MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If $\lim\limits_{x\to0}\dfrac{\cos(2x)+a\cos(4x)-b}{x^4}$ is finite, then $(a+b)$ is equal to :

(1) $\dfrac{1}{2}$ (2) $\dfrac{3}{4}$

(3) -1 (4) 0

Answer (1)

Sol. $\lim\limits_{x\to0}\dfrac{\cos(2x)+a\cos(4x)-b}{x^4}$

$$\lim_{x\to0}\frac{\left(1-\frac{4x^2}{2!}+\frac{(2x)^4}{4!}-....\right)+a\left(1-\frac{(2x)^2}{2!}+\frac{(4x)^4}{4!}-...\right)-b}{x^4}$$

For limit to be finite

$1 + a - b = 0$

And $-2 - 8a = 0$

$\Rightarrow \quad 8a = -2$

$\Rightarrow \quad a = \dfrac{-1}{4}$

$\Rightarrow \quad b = 1 + a$

$= 1 - \dfrac{1}{4}$

$b = -\dfrac{3}{4}$

$a + b = \dfrac{-1}{4} + \dfrac{3}{4} = \dfrac{1}{2}$

Option (1) is correct

2. If $\sum\limits_{r=0}^{10}\left(\dfrac{10^{r+1}-1}{10^r}\right)\cdot{}^{11}C_{r+1}=\dfrac{\alpha^{11}-11^{11}}{10^{10}}$, then α is equal to:

(1) 20

(2) 24

(3) 15

(4) 11

Answer (1)

Sol. $\sum\limits_{r=0}^{10}\left(\dfrac{10^{r+1}-1}{10^r}\right)\cdot{}^{11}C_{r+1}$

$$=(10-1)^{11}C_1+\left(\frac{10^2-1}{10}\right){}^{11}C_2$$

$$+...+\left(\frac{10^{11}-1}{10^{10}}\right){}^{11}C_{11}$$

$$=10\left({}^{11}C_1+{}^{11}C_2+...+{}^{11}C_{11}\right)$$

$$-\left({}^{11}C_1+\frac{{}^{11}C_2}{10}+...+\frac{{}^{11}C_{11}}{10^{10}}\right)$$

$$=10\left(2^{11}-1\right)-10\left(\frac{{}^{11}C_1}{10}+\frac{{}^{11}C_2}{10^2}+...+\frac{{}^{11}C_{11}}{10^{11}}\right)$$

$$=10\left(2^{11}-1\right)-10\left(\left(1+\frac{1}{10}\right)^{11}-1\right)$$

$$=10\left(2^{11}-1\right)-10\left(\frac{11^{11}-10^{11}}{10^{11}}\right)$$

$$= 10\left(2^{11}-1\right) - \left(\frac{11^{11}-10^{11}}{10^{10}}\right)$$

$$= \frac{10^{11}\left(2^{11}-1\right) - 11^{11} + 10^{11}}{10^{10}}$$

$$= \frac{2^{11}\left(10^{11}\right) - 11^{11}}{10^{10}} = \frac{20^{11}-11^{11}}{10^{10}}$$

$\Rightarrow \alpha = 20$

3. Let (a, b) be the point of intersection of the curve $x^2 = 2y$ and the straight line $y - 2x - 6 = 0$ in the second quadrant. Then the integral $I = \int_a^b \frac{9x^2}{1+5^x}dx$ is equal to:

(1) 21 (2) 27

(3) 24 (4) 18

Answer (3)

Sol. $x^2 = 2y$ and $y - 2x - 6 = 0$

$$\frac{x^2}{2} - 2x - 6 = 0$$

$$x^2 - 4x - 12 = 0$$

$$x^2 - 6x + 2x - 12 = 0$$

$$x(x-6) + 2(x-6) = 0$$

$$(x-6)(x+2) = 0$$

Point of intersection are (6, 18) and (–2, 2)

(–2, 2) is in second quadrant

$a = -2, b = 2$

$$I = \int_{-2}^{2} \frac{9x^2}{1+5^x}dx \qquad \ldots(i)$$

$$I = \int_{-2}^{2} \frac{9x^2}{1+5^{-x}}dx \qquad \ldots(ii)$$

Adding (i) and (ii)

$$2I = \int_{-2}^{2} 9x^2 dx$$

$$I = 9\int_0^2 x^2 dx$$

$$I = 9\left(\frac{x^3}{3}\right)_0^2 \Rightarrow I = 24$$

4. If $\theta \in \left[-\frac{7\pi}{6}, \frac{4\pi}{3}\right]$, then the number of solutions of $\sqrt{3}\text{cosec}^2\theta - 2\left(\sqrt{3}-1\right)\text{cosec}\theta - 4 = 0$, is equal to

(1) 8 (2) 10

(3) 7 (4) 6

Answer (4)

Sol.

$$\text{cosec}\theta = \frac{2\sqrt{3} - 2 \pm \sqrt{\left(2\left(\sqrt{3}-1\right)\right)^2 + 4\sqrt{3}(4)}}{2\sqrt{3}}$$

$$= \frac{2\sqrt{3} - 2 \pm \sqrt{4\left(3+1-2\sqrt{3}\right) + 16\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{2\sqrt{3} - 2 \pm 2\left(\sqrt{3}+1\right)}{2\sqrt{3}}$$

$$\Rightarrow \text{cosec}\theta = \frac{-2}{\sqrt{3}}, 2$$

$$\theta = \frac{-7\pi}{6}, \frac{-2\pi}{3}, \frac{-\pi}{3}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{4\pi}{3}$$

5. If the mean and the variance of 6, 4, a, 8, b, 12, 10, 13 are 9 and 9.25 respectively, then $a + b + ab$ is equal to :

(1) 103 (2) 100

(3) 106 (4) 105

Answer (1)

Sol. Mean = $\frac{6+4+a+8+b+12+10+13}{8} = 9$

$53 + a + b + = 72$

$\Rightarrow$ $a + b = 19$ and

And

$$\frac{36+16+a^2+64+b^2+144+100+169}{8} - 9^2 = 9.25$$

$$\Rightarrow \frac{529+a^2+b^2}{8} = 90.25$$

$\Rightarrow$ $a^2 + b^2 = 193$

$\therefore$ $a = 7, b = 12$

Hence, $a + b + ab = 19 + 84 = 103$

6. If the domain of the function $f(x) = \frac{1}{\sqrt{10+3x-x^2}} + \frac{1}{\sqrt{x+|x|}}$ is (a, b), then $(1 + a)^2 + b^2$ is equal to

(1) 26 (2) 30

(3) 29 (4) 25

Answer (1)

Sol. $x + |x| = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$

$\Rightarrow$ $\frac{1}{\sqrt{x+|x|}}$, domain is $x > 0$, as $2x \neq 0$

Similarly,

$\frac{1}{\sqrt{3x+10-x^2}}$ is defined when $3x + 10 - x^2 > 0$

$\Rightarrow$ $x^2 - 3x - 10 < 0$

$(x - 5)(x + 2) < 0$

$\Rightarrow$ $x \in (-2, 5)$

$\Rightarrow$ Domain will be $(0, \infty) \cap (-2, 5) = (0, 5)$

$\Rightarrow$ $(1 + a)^2 + b^2 = 1 + 25 = 26$

7. Given three indentical bags each containing 10 balls, whose colours are as follows :

	Red	Blue	Green
Bag I	3	2	5
Bag II	4	3	3
Bag III	5	1	4

A person chooses a bag at random and takes out a ball. If the ball is Red, the probability that it is from bag I is p and if the ball is Green, the probability that it is from bag III is q, then the value of $\left(\frac{1}{p} + \frac{1}{q}\right)$ is :

(1) 8 (2) 9

(3) 7 (4) 6

Answer (3)

Sol. $p(B_1/R) = \frac{p(B_1) \cdot p(R/B_1)}{p(R)}$

$$= \frac{\frac{1}{3} \times \frac{3}{10}}{\frac{1}{3} \times \frac{3}{10} + \frac{1}{3} \times \frac{4}{10} + \frac{1}{3} \times \frac{5}{10}}$$

$$= \frac{1}{4} = p$$

$$p(B_3 / G) = \frac{p(B_3) \cdot p(G / B_3)}{p(G)}$$

$$= \frac{\frac{1}{3} \times \frac{4}{10}}{\frac{1}{3} \times \frac{5}{10} + \frac{1}{3} \times \frac{3}{10} + \frac{1}{3} \times \frac{4}{10}}$$

$$= \frac{1}{3} = q$$

$$\therefore \quad \left(\frac{1}{p} + \frac{1}{q}\right) = 7$$

8. The line L_1 is parallel to the vector $\vec{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through the point (7, 6, 2) and the line L_2 is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point (5, 3, 4). The shortest distance between the lines L_1 and L_2 is :

(1) $\frac{21}{\sqrt{38}}$ (2) $\frac{23}{\sqrt{38}}$

(3) $\frac{21}{\sqrt{57}}$ (4) $\frac{23}{\sqrt{57}}$

Answer (2)

Sol. Eqn. of L_1 : $7\hat{i} + 6\hat{j} + 2\hat{k} + \lambda(-3\hat{i} + 2\hat{j} + 4\hat{k})$

Eqn. of L_2 : $5\hat{i} + 3\hat{j} + 4\hat{k} + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$

Shortest distance between L_1 and L_2

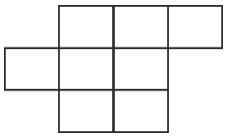
$$= \left|\frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$= \left|\frac{(-2\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} + 17\hat{j} - 7\hat{k})}{\sqrt{2^2 + (-17)^2 + (-7)^2}}\right|$$

$$= \left|\frac{-4 - 51 - 14}{\sqrt{4 + 289 + 49}}\right|$$

$$= \left|\frac{-69}{\sqrt{342}}\right| = \frac{23}{\sqrt{38}}$$

9. The number of ways, in which the letters A, B, C, D, E can be placed in the 8 boxes of the figure below so that no row remains empty and at most one letter can be placed in a box, is:

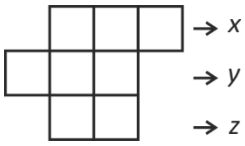


(1) 960 (2) 5760

(3) 840 (4) 5880

Answer (2)

Sol.



