JEE (MAIN)-2025 (Online)

Mathematics Memory Based
Answer & Solutions

EVENING SHIFT

DATE : 04-04-2025

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MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – APRIL, 2025

(Held On Friday 4th April, 2025) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

1. $\cot^{-1}\left(\frac{7}{4}\right) + \cot^{-1}\left(\frac{19}{4}\right) + \cot^{-1}\left(\frac{39}{4}\right) + \cdots\infty$

is equal to

(1) $\cot^{-1}(2)$

(2) $\cot^{-1}\left(\frac{1}{2}\right)$

(3) $\cot^{-1}\left(\frac{1}{3}\right)$

(4) $\cot^{-1}(3)$

Ans. **(2)**

Sol. $\tan^{-1}\left(\frac{4}{7}\right) + \tan^{-1}\left(\frac{4}{19}\right) + \tan^{-1}\left(\frac{4}{39}\right) + \ldots\ldots\infty$

$$\Rightarrow \sum_{r=1}^{\infty} \tan^{-1}\left(\frac{4}{(2r)^2+3}\right) = \sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{r^2+3/4}\right)$$

$$\Rightarrow \sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{1+\left(r^2-\frac{1}{4}\right)}\right)$$

$$\Rightarrow \sum_{r=1}^{\infty} \tan^{-1}\left(\frac{\left(r+\frac{1}{2}\right)-\left(r-\frac{1}{2}\right)}{1\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}\right)$$

$$\Rightarrow \sum_{r=1}^{n} \left(\tan^{-1}\right)\left(r+\frac{1}{2}\right) - \tan^{-1}\left(r-\frac{1}{2}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{3}{2}\right) - \tan^{-1}\left(\frac{1}{2}\right)$$

$$+\tan^{-1}\left(\frac{5}{2}\right) - \tan^{-1}\left(\frac{3}{2}\right)$$

$$+\tan^{-1}\left(\frac{5}{2}\right) - \tan^{-1}\left(\frac{3}{2}\right) + \tan^{-1}\left(n+\frac{1}{2}\right) - \tan^{-1}\left(n-\frac{1}{2}\right)$$

$$+\ldots\ldots\ldots\infty$$

$$\Rightarrow \tan^{-1}(\infty) - \tan^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)$$

$$= \cot^{-1}\left(\frac{1}{2}\right)$$

2. $\displaystyle\sum_{k=1}^{n}\left(\alpha^k + \frac{1}{\alpha^k}\right)^2 = 20,$

α is one of the root of $x^2 + x + 1 = 0$, then n is equal to

Ans. **(11)**

Sol.

$$x^2 + x + 1 = 0 \begin{cases} \omega \\ \omega_2 \end{cases}$$

Let $\alpha = \omega$

$$\sum_{k=1}^{n}\left(\omega^k + \frac{1}{\omega^k}\right)^2 = \left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega^2 + \frac{1}{\omega^2}\right)^2 + \left(\omega^3 + \frac{1}{\omega^3}\right)^2 + \ldots\ldots.$$

$$= (-1)^2 + (-1)^2 + (2)^2 + \ldots\ldots$$

$= (1 + 1 + 4) + (1 + 1 + 4) + (1 + 1 + 4) + 1 + 1$

$= 20$

So, $(9 + 2) = 11$ terms

So, $n = 11$

3. $\displaystyle\int \frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^{9}}dx = \frac{1}{m}\left[\left(\sqrt{1+x^2} + x\right)^n\left(n\sqrt{1+x^2} - x\right)\right] + c,$

where c is the constant of integration and $m, n \in \mathbb{N}$, then $m + n$ is equal to

