MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let $\left|\frac{\bar{z}-i}{2\bar{z}+i}\right|=\frac{1}{3}$, $z \in C$, be the equation of a circle with center at C. If the area of the triangle, whose vertices are at the points (0, 0), C and (α, 0) is 11 square units, then α^2 equals:

(1) $\frac{81}{25}$ (2) 50

(3) 100 (4) $\frac{121}{25}$

Answer (3)

Sol. Let $z = x + iy \Rightarrow \bar{z} = x - iy$

$3|\bar{z}-i| = 1|2\bar{z}+i|$

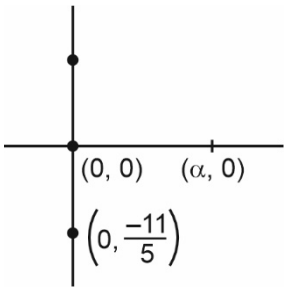
$= 3|(x-(y+1)i| = |2x+i(1-2y)|$

$= 3\sqrt{x^2+(y+1)^2} = \sqrt{(2x)^2+(1-2y)^2}$

$= 9(x^2+y^2+2y+1) = 4x^2+4y^2-4y+1$

$\Rightarrow 5x^2+5y^2+22y+8=0$

$\Rightarrow$ Centre $\equiv \left(0,-\frac{11}{5}\right)$



Area of Δ

$= \frac{1}{2}|\alpha|\left|\frac{-11}{5}\right| = 11$

$\Rightarrow |\alpha| = 10$

$\Rightarrow \alpha^2 = 100$

2. Let $f(x) = \log_e x$ and $g(x)=\frac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}$.

Then the domain of fog is

(1) $[1, \infty)$ (2) $(0, \infty)$

(3) $\mathbb{R}$ (4) $[0, \infty)$

Answer (3)

Sol. $f(g(x)) = \ln\left(\frac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}\right)$

Since $2x^2-2x+1>0 \quad \forall x \in \mathbb{R} \because (-2)^2-4(2)<0$

Consider

$x^4-2x^3+3x^2-2x+2$

$= (x^4-2x^3+x^2)+(x^2-2x+1)+(1+x^2)$

$= x^2(x-1)^2+(x-1)^2+(x^2+1)>0 \forall x \in \mathbb{R}$

$\Rightarrow g(x)>0 \ \forall x \in \mathbb{R}$

$\Rightarrow \ln f((x)),\ f(x)>0 \ \forall x \in \mathbb{R}$

$\Rightarrow x \in \mathbb{R}$ is domain

3. The number of words, which can be formed using all the letters of the word "DAUGHTER", so that all the vowels never come together, is

(1) 35000 (2) 37000

(3) 34000 (4) 36000

Answer (4)

Sol. E : all vowel never come together

$\bar{E}$: all vowel come together

$n(E) = n(U) - n(\bar{E})$ $\boxed{AUE}$ DGHTR

= 8! – (6!)(3!)

= 6!(7 × 8 – 6) = 720 (56 – 6)

= 720 × 50 = 36000

4. One die has two faces marked 1, two faces marked 2, one face marked 3 and one face marked 4. Another die has one face marked 1, two faces marked 2, two faces marked 3 and one face marked 4. The probability of getting the sum of numbers to be 4 or 5, when both the dice are thrown together, is

(1) $\frac{2}{3}$ (2) $\frac{4}{9}$

(3) $\frac{3}{5}$ (4) $\frac{1}{2}$

Answer (4)

Sol. Die 1 : {1, 1, 2, 2, 3, 4}

Die 2 : {1, 2, 2, 3, 3, 4}

(die 1, die 2) = {(1, 3), (3, 1), (2, 2), (2, 3), (3, 2), (1, 4), (4, 1)}

$$=\frac{2}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{1}{6}$$

$$\times\frac{2}{6}+\frac{2}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{2}{6}=\frac{1}{2}$$

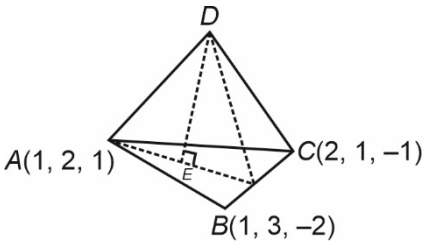
5. Let the position vectors of the vertices A, B and C of a tetrahedron $ABCD$ be $\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+3\hat{j}-2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E. If the length of AD is $\frac{\sqrt{110}}{3}$ and the volume of the tetrahedron is $\frac{\sqrt{805}}{6\sqrt{2}}$, then the position vector of E is

(1) $\frac{1}{12}(7\hat{i}+4\hat{j}+3\hat{k})$ (2) $\frac{1}{6}(7\hat{i}+12\hat{j}+\hat{k})$

(3) $\frac{1}{6}(12\hat{i}+12\hat{j}+\hat{k})$ (4) $\frac{1}{2}(\hat{i}+4\hat{j}+7\hat{k})$

Answer (2)

Sol.



Volume = $\frac{1}{3}\times\text{Base}\times h=\frac{\sqrt{805}}{6\sqrt{2}}$

Base area = $\frac{1}{2}\left|\overline{AB}\times\overline{AC}\right|$

$$=\frac{1}{2}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&-3\\1&-1&-2\end{vmatrix}=\frac{1}{2}\left|5\hat{i}+3\hat{j}+\hat{k}\right|=\frac{1}{2}\sqrt{35}$$

$$\frac{1}{3}\times\frac{1}{2}\sqrt{35}\times h=\frac{\sqrt{805}}{6\sqrt{2}}$$

$$\Rightarrow\quad h=\sqrt{\frac{23}{2}}$$

Now $AE^2 = AD^2 - DE^2$

$$=\frac{110}{9}-\frac{23}{2}=\frac{13}{18}$$

$$AE=\sqrt{\frac{13}{18}}$$

$$\overline{AE}=|AE|\left(\frac{\hat{i}-5\hat{k}}{\sqrt{26}}\right)=\frac{\hat{i}-5\hat{k}}{6}$$

Position vector of E = $\frac{\hat{i}-5\hat{k}}{6}+\hat{i}+2\hat{j}+\hat{k}$

$$=\frac{1}{6}(7\hat{i}+12\hat{j}+\hat{k})$$

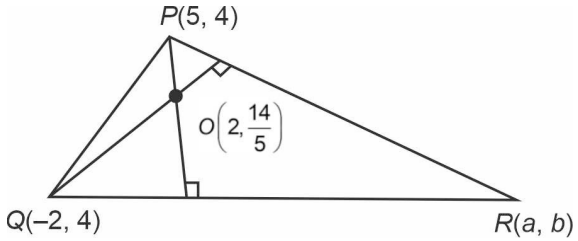
6. Let the area of a ΔPQR with vertices $P(5, 4)$, $Q(-2, 4)$ and $R(a, b)$ be 35 square units. If its orthocentre and centroid are $O\left(2, \frac{14}{5}\right)$ and $C(c, d)$ respectively, then $c + 2d$ is equal to

(1) $\frac{7}{3}$ (2) 2

(3) 3 (4) $\frac{8}{3}$

Answer (3)

Sol.