Let x, y, z be the number of box which are filled

$\Rightarrow \quad 1 \le x \le 3, 1 \le y \le 3, 1 \le z \le 2$

x	y	z	Number of ways
3	1	1	${}^3C_3 \cdot {}^3C_1 \cdot {}^2C_1 = 6$
2	2	1	${}^3C_2 \cdot {}^3C_2 \cdot {}^2C_1 = 18$
1	3	1	${}^3C_1 \cdot {}^3C_3 \cdot {}^2C_1 = 6$
2	1	2	${}^3C_2 \cdot {}^3C_1 \cdot {}^2C_2 = 9$
1	2	2	${}^3C_1 \cdot {}^3C_2 \cdot {}^2C_2 = 9$

Total ways = (48) to fill boxes

Now to arrange *a, b, c, d* and *e*

Number of ways will be 48.5! = 5760

10. Let $f : [1, \infty) \to [2, \infty)$ be a differentiable function. If $10\int_1^x f(t)dt = 5xf(x) - x^5 - 9$ for all $x \ge 1$, then the value of $f(3)$ is :

(1) 26 (2) 32

(3) 18 (4) 22

Answer (2)

Sol. $10\int_1^x f(t)dt = 5xf(x) - x^5 - 9$

differentiate w.r.t. x both sides

$10f(x) = 5f(x) + 5xf'(x) - 5x^4$

$\Rightarrow 5f(x) = 5xf'(x) - 5x^4$

Let $y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)$

$5y = 5x\frac{dy}{dx} - 5x^4 \Rightarrow \frac{dy}{dx} - \frac{y}{x} = x^3$

I.F. of this linear differential equation

$\Rightarrow$ I.F. $= e^{\int \frac{-1}{x}dx} = e^{-\ln x} = e^{\ln\left(\frac{1}{x}\right)} = \frac{1}{x}$

$\Rightarrow y\left(\frac{1}{x}\right) = \int x^3 \cdot \frac{1}{x}dx = \frac{x^3}{3} + C$

$\Rightarrow y = \frac{x^4}{3} + Cx$, at $x = 1$

$10\int_1^1 f(t)dt = 5f(1) - 1 - 9 \Rightarrow f(1) = 2$

$\Rightarrow f(1) = \frac{1}{3} + C = 2 \Rightarrow C = \frac{5}{3}$

$\Rightarrow f(3) = \frac{3^4}{3} + \frac{5}{3}\cdot 3 = 3^3 + 5 = 32$

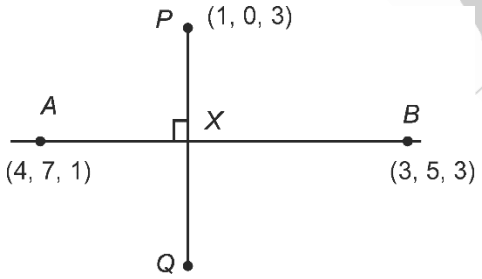
11. If the image of the point $P(1, 0, 3)$ in the line joining the points $A(4, 7, 1)$ and $B(3, 5, 3)$ is $Q(\alpha, \beta, \gamma)$, then $\alpha + \beta + \gamma$ is equal to :

(1) $\frac{46}{3}$ (2) $\frac{47}{3}$

(3) 18 (4) 13

Answer (1)

Sol.



Let X be mid point of P and Q, which would be also feet of perpendicular.

Let X divides A and B in $\lambda : 1$, $\lambda \neq -1$

$X = \left(\frac{3\lambda+4}{\lambda+1}, \frac{5\lambda+7}{\lambda+1}, \frac{3\lambda+1}{\lambda+1}\right)$

Now $PX \perp AB \Rightarrow \overrightarrow{PX}\cdot\overrightarrow{AB} = 0$

$\left(\frac{3\lambda+4}{\lambda+1} - 1\right)\cdot(4-3) + \left(\frac{5\lambda+7}{\lambda+1} - 0\right)(7-5)$

$+ \left(\frac{3\lambda+1}{\lambda+1} - 3\right)\cdot(1-3) = 0$

$\frac{2\lambda+3}{\lambda+1} + \frac{10\lambda+14}{\lambda+1} + \frac{4}{\lambda+1} = 0$

$\Rightarrow \frac{12\lambda+21}{\lambda+1} = 0 \Rightarrow \lambda = \frac{-7}{4}$

$X = \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

X is mid point of PQ

$Q \equiv \left(2\cdot\frac{5}{3} - 1, 2\cdot\frac{7}{3} - 0, 2\cdot\frac{17}{3} - 3\right) \equiv (\alpha, \beta, \gamma)$

$\Rightarrow \alpha + \beta + \gamma = \frac{2(5+7+17)}{3} - 4 = \frac{58}{3} - 4 = \frac{46}{3}$

12. Let $A = \{1, 2, 3, ..., 100\}$ and R be a relation on A such that $R = \{(a, b): a = 2b + 1\}$. Let (a_1, a_2), (a_2, a_3), (a_3, a_4), …, (a_k, a_{k+1}) be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k, for which such a sequence exists, is equal to:

(1) 6 (2) 8

(3) 5 (4) 7

Answer (3)

Sol. The sequence with k terms be

$(a_1, a_2), (a_2, a_3), \ldots, (a_k, a_{k+1})$

Where $a_i = 2a_{i+1} + 1$, using A on A relation

$a_1 = 2a_2 + 1$, a_1 will be odd.

$a_2 = 2a_3 + 1 \Rightarrow a_1 = 2(2a_3 + 1) + 1 = 4a_3 + 3$

$a_3 = 2a_4 + 1 \quad a_1 = 4(2a_4 + 1) + 3 = 8a_4 + 7$

$\vdots \qquad\qquad = 2^3\cdot a_4 + 7$

$a_k = 2a_{k+1} + 1 \Rightarrow a_1 = 2^k\cdot a_{k+1} + (2^k - 1) \in A$

$\frac{a_1+1-2^k}{2^k} = a_{k+1} \Rightarrow 2^k \mid (a_1+1)$, we need to find highest k. $a_1 + 1 \in \{2, \ldots, 101\}$

k maximum when $k = 6$, as at $k = 7$, $2^k = 128$

$128 \mid e_i \; \forall \; e_i \in A \Rightarrow a_1 = 95$ and $k = 6$

(95, 47), (47, 23), (23, 11), (11, 5), (5, 2) will be sequence.

13. Let the point P of the focal chord PQ of the parabola $y^2 = 16x$ be (1, –4). If the focus of the parabola divides the chord PQ in the ratio $m : n$, $gcd(m, n) = 1$, then $m^2 + n^2$ is equal to:

(1) 17 (2) 26

(3) 10 (4) 37

Answer (1)

Sol. $P(at^2, 2at) \equiv P(4t^2, 8t) \equiv (1, -4)$

$8t = -4 \Rightarrow t = -\frac{1}{2}$

$\therefore \; Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right) \quad (t_1 \cdot t_2 = -1)$

$S(4, 0)$ is the focus

$PS = a + at^2$

$QS = a + \frac{a}{t^2} = \frac{at^2 + a}{t^2}$

$\frac{PS}{QS} = t^2 = \frac{1}{4} = \frac{m}{n}$

$\therefore \; m^2 + n^2 = 17$

14. If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is:

(1) $\frac{\sqrt{5}}{7}$ (2) $\frac{4}{\sqrt{17}}$

(3) $\frac{3}{\sqrt{19}}$ (4) $\frac{\sqrt{3}}{16}$

Answer (2)

Sol. $2b = \frac{1}{4}(2ae)$

$\Rightarrow b = \frac{ae}{4}$

$\Rightarrow \frac{b^2}{a^2} = \frac{e^2}{16}$

$\Rightarrow \frac{17e^2}{16} = 1 \Rightarrow e = \frac{4}{\sqrt{17}}$

15. Let $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ and a vector $\vec{c}$ be such that $(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$ and $\vec{a} \cdot \vec{c} = 3$. If $\vec{b} \times \vec{c} = \vec{d}$, then $|\vec{a} \cdot \vec{d}|$ is equal to :

(1) 9 (2) 12

(3) 18 (4) 15

Answer (4)

Sol. $(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$

$\vec{a} \times \vec{b} - \vec{c} \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$

$\Rightarrow \vec{a} \times \vec{b} + \vec{d} = -18\hat{i} - 3\hat{j} + 12\hat{k}$