Ans. **(379)**

Sol. $I = \int\left(\sqrt{1+x^2} + x\right)^{19} dx$

Let $\sqrt{1+x^2} + x = t \Rightarrow \sqrt{1+x^2} = t - x$

$$\Rightarrow 1 + x^2 = t^2 + x^2 - 2tx$$

$$\Rightarrow 1 = t^2 - 2tx \Rightarrow x = \frac{t^2-1}{2t}$$

$$\Rightarrow x = \frac{1}{2}\left(t - \frac{1}{t}\right)$$

$$\Rightarrow dx = \frac{1}{2}\left(1 + \frac{1}{t^2}\right)dt$$

$$I=\int t^{19}\left(\frac{1}{2}\left(1+\frac{1}{t^2}\right)\right)dt$$

$$I=\frac{1}{2}\int\left(t^{19}+t^{17}\right)dt$$

$$=\frac{1}{2}\frac{t^{20}}{20}+\frac{1}{2}\frac{t^{18}}{18}+c$$

$$=\frac{t^{19}}{4}\left(\frac{t}{10}+\frac{1}{t\cdot 9}\right)+c$$

$$=\frac{t^{19}}{360}\left(9t+\frac{10}{t}\right)+c$$

$$=\frac{t^{19}}{360}\left(9\left(x+\sqrt{1+x^2}\right)+10\left(\sqrt{1+x^2}-x\right)\right)+c$$

$$=\frac{t^{19}}{360}\left(19\sqrt{1+x^2}-x\right)$$

⇒ n = 19

⇒ m = 360

So, m + n = 379

4. Let the mean & variance of observation 2, 3, 3, 4, 5, 7, a, b is 4 and 2, then mean deviation about mode of the observation is

Ans. **(1)**

Sol. $\bar{n}=\dfrac{2+3+3+4+5+7+a+b}{8}=4$

⇒ a + b = 8 …(1)

Also,

$$\sigma^2=\frac{4+9+9+16+25+49+a^2+b^2}{8}-(4)^2=2$$

⇒ $a^2 + b^2 + 112 = 18 \times 8$

⇒ $a^2 + b^2 = 32$ …(2)

From (1) & (2) ⇒ a = b = 4

Now, mode z = 4

$$M.D.=\frac{2+1+1+0+1+3+0+0}{8}=1$$

5. If $1^2\cdot{}^{15}C_1+2^2\cdot{}^{15}C_2+3^2\cdot{}^{15}C_3+\cdots+15^2\cdot{}^{15}C_{15}=2^m\cdot3^n\cdot5^k$, then $m+n+k$ is equal to

(1) 19 (2) 20

(3) 21 (4) 18

Ans. **(1)**

Sol. $\displaystyle\sum_{r=1}^{15}r^2.\,{}^{15}C_r=\sum r.15\,{}^{14}C_{r-1}$

$\Rightarrow 15\left(\sum\{(r-1)+1\}^{14}C_{r-1}\right)$

$\Rightarrow 15\left(\sum(r-1).\dfrac{14}{(r-1)}\,{}^{13}C_{r-2}+\sum{}^{14}C_{r-1}\right)$

$\Rightarrow 15(14.\,2^{13}+2^{14})$

$\Rightarrow 15\times2^{13}(16)$

$\Rightarrow 3\times5\times2^{17}$

$=2^m\times3^n.\,5^k$ ⇒ m = 17, n = 1, k = 1

m + n + k = 19.

6. Let domain of

$f(x)=\log_4\log_7(8-\log_2(x^2+2x+2))$

is (α,β) & domain of

$g(x)=\sin^{-1}\left(\frac{7x+10}{x-2}\right)$ is $[\gamma,\ \delta]$,

then find the value of $\alpha^2+\beta^2+\gamma^2+\delta^2$.

Ans. **(261)**

Sol. For domain of f(x)

$\log_7\left(8-\log_2\left(x^2+2x+2\right)\right)>0$

$\Rightarrow 8-\log_2\left(x^2+2x+2\right)>1$

$\Rightarrow 7>\log_2\left(x^2+2x+2\right)$

$\Rightarrow x^2+2x+2<128$

$\Rightarrow (x+1)^2-127<0$

$x\in(\alpha,\beta)$

$\alpha+\beta=-2;\alpha$

$x^2+2x-126=0$

$\alpha^2+\beta^2=126-2\alpha+126-2\beta$

$=252-2(\alpha+\beta)$

$=256$

and $8-\log_2\left(x^2+2x+2\right)>0$

$\Rightarrow 256>x^{2+2}+2x+2$

$\Rightarrow (x+1)^2-255<0$

this inequality gives us domain as a superset of x ∈ (α, β) so, ultimately domain for f(x) is x ∈ (α, β)