$QR: y-4=-\left(\frac{5-2}{4-\frac{14}{5}}\right)(x+2)$

$QR: 5x+2y+2=0 \quad \ldots(i)$

$PR: y-4=\frac{4}{\frac{14}{5}-4}(x-4)$

$PR: 10x-3y-38=0 \quad \ldots(ii)$

Solving (i) & (ii)

$R\ (2, -6)$

Centroid $C=\left(\frac{5-2+2}{3}, \frac{4+4-6}{3}\right) \equiv \left(\frac{5}{3}, \frac{2}{3}\right)$

$\Rightarrow \quad c=\frac{5}{3},\ d=\frac{2}{3}$

$c+2d=\frac{5}{3}+\frac{4}{3}=3$

7. Let $R = \{(1,2), (2,3), (3,3)\}$ be a relation defined on the set $\{1, 2, 3, 4\}$. Then the minimum number of elements, needed to be added in R so that R becomes an equivalence relation, is:

(1) 8 (2) 7

(3) 9 (4) 10

Answer (2)

Sol. $R = \{(1,2), (2,3), (3,3)\}$ be relation defined on set $\{1,2,3,4\}$ then the elements added to make this equivalence relation are (1,1), (2,2), (2,1), (3,2), (3,1), (1,3), (4, 4).

Total number of elements added = 7.

8. Let $I(x)=\int \frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$. If $I(37)-I(24)=\frac{1}{4}\left(\frac{1}{b^{\frac{1}{13}}}-\frac{1}{c^{\frac{1}{13}}}\right)$, $b, c \in N$, then $3(b + c)$ is equal to

(1) 22 (2) 40

(3) 26 (4) 39

Answer (4)

Sol. $I(x)=\int \frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$

$= =\int \frac{dx}{(x-11)^2\left(\frac{x+15}{x-11}\right)^{\frac{15}{13}}}$

Let $\frac{x+15}{x-11}=t \Rightarrow \frac{-26}{(x-11)^2}dx=dt$

$\therefore \quad I(x)=\int \frac{-\frac{1}{26}dt}{t^{\frac{15}{13}}}$

$=\frac{1}{4}t^{-\frac{2}{13}}+c=\frac{1}{4}\left(\frac{x-11}{x+15}\right)^{\frac{2}{13}}+c$

$\therefore \quad I(37)-I(24)=\frac{1}{4}\left(\left(\frac{1}{2}\right)^{\frac{2}{13}}-\left(\frac{1}{3}\right)^{\frac{2}{13}}\right)$

$\therefore \quad b = 4, c = 9 \Rightarrow 3(b + c) = 39$

9. If the system of equations

$(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$

$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$

$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$

has infinitely many solutions, then $\lambda^2 + \lambda$ is equal to

(1) 12 (2) 10

(3) 20 (4) 6

Answer (1)

Sol. The given system of equation are

$(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$

$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$

$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$

has infinite many solutions, then

$D = D_1 = D_2 = D_3 = 0$

$$\therefore \begin{vmatrix} \lambda-1 & \lambda-4 & \lambda \\ \lambda & \lambda-1 & \lambda-4 \\ \lambda+1 & \lambda+2 & -(\lambda+2) \end{vmatrix} = 0$$

$$\text{or, } \begin{vmatrix} -1 & -3 & 4 \\ -1 & -3 & 2\lambda-2 \\ \lambda+1 & \lambda+2 & -\lambda-2 \end{vmatrix} = 0$$

$$\text{or, } \begin{vmatrix} -1 & -3 & 4 \\ 0 & 0 & 2\lambda-6 \\ \lambda+1 & \lambda+2 & -\lambda-2 \end{vmatrix} = 0$$

$\therefore \quad (6 - 2\lambda)(2\lambda + 1) = 0$

$\therefore \quad \lambda = 3 \text{ or } -\frac{1}{2}$

But $\lambda = 3$ satisfies all other conditions.

$\therefore \quad \lambda^2 + \lambda = 12$

10. If A, B and $(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))$ are non-singular matrices of same order, then the inverse of $A(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))^{-1} B$, is equal to

(1) $\text{adj}(B^{-1}) + \text{adj}(A^{-1})$

(2) $\frac{1}{|AB|}(\text{adj}(B) + \text{adj}(A))$

(3) $AB^{-1} + A^{-1}B$

(4) $\frac{AB^{-1}}{|A|} + \frac{BA^{-1}}{|B|}$

Answer (2)

Sol. $[A(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))^{-1}B]^{-1}$

$= B^{-1}(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))A^{-1}$

$$= B^{-1}\left[\frac{A}{|A|} + \frac{B}{|B|}\right]A^{-1}$$

$$= \left[\frac{B^{-1}A}{|A|} + \frac{I}{|B|}\right]A^{-1}$$

$$= \frac{B^{-1}}{|A|} + \frac{A^{-1}}{|B|}$$

$$= \frac{\text{adj}(B)}{|AB|} + \frac{\text{adj}(A)}{|AB|}$$

$$= \frac{1}{|AB|}(\text{adj}(B) + \text{adj}(A))$$

11. If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

(1) -1080 (2) -1200

(3) -120 (4) -1020

Answer (1)

Sol. Sum of first 4 terms

$= \frac{1}{5} \times$ (sum of next 4 terms)

$$\frac{4}{2}(2a + 3d) = \frac{1}{5}(4a + 22d)$$

$$\Rightarrow \quad 16a = -8d \quad \Rightarrow \quad a = -\frac{d}{2}$$

$a = 3, d = -6$

$$S_{20} = \frac{20}{2}[2(3) + 19(-6)]$$

$= -10(18.6)$

$= -1080$

12. Let the arc AC of a circle subtend a right angle at the centre O. If the point B on the are AC, divides the arc AC such that $\frac{\text{length of arc } AB}{\text{length of arc } BC} = \frac{1}{5}$, and $\overrightarrow{OC} = \alpha\overrightarrow{OA} + \beta\overrightarrow{OB}$, then $\alpha + \sqrt{2}\left(\sqrt{3} - 1\right)\beta$ is equal to

(1) $5\sqrt{3}$ (2) $2 + \sqrt{3}$

(3) $2\sqrt{3}$ (4) $2 - \sqrt{3}$

Answer (4)

Sol. Let the radius be r

B divides the arc in 1 : 5, so AB would subtend an angle of 15° at origin O and similarly, BC would subtend an angle of 75° at origin.