Take dot with $\vec{a}$

$\vec{a} \cdot \vec{d} = -36 + 9 + 12 = -15$

$\therefore \; |\vec{a} \cdot \vec{d}| = 15$

16. $4\int_0^1 \left(\frac{1}{\sqrt{3+x^2} + \sqrt{1+x^2}}\right) dx - 3\log_e\left(\sqrt{3}\right)$ is equal to:

(1) $2 + \sqrt{2} - \log_e\left(1 + \sqrt{2}\right)$

(2) $2 - \sqrt{2} - \log_e\left(1 + \sqrt{2}\right)$

(3) $2 + \sqrt{2} + \log_e\left(1 + \sqrt{2}\right)$

(4) $2 - \sqrt{2} + \log_e\left(1 + \sqrt{2}\right)$

Answer (3)

Sol. $I = 4\int_0^1 \frac{1}{\sqrt{3+x^2} + \sqrt{1+x^2}} dx$

$= 2\int_0^1 \sqrt{3+x^2} - \sqrt{1+x^2} \; dx$

$$= 2\left[\int_0^1 \sqrt{3+x^2}\,dx - \int_0^1 \sqrt{1+x^2}\,dx\right]$$

$$= 2\left[\left(\frac{1}{2}x\sqrt{x^2+3} + \frac{3}{2}\ln\left|\sqrt{3+x^2}+x\right|\right) - \left(\frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\ln\left|\sqrt{1+x^2}+x\right|\right)\right]_0^1$$

$$= 2\left[\left(1+\frac{3}{2}\ln 3 - \frac{3}{2}\ln\sqrt{3}\right) - \left(\frac{\sqrt{2}}{2} + \frac{1}{2}\ln\left(\sqrt{2}+1\right)\right)\right]$$

$$= 2\left(1+\frac{3}{4}\ln 3 - \frac{1}{\sqrt{2}} - \frac{1}{2}\ln\left(\sqrt{2}+1\right)\right)$$

$$= 3\ln\sqrt{3} + 2 - \sqrt{2} - \ln\left(\sqrt{2}+1\right)$$

$$I - 3\ln\sqrt{3} = 2 - \sqrt{2} - \ln\left(\sqrt{2}+1\right)$$

17. The number of terms of an A.P. is even, the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is:

(1) 10

(2) 6

(3) 4

(4) 8

Answer (4)

Sol. Let the number of terms be $2n$

$$\begin{array}{r} T_1 + T_3 + T_5 \ldots T_{2n-1} = 24 \\ T_2 + T_4 + T_6 \ldots T_{2n} = 30 \\ \hline T_2 - T_1) + (T_4 - T_3) + \ldots (T_{2n} - T_{2n-1}) = 6 \end{array}$$

$nd = 6$

$$(a + (2n+1)d) - a = \frac{21}{2}$$

$$\Rightarrow 2nd - d = \frac{21}{2}$$

$$\Rightarrow 12 - \frac{21}{2} = d$$

$$\Rightarrow d = \frac{3}{2}$$

$\therefore n = 4$

$\therefore$ Total terms = 8

18. If the system of equations

$2x + \lambda y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \mu z = 9$

has infinitely many solutions, then $(\lambda^2 + \mu^2)$ is equal to

(1) 30

(2) 18

(3) 26

(4) 22

Answer (3)

Sol. $2x + \lambda y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \mu z = 9$

For infinite solutions $\Rightarrow \Delta = 0 = \Delta_1 = \Delta_2 = \Delta_3$

$$\Delta = \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$$

$\Rightarrow -4\lambda - 3\lambda\mu + 4\mu + 31 = 0$

$$\Delta_1 = \begin{vmatrix} 5 & \lambda & 3 \\ 7 & 2 & -1 \\ 9 & 5 & \mu \end{vmatrix} = 0 \Rightarrow -9\lambda - 7\lambda\mu + 10\mu + 76 = 0$$

$$\Delta_2 = \begin{vmatrix} 2 & 3 & 5 \\ 3 & -1 & 7 \\ 4 & \mu & 9 \end{vmatrix} = 0 \Rightarrow \mu + 5 = 0 \Rightarrow \mu = -5$$

$$\Delta_3 = \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0 \Rightarrow \lambda + 1 = 0 \Rightarrow \lambda = -1$$

$\therefore$ For infinite solution $\mu = -5$ and $\lambda = -1$

Now $\mu^2 + \lambda^2 = 25 + 1$

$= 26$

19. Let A be a 3 × 3 real matrix such that $A^2(A - 2I) - 4(A - I) = 0$, where I and O are the identity and null matrices, respectively. If $A^5 = \alpha A^2 + \beta A + \gamma I$, where α, β and γ are real constants, then $\alpha + \beta + \gamma$ is equal to:

(1) 4 (2) 20

(3) 12 (4) 76

Answer (3)

Sol. $A^2(A - 2I) - 4(A - I) = 0$

$A^3 - 2A^2 - 4A + 4I = 0$

Multiply by A

$A^4 = 2A^3 + 4A^2 - 4A$

$A^4 = 2(2A^2 + 4A - 4I) + 4A^2 - 4A$

$A^4 = 8A^2 + 4A - 8I$

Multiply again by A

$\Rightarrow A^5 = 8A^3 + 4A^2 - 8A$

$\Rightarrow A^5 = 8(2A^2 + 4A - 4I) + 4A^2 - 8A$

$\Rightarrow A^5 = 20A^2 + 24A - 32I$

Comparing with $A^5 = \alpha A^2 + \beta A + \gamma I$

$\alpha = 20, \beta = 24, \gamma = -32$

$\therefore \alpha + \beta + \gamma = 20 + 24 - 32$

$= 44 - 32$

$= 12$

20. Let the area of the triangle formed by a straight line $L : x + by + c = 0$ with co-ordinate axes be 48 square units. If the perpendicular drawn from the origin to the line L makes an angle of 45° with the positive x-axis, then the value of $b^2 + c^2$ is:

(1) 97

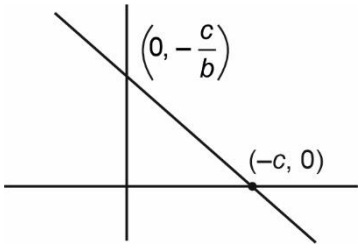
(2) 90

(3) 93

(4) 83

Answer (1)

Sol.



$L : x + by + c = 0$

$\because \frac{1}{2}\left|(-c)\cdot\left(\frac{-c}{b}\right)\right| = 48$

$\therefore \left|\frac{c^2}{b}\right| = 96$ …(i)

Slope of line $L = -\frac{1}{b}$

$\therefore$ Slope of line perpendicular to L is b.

$\therefore b = 1$

$\therefore c^2 = 96$

$\therefore b^2 + c^2 = 97$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If the sum of the first 10 terms of the series $\frac{4.1}{1+4.1^4} + \frac{4.2}{1+4.2^4} + \frac{4.3}{1+4.3^4} + \ldots\ldots$ is $\frac{m}{n}$, where gcd(m, n), then $m + n$ is equal to_____.

Answer (441)

Sol. $\frac{4.1}{1+4.1^4} + \frac{4.2}{1+4.2^4} + \frac{4.3}{1+4.3^4} + \ldots\ldots$

$T_r = \frac{4r}{1+4r^4} = \frac{4r}{4r^4 + 4r^2 + 1 - 4r^2}$

$= \frac{4r}{(2r^2+1)^2 - (2r)^2}$

$T_r = \frac{4r}{(2r^2 - 2r + 1)(2r^2 + 2r + 1)}$

$$T_r = \frac{(2r^2+2r+1)-(2r^2-2r+1)}{(2r^2-2r+1)(2r^2+2r+1)}$$

$$T_r = \left(\frac{1}{r^2+(r-1)^2} - \frac{1}{r^2+(r+1)^2}\right)$$

$$\sum_{r=1}^{10} T_r = \left(\frac{1}{0^2+1^2} - \frac{1}{1^2+2^2} + \frac{1}{1^2+2^2} - \frac{1}{2^2+3^2} + \ldots. \right.$$

$$\left. \frac{1}{9^2+10^2} - \frac{1}{10^2+11^2}\right)$$

$$= 1 - \frac{1}{221}$$

$$= \frac{220}{221}$$

$\therefore$ $m + n = 220 + 221$

$= 441$

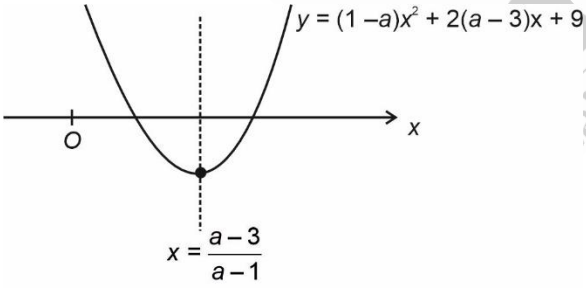
22. If the set of all $a \in R - \{1\}$, for which the roots of the equation $(1-a)x^2 + 2(a-3)x + 9 = 0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma)$, then $2\alpha + \beta + \gamma$ is equal to_____.

