The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.
- The number of students in Music club should be 20 more than half the number of students in Sports club.
- The total number of students to be allocated in all three clubs are 180.

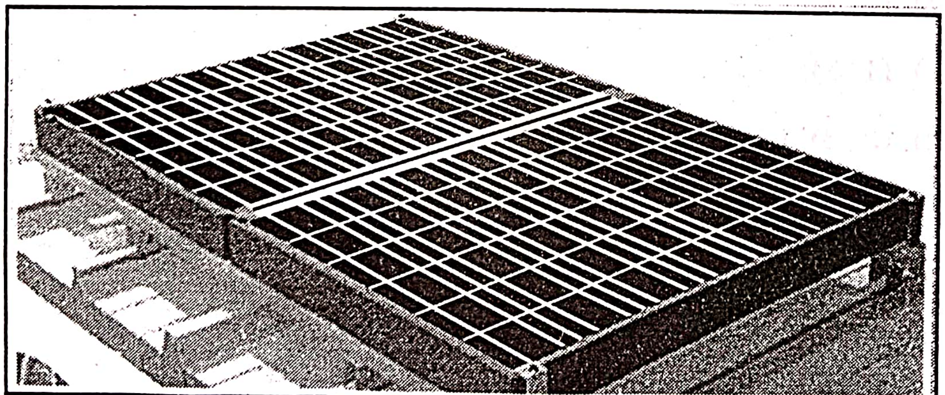
Find the number of students allocated to different clubs, using matrix method.

SECTION – E

This section comprises of 3 case study based questions of 4 marks each.

3 × 4 = 12

36.



A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres.

Based on this information, answer the following questions :

(i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y. **1**

(ii) Write the area of the solar panel as a function of x. **1**

(iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area. **2**

OR

(iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition. **2**

37. A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$:

$R_1 = \{(2, 3), (3, 2)\}$

$R_2 = \{(1, 2), (1, 3), (3, 2)\}$

$R_3 = \{(1, 2), (2, 1), (1, 1)\}$

$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$

$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$

The students are asked to answer the following questions about the above relations :

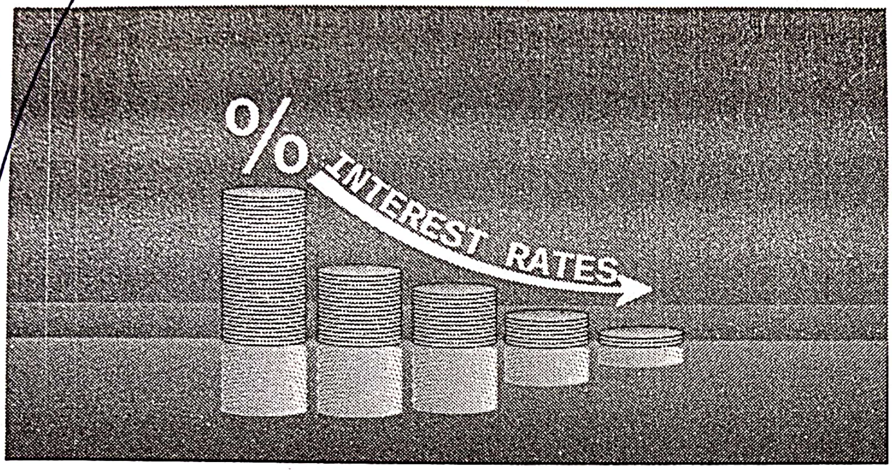
(i) Identify the relation which is reflexive, transitive but not symmetric.

(ii) Identify the relation which is reflexive and symmetric but not transitive.

(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive.

OR

(iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation ?

38. 

A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.

Based on the above information, answer the following :

(i) What is the probability that a customer after availing the loan will default on the loan repayment ? **2**

(ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest ? **2**