$\overrightarrow{OA}=\vec{a}, \overrightarrow{OB}=\vec{b}, \overrightarrow{OC}=\vec{c}$

$\vec{b}=r(\cos 75°)\hat{i}+r(\sin 75°)\hat{j}$

$$\vec{b}=\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)\vec{c}+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\vec{a}$$

$$2\sqrt{2}\vec{b}=\left(\sqrt{3}-1\right)\vec{c}+\left(\sqrt{3}+1\right)\vec{a}$$

$$\overrightarrow{OC}=-\underbrace{\frac{\left(\sqrt{3}+1\right)}{\left|\sqrt{3}-1\right|}}_{\alpha}\overrightarrow{OA}+\underbrace{\frac{2\sqrt{2}}{\left|\sqrt{3}-1\right|}}_{\beta}\overrightarrow{OB}$$

$$\alpha+\sqrt{2}\left(\sqrt{3}-1\right)\beta=2-\sqrt{3}$$

13. If the line $3x-2y+12=0$ intersects the parabola $4y=3x^2$ at the points A and B, then at the vertex of the parabola, the line segment AB subtends an angle equal to

(1) $\tan^{-1}\left(\frac{9}{7}\right)$ (2) $\tan^{-1}\left(\frac{11}{9}\right)$

(3) $\frac{\pi}{2}-\tan^{-1}\left(\frac{3}{2}\right)$ (4) $\tan^{-1}\left(\frac{4}{5}\right)$

Answer (1)

Sol. Line $L: 3x-2y+12=0$

Parabola $P: 4y=3x^2$

By putting $y=\frac{3x^2}{4}$ in equation of line

We get, $3x-2\left(\frac{3x^2}{4}\right)+12=0$

$\Rightarrow 6x-3x^2+24=0$

$\Rightarrow x^2-2x+8=0$

$\Rightarrow x=4,-2$

for $x=4$, we get $y=12$

for $x=-2$, we get $y=3$

So, points A and B are (4, 12) and (–2, 3)

Now, Vertex of parabola is (0, 0)

$$\Rightarrow \tan\theta=\left|\frac{3-\left(\frac{-3}{2}\right)}{1+3\left(\frac{-3}{2}\right)}\right|$$

$$\tan\theta=\frac{9}{7}$$

$$\Rightarrow \theta=\tan^{-1}\left(\frac{9}{7}\right)$$

$\Rightarrow$ Option (1) is correct

14. Let P be the foot of the perpendicular from the point $Q(10,-3,-1)$ on the line $\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}$. Then the area of the right-angled triangle PQR, where R is the point (3, –2, 1), is

(1) $3\sqrt{30}$ (2) $9\sqrt{15}$

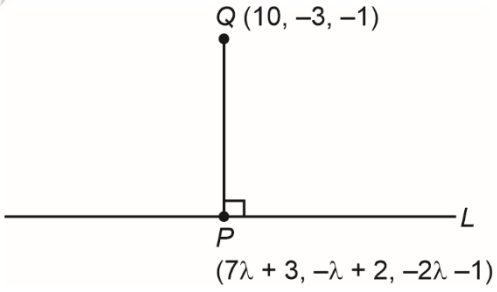
(3) $8\sqrt{15}$ (4) $\sqrt{30}$

Answer (1)

Sol. Q (10, –3, –1)

$$L: \frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}$$

General point on $L(7\lambda+3,-\lambda+2,-2\lambda-1)$



DR's of PQ are $7\lambda-7, -\lambda+5, -2\lambda$

$PQ \perp L$

$\Rightarrow (7\lambda-7)7+(-1)(-\lambda+5)+(-2\lambda)(-2)=0$

$\Rightarrow 49\lambda-49+\lambda-5+4\lambda=0$

$\Rightarrow 54\lambda - 54 = 0 \Rightarrow \lambda = 1$

$P(10, 1, -3)$ and $R(3, -2, 1)$

$\overrightarrow{PQ} = -4\hat{j} + 2\hat{k}$, $\overrightarrow{QR} = -7\hat{i} + \hat{j} + 2\hat{k}$,

$\overrightarrow{PR} = -7\hat{i} - 3\hat{j} + 4\hat{k}$

$\overrightarrow{PQ} \cdot \overrightarrow{QR} = 0 \Rightarrow$ right angle at Q

$\Rightarrow \text{ar}(\Delta PQR) = \frac{1}{2} \times PQ \times QR$

$= \frac{1}{2} \times \sqrt{20} \times \sqrt{54}$

$= \frac{1}{2} \times 2 \times 3\left(\sqrt{5} \times \sqrt{6}\right)$

$= 3\sqrt{30}$

$\Rightarrow$ Option (1) is correct

15. The value of $(\sin 70°)(\cot 10° \cot 70° - 1)$ is

(1) 0 (2) 2/3

(3) 3/2 (4) 1

Answer (4)

Sol. $(\sin 70°)(\cot 10° \cot 70° - 1)$

$= \sin 70° \cot 10° \cot 70° - \sin 70°$

$= \cot 10° \cos 70° - \sin 70°$

$= \frac{\cos 10° \cos 70° - \sin 70° \sin 10°}{\sin 10°}$

$= \frac{\cos(10° + 70°)}{\sin 10°}$

$= \frac{\cos 80°}{\sin 10°} = 1$

$\Rightarrow$ Option (4) is correct

16. Let a curve $y = f(x)$ pass through the points $(0, 5)$ and $(\log_e 2, k)$. If the curve satisfies the differential equation $2(3 + y)e^{2x}\,dx - (7 + e^{2x})dy = 0$, then k is equal to

(1) 8 (2) 4

(3) 32 (4) 16

Answer (1)

Sol. $\frac{dy}{dx} = \frac{2(3+y) \cdot e^{2x}}{7 + e^{2x}}$

$\frac{dy}{dx} - \frac{2ye^{2x}}{7 + e^{2x}} = \frac{6 \cdot e^{2x}}{7 + e^{2x}}$

I.F. $= e^{-\int \frac{2e^{2x}}{7+e^{2x}} dx}$

$\Rightarrow e^{-\ln(7+e^{2x})}$

$= \frac{1}{7 + e^{2x}}$

$y \cdot \frac{1}{7 + e^{2x}} = \int \frac{6e^{2x}}{\left(7 + e^{2x}\right)^2} dx$

$\frac{y}{7 + e^{2x}} = \frac{-3}{7 + e^{2x}} + C$

$\therefore y(0) = 5$

$\Rightarrow \frac{5}{8} = \frac{-3}{8} + C$

$\Rightarrow C = 1$

$\therefore y = -3 + 7 + e^{2x}$

$y = e^{2x} + 4$

$\therefore k = 8$

17. If $\frac{\pi}{2} \le x \le \frac{3\pi}{4}$, then $\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{13}\sin x\right)$ is equal to

(1) $x - \tan^{-1}\frac{4}{3}$ (2) $x + \tan^{-1}\frac{4}{5}$

(3) $x + \tan^{-1}\frac{5}{12}$ (4) $x - \tan^{-1}\frac{5}{12}$

Answer (4)

Sol. $\frac{12}{13}\cos x + \frac{5}{13}\sin x$

Let $\tan\alpha = \frac{5}{12}, \alpha \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow \sin\alpha = \frac{5}{13}, \cos\alpha = \frac{12}{13}$

$$\Rightarrow \frac{12}{13}\cos x + \frac{5}{13}\sin x = \cos\alpha\cos x + \sin\alpha\sin x$$

$$= \cos(x-\alpha)$$

$$\Rightarrow \cos^{-1}[\cos(x-\alpha)] = x-\alpha$$

$$= x - \tan^{-1}\left(\frac{5}{12}\right)$$

18. Marks obtains by all the students of class 12 are presented in a frequency distribution with classes of equal width. Let the median of this grouped data be 14 with median class interval 12-18 and median class frequency 12. If the number of students whose marks are less than 12 is 18, then the total number of students is

(1) 40 (2) 52

(3) 48 (4) 44

Answer (4)

