MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let f be a differentiable function on **R** such that $f(2) = 1$, $f'(2) = 4$. Let $\lim_{x\to 0}(f(2+x))^{3/x} = e^{\alpha}$. Then the number of times the curve $y = 4x^3 - 4x^2 - 4(\alpha - 7)x - \alpha$ meets x-axis is:

   (1) 3

   (2) 0

   (3) 2

   (4) 1

Answer (3)

Sol. $\lim_{x\to 0}(f(2+x))^{3/x} = (1^{\infty} \text{ form})$

$$e^{\lim_{x\to 0}\frac{3}{x}(f(2+x)-1)} = e^{\lim_{x\to} 3f'(2+x)}$$

$$= e^{3f'(2)}$$

$$= e^{12}$$

$$\Rightarrow \alpha = 12$$

$$y = 4x^3 - 4x^2 - 4(12-7)x - 12$$

$$y = 4x^3 - 4x^2 - 20x - 12$$

$$y = 4(x^3 - x^2 - 5x - 3)$$

$$= 4(x+1)^2(x-3)$$

It meets the x-axis at two points

2. Let the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ satisfy $A^n = A^{n-2} + A^2 - I$ for $n \ge 3$. Then the sum of all the elements of A^{50} is:

   (1) 39

   (2) 52

   (3) 44

   (4) 53

Answer (4)

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A^3 = A + A^2 - I$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$A^4 = A^2 + A^2 - I = 2A^2 - I$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \text{ and } A^5 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$A^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$$

Sum of elements = 53

3. Consider two sets A and B, each containing three numbers in A.P. let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of AP's in A and B respectively such that $D = d + 3$, $d > 0$. If $\frac{p+q}{p-q} = \frac{19}{5}$, then $p - q$ is equal to

(1) 630 (2) 540

(3) 450 (4) 600

Answer (2)

Sol. Let the terms in A be $a_1 - d$, a_1, $a_1 + d$

and in B be $a_2 - D$, a_2, $a_2 + D$

Now $3a_1 = 36$

$\Rightarrow a_1 = 12$

and $3a_2 = 36$

$\Rightarrow a_2 = 12$

Now $(12 - d)(12)(12 + d) = p$

and $(12 - D)(12)(12 + D) = q$

Also $\frac{p+q}{p-q} = \frac{19}{5}$

$\Rightarrow 12q = 7p$

$\Rightarrow 12(12 - D)(12)(12 + D) = 7(12 - d)(12)(12 + d)$

$\Rightarrow 12(9 - d)(12)(15 - d) = 7(12 - d)(12)(12 + d)$

$\Rightarrow 12(135 - d^2 - 6d) = 7(144 - d^2)$

$\Rightarrow d = 6, D = 9$

$p = 6 \times 12 \times 18 = 1296$

$q = 756$

$p - q = 540$

4. Let the product of $\omega_1 = (8+i)\sin\theta + (7+4i)\cos\theta$ and $\omega_2 = (1+8i)\sin\theta + (4+7i)\cos\theta$ be $\alpha + i\beta$, $i = \sqrt{-1}$. Let p and q be the maximum and the minimum values of $\alpha + \beta$ respectively. Then $p + q$ is equal to:

(1) 140 (2) 150

(3) 130 (4) 160

Answer (3)

Sol. $\omega_1 = (8\sin\theta + 7\cos\theta) + i(\sin\theta + 4\cos\theta)$

$\omega_2 = (\sin\theta + 4\cos\theta) + i(8\sin\theta + 7\cos\theta)$

$\alpha = (8\sin\theta + 7\cos\theta) + (\sin\theta + 4\cos\theta)$

$-(\sin\theta + 4\cos\theta) + (8\sin\theta + 7\cos\theta) = 0$

$\beta = (8\sin\theta + 7\cos\theta)^2 + (\sin\theta + 4\cos\theta)^2$

$= 65\sin^2\theta + 65\cos^2\theta + 56\sin 2\theta + 4\sin 2\theta$

$= 65 + 60\sin 2\theta$

$(\alpha + \beta)_{max} = 125 = p$

$(\alpha + \beta)_{min} = 5 = q$

$p + q = 130$

5. Let for two distinct values of p the lines $y = x + p$ touch the ellipse $E : \frac{x^2}{4^2} + \frac{y^2}{3^3} = 1$ at the points A and B. Let the line $y = x$ intersect E at the points C and D. Then the area of the quadrilateral $ABCD$ is equal to :

(1) 48 (2) 20

(3) 36 (4) 24

Answer (Bonus)

Sol. $E : \frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$

$T : y = mx \pm \sqrt{16m^2 + 9}$

$y = x + p$

$\Rightarrow \quad m = 1$

$\Rightarrow \quad p = \pm\sqrt{16+9}$

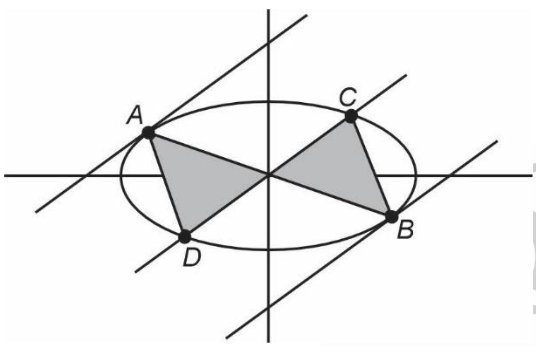
$= \pm 5$

$T : y = x \pm 5$ will to cut the E at $A\left(-\frac{16}{5}, \frac{9}{5}\right)$

$B\left(\frac{16}{5}, -\frac{9}{5}\right)$

Also, $y = x$ will cut the E at $C\left(\frac{12}{5}, \frac{12}{5}\right)$

$D\left(-\frac{12}{5}, -\frac{12}{5}\right)$



$ABCD$ in not give in cyclic order

$\therefore$ it does not form any quadrilateral

$\therefore$ No option should match

If order is not considered then

Area = 24 sq. unit.

6. If $1^2\cdot({}^{15}C_1) + 2^2\cdot({}^{15}C_2) + 3^2\cdot({}^{15}C_3) + \ldots + 15^2\cdot({}^{15}C_{15}) = 2^m \cdot 3^n \cdot 5^k$, where $m, n, k \in \mathbb{N}$, then $m + n + k$ is equal to :

(1) 18 (2) 19

(3) 21 (4) 20

Answer (2)

Sol. $\sum_{r=1}^{15} r^2 \cdot {}^{15}C_r \qquad (r\,{}^nC_r = n\,{}^{n-1}C_{r-1})$

$= 15\sum_{r=1}^{15} r \cdot {}^{14}C_{r-1}$

$= 15\sum_{r=1}^{15} (r-1+1)\,{}^{14}C_{r-1}$

$= 15 \cdot \sum_{r=1}^{15} (r-1)\,{}^{14}C_{r-1} + 15 \cdot \sum_{r=1}^{15} {}^{14}C_{r-1}$

$= 15 \cdot 14 \cdot 2^{13} + 15 \cdot 2^{14}$

$= 15 \cdot 2^{14}(7 + 1)$

$= 5 \cdot 3 \cdot 2^{17}$

$n + m + k = 17 + 1 + 1 = 19$

7. If a curve $y = y(x)$ passes through the point $\left(1, \frac{\pi}{2}\right)$ and satisfies the differential equation $(7x^4\cot y - e^x \operatorname{cosec} y)\frac{dx}{dy} = x^5$, $x \geq 1$, then at $x = 2$, the value of $\cos y$ is :

(1) $\frac{2e^2 - e}{64}$ (2) $\frac{2e^2 + e}{64}$

(3) $\frac{2e^2 - e}{128}$ (4) $\frac{2e^2 + e}{128}$

Answer (3)

Sol. $(7x^4 \cot y - e^x \operatorname{cosec} y)\frac{dx}{dy} = x^5$

$x^5\frac{dy}{dx} - 7x^4\cot y = -e^x \operatorname{cosec} y$

$\frac{dy}{dx} - \frac{7}{x}\cot y = -\frac{e^x}{x^5}\operatorname{cosec} y$

$\sin y\frac{dy}{dx} - \frac{7}{x}\cos y = -\frac{e^x}{x^5}$

Let $-\cos y = t$

$\sin y\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore \quad \frac{dt}{dx} + \frac{7}{x}t = -\frac{e^x}{x^5}$

$\therefore$ I.F. $= e^{\int \frac{7}{x}dx} = x^7$

$$t \cdot x^7 = \int \frac{-e^x}{x^5} \cdot x^7 dx$$

$$-\cos y \cdot x^7 = -\int e^x x^2 \, dx$$

$\cos y\, x^7 = e^x(x^2 - 2x + 2) + c$

$\because \quad x = 1$ then $y = \frac{\pi}{2} \Rightarrow c = -e$

$\therefore \quad \cos y \cdot x^7 = e^x(x^2 - 2x + 2) - e$

When $x = 2$ then $\cos y = \frac{2e^2 - e}{128}$

8. Let $f(x) + 2f\left(\frac{1}{x}\right) = x^2 + 5$ and $2g(x) - 3g\left(\frac{1}{2}\right) =$ x, x > 0. If $\alpha = \int_1^2 f(x)dx$, and $\beta = \int_1^2 g(x)\,dx$, then the value of $9\alpha + \beta$ is :

(1) 11 (2) 1

(3) 10 (4) 0

Answer (1)

Sol. $f(x) + 2f\left(\frac{1}{x}\right) = x^2 + 5$

$$2f\left(\frac{1}{x}\right) + 4f(x) = 2\left(\frac{1}{x^2} + 5\right)$$

$$3f(x) = \frac{2}{x^2} - x^2 + 5$$

$$f(x) = \frac{1}{3}\left(\frac{2}{x^2} - x^2 + 5\right)$$

$$2g(x) - 3g\left(\frac{1}{x}\right) = x$$

$$2g\left(\frac{1}{x}\right) - 3g(x) = \frac{1}{x}$$

Or $4g(x) - 6g\left(\frac{1}{x}\right) = 2x$

$$6g\left(\frac{1}{x}\right) - 9g(x) = \frac{3}{x}$$

$$-5g(x) = 2x + \frac{3}{x}$$

Or $g(x) = -\frac{1}{5}\left(2x + \frac{3}{x}\right)$

$$\int_1^2 f(x)dx = \int_1^2 \frac{1}{3}\left(\frac{2}{x^2} - x^2 + 5\right)dx$$

$$= \frac{1}{3}\left[-\frac{2}{x} - \frac{x^3}{3} + 5x\right]_1^2$$

$$= \frac{1}{3}\left[\left(-\frac{2}{2} - \frac{8}{3} + 10\right) - \left(-2 - \frac{1}{3} + 5\right)\right]$$

$$= \frac{1}{3}\left[-1 - \frac{8}{3} + 10 + 2 + \frac{1}{3} - 5\right]$$

$$\alpha = \frac{11}{9}$$

Now, $2g(x) = x + 3g\left(\frac{1}{2}\right)$

$$2g\left(\frac{1}{2}\right) = \frac{1}{2} + 3g\left(\frac{1}{2}\right)$$

$$g\left(\frac{1}{2}\right) = -\frac{1}{2}$$

$$\therefore \quad \beta = \int_1^2 g(x)dx$$

$$= \frac{1}{2}\int_1^2 \left(x + 3g\left(\frac{1}{2}\right)\right)dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2} + 3g\left(\frac{1}{2}\right)x\right]_1^2$$