Answer (7)

Sol. $f(x) = (1-a)x^2 + 2(a-3)x + 9$, $f(0) = 9 > 0$

$D \geq 0 \Rightarrow 4(a-3)^2 \geq 4(1-a)\cdot 9$

$$\Rightarrow \boxed{a \in (-\infty, -3] \cup [0, \infty)} \quad \ldots(i)$$



$$x_1 + x_2 = \frac{-2(a-3)}{1-a}, x_1 x_2 = \frac{9}{1-a}$$

$$x_1 + x_2 > 0 \Rightarrow \frac{a-3}{a-1} > 0 \Rightarrow \boxed{a \in (-\infty, 1) \cup (3, \infty)} \ldots(ii)$$

$x_1 x_2 > 0 \Rightarrow 1 - a > 0 \Rightarrow \boxed{a \in (-\infty, 1)} \ldots(iii)$

$\Rightarrow$ Interaction of (i), (ii) and (iii)

$a \in (-\infty, -3] \cup [0, 1)$

$\Rightarrow$ $\alpha = 3$, $\beta = 0$, $\gamma = 1 \Rightarrow 2\alpha + \beta + \gamma = 7$

23. If $y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$, then $(x-y)^2 + 3y^2$ is equal to

Answer (3)

Sol. $y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$

$$= \cos\left(\frac{\pi}{3}\right)\cos\left(\cos^{-1}\left(\frac{x}{2}\right)\right) - \sin\left(\frac{\pi}{3.}\right)\sin\left(\cos^{-1}\left(\frac{x}{2}\right)\right)$$

$$= \frac{1}{2}\cdot\frac{x}{2} - \frac{\sqrt{3}}{2}\cdot\sqrt{1-\frac{x^2}{4}}$$

$\Rightarrow$ $4y = x - \sqrt{3}\sqrt{4-x^2}$

$\Rightarrow$ $(4y - x)^2 = 3(4 - x^2)$

$\Rightarrow$ $16y^2 + x^2 - 8xy = 12 - 3x^2$

$x^2 + 4y^2 - 2xy = 3$

$(x-y)^2 + 3y^2 = 3$

24. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 2y\sec^2 x = 2\sec^2 x + 3\tan x \cdot \sec^2 x$ such that $y(0) = \frac{5}{4}$. Then $12\left(y\left(\frac{\pi}{4}\right) - e^{-2}\right)$ is equal to_____.

Answer (21)

Sol. $\frac{dy}{dx} + 2y\sec^2 x = 2\sec^2 x + 3\tan x\sec^2 x$

I.F. = $e^{\int 2\sec^2 x dx}$

I.F. = $e^{2\tan x}$

$$y \cdot e^{2\tan x} = \int e^{2\tan x}\left(2+3\tan x\right)\sec^2 x\,dx$$

Put $\tan x = u$

$\sec^2 x\,dx = du$

$$y \cdot e^{2u} = \int e^{2u}\left(2+3u\right)du$$

$$y \cdot e^{2u} \Rightarrow \frac{2e^{2u}}{2} + 3\int e^{2u} \cdot u\,du$$

$$y \cdot e^{2u} = e^{2u} + 3\left[\frac{ue^{2u}}{2} - \int\frac{e^{2u}}{2}\right]$$

$$ye^{2u} = e^{2u} + 3\left[\frac{ue^{2u}}{2} - \frac{e^{2u}}{4}\right] + C$$

$$ye^{2\tan x} = e^{2\tan x} + 3\left[\frac{\tan x e^{2\tan x}}{2} - \frac{e^{2\tan x}}{4}\right] + C$$

$F(0) = \frac{5}{4}$

$$\frac{5}{4} = 1 - \frac{3}{4} + C$$

$$\frac{5}{4} - \frac{1}{4} = C$$

$1 = C$

$$y = 1 + 3\left(\frac{\tan x}{2} - \frac{1}{4}\right) + 1\cdot e^{-2\tan x}$$

$$y\left(\frac{\pi}{4}\right) = 1 + 3\left(\frac{1}{2} - \frac{1}{4}\right) + \frac{1}{e^2}$$

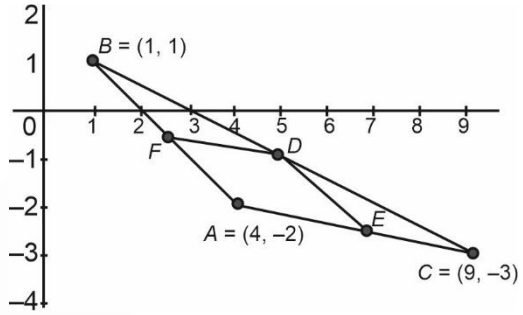
$$y\left(\frac{\pi}{4}\right) = \frac{7}{4} + \frac{1}{e^2}$$

$$12\left(y\left(\frac{x}{4}\right) - \frac{1}{e^2}\right) = 12\left(\frac{7}{4} + \frac{1}{e^2} - \frac{1}{e^2}\right) = 21$$

25. Let $A(4, -2)$, $B(1, 1)$ and $C(9, -3)$ be the vertices of a triangle ABC. Then the maximum area of the parallelogram $AFDE$, formed with vertices D, E and F on the sides BC, CA and AB of the triangle ABC respectively, is_____.

Answer (3)

Sol. The maximum area of such a parallelogram $AFDE$, with one vertex fixed at A and the other three points lying on the sides of triangle ABC, is half the area of triangle ABC.



Using the determinant formula for area of triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$:

Area ΔABC

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

Substitute the coordinates:

$$= \frac{1}{2}\left|4(1 - (-3)) + 1((-3) - (-2)) + 9((-2) - 1)\right|$$

$$= \frac{1}{2}\left|4(4) + 1(-1) + 9(-3)\right|$$

$$= \frac{1}{2}\left|16 - 1 - 27\right| = \frac{1}{2}\left|-12\right| = \frac{12}{2} = 6$$

Maximum area of parallelogram $AFDE$

$$= \frac{1}{2} \times \text{area of triangle} = \frac{1}{2} \times 6 = 3$$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answers :

26. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Net dipole moment of a polar linear isotropic dielectric substance is not zero even in the absence of an external electric field.

Reason (R) : In absence of an external electric field, the different permanent dipoles of a polar dielectric substance are oriented in random directions.

In the light of the above statements, choose the **most appropriate answer** from the options given below.

(1) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

(2) **(A)** is correct but **(R)** is not correct

(3) **(A)** is not correct but **(R)** is correct

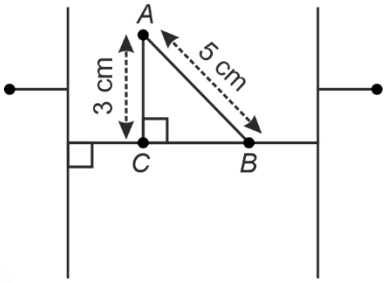
(4) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

Answer (3)

Sol. Net dipole moment of polar dielectric substance is zero in absence of electric field as dipoles are randomly oriented.

(A) is not correct but **(R)** is correct.

27. Two large plane parallel conducting plates are kept 10 cm apart as shown in figure. The potential difference between them is V. The potential difference between the points A and B (shown in the figure) is



(1) $\frac{3}{4}V$ (2) $1V$

(3) $\frac{2}{5}V$ (4) $\frac{1}{4}V$

Answer (3)

Sol. Field lines are perpendicular to the plates

$$V_{AB} = V_{CB} = \left(\frac{4}{10}\right)V = \frac{2V}{5}$$

28. Energy released when two deuterons ($_1H^2$) fuse to form a helium nucleus ($_2He^4$) is (Given: Binding energy per nucleon of $_1H^2$ = 1.1 MeV and binding energy per nucleon of $_2He^4$ = 7.0 MeV)

(1) 23.6 MeV

(2) 5.9 MeV

(3) 8.1 MeV

(4) 26.8 MeV

Answer (1)

Sol. $2\ {}_1H^2 \rightarrow {}_2He^4$

$Q = (4 \times 7 - 4 \times 1.1)$ MeV

$= 23.6$ MeV

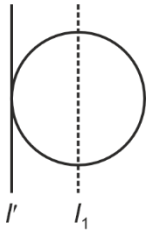
29. The moment of inertia of a circular ring of mass M and diameter r about a tangential axis lying in the plane of the ring is

(1) $\frac{3}{8}Mr^2$ (2) $2Mr^2$

(3) $\frac{3}{2}Mr^2$ (4) $\frac{1}{2}Mr^2$

Answer (1)

Sol.



$I_1 = \frac{MR^2}{2}$

$I' = I_1 + MR^2$

$= \frac{3}{2}MR^2$

$r = 2R$

$\therefore \quad I' = \frac{3}{8}Mr^2$

30. Consider a circular loop that is uniformly charged and has a radius $a\sqrt{2}$. Find the position along the positive z-axis of the cartesian coordinate system where the electric field is maximum if the ring was assumed to be placed in xy plane at the origin

(1) a (2) $a/2$

(3) $\frac{a}{\sqrt{2}}$ (4) 0

Answer (1)

Sol. $E = \frac{KQx}{\left(R^2 + x^2\right)^{3/2}}$

For E_{max}, $\frac{dE}{dx} = 0$

$x = \frac{R}{\sqrt{2}}$

$R = a\sqrt{2}$

$\therefore \quad x = a$

31. Two water drops each of radius 'r' coalesce to form a bigger drop. If 'T' is the surface tension, the surface energy released in this process is

(1) $4\pi r^2 T\left[\sqrt{2} - 1\right]$

(2) $4\pi r^2 T\left[2 - 2^{\frac{2}{3}}\right]$

(3) $4\pi r^2 T\left[2 - 2^{\frac{1}{3}}\right]$

(4) $4\pi r^2 T\left[1 + \sqrt{2}\right]$

Answer (2)

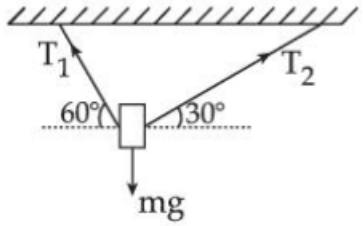
Sol. $2 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$R = (2)^{1/3} r$

$\Delta Q = E_i - E_f$

$= 2 \times 4\pi r^2 T - 4\pi (2)^{2/3} r^2 T$

$= 4\pi R^2 T\left(2 - (2)^{2/3}\right)$

32. 