Sol. $M = L + \frac{\frac{n}{2} - Cf}{f} \times h$

$$14 = 12 + \frac{\frac{n}{2} - 18}{12} \times 6$$

$$2 \times 2 = \frac{n}{2} - 18$$

$$\frac{n}{2} = 4 + 18$$

$n = 44$

19. If the function

$$f(x) = \begin{cases} \frac{2}{x}\{\sin(k_1+1)x + \sin(k_2-1)x\}, & x < 0 \\ 4, & x = 0 \\ \frac{2}{x}\log_e\left(\frac{2+k_1x}{2+k_2x}\right), & x > 0 \end{cases}$$

is continuous at $x = 0$ then $k_1^2 + k_2^2$ is equal to

(1) 20 (2) 5

(3) 8 (4) 10

Answer (4)

Sol. $\lim\limits_{x\to0^-} f(x) = \lim\limits_{x\to0^-} 2\left(\frac{\sin(k_1+1)x}{x} + \frac{\sin(k_2-1)x}{x}\right)$

$= 2((k_1+1) + (k_2-1))$

$= 2(k_1 + k_2)$ ...(i)

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} 2\left(\frac{\ln\left(1+\frac{k_1}{2}x\right)}{x} - \frac{\ln\left(1+\frac{k_2}{2}x\right)}{x}\right)$$

$= k_1 - k_2$ ...(ii)

For continuity at $x = 0$

$2(k_1 + k_2) = 4$ and $k_1 - k_2 = 4$

$\Rightarrow k_1 + k_2 = 2$ and $k_1 - k_2 = 4$

$\Rightarrow k_1 = 3$ & $k_2 = -1$

$k_1^2 + k_2^2 = 10$

20. The value of

$$\int_{e^2}^{e^4} \frac{1}{x}\left(\frac{e^{\left((\log_e x)^2+1\right)^{-1}}}{e^{\left((\log_e x)^2+1\right)^{-1}} + e^{\left((6-\log_e x)^2+1\right)^{-1}}}\right)dx \text{ is}$$

(1) 1 (2) e^2

(3) 2 (4) $\log_e 2$

Answer (1)

Sol. Put $\ln x = t \Rightarrow \frac{1}{x}dx = dt$

x	t
e^2	2
e^4	4

$$I = \int_2^4 \frac{e^{(t^2+1)^{-1}}}{e^{(t^2+1)^{-1}} + e^{\left((6-t)^2+1\right)^{-1}}}dt \quad ...(i)$$

$$I = \int_2^4 \frac{e^{\left((6-t)^2+1\right)^{-1}}}{e^{\left((6-t)^2+1\right)^{-1}} + e^{(t^2+1)^{-1}}}dt \quad ...(ii)$$

$\left\{\text{Using}\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right\}$

Adding (i) and (ii) gives

$2I = \int dt \Rightarrow I = 1$

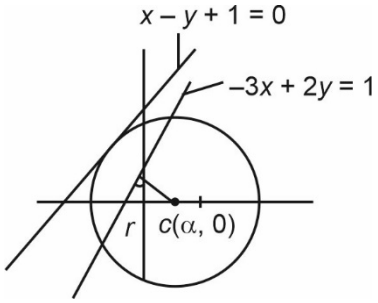
SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let the circle C touch the line $x - y + 1 = 0$, have the centre on the positive x-axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line $-3x + 2y = 1$. Let H be the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C. Then $2\alpha^2 + 3\beta^2$ is equal to _____.

Answer (19)

Sol.



$$r = \left|\frac{a+1}{\sqrt{2}}\right| \Rightarrow (a+1)^2 = 2r^2$$

Also $\left(\frac{3a-1}{\sqrt{13}}\right)^2 + \left(\frac{2}{\sqrt{13}}\right)^2 = r^2$

$$\Rightarrow \left(\frac{3a-1}{\sqrt{13}}\right)^2 + \frac{4}{13} = \frac{(a+1)^2}{2}$$

$5a^2 - 14a - 3 = 0$

$\therefore \quad a = -\frac{1}{5}, 3$

$\because \quad a \neq -\frac{1}{5} \Rightarrow a = 3$

$\Rightarrow \quad r = 2\sqrt{2}$

$\because$ One focus of $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ is $(3, 0)$

$\Rightarrow \quad \alpha e = 3$ and $2\alpha = 4\sqrt{2}$

$\Rightarrow \quad \alpha = 2\sqrt{2} \Rightarrow \alpha^2 = 8$

$$\alpha^2\left[1 + \frac{\beta^2}{\alpha^2}\right] = 9$$

$\alpha^2 + \beta^2 = 9$

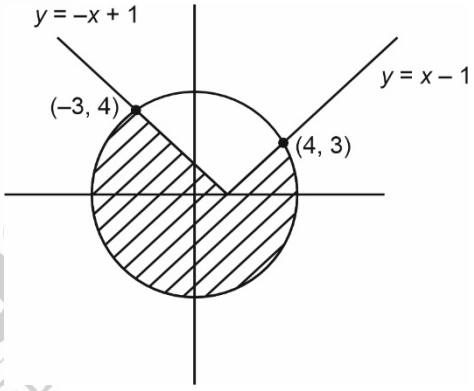
$\Rightarrow \quad \beta^2 = 1$

$\therefore \quad 2\alpha^2 + 3\beta^2 = 19$

22. If the area of the larger portion bounded between the curves $x^2 + y^2 = 25$ and $y = |x - 1|$ is $\frac{1}{4}(b\pi + c)$, $b, c \in \mathbb{N}$, then $b + c$ is equal to _____.

Answer (77)

Sol.



Area of shaded region

$$= 25\pi - \left[\int_{-3}^{4}\left(\sqrt{25 - x^2} - |x - 1|\right)dx\right]$$

$$= 25\pi - \left[\frac{1}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{-3}^{4} + \left(8 + \frac{9}{2}\right)$$

$$= \frac{1}{4}[75\pi + 2]$$

$\Rightarrow \quad b = 75, c = 2$

$\therefore \quad b + c = 77$

23. The sum of all rational terms in the expansion of $\left(1+2^{\frac{1}{3}}+3^{\frac{1}{2}}\right)^6$ is equal to _____

Answer (612)

Sol. The general term of multinomial expansion is

$$\frac{6!}{\alpha!\,\beta!\,\gamma!}(1)^{\alpha}\left(2^{\frac{1}{3}}\right)^{\beta}\left(3^{\frac{1}{2}}\right)^{\gamma}$$

For terms to be rational $3|\beta$ and $2|\gamma$

β	γ	α	Term
0	0	6	1
0	2	4	$15.3 = 45$
0	4	2	$15.3^2 = 135$
0	6	0	$1.3^3 = 27$
3	0	3	$20.2 = 40$
3	2	1	$60.2.3 = 360$
6	0	0	$1.4 = 4$

$\Rightarrow$ Sum of rational terms

$= 1 + 45 + 135 + 27 + 40 + 360 + 4 = 612$

24. If the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ has equal roots, where $a + c = 15$ and $b = \frac{36}{5}$, then $a^2 + c^2$ is equal to _____.

Answer (117)

Sol. Clearly one root is 1, another root is also 1.

