JEE (MAIN)-2025 (Online)

Mathematics Memory Based Answer & Solutions

MORNING SHIFT | **DATE : 04-04-2025**

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MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – APRIL, 2025

(Held On Friday 4th April, 2025) TIME : 9 : 00 AM to 12 : 00 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

1. Foci of ellipse are (2,5) and (2, –3), eccentricity is $\frac{4}{5}$. Find the length of latus rectum.

Ans. $\frac{18}{5}$

Sol. 2ae = 5 + 3 = 8

$e=\frac{4}{5} \Rightarrow a = 5$

$b^2 = 25\left(1-\frac{16}{25}\right) = 9 \Rightarrow b = 3$

$L=\frac{2b^2}{a}=\frac{18}{5}$

2. Solve $\int_{-1}^{1} \frac{1+2x}{e^{-x}+e^{x}} dx$

(1) $2\left(\tan^{-1} e-\frac{\pi}{4}\right)$ (2) $2\left(\tan^{-1} e-\frac{\pi}{3}\right)$

(3) $2\left(\tan^{-1} e-\frac{\pi}{2}\right)$ (4) $2\left(\frac{\pi}{2}-\tan^{-1} e\right)$

Ans. **(1)**

Sol. $I=\int_{-1}^{1}\frac{1+2x}{e^{-x}+e^{x}}dx$

Apply Kings rule,

$I=\int_{-1}^{1}\frac{1-2x}{e^{x}+e^{-x}}dx$

$2I=\int_{-1}^{1}\frac{(1+2x)+(1-2x)}{e^{x}+e^{-x}}dx$

$2I=2\int_{-1}^{1}\frac{dx}{e^{x}+e^{-x}}$

$I=2\int_{0}^{1}\frac{dx}{e^{x}+e^{-x}}$

$\Rightarrow 2\int_{0}^{1}\frac{ex}{\left(e^{x}\right)^2+1}dx$

$e^x = t$

$=2\int_{1}^{e}\frac{dt}{t^2+1}$

$=2\left[\tan^{-1}\right]_1^e$

$\Rightarrow 2\left(\tan^{-1} e-\frac{\pi}{4}\right)$

3. The sum of the series $1 + 3 + 5^2 + 7 + 9^2 + \ldots$ upto 40 terms is

(1) 41880 (2) 42880

(3) 41860 (4) 40860

Ans. **(1)**

Sol. $\left(1^2+5^2+9^2+....20 \text{ terms}\right)$

$+\left(3+7+11+....+20 \text{ terms}\right)$

$\Rightarrow \sum(4n-3)^2+\sum(4n-1)$

$\sum\left(16n^2-24n+9\right)+\sum 4n-1$

$\Rightarrow 16\sum n^2-20\sum n+8\sum 1$

$= 16\left\{\frac{20\times21\times41}{6}\right\}-20\left\{\frac{20\times21}{2}\right\}+8\times20$

$= 16\{2870\}-4200+160$

= 41880

4. Let there be two A.P′s with each having 2025 terms. Find the number of distinct terms in union of the two A.P′ s i.e., $A\cup B$ if first A.P. is 1,6,11,…and second A.P. is 9,16,23,…

(1) 3022 (2) 2025

(3) 4035 (4) 3761

Ans. **(4)**

Sol. Total number of terms : n(A) + n(B) = 2025 + 2025 = 4050

Now $n(A \cap B)$ = common terms of both A.Ps

Common difference = 35 (LCM of both A.P.)

Common A.P. : 16, 51, 86, ……

$\Rightarrow$ last term of smaller AP : 1 + (2024) (5)

= 10120

Now, 16 + (n – 1) (35) ≤ 10120

$(n-1) \le \frac{10104}{35}$

$(n-1)\le 288.68$

$n\le 289.68$

$\Rightarrow$ n = 289

Required ans.: 4050 - $n(A \cap B)$ = 4050 – 289

= 3761

5. If 10 $\sin^4\theta$ + 15 $\cos^4\theta$ = 6, then find the value of

$$\frac{27\operatorname{cosec}^6\theta+8\sec^6\theta}{16\sec^8\theta}$$

Ans. $\frac{2}{5}$

Sol. Let $\sin^2\theta = t$

$10t^2 + 15(1 + t^2 - 2t) = 6$

$25t^2 - 30t + 9 = 0$

$(5t - 3)^2 = 0$

$\Rightarrow \sin^2\theta = t = \frac{3}{5}$

$\cos^2\theta = \frac{2}{5}$

$$\frac{27\operatorname{cosec}^2\theta+8\sec^6\theta}{16\sec^8\theta}=\frac{27\left(\frac{5}{3}\right)^3+8\left(\frac{5}{2}\right)^3}{16\left(\frac{5}{2}\right)^4}=\frac{125+125}{625}=\frac{2}{5}$$

6. Consider a committee of 12 memebers is formed randomly out of 4 Engineers, 2 Doctors and 10 Professors. Find the probability that the committee has exactly 3 Engineers and 1 Doctor.