A body of mass 1 kg is suspended with the help of two strings making angles as shown in figure. Magnitudes of tensions T_1 and T_2, respectively, are (in N)

(Take acceleration due to gravity 10 m/s^2)

(1) $5\sqrt{3}$, 5

(2) $5\sqrt{3}$, $5\sqrt{3}$

(3) 5, $5\sqrt{3}$

(4) 5, 5

Answer (1)

Sol. $T_1\sin60 + T_2\sin30 = mg$ …(i)

$T_1\cos60 = T_2\cos30$ …(ii)

$T_1 = \sqrt{3}T_2$

$$\sqrt{3}T_2 \times \frac{\sqrt{3}}{2} - \frac{T_2}{2} = mg$$

$$T_2 = \frac{mg}{2} = 5 \text{ N}$$

$$T_1 = \frac{\sqrt{3}mg}{2} = 5\sqrt{3} \text{ N}$$

33. If μ_0 and ε_0 are the permeability and permittivity of free space, respectively, then the dimension of $\left(\frac{1}{\mu_0\varepsilon_0}\right)$ is

(1) T^2/L^2

(2) L^2/T^2

(3) T^2/L

(4) L/T^2

Answer (2)

Sol. $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\mu_0\varepsilon_0 \equiv \frac{1}{C^2} \equiv [L^{-2}T^2]$$

$$\therefore \quad \left[\frac{1}{\mu_0\varepsilon_0}\right] = \frac{L^2}{T^2}$$

34. Match **List - I** with **List - II**.

List - I	**List - II**
(A) Heat capacity of body (I)	J kg^{-1}
(B) Specific heat capacity of body (II)	J k^{-1}
(C) Latent heat	(III) J kg^{-1} K^{-1}
(D) Thermal conductivity	(IV) J m^{-1} K^{-1} s^{-1}

Choose the **correct** answer from the options given below.

(1) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(2) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(3) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Answer (1)

Sol. Heat capacity = J/K, A $\rightarrow$ II

Specific heat capacity = J/kg K, B $\rightarrow$ III

Latent heat = J/kg, C $\rightarrow$ I

Thermal conductivity

$$k \equiv \frac{Q}{A}\frac{\Delta x}{\Delta T} \equiv \text{J/mKs}, \quad \text{D} \rightarrow \text{IV}$$

35. A bi-convex lens has radius of curvature of both the surfaces same as 1/6 cm. If this lens is required to be replaced by another convex lens having different radii of curvatures on both sides ($R_1 \neq R_2$), without any change in lens power then possible combination of R_1 and R_2 is

(1) $\frac{1}{5}$ cm and $\frac{1}{7}$ cm (2) $\frac{1}{3}$ cm and $\frac{1}{7}$ cm

(3) $\frac{1}{6}$ cm and $\frac{1}{9}$ cm (4) $\frac{1}{3}$ cm and $\frac{1}{3}$ cm

Answer (1)

Sol. $\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right) = (\mu - 1)12$

$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$f' = f$

$12 = \frac{1}{R_1} + \frac{1}{R_2}$

36. Given a charge q, current I and permeability of vacuum μ_0. Which of the following quantity has the dimension of momentum?

(1) $q\mu_0/I$

(2) qI/μ_0

(3) $q\mu_0 I$

(4) $q^2\mu_0 I$

Answer (3)

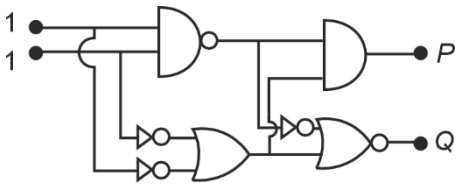
Sol. Momentum = mv

$F \equiv Bvq$

$\frac{mv}{t} \equiv \frac{\mu_0 Ivq}{r}$

$mv \equiv \mu_0 Iq$

37. In the digital circuit shown in the figure, for the given inputs the P and Q values are



(1) $P = 1, Q = 0$ (2) $P = 1, Q = 1$

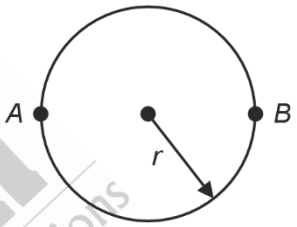
(3) $P = 0, Q = 0$ (4) $P = 0, Q = 1$

Answer (3)

Sol. $P = \overline{AB}\left(\overline{A} + \overline{B}\right) \Rightarrow 0$

$Q = \overline{\left(\overline{A} + \overline{B}\right) + AB} = 0$

38. A sportsman runs around a circular track of radius r such that he traverses the path $ABAB$. The distance travelled and displacement, respectively, are



(1) $3\pi r$, $2r$ (2) $3\pi r$, πr

(3) $2r$, $3\pi r$ (4) πr, $3r$

Answer (1)

Sol. Distance = $3 \times \pi r = 3\pi r$

Displacement = $2r$

39. An electron with mass 'm' with an initial velocity $(t = 0)\ \vec{v} = v_0\hat{i}(v_0 > 0)$ enters a magnetic field $\vec{B} = B_0\hat{j}$. If the initial de-Broglie wavelength at $t = 0$ is λ_0 then its value after time 't' would be:

(1) $\frac{\lambda_0}{\sqrt{1 - \frac{e^2B_0^2t^2}{m^2}}}$ (2) $\frac{\lambda_0}{\sqrt{1 + \frac{e^2B_0^2t^2}{m^2}}}$

(3) $\lambda_0\sqrt{1 + \frac{e^2B_0^2t^2}{m^2}}$ (4) λ_0

Answer (4)

Sol. The speed of the electron will not change in presence of magnetic field.

$\therefore \quad \lambda = \lambda_0$

40. A solenoid having area A and length l is filled with a material having relative permeability 2. The magnetic energy stored in the solenoid is:

(1) B^2Al (2) $\frac{B^2Al}{\mu_0}$

(3) $\frac{B^2Al}{2\mu_0}$ (4) $\frac{B^2Al}{4\mu_0}$

Answer (4)

Sol. $E = \left(\frac{B^2}{2\mu}\right) \times \text{Volume}$

$\mu = 2\mu_0$

$= \frac{B^2Al}{4\mu_0}$

41. Two identical objects are placed in front of convex mirror and concave mirror having same radii of curvature of 12 cm, at same distance of 18 cm from the respective mirrors. The ratio of sizes of the images formed by convex mirror and by concave mirror is:

(1) 2 (2) 3

(3) $\frac{1}{3}$ (4) $\frac{1}{2}$

Answer (4)

Sol. $m = \left(\frac{f}{f-u}\right)$

$h_i = mh_0$

$\frac{h_{i_1}}{h_{i_2}} = \left(\frac{f_1}{f_1-u}\right)\left(\frac{f_2-u}{f_2}\right)$

$f_1 = 6, f_2 = -6$

$u = -18$

$= \frac{6}{24} \times \frac{(12)}{(-6)}$

$\left|\frac{h_{i_1}}{h_{i_2}}\right| = \left|\frac{-1}{2}\right| = \frac{1}{2}$

42. A sinusoidal wave of wavelength 7.5 cm travels a distance of 1.2 cm along the x-direction in 0.3 sec. The crest P is at $x = 0$ at $t = 0$ sec and maximum displacement of the wave is 2 cm. Which equation correctly represents this wave?

(1) $y = 2\cos(3.35x - 0.83t)$ cm

(2) $y = 2\cos(0.83x - 3.35t)$ cm

(3) $y = 2\sin(0.83x - 3.5t)$ cm

(4) $y = 2\cos(0.13x - 0.5t)$ cm

Answer (2)

Sol. $\lambda = 7.5$ cm

$\lambda = \frac{2\pi}{K}$

$K = \frac{2\pi}{7.5} = 0.83 \text{ cm}^{-1}$

$v = \frac{1.2}{0.3} = \frac{\omega}{K}$

$\omega = 4 \times 0.83 = 3.35$

At $t = 0$, $x = 0$, there is a crest

$\therefore \quad y = 2\cos(0.83x - 3.35t)$ cm

43. In a moving coil galvanometer, two moving coils M_1 and M_2 have the following particulars :

$R_1 = 5\Omega$, $N_1 = 15$, $A_1 = 3.6 \times 10^{-3}\text{m}^2$, $B_1 = 0.25$ T

$R_2 = 7\Omega$, $N_2 = 21$, $A_2 = 1.8 \times 10^{-3}$ m^2, $B_2 = 0.50$ T

Assuming that torsional constant of the springs are same for both coils, what will be the ratio of voltage sensitivity of M_1 and M_2 ?