Product of roots = 1

$$\frac{c(a-b)}{a(b-c)} = 1$$

$c(a-b) = a(b-c)$

$ac - bc = ab - ac$

$2ac = b(a+c)$

$$2ac = \frac{36}{5}(15)$$

$ac = 18 \times 3 = 54$

$\therefore\ a^2 + c^2 = (a+c)^2 - 2ac$

$= (15)^2 - 2(54)$

$= 225 - 108$

$= 117$

25. If the set of all values of a, for which the equation $5x^3 - 15x - a = 0$ has three distinct real roots, is the interval (α, β), then $\beta - 2\alpha$ is equal to _____.

Answer (30)

Sol. $5x^3 - 15x - a = 0$ has 3 distinct real solution

$\therefore$ Let $f(x) = 5x^3 - 15x - a$

$f'(x) = 15x^2 - 15 = 0$

$15x^2 = 15$

$x^2 = 1$

$\boxed{x = \pm 1}$

$\therefore\ f(1)\, f(-1) < 0$

$(a - 10)(a + 10) < 0$

$a \in (-10, 10)$

$\therefore\ \beta - 2\alpha = 10 + 2(10) = 30$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. A solid sphere of mass 'm' and radius 'r' is allowed to roll without slipping from the highest point of an inclined plane of length 'L' and makes an angle 30° with the horizontal. The speed of the particle at the bottom of the plane is v_1. If the angle of inclination is increased to 45° while keeping L constant. Then the new speed of the sphere at the bottom of the plane is v_2. The ratio $v_1^2 : v_2^2$ is

(1) 1 : 3

(2) $1:\sqrt{2}$

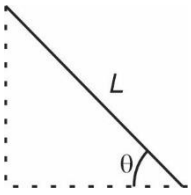
(3) 1 : 2

(4) $1:\sqrt{3}$

Answer (2)

Sol. Loss in P.E. = gain in K.E.

$$mgL\sin\theta = \frac{1}{2}\left(\frac{7}{5}\right)mv^2$$



$$\Rightarrow \quad v^2 \propto \sin\theta$$

$$\frac{v_1^2}{v_2^2} = \frac{\sin 30^\circ}{\sin 45^\circ} = \frac{1}{\sqrt{2}}$$

27. A gun fires a lead bullet of temperature 300 K into a wooden block. The bullet having melting temperature of 600 K penetrates into the block and melts down. If the total heat required for the process is 625 J, then the mass of the bullet is _____ grams.

(Latent heat of fusion of lead = 2.5×10^4 JKg^{-1} and specific heat capacity of lead 125 $JKg^{-1}\,K^{-1}$)

(1) 20 (2) 5

(3) 10 (4) 15

Answer (3)

Sol. $Q = ms\Delta T + mL$

$625 = m \times 125 \times 300 + m \times 2.5 \times 10^4$

$625 = m\{3.75 + 2.5\} \times 10^4$

$$\Rightarrow \frac{625}{6.25}\times 10^{-4}\ \text{kg} = 10\,\text{g} = m$$

28. A radioactive nucleus n_2 has 3 times the decay constant as compared to the decay constant of another radioactive nucleus n_1. If initial number of both nuclei are the same, what is the ratio of number of nuclei of n_2 to the number of nuclei of n_1, after one half-life of n_1?

(1) 1/8 (2) 1/4

(3) 4 (4) 8

Answer (2)

Sol. $\lambda_2 = 3\lambda \qquad \lambda_1 = \lambda$

$$N = N_0 e^{-\lambda t}$$

$$N_1 = N_0 e^{-\lambda t}$$

$$N_2 = N_0 e^{-3\lambda t}$$

$$\frac{N_2}{N_1} = e^{-2\lambda t}$$

for $t = \frac{\ln 2}{\lambda}$

$\Rightarrow \frac{N_2}{N_1} = e^{-2\lambda\frac{\ln 2}{\lambda}}$

$\frac{N_2}{N_1} = \frac{1}{4}$

29. The electric flux is $\phi = \alpha\sigma + \beta\lambda$

Where λ and σ are linear and surface charge density, respectively. $\left(\frac{\alpha}{\beta}\right)$ represents

(1) Area (2) Displacement

(3) Electric field (4) Charge

Answer (2)

Sol. $\alpha \equiv \frac{\phi}{\sigma}$

$\beta = \frac{\phi}{\lambda}$

$\frac{\alpha}{\beta} \equiv \frac{\lambda}{\sigma} \equiv$ displacement

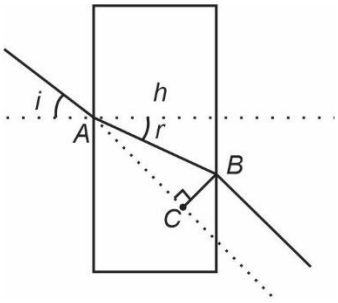
30. What is the lateral shift of a ray refracted through a parallel-sided glass slab of thickness '*h*' in terms of the angle of incidence '*i* ' and angle refraction '*r*', if the glass slab is placed in air medium?

(1) h (2) $\frac{h\sin(i-r)}{\cos r}$

(3) $\frac{h\tan(i-r)}{\tan r}$ (4) $\frac{h\cos(i-r)}{\sin r}$

Answer (2)

Sol.



$AB = h \sec r$

$BC = h \sec r \sin(i - r)$

$BC = \frac{h\sin(i-r)}{\cos r}$

31. Match the **List-I** with **List-II**

List-I		List-II	
A.	Pressure varies inversely with volume of an ideal gas.	I.	Adiabatic process
B.	Heat absorbed goes partly to increase internal energy and partly to do work.	II.	Isochoric process
C.	Heat is neither absorbed nor released by a system.	III.	Isothermal process
D.	No work is done on or by a gas.	IV.	Isobaric process

Choose the correct answer from the options given below:

(1) A-III, B-I, C-IV, D-II (2) A-I, B-IV, C-II, D-III

(3) A-III, B-IV, C-I, D-II (4) A-I, B-III, C-II, D-IV

Answer (3)

Sol. $PV = \mu RT$

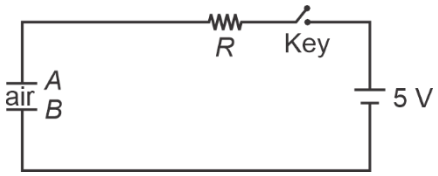
(A) $P \propto \frac{1}{V} \Rightarrow$ isothermal

(B) $\Delta Q = \Delta V + W \Rightarrow$ isobaric

(C) $\Delta Q = 0 \Rightarrow$ adiabatic

(D) $W = 0 \Rightarrow$ isochoric

32. Identify the valid statements relevant to the given circuit at the instant when the key is closed.



A. There will be no current through resistor R.

B. There will be maximum current in the connecting wires.

C. Potential difference between the capacitor plates A and B is minimum.

D. Charge on the capacitor plates is minimum.

Choose the correct answer from the options given below:

(1) C, D only (2) A, B, D only

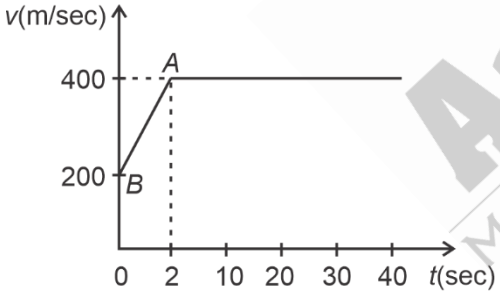
(3) A, C only (4) B, C, D only

Answer (4)

Sol. Capacitor behaves like closed circuit at $t = 0$ and charge is zero.