$= 0$

$\therefore \quad 9\alpha + \beta = 11$

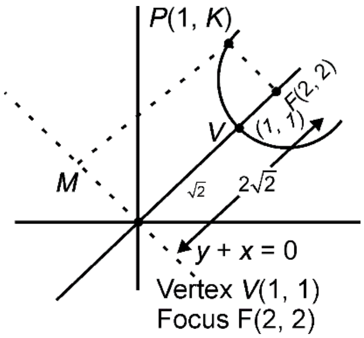
9. The axis of a parabola is the line $y = x$ and its vertex and focus are in the first quadrant at distances $\sqrt{2}$ and $2\sqrt{2}$ units from the origin, respectively. If the point $(1, k)$ lies on the parabola, then a possible value of k is:

(1) 3 (2) 4

(3) 8 (4) 9

Answer (4)

Sol.



Equation of directrix

$\Rightarrow \ y = -x$

By definition of parabola,

$PM = PF$

$$\left|\frac{1+K}{\sqrt{2}}\right| = \sqrt{(1-2)^2+(K-2)^2}$$

$$\frac{(1+K)^2}{2} = 1+K^2+4-4K$$

$$1+K^2+2K = 10+2K^2-8K$$

$K^2 - 10K + 9 = 0$

$(K-9)(K-1) = 0$

$\therefore \ K = 1$ or $K = 9$

10. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and R be a relation on A defined by xRy if and only if $2x - y \in \{0, 1\}$. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + m + n$ is equal to:

(1) 18 (2) 15

(3) 17 (4) 16

Answer (3)

Sol. $xRy \Leftrightarrow 2x - y \in \{0, 1\}$

$\Rightarrow \ y = 2x$ or $y = 2x - 1$

$A = \{-3, -2, -1, 0, 1, 2, 3\}$

$R = \{(-1, -2), (0, 0), (1, 2), (-1, -3), (0, -1), (1, 1), (2, 3)\}$

$\Rightarrow \ l = 7$

For R to be reflexive $(0, 0), (1, 1) \in R$

But other (a, a) such that $2a - a \in \{0, 1\}$

$\Rightarrow \ a \in \{0, 1\}$

5 other pairs needs to be added $\Rightarrow m = 5$

$xRy \Rightarrow yRx$ to be symmetric

$(-1, -2) \Rightarrow (-2, -1)$

$(1, 2) \Rightarrow (2, 1)$

$(-1, -3) \Rightarrow (-3, -1)$

$(0, -1) \Rightarrow (-1, 0)$

$(2, 3) \Rightarrow (3, 2) \Rightarrow$ 5 needs to be added, $n = 5$

$\Rightarrow \ l + m + n = 17$

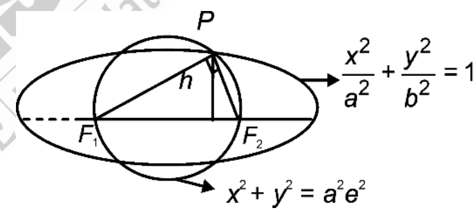
11. The centre of a circle C is at the centre of the ellipse $E : \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1,\ a > b$. Let C pass through the foci F_1 and F_2 of E such that the circle C and the ellipse E intersect at four points. Let P be one of these four points. If the area of the triangle PF_1F_2 is 30 and the length of the major axis of E is 17, then the distance between the foci of E is:

(1) 13 (2) 12

(3) $\dfrac{13}{2}$ (4) 26

Answer (1)

Sol.



$$x^2 + \frac{a^2y^2}{b^2} = a^2$$

$$\Rightarrow \ y^2\left(1-\frac{a^2}{b^2}\right) = a^2(e^2-1) = a^2\left(1-\frac{b^2}{a^2}-1\right)$$

$$= -b^2$$

$$\Rightarrow \ \frac{y^2(b^2-a^2)}{b^2} = -b^2 \ \Rightarrow \ y^2 = \frac{b^4}{(a^2-b^2)}$$

$$\text{Height} = |y| = \frac{b^2}{\sqrt{a^2-b^2}}$$

Area = $(2ae)\times\frac{1}{2}\times\frac{b^2}{\sqrt{a^2-b^2}}=30$

$=\frac{ab^2e}{a\sqrt{1-\frac{b^2}{a^2}}}=b^2,\ a=\frac{17}{2}$

Distance between foci = $2ae$

$=17\sqrt{1-\frac{b^2}{a^2}}=17\sqrt{1-\frac{30\times4}{289}}=13$

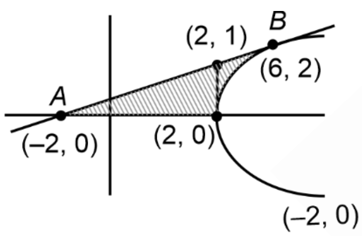
12. A line passing through the point $A(-2, 0)$, touches the parabola $P : y^2 = x - 2$ at the point B in the first quadrant. The area, of the region bounded by the line AB, parabola P and the x-axis, is:

(1) $\frac{8}{3}$ (2) 3

(3) 2 (4) $\frac{7}{3}$

Answer (1)

Sol.



$y^2=4\left(\frac{1}{4}\right)(x-2)$

$y=m(x-2)+\frac{1}{4m}$ passes through $(-2, 0)$

$\Rightarrow 0=-4m+\frac{1}{4m}\Rightarrow 16m^2=1$

$\Rightarrow\quad m=\pm\frac{1}{4}$

$m=\frac{1}{4}$ in first quadrant $\Rightarrow$ contact point $(6, 2)$

$\Rightarrow$ Area = $\frac{1}{2}\times(1)\times4+\int_2^6\left[\left(\frac{x+2}{4}\right)-\sqrt{x-2}\right]dx$

$=2+\frac{2}{3}=\frac{8}{3}$

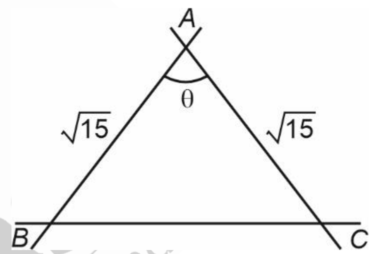
13. Let A be the point of intersection of the lines $L_1:\frac{x-7}{1}=\frac{y-5}{0}=\frac{z-3}{-1}$ and $L_2:\frac{x-1}{3}=\frac{y+3}{4}=\frac{z+7}{5}$. Let B and C be the points on the lines L_1 and L_2 respectively such that $AB = AC = \sqrt{15}$. Then the square of the area of the triangle ABC is:

(1) 57 (2) 63

(3) 60 (4) 54

Answer (4)

Sol. $L_1:\frac{x-7}{1}=\frac{y-5}{0}=\frac{z-3}{-1}$; $L_2:\frac{x-1}{3}=\frac{y+3}{4}=\frac{z+7}{5}$



$\cos\theta=\left|\frac{3+0-5}{\sqrt{2}\times\sqrt{50}}\right|$

$=\frac{2}{10}=\frac{1}{5}$

$\therefore\ \sin\theta=\frac{2\sqrt{6}}{5}$

Area = $\frac{1}{2}ab\sin\theta$

$=\frac{1}{2}\times\sqrt{15}\times\sqrt{15}\times\frac{2\sqrt{6}}{5}$

$=3\sqrt{6}$

$(\text{Area})^2 = 9 \times 6 = 54$

14. Let the values of p, for which the shortest distance between the lines $\frac{x+1}{3}=\frac{y}{4}=\frac{z}{5}$ and $\vec{r}=\left(p\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$ is $\frac{1}{\sqrt{6}}$, be a, b, $(a<b)$. Then the length of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is:

(1) 18 (2) 9

(3) $\frac{3}{2}$ (4) $\frac{2}{3}$

Answer (4)

Sol. $\frac{x+1}{3}=\frac{y}{4}=\frac{z}{5}$; $\left(p\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$

$$d=\left|\frac{\left(\vec{a}-\vec{b}\right)\cdot\left(\vec{p}_1\times\vec{p}_2\right)}{\left|\vec{p}_1\times\vec{p}_2\right|}\right|=\frac{1}{\sqrt{6}}$$

$$\vec{a}-\vec{b}=(p+1)\hat{i}+2\hat{j}+\hat{k}$$

$$\vec{p}_1\times\vec{p}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&4&5\\2&3&4\end{vmatrix}=\hat{i}-2\hat{j}+\hat{k}$$

$$\frac{1}{\sqrt{6}}=\left|\frac{(p+1)-4+1}{\sqrt{6}}\right|$$

$$=|p-2|=1\Rightarrow\ p=3,1$$

$a=1$, $b=3$

$$\frac{x^2}{1}+\frac{y^2}{9}=1$$

Length of $LR=\frac{2a^2}{b}=\frac{2}{3}$

15. If the sum of the first 20 terms of the series $\frac{4\cdot1}{4+3\cdot1^2+1^4}+\frac{4\cdot2}{4+3\cdot2^2+2^4}+\frac{4\cdot3}{4+3\cdot3^2+3^4}+\frac{4\cdot4}{4+3\cdot4^2+4^4}+\ldots$ is $\frac{m}{n}$, where m and n are coprime, then $m+n$ is equal to :

(1) 420 (2) 423

(3) 421 (4) 422

Answer (3)