(1) 1 : 2 (2) 1 : 3

(3) 1 : 4 (4) 1 : 1

Answer (4)

Sol. $K\theta = NIAB$

$$\frac{\theta}{I} = \frac{NAB}{K}$$

$$\frac{\theta}{V} = \frac{\theta}{RI} = \frac{NAB}{KR}$$

Ratio of voltage sensitivity $= \left(\frac{N_1A_1B_1}{R_1}\right)\left(\frac{R_2}{N_2A_2B_2}\right)$

$$= \frac{5}{7} \times \frac{2}{1} \times \frac{1}{2} \times \frac{7}{5} = 1$$

44. Identify the characteristics of an adiabatic process in a monoatomic gas.

(A) Internal energy is constant.

(B) Work done in the process is equal to the change in internal energy.

(C) The product of temperature and volume is a constant.

(D) The product of pressure and volume is a constant.

(E) The work done to change the temperature from T_1 to T_2 is proportional to $(T_2 - T_1)$.

Choose the **correct** answer from the options given below :

(1) (A), (C), (E) only

(2) (B), (E) only

(3) (A), (C), (D) only

(4) (B), (D) only

Answer (2)

Sol. For adiabatic process

$Q = 0$

$\therefore \quad \Delta U = -W$

B, E are correct

45. Assuming the validity of Bohr's atomic model for hydrogen like ions the radius of Li^{++} ion in its ground state is given by $\frac{1}{X}a_0$, where X = _____.

(Where a_0, is the first Bohr's radius.)

(1) 2 (2) 9

(3) 3 (4) 1

Answer (3)

Sol. $r = \frac{a_0 n^2}{z}$

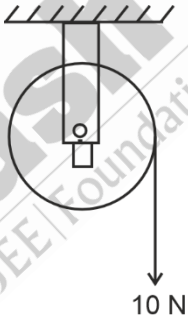
$\therefore \quad r_{Li}(n=1) = \frac{a_0(1)}{3}$

$\therefore \quad X = 3$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46.



A wheel of radius 0.2 m rotates freely about its center when a string that is wrapped over its rim is pulled by force of 10 N as shown in figure. The established torque produces an angular acceleration of 2 rad/s^2. Moment of inertia of the wheel is _____ kgm^2.

(Acceleration due to gravity =10 m/s^2)

Answer (1)

Sol. $\tau = FR = I\alpha$

$$I = \frac{FR}{\alpha} = \frac{10 \times 0.2}{2} = 1$$

47. The length of a light string is 1.4 m when the tension on it is 5 N. If the tension increases to 7 N, the length of the string is 1.56 m. The original length of the string is _____ m.

Answer (1)

Sol. $l = l_0 + \Delta l = l_0 + \frac{Tl_0}{AY}$

$l = l_0(1 + KT)$

$1.4 = l_0(1 + 5K)$ …(i)

$1.56 = l_0(1 + 7K)$ …(ii)

On solving

$l_0 = 1$

48. The internal energy of air in 4 m × 4 m × 3 m sized room at 1 atmospheric pressure will be _____ × 10^6 J.

(Consider air as diatomic molecule)

Answer (12)

Sol. $U = nC_vT$

$= n \times \frac{5R}{2}T$

$= \frac{5}{2}(nRT) = \frac{5}{2}PV$

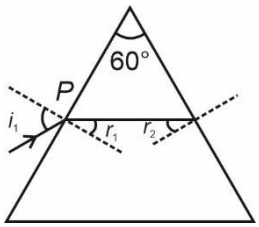
$U = \frac{5}{2} \times 10^5 \times 4 \times 4 \times 3$

$= 12 \times 10^6$ J

49. A ray of light suffers minimum deviation when incident on a prism having angle of the prism equal to 60°. The refractive index of the prism material is $\sqrt{2}$. The angle of incidence (in degrees) is _____.

Answer (45)

Sol.



For minimum deviation $r_1 = r_2 = \frac{A}{2} = 30°$

At P

$1 \times \sin i = \mu \times \sin 30$

$\sin i = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$

$i = 45°$

50. A satellite of mass 1000 kg is launched to revolve around the earth in an orbit at a height of 270 km from the earth's surface. Kinetic energy of the satellite in this orbit is _____ × 10^{10} J.

(Mass of earth = 6 × 10^{24} kg, Radius of earth = 6.4 × 10^6 m, Gravitational constant = 6.67 × 10^{-11} Nm^2kg^{-2})

Answer (3)

Sol. $K = -\frac{U}{2} = \frac{GMm}{2r}$

$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 10^3}{2 \times 6670 \times 10^3}$

$= 3 \times 10^{10}$ J

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. Arrange the following in order of magnitude of work done by the system/on the system at constant temperature.

(a) $|w_{reversible}|$ for expansion in infinite stages

(b) $|w_{irreversible}|$ for expansion in single state

(c) $|w_{reversible}|$ for compression in infinite stages

(d) $|w_{irreversible}|$ for compression in single stage

Choose the **correct** answer from the options given below.

(1) c = a > d > b (2) d > c = a > b

(3) a > c > b > d (4) a > b > c > d

Answer (2)

Sol. $|w_{reversible\ expansion}| > |w_{irreversible\ expansion}|$

$|w_{reversible\ expansion}| = |w_{reversible\ compression}|$

$|w_{irreversible\ compression}| > |w_{reversible\ compression}|$

52. Which among the following molecules is (a) involved in sp^3d hybridization, (b) has different bond lengths and (c) has lone pair of electrons on the central atom?

(1) XeF_4 (2) XeF_2

(3) PF_5 (4) SF_4

Answer (4)

Sol.

$(S-F)_a > (S-F)_e$

SF_4 has 4 bond pairs of electrons and 1 lone pair of electrons.

Since, steric number is 5,

Hybridisation of $SF_4 = sp^3d$

53. Match **List-I** with **List-II**.

List-I (Purification technique)	**List-II (Mixture of organic compounds)**
(A) Distillation (simple)	(I) Diesel + Petrol
(B) Fractional distillation	(II) Aniline + Water
(C) Distillation under reduced pressure	(III) Chloroform + Aniline
(D) Steam distillation	(IV) Glycerol + Spent-lye

Choose the **correct** answer from the options given below.

(1) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(2) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(3) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Answer (2)

Sol. Boiling point of aniline is 547 K and B.P. of $CHCl_3$ is 334 K. So, they are separated by simple distillation.

54. Formation of $Na_4[Fe(CN)_5NOS]$, a purple coloured complex formed by addition of sodium nitroprusside in sodium carbonate extract of salt indicates the presence of

(1) Sulphate ion (2) Sulphide ion

(3) Sulphite ion (4) Sodium ion

Answer (2)

Sol. Test for sulphide ion

$$S^{2-} + Na_2[Fe(CN)_5NO] \longrightarrow \underset{\text{Deep violet compound}}{[Fe(CN)_5NOS]^{4+}}$$

55. Electronic configuration of four elements A, B, C and D are given below :

(A) $1s^22s^22p^3$ (B) $1s^22s^22p^4$

(C) $1s^22s^22p^5$ (D) $1s^22s^22p^2$

Which of the following is the correct order of increasing electronegativity (Pauling's scale)?

(1) A < C < B < D (2) A < B < C < D

(3) D < A < B < C (4) A < D < B < C

Answer (3)

Sol. (A) $1s^2 2s^2 2p^3 \Rightarrow N$

(B) $1s^2 2s^2 2p^4 \Rightarrow O$

(C) $1s^2 2s^2 2p^5 \Rightarrow F$

(D) $1s^2 2s^2 2p^2 \Rightarrow C$

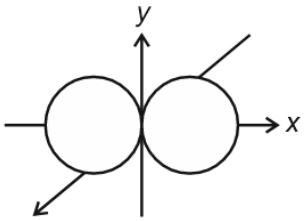
Order of electronegativity

F > O > N > C

C > B > A > D

56. Which of the following statements are **true**?

(A) The subsidiary quantum number l describes the shape of the orbital occupied by the electron.

(B) 

is the boundary surface diagram of the $2p_x$ orbital.

(C) The + and – signs in the wave function of the $2p_x$ orbital refer to charge.

(D) The wave function of $2p_x$ orbital is zero everywhere in the *xy* plane.

(1) (B) and (D) only (2) (C) and (D) only

(3) (A) and (B) only (4) (A), (B) and (C) only

Answer (3)

Sol. Subsidiary quantum number or azimuthal quantum number (l) describes shape of orbital.

The given surface diagram is of the $2p_x$ orbital.

57. A tetrapeptide, "x" on complete hydrolysis produced glycine (Gly), alanine (Ala), valine (Val), leucine (Leu) in equimolar proportion each. The number of tetrapeptides (sequences) possible involving each of these amino acids is:

(1) 32 (2) 24

(3) 8 (4) 16

Answer (2)

Sol. Let A = Gly

B = Ala

C = Val

D = Leu

Total 24 tetrapeptides are formed. The 24 tetrapeptides formed including all the four amino acids are

ABCD	BACD	CABD	DABC
ABDC	BADC	CADB	DACB
ACBD	BDAC	CBAD	DBAC
ACDB	BDCA	CBDA	DBCA
ADBC	BCAD	CDAB	DCAB
ADCB	BCDA	CDBA	DCBA

Total 24

58. Given below are two statements:

Statement (I): Neopentane forms only one monosubstituted derivative.

Statement (II): Melting point of neopentane is higher than n-pentane.

In the light of the above statements, choose the **most appropriate answer** from the options given below:

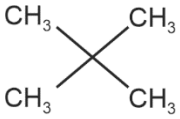
(1) **Statement I** is incorrect but **Statement II** is correct

(2) Both **Statement I** and **Statement II** are incorrect

(3) Both **Statement I** and **Statement II** are correct

(4) **Statement I** is correct but **Statement II** is incorrect

Answer (3)

Sol.  neopentane has only one type of hydrogen so form only one monosubstituted derivative.