A ✗, B ✓, C ✓, D ✓

33. The motion of an airplane is represented by velocity-time graph as shown below. The distance covered by airplane in the first 30.5 second is _____ km.



(1) 12 (2) 3

(3) 6 (4) 9

Answer (1)

Sol. Distance = area under the graph

$d = 300 \times 2 + 400 \times 28.5$

$= 600 + 114000$

$= 12000$ m

34. Given below are two statements:

Statement I: The hot water flows faster than cold water.

Statement II: Soap water has higher surface tension as compared to fresh water.

In the light of above statements, choose the **correct** answer from the options given below

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

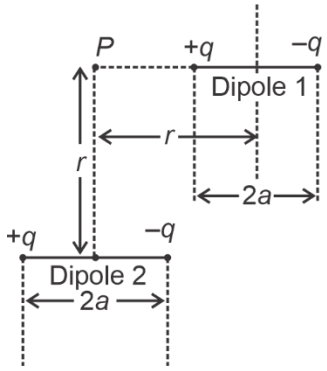
(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Answer (4)

Sol. Hot water flows faster because of less viscosity (✓) and soap water has less surface tension because bubbles are easily formed (✗)

35. A point particle of charge Q is located at P along the axis of an electric dipole 1 at a distance r as shown in the figure. The point P is also on the equatorial plane of a second electric dipole 2 at a distance r. The dipoles are made of opposite charge q separated by a distance $2a$. For the charge particle at P not to experience any net force, which of the following correctly describes the situation?

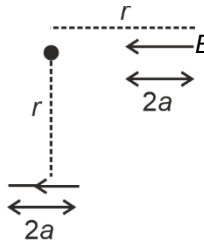


(1) $\frac{a}{r} \sim 3$ (2) $\frac{a}{r} \sim 0.5$

(3) $\frac{a}{r} \sim 10$ (4) $\frac{a}{r} \sim 20$

Answer (1)

Sol.



$E_1 = E_2$, taking $kq = 1$

$$\Rightarrow \frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} = \frac{2a}{\left(a^2+r^2\right)^{3/2}}$$

$$\frac{4ar}{\left(r^2-a^2\right)^2} = \frac{2a}{\left(a^2+r^2\right)^{3/2}}$$

$$\left(r^2-a^2\right)^2 = 2r\left(a^2+r^2\right)^{3/2}$$

$$\left(1-\frac{a^2}{r^2}\right)^2 = 2\left(1+\frac{a^2}{r^2}\right)^{3/2}$$

$$\left(1-x^2\right)^2 = 2\left(1+x^2\right)^{3/2} \quad \left(x = \frac{a}{i}\right)$$

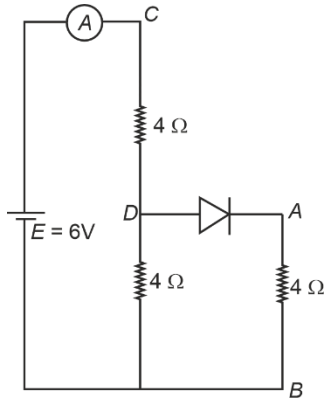
$$\frac{\left(1-x^2\right)^2}{\left(1+x^2\right)^{3/2}} = 2$$

Now for x = 3

We get $\frac{64}{10\sqrt{10}} \approx 2 \Rightarrow \frac{a}{r} \approx 3$

[But for $a > r$ point charge will between the dipole where $\vec{E} \neq 0$]

36. Refer to the circuit diagram given in the figure. which of the following observations are correct?



A. Total resistance of circuit is 6 Ω

B. Current in Ammeter is 1 A

C. Potential across *AB* is 4 volts.

D. Potential across *CD* is 4 volts

E. Total resistance of the circuit is 8 Ω.

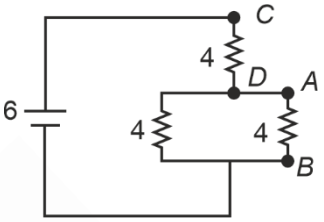
Choose the correct answer from the options given below:

(1) A, B and D Only (2) B, C and E Only

(3) A, B and C Only (4) A, C and D Only

Answer (1)

Sol.



R_{eq} (4 ↑↑ 4) + 4 = 6 Ω

$$i = \frac{6}{6}\text{A}$$

$$V_{CD} = 1 \times 4 = 4\text{V} \Rightarrow V_{AB} = 2\text{V}$$

(A, B, D)

37. Given a thin convex lens (refractive index μ_2), kept in a liquid (refractive index μ_1, $\mu_1 < \mu_2$) having radii of curvatures $|R_1|$ and $|R_2|$. Its second surface is silver polished. Where should an object be placed on the optic axis so that a real and inverted image is formed at the same place?

(1) $\frac{(\mu_2+\mu_1)|R_1|}{(\mu_2-\mu_1)}$

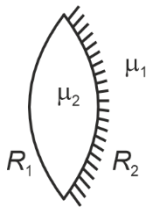
(2) $\frac{\mu_1|R_1|.|R_2|}{\mu_2(|R_1|+|R_2|)-\mu_1|R_1|}$

(3) $\frac{\mu_1|R_1|.|R_2|}{\mu_2(|R_1|+|R_2|)-\mu_1|R_2|}$

(4) $\frac{\mu_1|R_1|.|R_2|}{\mu_2(2|R_1|+|R_2|)-\mu_1\sqrt{|R_1|.|R_2|}}$

Answer (3)

Sol.



$$\frac{1}{f_l}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$f_m=-\frac{R_2}{2}$$

$$p_{eq}=2p_l+pm$$

$$\frac{1}{|f|}=\frac{2(\mu_2-\mu_1)}{\mu_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{2}{R_2}$$

$$=\frac{2(\mu_2-\mu_1)(R_1+R_2)}{\mu_1(R_1R_2)}+\frac{2}{R_2}$$

$$\frac{1}{f}=\frac{2\mu_2R_1+2\mu_2R_2-2\mu_1R_1-2\mu_1R_2+2}{\mu_1R_1R_2}$$

$$\Rightarrow\quad f=\frac{\mu_1R_1R_2}{2\mu_2R_1+2\mu_2R_2-2\mu_1R_2}$$

Required distance = 2f.

38. A sub-atomic particle of mass 10^{-30} kg is moving with a velocity 2.21×10^{6} m/s. Under the matter wave consideration, the particle will behave closely like _____.