For domain of g(x)

$$-1 \le \frac{7x+10}{x-2} \le 1 \Rightarrow x \in [-2,-1]$$

$$\gamma + \delta = -3, \gamma = -2$$

$$\delta = -1$$

$$\Rightarrow \gamma^2 + \delta^2 = 5$$

So, $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 261$

7. If the sum of first 20 terms of series

$$\frac{4.1}{4+3.1^2+1^4} + \frac{4.2}{4+3.2^2+2^4} + \frac{4.3}{4+3.3^2+3^4} + \frac{4.4}{4+3.4^2+4^4} + \cdots \text{is} \frac{m}{n},$$

where $m,\ n$ are co-primes, then $m+n$ is equal to

(1) 420 (2) 421

(3) 422 (4) 423

Ans. **(2)**

Sol.

$$s = \sum_{r=1}^{n} \frac{4r}{4+3r^2+r^4}$$

$$2\sum_{r=1}^{n} \frac{2r}{(r^2+2)^2 - r^2}$$

$$2\sum_{r=1}^{n} \frac{(r^2+2+r)-(r^2+2-r)}{(r^2+2+r)(r^2+2-r)}$$

$$2\sum_{r=1}^{n} \left(\frac{1}{r^2+2-r} - \frac{1}{r^2+2+r}\right)$$

$$s_n = 2\left[\begin{array}{l} \frac{1}{2} - \frac{1}{4} \\ \frac{1}{4} - \frac{1}{8} \\ \frac{1}{n^2+2-n} - \frac{1}{n^2+2+n} \end{array}\right]$$

$$s_n = 2\left[\frac{1}{2} - \frac{1}{n^2+2+n}\right]$$

$$s_n = \frac{n^2+n}{n^2+n+2}$$

$$s_{20} = \frac{400+20}{400+20+2} = \frac{210}{211} = \frac{m}{n}$$

m + n = 210 + 211

= 421.

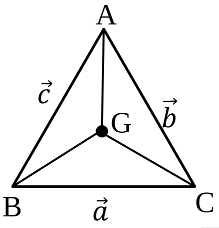
8. Let the three sides of a triangle ABC is given by vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}, \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and

$3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$, let G be the centroid of triangle ABC, then

$6\left(|\overrightarrow{AG}|^2 + |\overrightarrow{BG}|^2 + |\overrightarrow{CG}|^2\right)$ is equal to

Ans. **(164)**

Sol.



Let ℓ_1, ℓ_2, ℓ_3 be medians then

$$\ell_1^2 + \ell_2^2 + \ell_3^2 = \frac{3}{4}(a^2+b^2+c^2)$$

$$= \frac{3}{4}(6+35+41) = \frac{3\times 41}{2} = \frac{123}{2}$$

Now, $\left|\overrightarrow{AG}\right|^2 + \left|\overrightarrow{BG}\right|^2 + \left|\overrightarrow{CG}\right|^2 = \frac{4}{9}(\ell_1^2 + \ell_2^2 + \ell_3^2)$

$$= \frac{4}{9} \times \frac{123}{2}$$

So, $6\left(\left|\overrightarrow{AG}\right|^2 + \left|\overrightarrow{BG}\right|^2 + \left|\overrightarrow{CG}\right|^2\right) = \frac{4}{9} \times \frac{123}{2} \times 6$

= 164

9. Consider two sets A & B containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of B 36 and q respectively. Let d and D be the common difference of A.P′s in A & B respectively such that $D = d + 3, d > 0$. If $\frac{p+q}{p-q} = \frac{19}{5}$, then $p - q$ is equal to

Ans. **(540)**

Sol. A = {12 – d, 12, 12 + d}

B = {12 – 0, 12, 12 + D}

p = 12 (144 – d^2)

q = 12(144 – D^2)

$$\frac{p+q}{p-q} = \frac{19}{5} \Rightarrow \frac{p}{q} = \frac{12}{7}$$

$$\frac{144-d^2}{144-(d+3)^2}=\frac{12}{7}$$

$\Rightarrow\quad 5d^2 + 72d - 612 = 0$

$5d^2 - 30d + 102d - 612 = 0$

$(5d + 106)(d - 6) = 0$

$\Rightarrow\quad d = 6$

$p = 12(144 - 36) = 12 \times 108$

$q = 12(144 - 81) = 12 \times 63$

$p - q = 12(108 - 63)$

$= 12 \times 45$

$p - q = 540.$

10. Let $f(x)$ and $g(x)$ satisfies the functional equation $2g(x) + 3g\left(\frac{1}{x}\right) = x$ and

$2f(x) + 3f\left(\frac{1}{x}\right) = x^2 + 5$. If $\alpha = \int_1^2 f(x)dx$ and $\beta = \int_1^2 g(x)dx$, then $(9\alpha + \beta)$ is equal to