59. The nature of oxide (TeO_2) and hydride (TeH_2) formed by Te, respectively are:

(1) Oxidising and basic (2) Reducing and acidic

(3) Oxidising and acidic (4) Reducing and basic

Answer (3)

Sol. TeO_2 is oxidising in nature TeH_2 is reducing in nature and acidic in nature.

60. The type of hybridization and the magnetic property of $[MnCl_6]^{3-}$ are,

(1) sp^3d^2, paramagnetic with two unpaired electrons.

(2) d^2sp^3, paramagnetic with two unpaired electrons.

(3) sp^3d^2, paramagnetic with four unpaired electrons.

(4) d^2sp^3, paramagnetic with four unpaired electrons.

Answer (3)

Sol. Cl^- with Mn^{3+} is a weak field ligand

$Mn^{3+} \Rightarrow 3d^4 \Rightarrow t_{2g}^3\, e_g^1$

Hybridisation = sp^3d^2 with 4 unpaired electron and paramagnetic.

61. The *d*-orbital electronic configuration of the complex among $[Co(en)_3]^{3+}$, $[CoF_6]^{3-}$, $[Mn(H_2O)_6]^{2+}$ and $[Zn(H_2O)_6]^{2+}$ that has the highest CFSE is

(1) $t_{2g}^3e_g^2$

(2) $t_{2g}^6e_g^4$

(3) $t_{2g}^6e_g^0$

(4) $t_{2g}^4e_g^2$

Answer (3)

Sol. $[Co(en)_3]^{3+}$ has highest CFSE

Co^{3+} with SFL (en)

$= t_{2g}^6e_g^0$

CFSE = $(6 \times (-0.4) + 0 \times 0.6)\Delta_0$

$= -2.4\ \Delta_0$

62. In Dumas' method for estimation of nitrogen, 0.5 gram of an organic compound gave 60 mL of nitrogen collected at 300 K temperature and 715 mm Hg pressure. The percentage composition of nitrogen in the compound (Aqueous tension at 300 K = 15 mm Hg) is _____%.

(1) 12.57 (2) 20.87

(3) 1.257 (4) 18.67

Answer (1)

Sol. $P_{N_2} = P_{\text{moist gas}} - \text{aqueous tension}$

= (715 – 15)mm Hg

= 700 mm Hg

$V_{N_2} = 60$ mL

= 0.060 L

T = 300 K

$$n_{N_2} = \frac{PV}{RT} = \frac{700 \times 0.06}{760 \times 0.0821 \times 300}$$

= 0.022 mol

Mass of $N_2 = 0.0224 \times 28 = 0.0628$

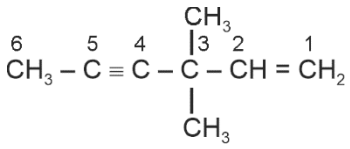
$$\%\, N_2 = \frac{0.0628}{0.5} \times 100 = 12.58\%$$

63. In 3,3-dimethylhex-1-en-4-yne, there are _____ sp^3, _____ sp^2 and ____*sp* hybridised carbon atoms respectively.

(1) 2, 4, 2 (2) 2, 2, 4

(3) 3, 3, 2 (4) 4, 2, 2

Answer (4)

Sol. 

sp^3 C = 4

sp^2 C = 2

sp C = 2

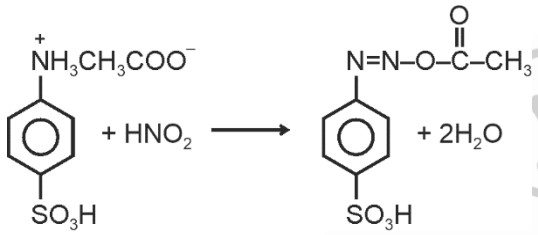
64. When a concentrated solution of sulphanilic acid and 1-naphthylamine is treated with nitrous acid (273 K) and acidified with acetic acid, the mass (g) of 0.1 mole of product formed is

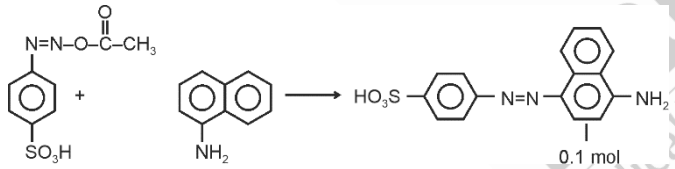
(Given molar mass in g mol^{-1} H : 1, C : 12, N : 14, O : 16, S : 32)

(1) 343 (2) 33

(3) 66 (4) 330

Answer (2)

Sol. 



Molar mass of product formed

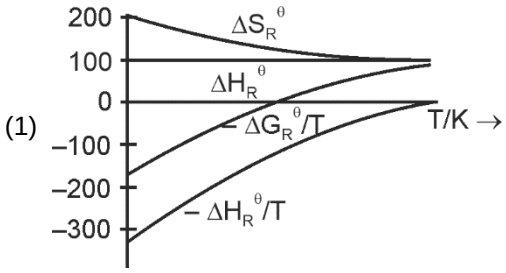
= (16 × 12) + (14 × 3) + (16 × 3) + 32 + 13

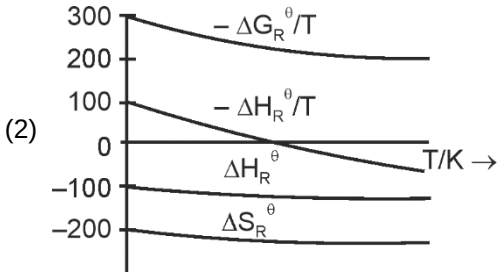
= 327 g

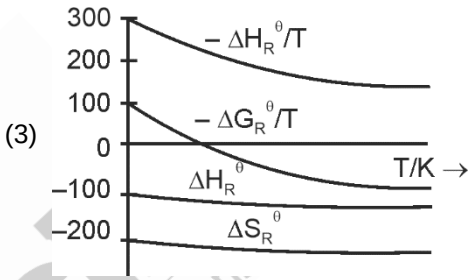
Mass of 0.1 mol product

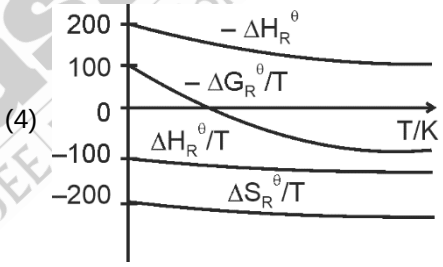
= 32.7 g ≈ 33 g

65. Which of the following graphs correctly represents the variation of thermodynamic properties of Haber's process?

(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

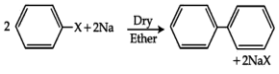
ΔS_r = –ve

ΔH_r = –ve

$\frac{-\Delta H_r^\circ}{T}$ ⇒ first decreases then constant

$\frac{-\Delta G_R^\circ}{T}$ ⇒ first decreases then constant

66. Match **List -I** with **List-II.**

	List-I (Reaction)		**List-II (Name of Reaction)**
(A)		(I)	Lucas reaction
(B)	$ArN_2^+X^- \xrightarrow[HCl]{Cu} ArCl + N_2 \uparrow + CuX$	(II)	Finkelstein reaction
(C)	$C_2H_5Br + NaI \xrightarrow[Acetone]{Dry} C_2H_5I + NaBr$	(III)	Fittig reaction
(D)	$CH_3C(OH)(CH_3)CH_3 \xrightarrow[ZnCl_2]{HCl} CH_3C(Cl)(CH_3)CH_3$	(IV)	Gatterman reaction

Choose the **correct** answer from the options given below:

(1) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

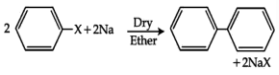
(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(4) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Answer (1)

Sol.

(A)		Fittig reaction
(B)	$ArN_2^+X^- \xrightarrow[HCl]{Cu} ArCl + N_2 \uparrow + CuX$	Gatterman reaction
(C)	$C_2H_5Br + NaI \xrightarrow[Acetone]{Dry} C_2H_5I + NaBr$	Finkelstein reaction
(D)	$CH_3C(OH)(CH_3)CH_3 \xrightarrow[ZnCl_2]{HCl} CH_3C(Cl)(CH_3)CH_3$	Lucas reaction

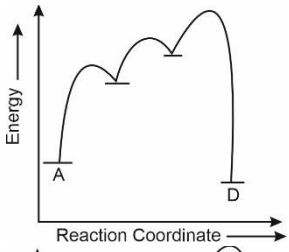
67. Reactant A converts to product D through the given mechanism (with the net evolution of heat) :

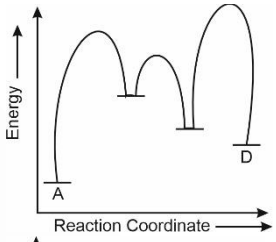
$A \rightarrow B$ Slow; ΔH = +ve

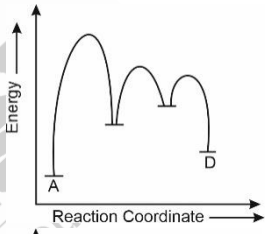
$B \rightarrow C$ Fast; ΔH = –ve

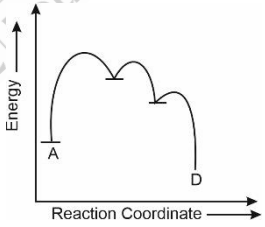
$C \rightarrow D$ Fast; ΔH = –ve

Which of the following represents the above reaction mechanism?