($h = 6.63\times10^{-34}$ J.s)

(1) Gamma rays (2) X-rays

(3) Visible radiation (4) Infra-red radiation

Answer (2)

Sol. $$\lambda=\frac{6.63\times10^{-34}}{10^{-30}\times2.21\times10^{6}}$$

$\lambda = 3\times10^{-10}$

$\lambda = 3$ Å

$\Rightarrow$ X Ray

39. Regarding self-inductance:

A. The self-inductance of the coil depends on its geometry.

B. Self-inductance does not depend on the permeability of the medium.

C. Self-induced e.m.f. opposes any change in the current in a circuit.

D. Self-inductance is electromagnetic analogue of mass in mechanics.

E. Work needs to be done against self-induced e.m.f. in establishing the current.

Choose the correct answer from the options given below:

(1) A, C, D, E only (2) A, B, C, E only

(3) A, B, C, D only (4) B, C, D, E only

Answer (1)

Sol. (A) $L = \mu_r\mu_0n^2$ Al ✓

(B) $L = \mu_r\ \mu_vn^2$ Al ✗

(C) ✓

(D) ✓

(E) ✓

40. Consider a moving coil galvanometer (MCG):

A. The torsional constant in moving coil galvanometer has dimentions $[ML^2T^{-2}]$

B. Increasing the current sensitivity may not necessarily increase the voltage sensitivity.

C. If we increase number of turns (N) to its double (2N), then the voltage sensitivity doubles.

D. MCG can be converted into an ammeter by introducing a shunt resistance of large value in parallel with galvanometer.

E. Current sensitivity of MCG depends inversely on number of turns of coil.

Choose the correct answer from the options given below:

(1) A, D Only (2) A, B Only

(3) B, D, E Only (4) A, B, E Only

Answer (2)

Sol. (A) $\tau = C\theta$

$C \equiv ML^2T^{-2}$ ✓

(B) $C\theta = iANB$

$\frac{\theta}{i} = \frac{ANB}{C}$ here $N \uparrow, \frac{\theta}{i} \uparrow$

$\frac{\theta}{v} = \frac{ANB}{CR}$ here $\frac{N}{R}$ cannot increase

✓

(C) X; Theoretical

(D) Shunt of law of resistance X

(E) X; explained in [B]

41. A light hollow cube of side length 10 cm and mass 10 g, is floating in water. It is pushed down and released to execute simple harmonic oscillations. The time period of oscillations is $y\pi \times 10^{-2}$ s, where the value of y is

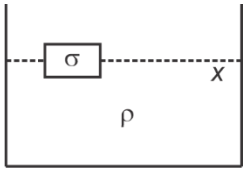
(Acceleration due to gravity, g = 10 m/s^2, density of water = 10^3 kg/m^3)

(1) 2 (2) 6

(3) 4 (4) 1

Answer (1)

Sol. Additional buoyant force



$g\rho\ a^2x = \sigma a^3 A$

$A = \frac{\rho}{\sigma}\frac{g}{a}x$

$T = 2\pi\sqrt{\frac{\sigma a}{\rho g}}$

Now, $\sigma = \frac{10\times10^{-3}}{10^{-3}} = 10$

$\Rightarrow T = 2\pi\sqrt{\frac{10\times0.1}{10^3\times10}}$

$= 2\pi\times10^{-2}$

42. The position of a particle moving on x-axis is given by $x(t) = A\sin t + B\cos^2 t + Ct^2 + D$, where t is time. The dimension of $\frac{ABC}{D}$ is

(1) L^2T^{-2} (2) L^3T^{-2}

(3) L (4) L^2

Answer (1)

Sol. $A \equiv L$

$B \equiv L$

$C \equiv LT^{-2}$

$D \equiv L$

$\Rightarrow \frac{ABC}{D} = \frac{L\times L\times LT^2}{L} = L^2T^{-2}$

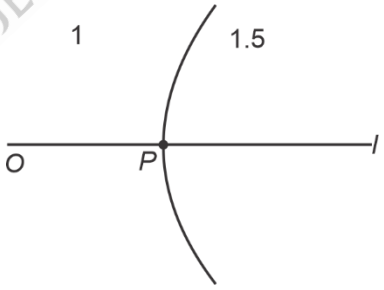
43. A spherical surface of radius of curvature R, separates air from glass (refractive index = 1.5). The centre of curvature is in the glass medium. A point object 'O' placed in air on the optic axis of the surface, so that its real image is formed at 'I' inside glass. The line OI intersects the spherical surface at P and $PO = PI$. The distance PO equals to

(1) $3R$ (2) $2R$

(3) $1.5R$ (4) $5R$

Answer (4)

Sol.



$PO = PI$

$\frac{1.5}{x} - \frac{1}{-x} = \frac{0.5}{R}$

$\frac{2.5}{x} = \frac{0.5}{R}$

$x = 5R$

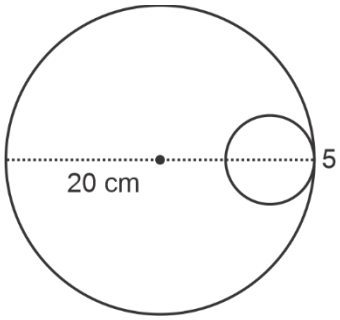
44. Consider a circular disc of radius 20 cm with centre located at the origin. A circular hole of radius 5 cm is cut from this disc in such a way that the edge of the hole touches the edge of the disc. The distance of centre of mass of residual or remaining disc from the origin will be

(1) 1.5 cm (2) 0.5 cm

(3) 1.0 cm (4) 2.0 cm

Answer (3)

Sol.



$M \propto A$

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{A(0) + \left(-\frac{A}{16}\right)(15)}{A - \frac{A}{16}}$$

$$= \frac{-15}{16-1} = -1\,\text{cm}$$

45. The electric field of an electromagnetic wave in free space is

$\vec{E} = 57\cos\left[7.5\times10^6 t - 5\times10^{-3}(3x+4y)\right]$

$\left(4\hat{i} - 3\hat{j}\right)$N/C. The associated magnetic field in Tesla is

(1) $\vec{B} = -\frac{57}{3\times10^8}\cos\left[7.5\times10^6 t - 5\times10^{-3}(3x+4y)\right]\left(\hat{k}\right)$

(2) $\vec{B} = \frac{57}{3\times10^8}\cos\left[7.5\times10^6 t - 5\times10^{-3}(3x+4y)\right]\left(5\hat{k}\right)$

(3) $\vec{B} = -\frac{57}{3\times10^8}\cos\left[7.5\times10^6 t - 5\times10^{-3}(3x+4y)\right]\left(5\hat{k}\right)$

(4) $\vec{B} = \frac{57}{3\times10^8}\cos\left[7.5\times10^6 t - 5\times10^{-3}(3x+4y)\right]\left(\hat{k}\right)$

Answer (2)

Sol. $E = CB$

and $\vec{E}\times\vec{B} \uparrow\uparrow \vec{C}$

$$B = \frac{57\times5}{3\times10^8}$$

$3\hat{i} + 4\hat{j}$ is direction of propagation

As, $\vec{E}\times\vec{B} \uparrow\uparrow 3\hat{i} + 4\hat{j}$

$$\Rightarrow \left(4\hat{i} - 3\hat{j}\right)\times\left(-\hat{k}\right) = 4\hat{j} + 3\hat{i}$$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. A positive ion A and a negative ion B has charges 6.67×10^{-19} C and 9.6×10^{-10} C, and masses 19.2×10^{-27} kg and 9×10^{-27} kg respectively. At an instant, the ions are separated by a certain distance r. At that instant the ratio of the magnitudes of electrostatic force to gravitational force is $P \times 10^{-23}$, where the value of P is _____.