(1) $\frac{27+6\,ln2}{10}$

(2) $\frac{27-6\,ln2}{10}$

(3) $\frac{3}{5}ln2$

(4) $\frac{3}{5}ln2 + \frac{7}{30}$

Ans. **(1)**

Sol.

$$\left.\begin{aligned}&\left(2g(x)+3g\left(\frac{1}{x}\right)=x\right)2\\&\left(2f(x)+3f\left(\frac{1}{x}\right)=x^2+5\right)2\\&\left(2f\left(\frac{1}{x}\right)+3f(x)=\frac{1}{x^2}+5\right)3\\&\left(2g\left(\frac{1}{x}\right)+3g(x)=\frac{1}{x}\right)3\end{aligned}\right\}$$

$$4g(x)+6g\left(\frac{1}{x}\right)=2x \qquad \ldots\text{(i)}$$

$$6g\left(\frac{1}{x}\right)+9g(x)-\frac{3}{x} \qquad \ldots\text{(ii)}$$

Solving (i) and (ii) we get

$$-5g(x)=2x-\frac{3}{x}$$

$$\Rightarrow g(x)=\frac{1}{5}\left(\frac{3}{x}-2x\right)$$

$$4f(x)+6f\left(\frac{1}{x}\right)=2x^2+10 \qquad \ldots\text{(iii)}$$

$$9f(x)+6f\left(\frac{1}{x}\right)=\frac{3}{x^2}+15 \qquad \ldots\text{(iv)}$$

Solving (iii) and (iv) we get

$$-5f(x)=2x^2-\frac{3}{x^2}-5$$

$$\Rightarrow f(x)=\frac{1}{5}\left(\frac{3}{x^2}+5-2x^2\right)$$

$$\beta=\frac{1}{5}\left[3\ell nx-\frac{2x^2}{2}\right]_1^2$$

$$=\frac{1}{5}\left[(3\ell n2-4)-(0-1)\right]$$

$$=\frac{1}{5}(3\ell n2-3)$$

$$\alpha=\frac{1}{5}\left[\left(-\frac{3}{x}+5x-\frac{2x^3}{3}\right)\right]_1^2$$

$$\alpha=\frac{1}{5}\left[\left(\frac{-3}{2}+10-\frac{16}{3}\right)-\left(-3+5-\frac{2}{3}\right)\right]$$

$$\alpha=\frac{1}{5}\left[\frac{19}{6}-\frac{4}{3}\right]$$

$$9\alpha=\frac{9}{5}\left[\frac{11}{6}\right]=\frac{33}{10}$$

$$\text{So, } 9\alpha+\beta=\frac{6\ell n2+27}{10}$$

11. Let the matrix $A=\begin{bmatrix}1&0&0\\1&0&1\\0&1&0\end{bmatrix}$ satisfy

$A^n = A^{n-2} + A^2 - I$ for $n \geq 3$,

then the sum of all the elements of A^{50} is equal to

Ans. **(53)**

Sol.

$A^n - A^{n-2} = A^2 - I$

$A^{50} - A^{48} = A^2 - I$

$A^{48} - A^{46} = A^2 - I$

$\vdots$

$A^4 - A^2 = A^2 - I$

$A^{50} - A^2 = 24(A^2 - I)$

$A^{50} = 25A^2 - 24\,I$

$$A^2=\begin{pmatrix}1&0&0\\1&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}1&0&0\\1&0&1\\0&1&0\end{pmatrix}=\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix}$$

$$\text{So, } 25A^2 - 24I = \begin{pmatrix}25 & 0 & 0\\ 25 & 25 & 0\\ 25 & 0 & 25\end{pmatrix} - \begin{pmatrix}24 & 0 & 0\\ 0 & 24 & 0\\ 0 & 0 & 24\end{pmatrix}$$

$$= \begin{pmatrix}1 & 0 & 0\\ 25 & 1 & 0\\ 25 & 0 & 1\end{pmatrix}$$

Sum of all elements = 1 + 25 + 25 + 1 + 1 = 53.