(1) $\frac{15}{91}$ (2) $\frac{18}{71}$

(3) $\frac{18}{91}$ (4) $\frac{17}{91}$

Ans. **(3)**

Sol. Total cases = ${}^{16}C_{12}$

Favourable = ${}^{4}C_3 \times {}^{2}C_1 \times {}^{10}C_8$

$$P=\frac{4\times 2\times\frac{10\times 9}{2}\times 4!}{16\times 15\times 14\times 13}=\frac{18}{91}$$

7. The number of integral values of $n \in N$ for which the equation $x^2 + 4x - n = 0, n \in [20,100]$ have integral roots, is

(1) 7 (2) 5

(3) 4 (4) 6

Ans. **(4)**

Sol. $x^2 + 4x - n = 0$

$(x + 2)^2 - 4 - n = 0$

$(x + 2)^2 = 4 + n$

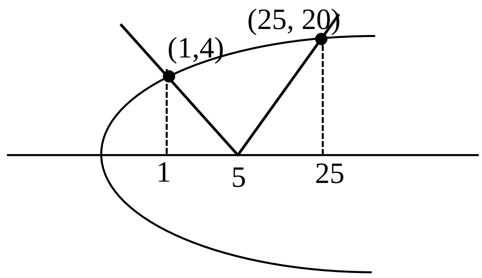
$x+2=\pm\sqrt{n+4}$

N = 21, 32, 45, 60, 77, 96

Six values.

8. Let $|x-5| \le y \le 4\sqrt{x}$. If the area enclosed is A, then 3A equal to

Ans. **(368)**



Sol.

$y^2 = 16x = (x-5)^2$

$\Rightarrow x^2 - 26x + 25 = 0$

$\Rightarrow x = 1, 25$

$$A=\int_1^{25}4\sqrt{x}dx-\frac{1}{2}\cdot 4\cdot 4-\frac{1}{2}\cdot 20\cdot 20$$

$$=\frac{8}{3}\left[x^{\frac{3}{2}}\right]_1^{25}-\frac{1}{2}(416)$$

$$=\frac{8}{3}(125-1)-208$$

$$=\frac{8}{3}\times 124-208$$

$3A = 992 - 624 = 368$

9. In 10 balls, 3 are defective. If 2 are chosen at random, find variance (σ^2) of the defective balls.

Ans. $\frac{28}{75}$

Sol.

0	1	2
$\frac{{}^7C_2}{{}^{10}C_2}$	$\frac{{}^7C_1\cdot{}^3C_1}{{}^{10}C_2}$	$\frac{{}^3C_2}{{}^{10}C_2}$
$=\frac{7}{10}\cdot\frac{6}{9}$	$\frac{7.3.2}{10.9}$	$\frac{3}{10}.\frac{2}{9}=\frac{1}{15}$
$=\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

$\sigma^2 = \Sigma(x - \mu)^2P(x)$

$$=\left(\frac{3}{5}\right)^2.\frac{7}{15}+\left(\frac{2}{5}\right)^2.\frac{7}{15}+\left(\frac{7}{5}\right)^2.\frac{1}{15}$$

$$=\frac{63+28+49}{375}=\frac{140}{375}=\frac{28}{75}$$

$$\left\{\begin{array}{l}\sum x^2P(x)-\left(\sum xP(x)\right)^2\\ \sum (x-\mu)^2 P(x)\\ \mu = 0+\frac{7}{15}+\frac{2}{15}\\ =\frac{9}{15}=\frac{3}{5}\end{array}\right\}$$

10. Let $A = \begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}$. Here $A^2 = A^T$.
Then find trace $[(A+I)^3 + (A-I)^3 - 6A]$.