Sol. $S_n=\sum_{r=1}^{n}\frac{4r}{4+3r^2+r^4}$

$$=2\sum_{r=1}^{n}\frac{2r}{(r^2+2)^2-r^2}=2\sum_{r=1}^{n}\frac{(r^2+2+r)-(r^2+2-r)}{(r^2+2+r)(r^2+2-r)}$$

$$=2\sum_{r=1}^{n}\left(\frac{1}{r^2+2-r}-\frac{1}{r^2+2+r}\right)$$

$$S_{20}=2\left[\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{8}\right)+\ldots\right]$$

$$=2\left(\frac{1}{2}-\frac{1}{20^2+2+20}\right)$$

$$=2\left(\frac{1}{2}-\frac{1}{422}\right)$$

$$=2\left(\frac{422-2}{422\times2}\right)=\frac{420}{422}=\frac{210}{211}=\frac{m}{n}$$

$m+n=421$

16. Let the mean and the standard deviation of the observation 2, 3, 3, 4, 5, 7, a, b be 4 and $\sqrt{2}$ respectively. Then the mean deviation about the mode of these observations is :

(1) $\frac{3}{4}$ (2) 1

(3) $\frac{1}{2}$ (4) 2

Answer (2)

Sol. $\frac{2+3+3+4+5+7+a+b}{8}=4$

$\Rightarrow\ a+b=8$

$$\left(\sqrt{2}\right)^2=\frac{2^2+3^2+3^2+4^2+5^2+7^2+a^2+b^2}{8}-16$$

$112+a^2+b^2=18\times8$

$\Rightarrow\ a^2+b^2=32$

$\Rightarrow\ a=b=4$

Now numbers be

2, 3, 3, 4, 4, 4, 5, 7

Mode = 4

Mean deviation about mode :

$$\frac{|2-4|+|3-4|+|3-4|+0+0+0+|5-4|+|4-7|}{8}$$

$$=\frac{2+1+1+1+3}{8}=\frac{8}{8}=1$$

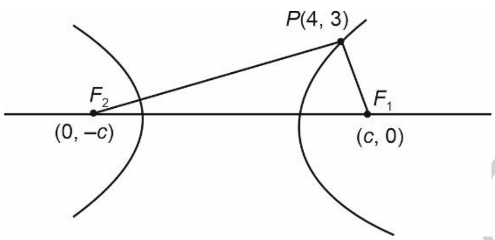
17. Let the sum of the focal distances of the point $P(4, 3)$ on the hyperbola $H : \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $8\sqrt{\frac{5}{3}}$. If for H, the length of the latus rectum is l and the product of the focal distance of the point P is m, then $9l^2 + 6m$ is equal to:

(1) 186 (2) 187

(3) 184 (4) 185

Answer (4)

Sol.



$$\sqrt{(c+4)^2+9}+\sqrt{(c-4)^2+9}=8\sqrt{\frac{5}{3}}$$

Solving, $c=\frac{5}{\sqrt{6}}=al \Rightarrow a^2\left(1+\frac{b^2}{a^2}\right)=\frac{25}{6} \Rightarrow a^2+b^2=\frac{25}{6}$

$$\frac{16}{a^2}-\frac{9}{b^2}=1$$

$16b^2 - 9d^2 = a^2b^2 \Rightarrow 16\left(\frac{25}{6}-a^2\right)-9a^2=9a^2b^2$

$$PF_1+PF_2=8\sqrt{\frac{5}{3}} \Rightarrow a^2=\frac{5}{2}, b^2=\frac{5}{3}$$

$|PF_1 - PF_2| = 2a$

$$\frac{64.5}{3}=4a^2+4m \Rightarrow m=\frac{80}{3}-a^2$$

$6m = 160 - 6a^2$

$$9\ell^2=9\left(\frac{2b^2}{a}\right)^2=\frac{36b^4}{a^2}$$

$$9\ell^2+6m=\frac{36\left(\frac{25}{9}\right)}{\frac{5}{2}}+160-6\left(\frac{5}{2}\right)$$

$$=\frac{72\times 5}{9}+160-15$$

$= 160 + 40 - 15 = 185$

18. Let $a > 0$. If the function $f(x) = 6x^3 - 45ax^2 + 108a^2x + 1$ attains its local maximum and minimum values at the points x_1 and x_2 respectively such $x_1x_2 = 54$, then $a + x_1 + x_2$ is equal to

(1) 15

(2) 13

(3) 18

(4) 24

Answer (3)

Sol. $f(x) = 6x^3 - 45ax^2 + 108a^2x + 1$

For maxima or minima $f'(x) = 0$

$$f'(x) = 18x^2 - 90x + 108a^2 = 0 \begin{cases} x_1 \\ x_2 \end{cases}$$

$$x_1x_2=\frac{108a^2}{18}=54$$

$\Rightarrow a^2 = 9 \Rightarrow a = 3$

Now, $a+x_1+x_2=3+\frac{90}{6}=3+15=18$

19. Let the domains of the functions $f(x) = \log_4\log_3\log_7(8 - \log_2(x^2 + 4x + 5))$ and $g(x)=\sin^{-1}\left(\frac{7x+10}{x-2}\right)$ be (α, β) and $[\gamma, \delta]$, respectively. Then $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is equal to:

(1) 14 (2) 16

(3) 13 (4) 15

Answer (4)

Sol. $f(x) = \log_4(\log_3(\log_7(8 - \log_2(x^2 + 4x + 5))))$

$\log_3(\log_1(8 - \log_2(x^2 + 4x + 5))) > 0$

$\log_7(8 - \log_2(x^2 + 4x + 5)) > 1$

$8 - \log_2(x^2 + 4x + 5) > 7$

$-\log_2(x^2 + 4x + 5) > -1$

$\log_2(x^2 + 4x + 5) < 1$

$x^2 + 4x + 5 < 2$

$x^2 + 4x + 3 < 0$

$\Rightarrow (x + 3)(x + 1) < 0 \qquad \ldots(1)$

$\log_7(8 - \log_2(x^2 + 4x + 5)) > 0$

$8 - \log_2(x^2 + 4x + 5) > 1$

$\log_2(x^2 + 4x + 5) < 9$

$x^2 + 4x + 5 < 2^9$

$x^2 + 4x + 5 < 512$

$\Rightarrow x^2 + 4x - 507 < 0$

$\Rightarrow x = -4 \pm \sqrt{16 + 2028}$

$$x = \frac{-4 \pm \sqrt{2044}}{2} \qquad \ldots(2)$$

$$\Rightarrow \left(x - \left(\frac{-4 + \sqrt{2044}}{2}\right)\right)\left(x - \left(\frac{-4 - \sqrt{2044}}{2}\right)\right) < 0$$

$x^2 + 4x + 5 > 0$

$D > 0$

$x \in R$

Also, $8 - \log_2(x^2 + 4x + 5) > 0$

$\log_2(x^2 + 4x + 5) < 8$

$x^2 + 4x + 5 < 256$

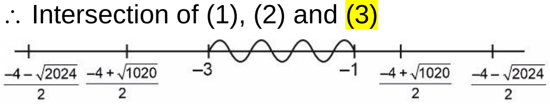
$\Rightarrow x^2 + 4x - 251 < 0$

$\Rightarrow x = -4 \pm \sqrt{16 + 1004}$

$$\Rightarrow x = \frac{-4 \pm \sqrt{1020}}{2}$$

$$\Rightarrow \left(x - \left(\frac{-4 + \sqrt{1020}}{2}\right)\right)\left(x - \left(\frac{-4 - \sqrt{1020}}{2}\right)\right) < 0$$

$\therefore$ Intersection of (1), (2) and (3)



$\therefore x \in (-3, -1)$

$$-1 \le \frac{7x + 10}{x - 2} \le 1$$

$\Rightarrow x \in [-2, -1]$

$\therefore \alpha^2 + \beta^2 + \gamma^2 + \delta^2 = (-3)^2 + (-1)^2 + (-2)^{-2} + (-1)^2$

$= 9 + 1 + 4 + 1$

$= 15$

20. The sum of the infinite series

$$\cot^{-1}\left(\frac{7}{4}\right) + \cot^{-1}\left(\frac{19}{4}\right) + \cot^{-1}\left(\frac{39}{4}\right) + \cot^{-1}\frac{67}{4} + \ldots \text{ is:}$$

(1) $\frac{\pi}{2} - \cot^{-1}\left(\frac{1}{2}\right)$ (2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)$

(3) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{2}\right)$ (4) $\frac{\pi}{2} + \cot^{-1}\left(\frac{1}{2}\right)$

Answer (2)

Sol. $\cot^{-1}\left(\frac{7}{4}\right) + \cot^{-1}\left(\frac{19}{4}\right) + \cot^{-1}\left(\frac{39}{4}\right) + \cot^{-1}\left(\frac{67}{4}\right) + \ldots$

$$T_r = \cot^{-1}\left(\frac{4r^2 + 3}{4}\right)$$

$$T_r = \tan^{-1}\left(\frac{1}{\left(\frac{3}{4} + r^2\right)}\right)$$

$$T_r = \tan^{-1}\left(\frac{\left(r + \frac{1}{2}\right) - \left(r - \frac{1}{2}\right)}{1 + r^2 - 1/4}\right)$$

$$T_r = \tan^{-1}\left(\frac{\left(r + \frac{1}{2}\right) - \left(r - \frac{1}{2}\right)}{1 + \left(r + \frac{1}{2}\right)\left(r - \frac{1}{2}\right)}\right)$$

$$T_r = \tan^{-1}\left(r + \frac{1}{2}\right) - \tan^{-1}\left(r - \frac{1}{2}\right)$$

$$T_1 = \tan^{-1}\left(\frac{3}{2}\right) - \tan^{-1}\left(\frac{1}{2}\right)$$

$$T_2 = \tan^{-1}\left(\frac{5}{2}\right) - \tan^{-1}\left(\frac{3}{2}\right)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$T_n = \tan^{-1}\left(\frac{2n+1}{2}\right) - \tan^{-1}\left(\frac{1}{2}\right)$$

$$\Sigma T_r = \tan^{-1}\left(\frac{2n+1}{2}\right) - \tan^{-1}\left(\frac{1}{2}\right)$$

$$\Sigma T_r = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)$$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If α is a root of the equation $x^2 + x + 1 = 0$ and $\sum_{k=1}^{n}\left(\alpha^k + \frac{1}{a^k}\right)^2 = 20$, then n is equal to ______.