(1) 

(2) 

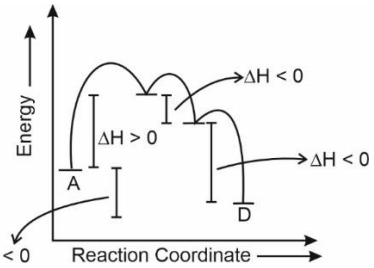
(3) 

(4) 

Answer (4)

Sol. As given net evolution of heat takes place.

$A \rightarrow D$ $\Delta H < 0$



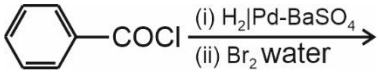
68. Consider the following reactions. Form these reactions which reaction will give carboxylic acid as a major product?

(A) $R-C\equiv N \xrightarrow[\text{mild condition}]{(i)H^+/H_2O}$

(B) $R-MgX \xrightarrow[(ii)H_3O^+]{(i)CO_2}$

(C) $R-C\equiv N \xrightarrow[(ii)H_3O^+]{(i)SnCl_2/HCl}$

(D) $R{\cdot}CH_2{\cdot}OH \xrightarrow{PCC}$

(E) 

Choose the **correct** answer from the options given below:

(1) B and E only (2) B, C and E only

(3) A and D only (4) A, B and E only

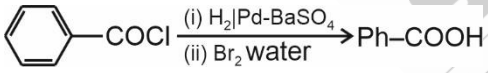
Answer (1)

Sol. (A) $R-C\equiv N \xrightarrow[\text{mild condition}]{(i)H^+/H_2O} R-\overset{\overset{O}{\|}}{C}-NH_2$

(B) $R-MgX \xrightarrow[(ii)H_3O^+]{(i)CO_2} R-COOH$

(C) $R-C\equiv N \xrightarrow[(ii)H_3O^+]{(i)SnCl_2/HCl} R-CHO$

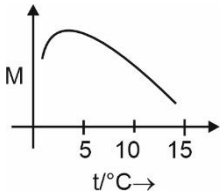
(D) $R{\cdot}CH_2{\cdot}OH \xrightarrow{PCC} R-CHO$

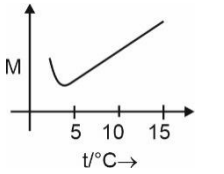
(E) 

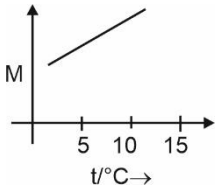
Br_2 water oxidises aldehyde to carboxylic acid.

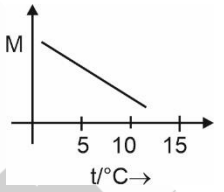
69. 'x' g of NaCl is added to water in a beaker with a lid. The temperature of the system is raised form 1°C to 25°C. Which out of the following plots, is best suited for the change in the molarity (M) of the solution with respect to temperature?

[Consider the solubility of NaCl remains unchanged over the temperature range]

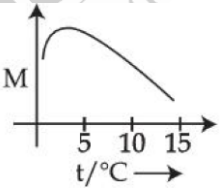
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

Volume is minimum at 4°C, so molarity will be maximum at 4°C.

70. Consider the following chemical equilibrium of the gas phase reaction at a constant temperature :

$$A(g) \rightleftharpoons B(g) + C(g)$$

If p being the total pressure, K_p is the pressure equilibrium constant and α is the degree of dissociation, then which of the following is true at equilibrium?

(1) If K_p value is extremely high compared to p, α becomes much less than unity

(2) When p increases α increases

(3) If p value is extremely high compared to K_p, $\alpha \approx 1$

(4) When p increases α decreases

Answer (4)

Sol. According to Le-Chatelier's principle as pressure increases in above reaction the reaction moves in backward direction hence α decreases.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. When 1 g each of compounds AB and AB_2 are dissolved in 15 g of water separately, they increased the boiling point of water by 2.7 K and 1.5 K respectively. The atomic mass of A (in amu) is _____ $\times 10^{-1}$ (Nearest integer)

(Given: Molal boiling point elevation constant is 0.5 K kg mol^{-1})

Answer (25)

Sol. For AB

$\Delta T_b = 2.7$

$K_b = 0.5$ K kg mol^{-1}

$$m = \frac{\Delta T_b}{K_b} = 5.4 \text{ m}$$

For AB_2

$\Delta T_b = 1.5$ K

$$m = \frac{1.5}{0.5} = 5.4 \text{ m}$$

Moles of AB = 0.081 mol

Moles of AB_2

= 0.045 mol

Molar mass of AB = 12.35 g/mol

Molar mass of AB_2 = 22.22 g/mol

Let atomic mass of A = a, B = b

a + b = 12.35

a + 2b = 22.22

On solving a = 2.48

$= 24.8 \times 10^{-1}$

72. 0.2% (w/v) solution of NaOH is measured to have resistivity 870.0 mΩ m. The molar conductivity of the solution will be _____ $\times 10^2$ mS dm^2 mol^{-1}. (Nearest integer)

Answer (23)

Sol. We have 0.2 g of NaOH in 100 mL solution

$$M = \frac{0.2}{40 \times 100} \times 1000$$

= 0.05 M

$$\kappa = \frac{1}{0.870} = 1.15 \text{ S m}^{-1}$$

$= 1.15 \times 10^{-2}$ S cm^{-1}

$$\wedge_m = \frac{\kappa \times 1000}{m} = \frac{1.15 \times 10^{-2} \times 1000}{0.05}$$

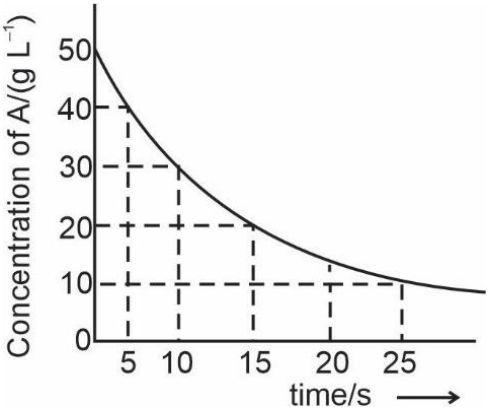
= 230 S cm^2 mol^{-1}

$= 23 \times 10^4$ m S cm^2 mol^{-1}

$= 23 \times 10^2$ m S dm^2 mol^{-1}

73. For the reaction A → B the following graph was obtained. The time required (in seconds) for the concentration of A to reduce to 2.5 g L^{-1} (if the initial concentration of A was 50 g L^{-1}) is _____. (Nearest integer)

Given: log 2 = 0.3010



Answer (47)

Sol. The graph given is for 1st order reaction

$$k = \frac{2.303}{t}\log\frac{[A_0]}{[A_t]}$$

$$k = \frac{2.303}{25}\log\frac{[50]}{[10]}$$

$$k = \frac{2.303}{25}\log 5$$

$$= \frac{2.303\times 0.7}{25}s^{-1}$$

$$= \frac{2.303\times 0.7}{25} = \frac{2.303}{t}\log\frac{50}{2.5}$$

$$\frac{0.699}{25} = \frac{1}{t}\log 20$$

$$t = \frac{1.301\times 25}{0.699}$$

= 46.53 s

≈ 47 s

74. The spin-only magnetic moment value of M^{n+} ion formed among Ni, Zn, Mn and Cu that has the least enthalpy of atomisation is _____. (in nearest integer) Here n is equal to the number of diamagnetic complexes among $K_2[NiCl_4]$, $[Zn(H_2O)_6]Cl_2$, $K_3[Mn(CN)_6]$ and $[Cu(PPh_3)_3I]$

Answer (0)

Sol. $K_2[NiCl_4] \Rightarrow Ni^{2+} \Rightarrow 3d^8 \Rightarrow sp^3 \Rightarrow$ Paramagnetic

$[Zn(H_2O)_6]Cl_2 \Rightarrow Zn^{2+} \Rightarrow 3d^{10} \Rightarrow$ diamagnetic

$K_3[Mn(CN)_6] \Rightarrow Mn^{3+} \Rightarrow 3d^4 \Rightarrow$ paramagnetic

$[Cu(PPh_3)I) \Rightarrow Cu^{+1} \Rightarrow 3d^{10} \Rightarrow$ diamagnetic

n = 2

Zn has least enthalpy of atomisation

$Zn^{2+} \Rightarrow \mu = 0$

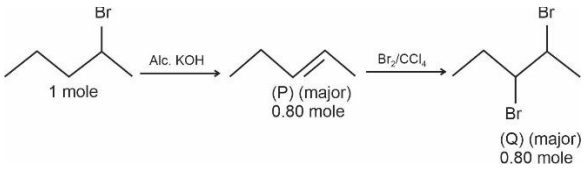
75. 

Consider the above sequence of reactions. 151 g of 2-bromopentane is made to react. Yield of major product P is 80% whereas Q is 100%.

Mass of product Q obtained is _____g.

(Given molar mass in g mol^{-1} H: 1, C: 12, O: 16, Br: 80)

Answer (184)

Sol. 

Molecular mass of Q = 230 g mol^{-1}

Mass of Q = 0.8 × 230 = 184 g

❑ ❑ ❑