Ans. **(6)**

Sol. Here, A is orthogonal matrix

So, $A^T = A^{-1}$

$\Rightarrow A^2 = A^T \Rightarrow A^2 = A^{-1} \Rightarrow A^3 = I$

$B = (A + I)^3 + (A - I)^3 - 6A$

$= 2(A^3 + 3A) - 6A$

$= 2A^3$

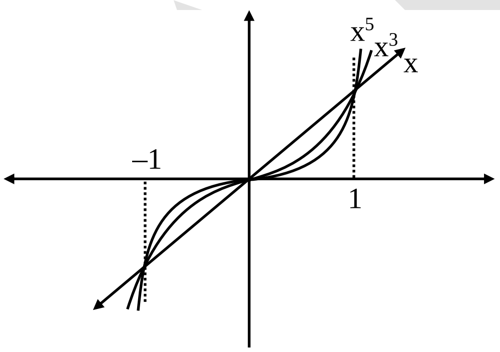
$= 2I$

$Tr(B) = 2 + 2 + 2 = 6$

11. $f(x) = \max\{x, x^3, x^5, \ldots, x^{21}\}$, if number of points where $f(x)$ is discontinuous $= p$ and number of points where $f(x)$ is not differentiable $= q$, then find the value of $p + q$

Ans. **(3)**

Sol.



$$f(x)=\begin{cases} x & : \quad x<-1\\ x^{21} & : \quad -1\le x<0\\ x & : \quad 0\le x<1\\ x^{21} & : \quad 1\le x\end{cases}$$

f(x) is always continuous $\quad \therefore$ p = 0

f(x) is not different at 3 points q = 3

$\therefore$ p + q = 3

12. $\lim_{x\to 1^+}\frac{(x-1)(6+\lambda\cos(x-1))+\mu\sin(1-x)}{(x-1)^3} = -1$,
where $\lambda, \mu \in R$. Then $\lambda + \mu$ is equal to

(1) 17 (2) 18

(3) 19 (4) 20

Ans. **(2)**

Sol. $\lim_{x\to 1^+}\frac{(x-1)(6+\lambda\cos(x-1))+\mu\sin(1-x)}{(x-1)^3} = -1$

$\Rightarrow \lim_{t\to 0^+}\frac{6t+\lambda t\cos t-\mu\sin t}{t^3} = -1$

$\Rightarrow \lim_{t\to 0^+}\frac{6t+\lambda t\left(1-\frac{t^2}{2}+\frac{t^4}{24}\right)-\mu\left(t-\frac{t^3}{6}+\frac{t^5}{120}\right)}{t^3} = -1$

so, $\lambda + 6 - \mu = 0$ ...(i)

and $\frac{\mu}{6}-\frac{\lambda}{2} = -1$ ...(ii)

solving (i) and (ii)

we get, $\mu = 12$, $\lambda = 6$

so, $\lambda + \mu = 18$

13. Let $f, g : (1, \infty) \to R$ be defined as $f(x) = \frac{2x+3}{5x+2}$ and $g(x) = \frac{2-3x}{1-x}$. If the range of the function $fog : [2,4] \to R$ is $[\alpha, \beta]$, then $\frac{1}{\beta-\alpha}$ is equal to

Ans. **(56)**

Sol. $g(2) = 4$

$g(4) = \frac{10}{3}$

$f(g(4)) = \frac{\frac{20}{3}+3}{\frac{50}{3}+2} = \frac{29}{56}$

$f(4) = \frac{11}{22} = \frac{1}{2}$

$f\left(\frac{10}{3}\right) = \frac{29}{56} = \beta$

So, $\frac{1}{\beta-\alpha} = \frac{1}{\left|\frac{29}{56}-\frac{1}{2}\right|} = \left|\frac{1}{\frac{29}{56}-\frac{1}{2}}\right| = 56$

14. Evaluate : $\int_{-1}^{1} \frac{[1+\sqrt{|x|-x}]e^{x}+(\sqrt{|x|-x})e^{-x}}{e^{x}+e^{-x}}dx$

Ans. $1+\frac{2\sqrt{2}}{3}$

Sol. Apply king property

$$I=\int_{-1}^{1}\frac{\left(1+\sqrt{|x|+x}\right)e^{-x}+\left(\sqrt{|x|+x}\right)e^{x}}{e^{x}+e^{-x}}dx$$

$$\therefore 2I=\int_{-1}^{1}\frac{\left(1+\sqrt{|x|+x}+\sqrt{|x|-x}\right)\left(e^{x}+e^{-x}\right)}{\left(e^{x}+e^{-x}\right)}dx$$

$$\Rightarrow 2I=\int_{-1}^{1}\left(1+\sqrt{|x|+x}+\sqrt{|x|-x}\right)dx$$

Apply odd even property

$$2I=2\int_{0}^{1}\left(1+\sqrt{|x|+x}+\sqrt{|x|-x}\right)dx$$

$$I=\int_{0}^{1}\left(1+\sqrt{2x}\right)dx$$

$$I=\left(x+\sqrt{2}\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_{0}^{1}$$

$$I=1+\frac{2\sqrt{2}}{3}$$

15. In the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^{n}$, $n\in N$. If the ratio of 15^{th} term from the beginning to the 15^{th} term from the end is $\frac{1}{6}$, then find the value of ${}^{n}C_{3}$