Answer (11)

Sol. α is root of equation $1 + x + x^2 = 0$, $\alpha = \omega$ or ω^2

$$\left(\alpha^k + \frac{1}{\alpha^k}\right)^2 = \alpha^{2k} + \frac{1}{\alpha^{2k}} + 2 = \omega^k + \frac{1}{\omega^k} + 2$$

$$\Rightarrow \quad \omega^k + \frac{1}{\omega^k} + 2 = \begin{cases} 4, & 3 \text{ divides } k \\ 1, & 3 \text{ does not divide } k \end{cases}$$

$$\therefore \quad \sum_{k=1}^{n}\left(\alpha^k + \frac{1}{\alpha^k}\right)^2 = 20$$

$\Rightarrow$ (1 + 1 + 4) + (1 + 1 + 4) + (1 + 1 + 4) + (1 + 1) = 20

$\Rightarrow$ $n = 11$

22. A card from a pack of 52 cards is lost. From the remaining 51 cards, n cards are drawn and are found to be spades. If the probability of the lost card to be a spade is $\frac{11}{50}$, then n is equal to ______.

Answer (2)

Sol. $P\left(\frac{\text{Lost}_{(\text{spade})}}{\text{n cards are spade}}\right)$

$$= \frac{P\left(\frac{n_s}{L_s}\right)P(L_s)}{P\left(\frac{n_s}{L_s}\right)P(L_s) + P\left(\frac{n_s}{\bar{L}_s}\right)P(\bar{L}_s)}$$

$$= \frac{\frac{^{12}C_n}{^{51}C_n} \times \frac{1}{4}}{\frac{^{12}C_n}{^{51}C_n} \times \frac{1}{4} + \frac{3}{4} \times \frac{^{13}C_n}{^{51}C_n}} = \frac{1}{1 + 3 \cdot \frac{^{13}C_n}{^{12}C_n}} = \frac{13-n}{52-n}$$

$$\Rightarrow \quad \frac{13-n}{52-n} = \frac{11}{50}$$

$\Rightarrow$ $n = 2$

23. If $\int \frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^9}dx$

$$= \frac{1}{m}\left(\left(\sqrt{1+x^2}+x\right)^n\left(n\sqrt{1+x^2}-x\right)\right) + C$$

where C is the constant of integration and m, $n \in \mathbb{N}$, then $m + n$ is equal to ______.

Answer (379)

Sol. $\sqrt{1+x^2} + x = \sec\theta + \tan\theta = t$

$$\sqrt{1+x^2} = \sec\theta = \frac{t^2+1}{2t}$$

$$x = \tan\theta = \frac{t^2-1}{2t}$$

The given expression becomes

$$\frac{1}{m}t^2\left(n \cdot \frac{t^2+1}{2t} - \frac{t^2-1}{2t}\right) = \frac{t^{n-1}}{2m}\left((n-1)t^2 + n + 1\right)$$

By compare

$n = 19$

$m = 360$

$\therefore$ $n + m = 379$

24. Let the three sides of a triangle ABC be given by the vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$. Let G be the centroid of the triangle ABC. Then $6\left(|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2\right)$ is equal to ______.

Answer (164)

Sol. Assuming Vertex A to be origin

$$\vec{A}=\vec{a}_1=\vec{0}$$

$$\vec{B}=\vec{a}_1+\vec{u}=\vec{u}=2\hat{i}-\hat{j}+\hat{k}$$

$$\vec{C}=\vec{a}_1+\vec{v}=\vec{v}=3\hat{i}-4\hat{j}-4\hat{k}$$

One solving

$\vec{A}=\vec{0}, \vec{B}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{C}=3\hat{i}-4\hat{j}-4\hat{k}$, are the position vector of vertices AB and C respectively.

$$\vec{G}=\frac{1}{3}(\vec{A}+\vec{B}+\vec{C})=\frac{1}{3}(\vec{0}+\vec{B}+\vec{C})=\frac{1}{3}(\vec{B}+\vec{C})$$

$$\Rightarrow \quad \vec{G}=\frac{5}{3}\hat{i}-\frac{5}{3}\hat{j}-\hat{k}$$

$$\overrightarrow{AG}=\vec{G}-\vec{A}=\vec{G}$$

$$|\overrightarrow{AG}|^2=\left(\frac{5}{3}\right)^2+\left(\frac{5}{3}\right)^2+(1)^2=\frac{25}{9}+\frac{25}{9}+1=\frac{50}{9}+1=\frac{59}{9}$$

$$\overrightarrow{BC}=\vec{G}-\vec{B}$$

$$\vec{B}=2\hat{i}-\hat{j}+\hat{k}$$

$$|\overrightarrow{BG}|^2=\left(\frac{1}{3}\right)^3+\left(\frac{2}{3}\right)^2+4=\frac{1}{9}+\frac{4}{9}+4=\frac{5}{9}+4=\frac{41}{9}$$

$$\overrightarrow{CG}=\vec{G}-\vec{C}$$

$$\vec{C}=3\hat{i}-4\hat{j}-4\hat{k}$$

$$|\overrightarrow{CG}|^2=\left(\frac{4}{3}\right)^2+\left(\frac{7}{3}\right)^2+9=\frac{16}{9}+\frac{49}{9}+9=\frac{65}{9}+9=\frac{65}{9}+\frac{81}{9}=\frac{146}{9}$$

$$6\left(|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2\right)=6\cdot\left(\frac{59}{9}+\frac{41}{9}+\frac{146}{9}\right)=6\cdot\frac{246}{9}=164$$

25. Let m and n, $(m < n)$, be two 2-digit numbers. Then the total number of pairs (m, n), such that $gcd\,(m, n) = 6$, is ______.

Answer (64)

Sol. $m = 6a$, $n = 6b$

So $gcd(m, n) = 6 \Rightarrow gcd(a, b) = 1$

$$m=6a\ge 10\Rightarrow a\ge\left[\frac{10}{6}\right]=2$$

$$m=6a\le 99\Rightarrow a\le\left[\frac{99}{6}\right]=16$$

So $a, b\in\{2, 3, ..., 16\}$, and we count how many coprime pairs (a, b) with $a < b$, $gcd(a, b) = 1$

$a = 2 \Rightarrow b = 3, 5, 7, 9, 11, 13, 15 \Rightarrow 7$

$a = 3 \Rightarrow b = 4, 5, 7, 8, 10, 11, 13, 14, 16 \Rightarrow 9$

$a = 4 \Rightarrow b = 5, 7, 9, 11, 13, 15 \Rightarrow 6$

$a = 5 \Rightarrow b = 6, 7, 8, 9, 11, 12, 13, 14, 16 \Rightarrow 9$

$a = 6 \Rightarrow b = 7, 11, 13 \Rightarrow 3$

$a = 7 \Rightarrow b = 8, 9, 10, 11, 12, 13, 15, 16 \Rightarrow 8$

$a = 8 \Rightarrow b = 9, 11, 13, 15 \Rightarrow 4$

$a = 9 \Rightarrow b = 10, 11, 13, 14, 16 \Rightarrow 5$

$a = 10 \Rightarrow b = 11, 13 \Rightarrow 2$

$a = 11 \Rightarrow b = 12, 13, 14, 15, 16 \Rightarrow 5$

$a = 12 \Rightarrow b = 13, 17 \times \rightarrow$ only 13 is valid $\Rightarrow 1$

$a = 13 \Rightarrow b = 14, 15, 16 \Rightarrow 3$

$a = 14 \Rightarrow b = 15, \Rightarrow 1$

$a = 15 \Rightarrow b = 16 \Rightarrow 1$

Total = 7 + 9 + 6 + 9 + 3 + 8 + 4 + 5 + 2 + 5 + 1 + 3 + 1 + 1 = 64

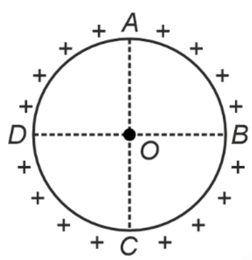
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answers :

26. A metallic ring is uniformly charged as shown in figure. *AC* and *BD* are two mutually perpendicular diameters. Electric field due to arc *AB* at '*O*' is '*E*' in magnitude. What would be the magnitude of electric field at '*O*' due to arc *ABC*?



(1) $\sqrt{2}E$

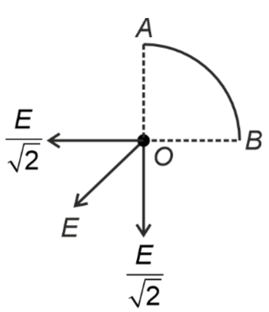
(2) Zero

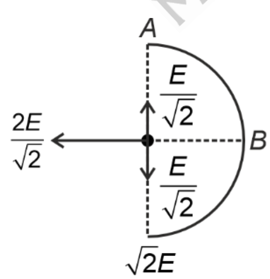
(3) $2E$

(4) $E/2$

Answer (1)

Sol.





27. A finite size object is placed normal to the principal axis at a distance of 30 cm from a convex mirror of focal length 30 cm. A plane mirror is now placed in such a way that the image produced by both the mirrors coincide with each other. The distance between the two mirrors is :

(1) 7.5 cm

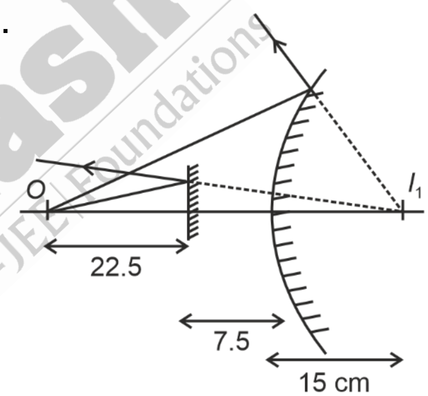
(2) 22.5 cm

(3) 45 cm

(4) 15 cm

Answer (1)

Sol.