12. Let the values of P for which shortest distance between the lines

$\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}$ &

$\vec{r} = (p\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ is

$\frac{1}{\sqrt{6}}$ be $a, b (a < b)$, then the length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to

Ans. **(2/3)**

Sol. $$\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}$$

$$\frac{x-p}{2} = \frac{y-2}{3} = \frac{z-1}{4}$$

So, from the above two equations we have

$$\vec{a}_1 = (-1, 0, 0)$$

$$\vec{a}_2 = (p, 2, 1)$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 4 & 5\\ 2 & 3 & 4\end{vmatrix}$$

$$= \hat{i} - 2\hat{j} + \hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{6}$$

$$S.D. = \begin{vmatrix}p+1 & 2 & 1\\ 3 & 4 & 5\\ 2 & 3 & 4\end{vmatrix} \div \sqrt{6}$$

$$= \frac{(p+1)(1) - 2(2) + (1)}{\sqrt{6}} = \pm\frac{1}{\sqrt{6}}$$

⇒ p – 2 = ±1

⇒ p = 3, 1

⇒ a = 1 and b = 3

So, the ellipse is $\frac{x^2}{1} + \frac{y^2}{9} = 1$

⇒ length of latus rectum = $\frac{2a^2}{b} = \frac{2}{3}$

13. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and $xRy \Rightarrow 2x - y \in \{0, 1\}$. If l is number of elements in given relation, m and n are minimum number of elements to be added to make it reflexive and symmetric, respectively. Then $l + m + n$ is equal to

Ans. **(17)**

Sol. For 2x – y = 0 ⇒ y = 2x

x = –1, 0, 1

Ordered pairs ⇒ (–1, –2), (0, 0), (1, 2)

For 2x – y = 1 ⇒ y = 2x – 1

x = –1, 0, 1, 2

ordered pairs ⇒ (–1, –3), (0, –1), (1, 1), (2, 3)

l = 7

m = 5

n = 5 ⇒ l + m + n = 17.

14. Let the sum of the focal distance of the point P(4,3) on the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $8\sqrt{\frac{5}{3}}$, then if the length of the latus rectum is l then value of $9l^2$ is

Ans. **(40)**

Sol. $$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Passes (4, 3)

$$\frac{16}{a^2} - \frac{9}{b^2} = 1$$

$$\frac{16}{a^2} - \frac{9 \times 3}{2a^2} = 1$$

$$16 - \frac{27}{2} = a^2$$

$$a^2 = \frac{5}{2}$$

$$b^2 = \frac{2}{3} \times \frac{5}{2}$$

$$= \frac{5}{3}$$

$$\ell = \frac{2b^2}{a}$$

$$=\frac{2\times\frac{5}{3}}{\sqrt{\frac{5}{2}}}$$

$$=\frac{10}{3}\times\sqrt{\frac{2}{5}}$$

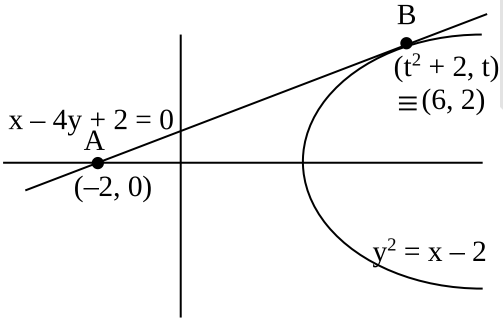
$$\ell^2=\frac{100}{9}\times\frac{2}{5}$$

$$=\frac{40}{9}$$

$$\left\{\begin{array}{l}(ex_1-a)+(ex_4+a)=8\sqrt{\frac{5}{3}}\\ 2e\times 4=8\frac{\sqrt{5}}{\sqrt{3}}\\ e=\sqrt{\frac{5}{3}}\\ b^2=a^2\left(\frac{5}{3}-1\right)\\ b^2=\frac{2}{3}a^2\end{array}\right\}$$

15. A line passing through the point A(-2, 0) touches the parabola $P: y^2 = x - 2$ at the point B in the first quadrant. The area of the region bounded by the line AB, parabola P and the x-axis is equal to

Ans. $\frac{8}{3}$



Sol.

$$m_T=\frac{1}{2t}$$

$$\Rightarrow\frac{t}{t^2+4}=\frac{1}{2t}$$

$$\Rightarrow 2t^2=t^2+4$$

$$\Rightarrow t^2=4$$

So, $A=\int_0^2\left(\left(y^2+2\right)-\left(4y-2\right)\right)dxy$

$$=\left.\frac{(y-2)^3}{3}\right|_0^2=\frac{8}{3}$$