Ans. **(2300)**

Sol. In the expansion of $(a+b)^{n}$

15^{th} term from beginning : $T_{15}={}^{n}C_{14}a^{n-14}b^{14}$

15^{th} term from the end : $T'_{15}={}^{n}C_{14}b^{n-14}a^{14}$

$$\therefore \frac{T_{15}}{T'_{15}}=\frac{1}{6}$$

$$\Rightarrow \frac{a^{n-14}b^{14}}{b^{n-14}a^{14}}=\frac{1}{6}$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n-28}=\frac{1}{6}$$

$$\left(6^{\frac{1}{3}}\right)^{n-28}=6^{-1}$$

$$\Rightarrow \frac{n-28}{3}=-1$$

$$\Rightarrow n-28=-3$$

n = 25

$$\therefore {}^{25}C_{3}=2300$$

16. If $f(x)=1-2x+\int_{0}^{x}e^{x-t}f(t)\,dt$, then the area bounded by the curve $y=f(x)$ and coordinate axes is (in square units)

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$

(3) 2 (4) 1

Ans. **(1)**

Sol.
$$y.e^{-x}=(1-2x)e^{-x}+\int_{0}^{x}e^{-t}f(t)dt$$

$$y.e^{-x}(-1)+e^{-x}.y'=-e^{-x}+2xe^{-x}-2e^{-x}+e^{-x}.y$$

$$e^{-x}.y'-2ye^{-x}=(2x-3)e^{-x}$$

$$y'-2y=(2x-3)$$

$$e^{-2x}.y=\int(2x-3)e^{-2x}dx$$

$$=(2x-3)\left(-\frac{e^{-2x}}{2}\right)-2.\left(\frac{e^{-2x}}{4}\right)+c$$

$$=\frac{e^{-2x}}{2}(3-2x-1)$$

$$\Rightarrow e^{-2x}.y=e^{-2x}(1-x)+c$$

When x = 0, y = 1

So, 1 = (1 – 0) + c

⇒ c = 0

So, y = 1– x

So area bounded = $\frac{1}{2}\times1\times1=\frac{1}{2}$

17. The value of

$\sin^{-1}\left(\frac{\sqrt{3}x}{2}+\frac{1}{2}\sqrt{1-x^{2}}\right),-\frac{1}{2}<x<\frac{1}{\sqrt{2}}$ is equivalent to

(1) $\frac{2\pi}{3}-\cos^{-1}x,-\frac{1}{2}<x<\frac{1}{\sqrt{2}}$

(2) $\pi-\cos^{-1}x,-\frac{1}{2}<x<\frac{1}{\sqrt{2}}$

(3) $\frac{\pi}{3}-\cos^{-1}x,-\frac{1}{2}<x<\frac{1}{\sqrt{2}}$

(4) $\frac{\pi}{2}-\cos^{-1}x,-\frac{1}{2}<x<\frac{1}{\sqrt{2}}$

Ans. **(1)**

Sol. Put x = sinθ

$$\sin^{-1}\left(\sin\theta\cos\frac{\pi}{6}+\cos\theta\sin\frac{\pi}{6}\right)$$

$$=\sin^{-1}\left[\sin\left(\theta+\frac{\pi}{6}\right)\right]$$

$$=\theta+\frac{\pi}{6}$$

$$=\sin^{-1}x+\frac{\pi}{6}$$

$$=\frac{\pi}{2}-\cos^{-1}x+\frac{\pi}{6}=\frac{2\pi}{3}-\cos^{-1}x$$

18. Let A and B two distinct points on the line $L:\frac{x-6}{3}=\frac{y-7}{3}=\frac{z-7}{-2}$. Both A and B are at a distance $2\sqrt{22}$ from the foot of the perpendicular drawn from the point (1, 2, 3)on the line L. If O is origin then $\overrightarrow{OA}\cdot\overrightarrow{OB}$ is equal to