$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}+\frac{1}{-30}=\frac{1}{30}$$

$v = 15$

Distance = 7.5 cm

28. Displacement of a wave is expressed as $x(t) = 5\cos\left(628t + \frac{\pi}{2}\right)$ m. The wavelength of the wave when its velocity is 300 m/s is :

(π = 3.14)

(1) 0.5 m
(2) 5 m
(3) 3 m
(4) 0.33 m

Answer (3)

Sol. $x = 5\cos\left(628t + \frac{\pi}{2}\right)$

$2\pi f = 628$

$6.28f = 628$

$f = 100\ \mathrm{H_2}$

$\lambda = \frac{v}{f} = \frac{300}{100} = 3$ m

29. In an electromagnetic system, a quantity defined as the ratio of electric dipole moment and magnetic dipole moment has dimension of [$M^P L^Q T^R A^S$]. The value of P and Q are :

(1) 1, –1
(2) –1, 0
(3) 0, –1
(4) –1, 1

Answer (3)

Sol. $E = \frac{1}{4\pi\varepsilon_0}\frac{P_E}{r^3}$

$B = \frac{\mu_0}{4\pi}\frac{P_m}{r^3}$

$\frac{E}{B} = \frac{1}{\mu_0\varepsilon_0}\cdot\frac{P_E}{P_m}$

$L^{-1}T^1 = \frac{P_E}{P_m}$

30. Consider a *n*-type semiconductor in which n_e and n_h are number of electrons and holes, respectively.

(A) Holes are minority carriers

(B) The dopant is a pentavalent atom

(C) $n_e n_h \neq n_i^2$

(where n_i is number of electrons or holes in semiconductor when it is intrinsic form)

(D) $n_e n_h \geq n_i^2$

(E) The holes are not generated due to the donors

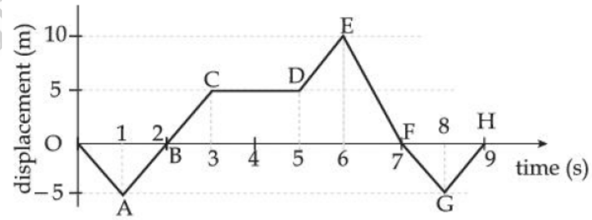
Choose the **correct** answer from the options given below:

(1) (A), (B), (C) only
(2) (A), (B), (E) only
(3) (A), (C), (E) only
(4) (A), (C), (D) only

Answer (2)

Sol. $n_e n_h = n_i^2$ always holds true so option C, D are incorrect.

31. The displacement *x* versus time graph is shown below.



(A) The average velocity during 0 to 3 s is 10 m/s

(B) The average velocity during 3 to 5 s is 0 m/s

(C) The instantaneous velocity at t = 2 s is 5 m/s

(D) The average velocity during 5 to 7 s and instantaneous velocity at t = 6.5 s are equal

(E) The average velocity from t = 0 to t = 9 s is zero

Choose the correct answer from the options given below

(1) (B), (D), (E) only

(2) (B), (C), (D) only

(3) (A), (D), (E) only

(4) (B), (C), (E) only

Answer (4)

Sol. V_{avg} $t = 0$ to $t = 3 \Rightarrow \frac{5}{3}$

V_{avg} $t = 3$ to $t = 5 \Rightarrow \frac{0}{2} = 0$

V at $t = 2$ is equal to slope = 5

From t = 5 to t = 7 sec slope is not constant so avg velocity is not equal to instantaneous velocity.

32. For the determination of refractive index of glass slab, a travelling microscope is used whose main scale contains 300 equal divisions equals to 15 cm. The vernier scale attached to the microscope has 25 divisions equals to 24 divisions of main scale. The least count (LC) of the travelling microscope is (in cm)

(1) 0.0005 (2) 0.001

(3) 0.002 (4) 0.0025

Answer (3)

Sol. $\text{MSD} = \frac{15}{300} = 0.05 \text{ cm}$

25VSD = 24 MSD

$$1\text{VSD} = \frac{24}{25}\text{MSD}$$

LC = 1MSD – 1VSD

$$= \left(1 - \frac{24}{25}\right)\text{MSD}$$

$$= \frac{1}{25} \times 0.05 \text{ cm}$$

= 0.002 cm

33. An object is kept at rest at a distance of 3R above the earth's surface where R is earth's radius. The minimum speed with which it must be projected so that it does not return to earth is

(Assume M = mass of earth, G = Universal gravitational constant)

(1) $\sqrt{\frac{2GM}{R}}$

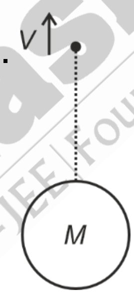
(2) $\sqrt{\frac{GM}{2R}}$

(3) $\sqrt{\frac{GM}{R}}$

(4) $\sqrt{\frac{3GM}{R}}$

Answer (2)

Sol.



$KE_i + PE_i = KE_g + PE_g$

$$\frac{1}{2}mv^2 - \frac{GMm}{4R} = 0 + 0$$

$$v^2 = \frac{GM}{R}\frac{1}{2}$$

$$\boxed{v = \sqrt{\frac{GM}{2R}}}$$

34. A wheel is rolling on a plane surface. The speed of a particle on the highest point of the rim is 8 m/s. The speed of the particle on the rim of the wheel at the same level as the centre of wheel, will be

(1) $8\sqrt{2}$ m/s

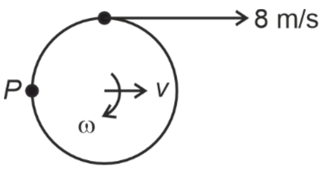
(2) $4\sqrt{2}$ m/s

(3) 8 m/s

(4) 4 m/s

Answer (2)

Sol.



$V + \omega R = 8$

$2V = 8$

$V = 4$

$V_P = \sqrt{2}V$

$= 4\sqrt{2}$

35. Two polarisers P_1 and P_2 are placed in such a way that the intensity of the transmitted light will be zero. A third polariser P_3 is inserted in between P_1 and P_2, at particular angle between P_2 and P_3. The transmitted intensity of the light passing the through all three polarisers is maximum. The angle between the polarisers P_2 and P_3 is

(1) $\frac{\pi}{3}$

(2) $\frac{\pi}{8}$

(3) $\frac{\pi}{6}$

(4) $\frac{\pi}{4}$

Answer (4)

Sol. $I = I_0\cos^2\theta$

Angle between P_1 and P_2 is 90°

$I = I_0\cos^2\theta \cdot \cos^2(90 - \theta)$

$I = I_0\cos^2\theta \cdot \sin^2\theta$

I will be maximum at $\theta = 45°$

36. Match **List - I** with **List - II**.

List - I	**List - II**
(A) Isobaric	(I) $\Delta Q = \Delta W$
(B) Isochoric	(II) $\Delta Q = \Delta U$
(C) Adiabatic	(III) ΔQ = zero
(D) Isothermal	(IV) $\Delta Q = \Delta U + P\Delta V$

ΔQ = Heat supplied

ΔW = Work done by the system

ΔU = Change in internal energy

P = Pressure of the system

ΔV = Change in volume of the system

Choose the **correct** answer from the options given below :

(1) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(2) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

(3) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(4) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

Answer (3)

Sol. Isobaric $\Rightarrow$ $\Delta Q = \Delta U + \int PdV$

$\underline{\Delta Q = \Delta U + P\Delta V}$

Isochoric $\Rightarrow \Delta Q = \Delta U$

Adiabatic $\Rightarrow \Delta Q = 0$

Isothermal $\Rightarrow \Delta Q = \Delta W$

37. Consider a rectangular sheet of solid material of length l = 9 cm and width d = 4 cm. The coefficient of linear expansion is α = 3.1 x 10^{-5} K^{-1} at room temperature and one atmospheric pressure. The mass of sheet m = 0.1 kg and the specific heat capacity C_v = 900 J $kg^{-1}K^{-1}$. If the amount of heat supplied to the material is 8.1 ×10^2 J then change in area of the rectangular sheet is

(1) 4.0×10^{-7} m^2
(2) 2.0×10^{-6} m^2
(3) 6.0×10^{-7} m^2
(4) 3.0×10^{-7} m^2

Answer (2)

Sol. $Q = mc\Delta T$

$8.1 \times 10^2 = 900 \times 0.1 \times \Delta T$

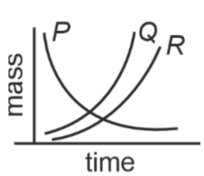
$\Delta T = 9K$

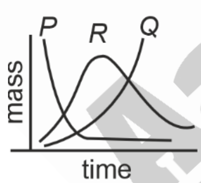
$\Delta A = A \cdot 2\alpha \Delta T$

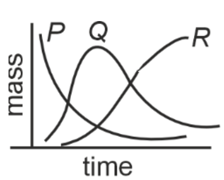
$= 36 \times 2 \times 3.1 \times 10^{-5} \times 9 \times 10^{-4}$

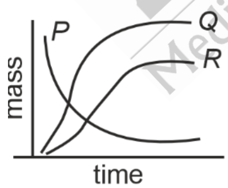
$= 2 \times 10^{-6}$ m^2

38. A radioactive material P first decays into Q and then Q decays to non-radioactive material R. Which of the following figure represents time dependent mass of P, Q and R?

(1)


(2)


(3)


(4)


Answer (3)

Sol. $P \rightarrow Q \rightarrow R$

Final mass of R will be equal to initial mass of P and mass of P is continuously decreasing with time

39. Given below are two statements :

Statement (I) : The dimensions of Planck's constant and angular momentum are same.

Statement (II) : In Bohr's model electron revolve around the nucleus only in those orbits for which angular momentum is integral multiple of Planck's constant.

In the light of the above statements, choose the **most appropriate answer** from the options given below.

(1) Both **Statement I** and **Statement II** are incorrect

(2) Both **Statement I** and **Statement II** are correct

(3) **Statement I** is incorrect but **Statement II** is correct

(4) **Statement I** is correct but **Statement II** is incorrect

Answer (4)

Sol. $h\nu = E$

$[h]T = ML^2T^{-2}$

$[h] = ML^2T^{-1}$

$L = mvr$

$[L] = MLT^{-1}L = ML^2T^{-1}$

$$mvr = \frac{nh}{2\pi}$$

Angular momentum is integral multiple of $\frac{h}{2\pi}$.

40. There are n number of identical electric bulbs, each is designed to draw a power p independently from the mains supply. They are now joined in series across the mains supply. The total power drawn by the combination is

(1) p
(2) np
(3) $\frac{p}{n}$
(4) $\frac{p}{n^2}$

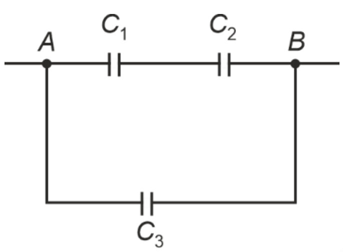
Answer (3)

Sol. $\frac{v^2}{R} = p$

$R_{eq} = nR$

$$p' = \frac{v^2}{R_{eq}} = \frac{v^2}{nR} \Rightarrow \frac{p}{n}$$

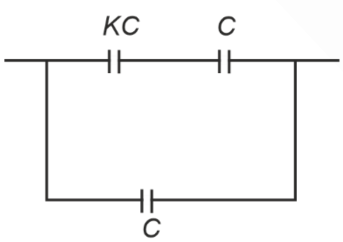
41. Three parallel plate capacitors C_1, C_2 and C_3 each of capacitance 5 μF are connected as shown in figure. The effective capacitance between points A and B, when the space between the parallel plates of C_1 capacitor is filled with a dielectric medium having dielectric constant of 4, is:



(1) 30 μF
(2) 9 μF
(3) 22.5 μF
(4) 7.5 μF

Answer (2)

Sol.



$$C_{eq} = \left(\frac{KC \cdot C}{KC + C}\right) + C$$

$$= \left(\frac{K}{K+1}\right)C + C$$

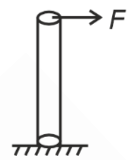
$$= \frac{4}{5} \times 5 + 5 = 9\ \mu F$$

42. A cylindrical rod of length 1 m and radius 4 cm is mounted vertically. It is subjected to a shear force of 10^5 N at the top. Considering infinitesimally small displacement in the upper edge, the angular displacement θ of the rod axis from its original position would be (shear moduli, $G = 10^{10}$ N/m^2)

(1) $\frac{1}{4\pi}$
(2) $\frac{1}{40\pi}$
(3) $\frac{1}{2\pi}$
(4) $\frac{1}{160\pi}$

Answer (4)

Sol.

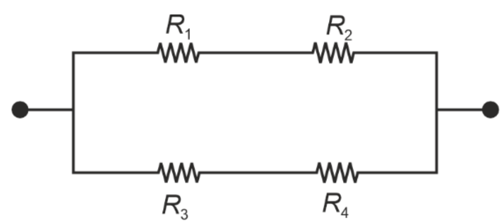


$$\frac{F}{A} = \eta\theta$$

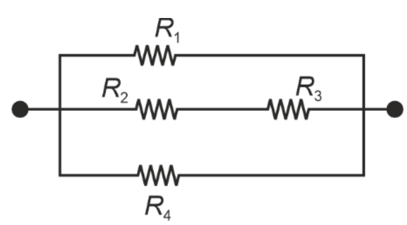
$$\frac{10^5}{\pi 16 \times 10^{-4} \times 10^{10}} = \theta$$

43. From the combination of resistors with resistances values $R_1 = R_2 = R_3 = 5\ \Omega$ and $R_4 = 10\ \Omega$, which of the following combination is the best circuit to get an equivalent resistance of 60 Ω ?

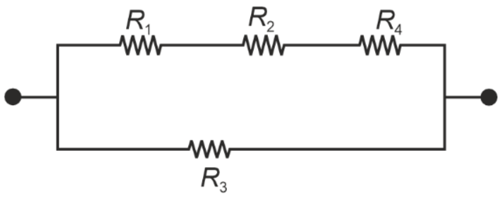
(1)



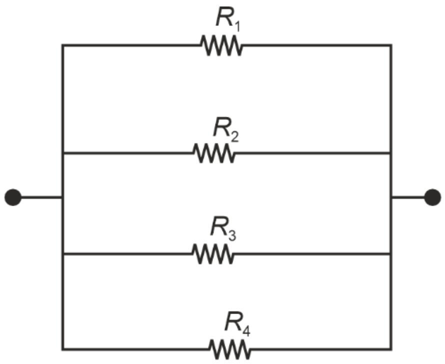
(2)



(3)



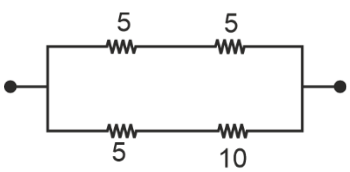
(4)



Answer (1)

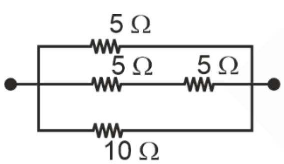
Sol. $R_1 = R_2 = R_3 = 5\ \Omega$ and $R_4 = 10\ \Omega$

(1)



$$R_{eq} = \frac{15 \times 10}{250} = 6\ \Omega$$

(2)



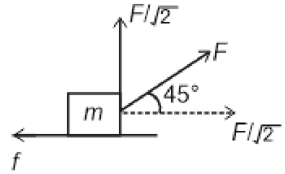
$$R_{eq} = 2.5\ \Omega$$

44. A block of mass 25 kg is pulled along a horizontal surface by a force at an angle 45° with the horizontal. The friction coefficient between the block and the surface is 0.25. The block travels at a uniform velocity. The work done by the applied force during a displacement of 5 m of the block is :

(1) 735 J
(2) 490 J
(3) 970 J
(4) 245 J

Answer (4)

Sol.



$$f = \frac{F}{\sqrt{2}}$$

$$\mu\left(mg - \frac{F}{\sqrt{2}}\right) = \frac{F}{\sqrt{2}}$$

$$\frac{1}{4}\left(245 - \frac{F}{\sqrt{2}}\right) = \frac{F}{\sqrt{2}}$$

$$245 = 5\frac{F}{\sqrt{2}}$$

$$\frac{F}{\sqrt{2}} = 45$$

$$W = \frac{F}{\sqrt{2}} \times 5$$

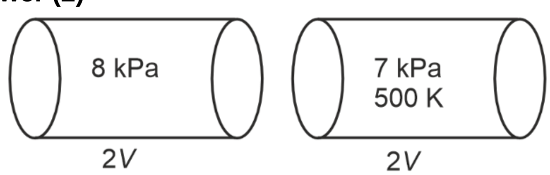
$= 245$ J

45. There are two vessels filled with an ideal gas where volume of one is double the volume of other. The large vessel contains the gas at 8 kPa at 1000 K while the smaller vessel contains the gas at 7 kPa at 500 K. If the vessels are connected to each other by a thin tube allowing the gas to flow and the temperature of both vessels is maintained at 600 K, at steady state the pressure in the vessels will be (in kPa).

(1) 18
(2) 6
(3) 24
(4) 4.4

Answer (2)

Sol.



$$\frac{PV}{T} = \frac{P_1V_1}{T1} + \frac{P_2V_2}{T_2}$$

$$\frac{P.3V}{600} = \frac{8 \times 2V}{1000} + \frac{7 \times V}{500}$$

P = 6 kPa

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. An inductor of self inductance 1 H is connected in series with a resistor of 100 π ohm and an ac supply of 100 π volt, 50 Hz. Maximum current flowing in the circuit is _______ A.

Answer (1)

Sol. $X_L = 2\pi \times 50 \times 1 = 100\ \pi\Omega$

$$Z = 100\pi\sqrt{2}\ \Omega$$

$$i_{\max} = \frac{100\pi\sqrt{2}}{100\pi\sqrt{2}} = 1\text{ A}$$

47. A particle of charge 1.6 μC and mass 16 μg is present in a strong magnetic field of 6.28 T. The particle is then fired perpendicular to magnetic field. The time required for the particle to return to original location for the first time is ___________ s. (π = 3.14)

Answer (0.01)

Sol. $w = \dfrac{2B}{m}$

$$T = \frac{2\pi m}{qB} = \frac{2 \times 3.14 \times 16 \times 10^{-9}}{1.6 \times 10^{-6} \times 6.28}$$

T = 0.01 sec

Answer is not integer

48. If an optical medium possesses a relative permeability of $\dfrac{10}{\pi}$ and relative permittivity of $\dfrac{1}{0.0885}$, then the velocity of light is greater in vacuum than that in this medium by ___________ times.

($\mu_0 = 4\pi \times 10^{-7}$ H/m, $\epsilon_0 = 8.85 \times 10^{-12}$ F/m, $c = 3 \times 10^8$ m/s)

Answer (6)

Sol. $\mu_r = \dfrac{10}{\pi}$ $\qquad \epsilon_r = \dfrac{1}{0.0885}$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$v = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}} = \frac{1}{\sqrt{\mu_r \epsilon_r}}c$$

$$v \simeq \frac{c}{6}$$

$$c \simeq 6v$$

49. A solid sphere with uniform density and radius R is rotating initially with constant angular velocity (ω_1) about its diameter. After some time during the rotation its starts loosing mass at a uniform rate, with no change in its shape. The angular velocity of the sphere when its radius become R/2 is $x\omega_1$. The value of x is ___________.

Answer (32)

Sol. Angular momentum will remain conserve.

$I_1\omega_1 = I_2\omega_2$

$$\frac{2}{5}MR^2\omega_1 = \frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R}{2}\right)^2\omega_2$$

$32\omega_1 = \omega_2$

= 32

50. In a Young's double slit experiment, two slits are located 1.5 mm apart. The distance of screen from slits is 2 m and the wavelength of the source is 400 nm. If the 20 maxima of the double slit pattern are contained within the central maximum of the single slit diffraction pattern, then the width of each slit is $x \times 10^{-3}$ cm, where x-value is ___________.

Answer (15)

Sol. d = 1.5 mm

D = 2 m

λ = 400 nm

$$\frac{20\ \lambda D}{d} = \frac{2\lambda}{a}$$

$$a = \frac{d}{10D} = \frac{1.5}{10}\text{mm}$$

$$= \frac{150 \times 10^{-3}\text{ cm}}{10}$$

$= 15 \times 10^{-3}$ cm

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. The incorrect relationship in the following pairs in relation to ionisation enthalpies is

(1) $Mn^+ < Mn^{2+}$ (2) $Mn^{2+} < Fe^{2+}$

(3) $Mn^+ < Cr^+$ (4) $Fe^{2+} < Fe^{3+}$

Answer (2)

Sol. I.E. of Mn^{2+} : 3260 kJ/mol

I.E. of Fe^{2+} : 2962 kJ/mol

Successive IE always increases

52. Consider the ground state of chromium atom (Z = 24). How many electrons are with Azimuthal quantum number l = 1 and l = 2 respectively?

(1) 16 and 4 (2) 12 and 4

(3) 12 and 5 (4) 16 and 5

Answer (3)

Sol. Cr[24] : $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^5$

l = 1 : p-orbital : $n_e = 12$

l = 2 : d-orbital : $n_e = 5$

53. A toxic compound "A" when reacted with NaCN in aqueous acidic medium yields an edible cooking component and food preservative "B". "B" is converted to "C" by diborane and can be used as an additive to petrol to reduce emission. "C" upon reaction with oleum at 140°C yields an inhalable anesthetic "D". Identify "A", "B", "C" and "D", respectively.