Ans. **(18)**

Sol. $\overrightarrow{PM}=3\lambda+5,3\lambda+5,-2\lambda+4$

$\vec{L}=(3,3,-2)$

$\overrightarrow{PM}.\vec{L}=0=9\lambda+15+9\lambda+15+4\lambda-8=0$

$22\lambda+22=0 \Rightarrow \lambda=-1$

$M(3,4,9)$

Now,

Let $A(3\mu+6,3\mu+7,-2\mu+7)$

MA = $2\sqrt{22}$

$\Rightarrow(3\mu+3)^2+(3\mu+3)^2+(-2\mu-2)^2=4\times22$

$\Rightarrow(\mu+1)^2(9+9+4)=4\times22$

$\Rightarrow\mu+1=\pm2$

$\Rightarrow\mu=1,-3$

$A(9,10,5)\qquad B(-3,-2,13)$

$\overrightarrow{OA}\cdot\overrightarrow{OB}=-27-20+65=18$

19. Given two lines $L_1:\frac{x-3}{3}=\frac{y-\alpha}{1}=\frac{z+2}{-2}$ and $L_2:\frac{x+1}{2}=\frac{y+2}{1}=\frac{z-\beta}{-1}$. If shortest distance between L_1 and L_2 is $30\sqrt{3}$, then find the value of $|\alpha+\beta|$.

Ans. **(90)**

Sol. Point A(3, α, –2) B(–1, –2, β)

A, b, c, = 3, 1, –2 & a_2, b_2, c_2 = 2, 1, –1

A.T.Q.

$$\frac{1}{\sqrt{(-1+2)^2+(-3+4)^2+(3-2)^2}}\begin{vmatrix}3+1 & \alpha+2 & -2-\beta\\ 3 & 1 & -2\\ 2 & 1 & -1\end{vmatrix}=30\sqrt{3}$$

$$\Rightarrow\frac{1}{\sqrt{1+1+1}}\begin{vmatrix}4 & \alpha+2 & -2-\beta\\ 3 & 1 & -2\\ 2 & 1 & -1\end{vmatrix}=30\sqrt{3}$$

$\Rightarrow$ 4(–1 + 2) – (α + 2) (–3 + 4) + (–2 – β) (3 – 2) = 30 × 3

$\Rightarrow$ 4 – α – 2 – 2 – β = 90

$\Rightarrow$ |α + β| = 90

20. If $\vec{v}=2\hat{\imath}+\hat{\jmath}-\lambda\hat{k},(\lambda>0),\vec{u}=3\hat{\imath}-\hat{\jmath}$ and $\vec{v}_1$ is parallel to $\vec{u}$, $\vec{v_2}$ is perpendicular to $\vec{u}$ and $\vec{v}=\vec{v_1}+\vec{v_2}$. If angle between $\vec{v}$ and $\vec{v_1}$ is $\cos^{-1}\left(\frac{\sqrt{5}}{2\sqrt{7}}\right)$, then $|\vec{v_1}|^2+|\vec{v_2}|^2$equals to

Ans. **(14)**

Sol. $|\vec{v}_1|=\lambda\vec{\mu}$

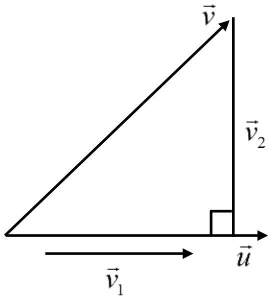
$\vec{v}_2\cdot\vec{\mu}=0$

$\vec{v}=\vec{v}_1+\vec{v}_2$

$\vec{v}_1=(\vec{v}.\hat{u})\hat{u}$

$\vec{v}_2=\vec{v}-(\vec{v}.\hat{u})\hat{u}$

$\frac{\vec{v}.\vec{u}}{|\vec{v}||\vec{u}|}=\frac{\sqrt{5}}{2\sqrt{7}}$



$$\Rightarrow\frac{5}{\sqrt{4+1+\lambda^2}\sqrt{10}}=\frac{\sqrt{5}}{2\sqrt{7}}$$

$$\Rightarrow\frac{5}{\sqrt{5+\lambda^2}}=\frac{5}{\sqrt{14}}$$

$\Rightarrow\lambda^2=9\Rightarrow\lambda=3\ ;\ \lambda>0$

$|\vec{v}_1|^2+|\vec{v}_2|^2=|\vec{v}|^2$

$=2^2+1^2+\lambda^2$

$=5+9$

$=14$