(1) Ethanol; acetonitrile; ethylamine; ethylene

(2) Methanol; formaldehyde; methyl chloride; chloroform

(3) Methanol; acetic acid; ethanol; diethyl ether

(4) Acetaldehyde; 2-hydroxypropanoic acid; propanoic acid; dipropyl ether

Answer (3)

Sol.

$$\underset{(A)}{CH_3OH} \xrightarrow{H^+} CH_3-\overset{\oplus}{O}H_2 \xrightarrow{CN^-} CH_3-CN \xrightarrow[H_2O]{H^+} \underset{(B)}{CH_3COOH}$$

$$\underset{(B)}{CH_3COOH} \xrightarrow{B_2H_6} \underset{(C)}{CH_3CH_2OH} \xrightarrow[140^\circ C]{oleum} CH_3-CH_2-O-CH_2-CH_3$$

54. Given below are two statements :

Statement I : Alcohols are formed when alkyl chlorides are treated with aqueous potassium hydroxide by elimination reaction.

Statement II : In alcoholic potassium hydroxide, alkyl chlorides form alkenes by abstracting the hydrogen from the β-carbon.

In the light of the above statements, choose the **most appropriate answer** from the options given below.

(1) Both **Statement I** and **Statement II** are correct

(2) **Statement I** is incorrect but **Statement II** is correct

(3) Both **Statement I** and **Statement II** are incorrect

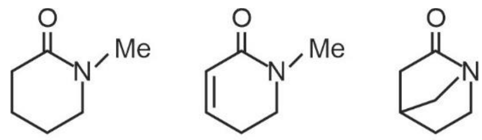
(4) **Statement I** is correct but **Statement II** is incorrect

Answer (2)

Sol. $R-Cl \xrightarrow{aq.\ KOH} R-OH$ (Substitution)

$$R-CH_2-CH_2-Cl \xrightarrow{alc.\ KOH} \underset{(Elimination)}{R-CH=CH_2}$$

55. The correct order of basicity for the following molecules is

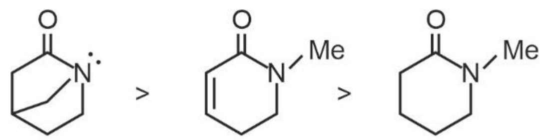


(P) (Q) (R)

(1) Q > P > R
(2) R > P > Q
(3) R > Q > P
(4) P > Q > R

Answer (3)

Sol. Basic strength of



(R) > (Q) > (P)

56. The IUPAC name of the following compound is :

$$HC \equiv C - CH_2 - \underset{}{\overset{OH}{\overset{|}{C}H}} - CH_2 - CH = CH_2$$

(1) 4-Hydroxyhept-6-en-1-yne
(2) Hept-6-en-1-yn-4-ol
(3) 4-Hydroxyhept-1-en-6-yne
(4) Hept-1-en-6-yn-4-ol

Answer (4)

Sol. $$\underset{7}{HC} \equiv \underset{6}{C} - \underset{5}{CH_2} - \underset{4}{\overset{OH}{\overset{|}{C}H}} - \underset{3}{CH_2} - \underset{2}{CH} = \underset{1}{CH_2}$$

Hept-1-en-6-yn-4-ol

57. 'X' is the number of electrons in t_{2g} orbitals of the most stable complex ion among $[Fe(NH_3)_6]^{3+}$, $[FeCl_6]^{3-}$, $[Fe(C_2O_4)_3]^{3-}$ and $[Fe(H_2O)_6]^{3+}$. The nature of oxide of vanadium of the type V_2O_X is:

(1) Amphoteric
(2) Acidic
(3) Neutral
(4) Basic

Answer (1)

Sol. Among the given complexes $[Fe(C_2O_4)_3]^{3-}$ is most stable due to chelation.

$[Fe(C_2O_4)_3]^{3-}$: Fe^{3+} : $t_{2g}^5 e_g^0$

x = 5

V_2O_5 is amphoteric

58. Given below are two statements:

Statement (I): Molal depression constant K_f is given by $\dfrac{M_1RT_f}{\Delta S_{fus}}$, where symbols have their usual meaning.

Statement (II): K_f for benzene is less than the K_f for water.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **Statement I** is correct but **Statement II** is incorrect
(2) Both **Statement I** and **Statement II** are correct
(3) **Statement I** is incorrect but **Statement II** is correct
(4) Both **Statement I** and **Statement II** are incorrect

Answer (1)

Sol. $\because \ \Delta S_{fus} = \dfrac{\Delta H_{fus}}{T}$

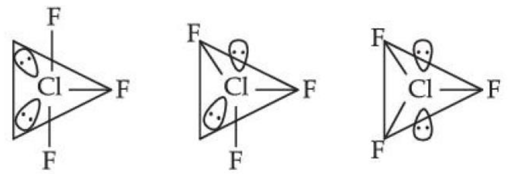
So, $K_f : \dfrac{MRT_f^2}{\Delta H_{fus}} = \dfrac{MRT_f}{\Delta S_{fus}}$

$K_f(H_2O) = 1.86$ K kg mol^{-1}

K_f(benzene) = 5.12 K mol^{-1}

59. Given below are two statements:

Statement (I): For $\ddot{\underset{\cdot\cdot}{Cl}}F_3$, all three possible structures may be drawn as follows.



I II III

Statement (II): Structure III is most stable, as the orbitals having the lone pairs are axial, where the lp-bp repulsion is minimum.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **Statement I** is incorrect but **Statement II** is correct

(2) Both **Statement I** and **Statement II** are incorrect

(3) Both **Statement I** and **Statement II** are correct

(4) **Statement I** is correct but **Statement II** is incorrect

Answer (4)

Sol. Lone pairs placed at equatorial position in the stable structure.

60. Half life of zero order reaction A $\rightarrow$ product is 1 hour, when initial concentration of reactant is 2.0 mol L^{-1}. The time required to decrease concentration of A from 0.50 to 0.25 mol L^{-1} is:

(1) 4 hour (2) 0.5 hour

(3) 60 min (4) 15 min

Answer (4)

Sol. For zero order reaction:

$C_t = C_0 - kt$ and $t_{1/2} = \frac{C_0}{2k}$

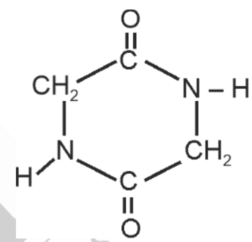
So, $k = \frac{2}{2\times 1} = 1\,\text{mol L}^{-1}\,\text{h}^{-1}$

$C_t = C_0 - kt$

$0.25 = 0.5 - 1t$

$t = 0.25$ h = 15 min

61. A dipeptide, "x" on complete hydrolysis gives "y" and "z". "y" on treatment with aq. HNO_2 produces lactic acid. On the other hand "z" on heating gives the following cyclic molecule.



Based on the information given, the dipeptide x is

(1) alanine-alanine

(2) alanine-glycine

(3) valine-leucine

(4) valine-glycine

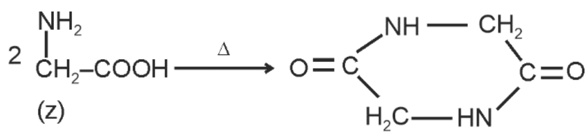
Answer (2)

Sol. x gives $\underset{(z)}{\overset{NH_2}{\overset{|}{CH_2}}-COOH}$ and $\underset{(y)}{CH_3-\overset{NH_2}{\overset{|}{CH}}-COOH}$

Upon hydrolysis

$\underset{(y)}{CH_3-\overset{NH_2}{\overset{|}{CH}}-COOH} \xrightarrow{HNO_2} CH_3-\underset{OH}{\underset{|}{CH}}-COOH$

(Lactic acid)



(Cyclic structure)

y = Alanine

z = Glycine

62. Which among the following compounds give yellow solid when reacted with NaOI/NaOH?

(A) $CH_3-CH(OH)-C_2H_5$

(B) $CH_3-CH_2-CH_2-OH$

(C) $CH_3-CO-C_2H_5$

(D) $CH_3-CO-OH$

(E) CH_3-CH_2-CHO

Choose the **correct** answer from the options given below

(1) (A), (C) and (D) only (2) (A) and (C) only

(3) (B), (C) and (E) only (4) (C) and (D) only

Answer (2)

Sol. Compounds having $CH_3-CH(OH)-$ group and $CH_3-C(=O)-$ group gives yellow ppt of CHI_3 on treatment with NaOI/NaOH.

63. The correct order of $[FeF_6]^{3-}$, $[CoF_6]^{3-}$, $[Ni(CO)_4]$ and $[Ni(CN)_4]^{2-}$ complex species based on the number of unpaired electrons present is

(1) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Ni(CN)_4]^{2-} = [Ni(CO)_4]$

(2) $[Ni(CN)_4]^{2-} > [FeF_6]^{3-} > [CoF_6]^{3-} > [Ni(CO)_4]$

(3) $[CoF_6]^{3-} > [FeF_6]^{3-} > [Ni(CO)_4] > [Ni(CN)_4]^{2-}$

(4) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Ni(CN)_4]^{2-} > [Ni(CO)_4]$

Answer (1)

Sol. $[FeF_6]^{3-}$

$Fe^{3+} \Rightarrow t_{2g}^3 e_g^2$

No. of unpaired electron = 5

$[CoF_6]^{3+}$

$Co^{3+} \Rightarrow t_{2g}^4 e_g^2$

No. of unpaired electron = 4

$Ni(CO)_4 \Rightarrow Ni^0 \Rightarrow 3d^{10}4s^0$ in presence of CO ligand

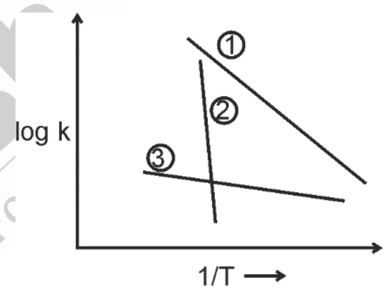
No. of unpaired electron = 0

$[Ni(CN)_6]^{2-}$

$Ni^{4+} \Rightarrow t_{2g}^6 e_g^0$

No. of unpaired electron = 0

64. Consider the following plots of log of rate constant k (log k) vs $\frac{1}{T}$ for three different reactions. The correct order of activation energies of these reactions is



(1) $Ea_3 > Ea_2 > Ea_1$ (2) $Ea_1 > Ea_3 > Ea_2$

(3) $Ea_2 > Ea_1 > Ea_3$ (4) $Ea_1 > Ea_2 > Ea_3$

Answer (3)

Sol. $\because k = Ae^{-E_a/RT}$

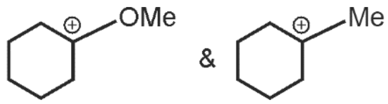
$$\log k = \frac{-E_a}{2.303\,R} \times \frac{1}{T} + \log A$$

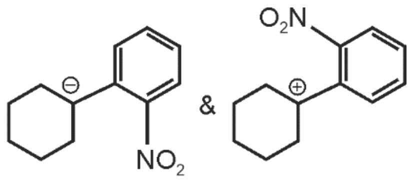
$$\text{slope} = \frac{-E_a}{2.303\,R}$$

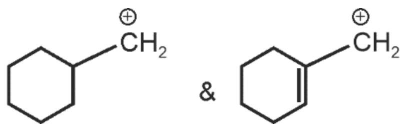
$\because$ slope of 2 < 1 < 3

Hence : $Ea_2 > Ea_1 > Ea_3$

65. In which pairs, the first ion is more stable than the second?

(A) 

(B) 

(C) 

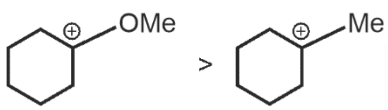
(D) 

(1) (B) and (D) only (2) (B) and (C) only

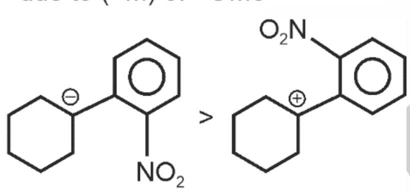
(3) (A) and (C) only (4) (A) and (B) only

Answer (4)

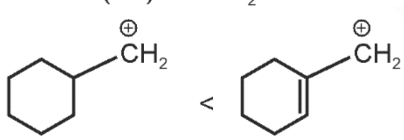
Sol. Stability order:

(A) 

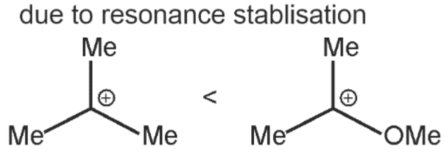
due to (+M) of –OMe

(B) 

due to (–M) of $–NO_2$

(C) 

due to resonance stablisation

(D) 

due to (+M) of –OMe

66. Match **List I** with **List - II.**

	List - I (Separation of)		**List - II (Separation Technique)**
(A)	Aniline from aniline-water mixture	(I)	Simple distillation
(B)	Glycerol from spent-lye in soap industry	(II)	Fractional distillation
(C)	Different fractions of crude oil in petroleum industry	(III)	Distillation at reduced pressure
(D)	Chloroform-Aniline mixture	(IV)	Steam distillation

Choose the correct answer from the options given below:

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(2) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(3) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Answer (2)

Sol.

(A)	Aniline from aniline-water mixture	(IV)	Steam distillation
(B)	Glycerol from spent-lye in soap industry	(III)	Distillation at reduced pressure
(C)	Different fractions of crude oil in petroleum industry	(II)	Fractional distillation
(D)	Chloroform-Aniline mixture	(I)	Simple distillation

67. Given below are two statements:

Statement (I) : The first ionisation enthalpy of group 14 elements is higher than the corresponding elements of group 13.

Statement (II) : Melting points and boiling points of group 13 elements are in general much higher than those of corresponding elements of group 14.

In the light of the above statements, choose the **most appropriate answer** from the options given below:

(1) Both **Statement I** and **Statement II** are incorrect

(2) **Statement I** is incorrect but **Statement II** is correct

(3) **Statement I** is correct but **Statement II** is incorrect

(4) Both **Statement I** and **Statement II** are correct

Answer (3)

Sol. On moving from left to right in periodic table, ionisation energy and melting/boiling point increases.

68. The elements of Group 13 with highest and lowest first ionisation enthalpies are respectively:

(1) B and Tl (2) Tl and B

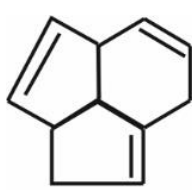
(3) B and Ga (4) B and In

Answer (4)

Sol. I.E. order for group 13 is :

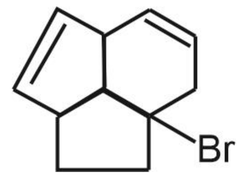
B > Tl > Ga > Al > In

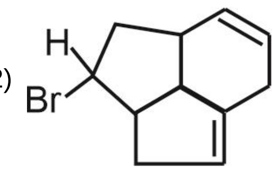
69. Consider the following molecule (X).

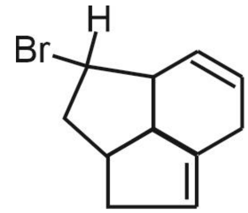
The structure of X is 

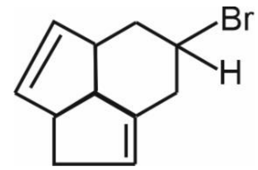
(X)

The major product formed when the given molecule (X) treated with HBr (1 eq) is :

(1) 

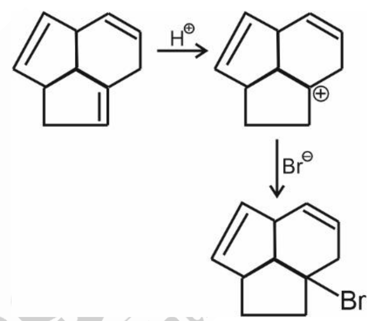
(2) 

(3) 

(4) 

Answer (1)

Sol. Among the given options, the major product decided by stability of carbocation formed in intermediate.



70. Consider the given data:

(a) $HCl(g) + 10\ H_2O(l) \rightarrow HCl.10\ H_2O\ \Delta H = -69.01$ kJ mol^{-1}

(b) $HCl(g) + 40\ H_2O(l) \rightarrow HCl.40\ H_2O\ \Delta H = -72.79$ kJ mol^{-1}

Choose the **correct** statement:

(1) Dissolution of gas in water is an endothermic process.

(2) The heat of solution depends on the amount of solvent.

(3) The heat of formation of HCl solution is represented by both (a) and (b).

(4) The heat of dilution for the HCl ($HCl.10\ H_2O$ to $HCl.40\ H_2O$) is 3.78 kJ mol^{-1}.

Answer (4)

Sol. $HCl(g) + \infty H_2O(l) \rightarrow HCl(aq)$

Heat released the above process is heat of solution.

From reaction (b) – (a) we get heat of dilution for HCl ($HCl.10\ H_2O$ to $HCl.40\ H_2O$) as 3.78 kJ mol^{-1}.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. Sea water, which can be considered as a 6 molar (6 M) solution of NaCl, has a density of 2 g mL^{-1}. The concentration of dissolved oxygen (O_2) in sea water is 5.8 ppm. Then the concentration of dissolved oxygen (O_2) in sea water, is $x \times 10^{-4}$ m.

x = __________. (Nearest integer)

Given: Molar mass of NaCl is 58.5 g mol^{-1}

Molar mass of O_2 is 32 g mol^{-1}

Answer (2)

Sol. Given 5.8 ppm of O_2, means 5.8 mg O_2 in 1L of sea water

or 5.8×10^{-3}g O_2 in 1L sea water

number of moles of $O_2 = \dfrac{5.8 \times 10^{-3}}{32}$ in 1L

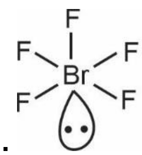
Molarity of $O_2 = \dfrac{5.8 \times 10^{-3}}{32} M = 1.8125 \times 10^{-4}\ M$

Since mass of solute is very less than solvent so molality = molarity

72. A metal complex with a formula $MCl_4.3NH_3$ is involved in sp^3d^2 hybridisation. It upon reaction with excess of $AgNO_3$ solution gives 'x' moles of AgCl. Consider 'x' is equal to the number of lone pairs of electron present in central atom of BrF_5. Then the number of geometrical isomers exhibited by the complex is ________.

Answer (2)

Sol.

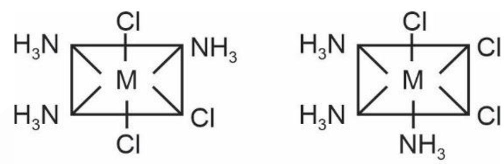


number of lone pair = 1

$AgNO_3$ + Complex $\rightarrow$ 1 mol AgCl

Complex ion should be $[M(NH_3)_3Cl_3]Cl$

Total number of geometrical isomers = 2



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73. x mg of $Mg(OH)_2$ (molar mass = 58) is required to be dissolved in 1.0 L of water to produce a pH of 10.0 at 298 K. The value of x is _________mg. (Nearest integer)

(Given: $Mg(OH)_2$ is assumed to dissociate completely in H_2O]

Answer (3)

Sol. For pH = 10, $[OH^-] = 10^{-4}$

$\because\ Mg(OH)_2 \longrightarrow Mg^{2+} + 2OH^-$

$[Mg(OH)_2] = 0.5 \times 10^{-4}$ M

Mass of $Mg(OH)_2 = 5 \times 10^{-5} \times 1 \times 58 = 2.9$ mg

≈ 3 mg

74. The amount of calcium oxide produced on heating 150 kg limestone (75% pure) is __________ kg. (Nearest integer)

Given: Molar mass (in g mol^{-1}) of Ca-40, O-16, C-12

Answer (63)

Sol. Mass of pure $CaCO_3 = \dfrac{150 \times 75}{100} = 112.5$ kg

Number of moles $= \dfrac{112.5}{100} \times 10^3 = 1125$ moles

$$CaCO_3(s) \longrightarrow CaO(s) + CO_2(g)$$

Moles of CaO formed = 1125 mol

mass of CaO $= \dfrac{1125 \times 56}{1000} = 63$ kg

75. The molar conductance of an infinitely dilute solution of ammonium chloride was found to be 185 S cm^2 mol^{-1} and the ionic conductance of hydroxyl and chloride ions are 170 and 70 S cm^2 mol^{-1}, respectively. If molar conductance of 0.02 M solution of ammonium hydroxide is 85.5 S cm^2 mol^{-1}, its degree of dissociation is given by x × 10^{-1}.

The value of x is _________. (Nearest integer)

Answer (3)

Sol. $\wedge_m^0(NH_4Cl) = 185 \text{ S cm}^2 \text{ mol}^{-1}$,

$\lambda_{eq.}(OH^-) = 170 \text{ S cm}^2 \text{ mol}^{-1}$,

$\lambda_{eq.}(Cl^-) = 70 \text{ S cm}^2 \text{ mol}^{-1}$,

$\wedge^0(NH_4OH) = \lambda^0(NH_4^+) + \lambda^0(OH^-)$

$= \wedge^0(NH_4Cl) - \lambda^0(Cl^-) + \lambda^0(OH^-)$

= (185 – 70) + 170

= 285 S cm^2 mol^{-1}

$\propto = \dfrac{85.5}{285} = 0.3 = 3 \times 10^{-1}$

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