(Take $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ Nm^{2}C^{-1} and universal gravitational constant as 6.67×10^{-11} Nm^{2}kg^{-2})

Answer (None)

Sol. $F_e = \frac{kq_1q_2}{r^2}$

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$\frac{F_e}{F_g} = \frac{kq_1q_2}{Gm_1m_2}$$

$$= \frac{9\times10^9 \times 6.67\times10^{-19} \times 9.6\times10^{-10}}{6.67\times10^{-11}\times19.2\times10^{-27}\times9\times10^{-27}}$$

$$= \frac{10^{-20}}{2\times10^{-65}}$$

***Answer does not match with description.**

47. An ideal gas initially at 0°C temperature, is compressed suddenly to one fourth of its volume. If the ratio of specific heat at constant pressure to that at constant volume is 3/2, the change in temperature due to the thermodynamic process is _____ K.

Answer (273)

Sol. $TV^{\gamma-1} = \text{constant}$

$$273 \times V^{0.5} = T\left(\frac{V}{4}\right)^{0.5}$$

$T = 546$

$\Delta T = 273$ K

48. A force $f = x^2y\hat{i} + y^2\hat{j}$ acts on a particle in a plane $x + y = 10$. The work done by this force during a displacement from (0, 0) to (4m, 2m) _____ Joule (round off to the nearest integer)

Answer (152)

Sol. $y = 10 - x$

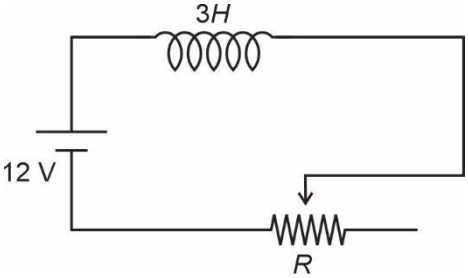
$$w = \int_0^4 x^2(10-x)dx + \int_0^2 y^2 dy$$

$$= \frac{10x^3}{3} - \frac{x^4}{4}\Big|_0^4 + \frac{y^3}{3}\Big|_0^2$$

$$= \frac{640}{3} - \frac{256}{4} + \frac{8}{3}$$

$= 216 \times 64$

$= 152$ J

49. 

In the given circuit the sliding contact is pulled outwards such that electric current in the circuit changes at the rate of 8 A/s. At an instant when R is 12 Ω, the value of the current in the circuit will be _____ A.

Answer (1)

Sol. $\varepsilon - \frac{Ldi}{dt} = iR = 0$

$12 - 3 \times 8 - i\,12 = 0$

$12 = i\,12$

$i = 1$

50. Two particles are located at equal distance from origin. The position vectors of those are represented by $\vec{A} = 2\hat{i} + 3n\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4p\hat{k}$, respectively. If both the vectors are at right angle to each other, the value of n^{-1} is _____.

Answer (3)

Sol. $\vec{A}\cdot\vec{B} = 0$ and $|\vec{A}| = |\vec{B}|$

$\Rightarrow$ $4 - 6n + 8p = 0$

$3n - 4p = 2$ …(i)

Also $4 + 9n^2 + 4 = 4 + 4 + 16p^2$

$3n = \pm 4p$ …(ii)

$\pm 4p - 4p = 2$

Taking –ve sign

$-8p = 2$

$p = -\frac{1}{4}$

$3n + 1 = 2$

$n = \frac{1}{3}$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. 2.8×10^{-3} mol of CO_2 is left after removing 10^{21} molecules from its 'x' mg sample. The mass of CO_2 taken initially is

Given : $N_A = 6.02 \times 10^{23}$ mol^{-1}

(1) 98.3 mg (2) 48.2 mg

(3) 150.4 mg (4) 196.2 mg

Answer (4)

Sol. Moles of removed $CO_2 = \frac{10^{21}}{6.02 \times 10^{23}}$ mol

$= 1.66 \times 10^{-3}$ mol

mole of CO_2 left = 2.8×10^{-3} moles

total moles of CO_2 taken initially

$= (2.8 + 1.66) \times 10^{-3}$ mol

mass of CO_2 taken initially

$= 4.46 \times 10^{-3} \times 44$

$= 196.24 \times 10^{-3}$ g

= 196.24 mg

52. $FeO_4^{2-} \xrightarrow{+2.0\,V} Fe^{3+} \xrightarrow{0.8\,V} Fe^{2+} \xrightarrow{-0.5\,V} Fe^0$

In the above diagram, the standard electrode potentials are given in volts (over the arrow).

The value of $E^{\ominus}_{FeO_4^{2-}/Fe^{2+}}$ is

(1) 1.7 V (2) 2.1 V

(3) 1.4 V (4) 1.2 V

Answer (1)

Sol. $FeO_4^{2-} + 3e^- \rightarrow Fe^{3+}$; $E^\circ = 2.0$ eV

$Fe^{3+} + e^- \rightarrow Fe^{2+}$; $E^\circ = 0.8$ eV

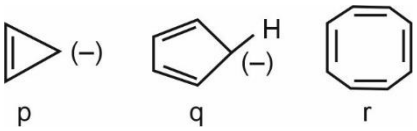
$FeO_4^{2-} + 4e^- \rightarrow Fe^{2+}$; $E^\circ = x$ (say)

$4 \times x = 3 \times 2 + 1 \times 0.8$

$x = \frac{6.8}{4}$ V

$= 1.7$ V

53. The correct stability order of the following species/molecules is



p q r

(1) q > p > r (2) q > r > p

(3) r > q > p (4) p > q > r

Answer (2)

Sol. q is aromatic

r is non-aromatic

p is antiaromatic

q > r > p (order of stability)

Aromatic > non-aromatic > antiaromatic

54. Ice at –5ºC is heated to become vapour with temperature of 110ºC at atmospheric pressure. The entropy change associated with this process can be obtained from

(1) $\int_{268K}^{273K} C_{p,m}dT + \frac{\Delta H_{m,fusion}}{T_f} + \frac{\Delta H_{m,vaporisation}}{T_b} + \int_{273K}^{373K} C_{p,m}dT + \int_{373K}^{383K} C_{p,m}dT$

(2) $\int_{268K}^{383K} C_p\,dT + \frac{\Delta H_{melting}}{273} + \frac{\Delta H_{boiling}}{373}$

(3) $\int_{268K}^{273K} \frac{C_{p,m}}{T}dT + \frac{\Delta H_{m,fusion}}{T_f} + \frac{\Delta H_{m,vaporisation}}{T_b} + \int_{273K}^{373K} \frac{C_{p,m}dT}{T} + \int_{373K}^{383K} \frac{C_{p,m}dT}{T}$

(4) $\int_{268K}^{383K} C_p dT + \frac{q_{rev}}{T}$

Answer (3)

Sol. $\underset{-5^\circ C}{H_2O(s)} \rightarrow \underset{0^\circ C}{H_2O(s)}\ ;\ \Delta S_1 = \int_{268K}^{273K} \frac{C_{p,\,m}dT}{T}$

$\underset{0^\circ C}{H_2O(s)} \rightleftharpoons \underset{0^\circ C}{H_2O(l)}\ ;\ \Delta S_2 = \frac{\Delta H_{m,\,fus}}{273}$

$\underset{0^\circ C}{H_2O(l)} \rightarrow \underset{100^\circ C}{H_2O(l)}\ ;\ \Delta S_3 = \int_{273}^{373} \frac{C_{p,\,m}dt}{T}$

$\underset{100^\circ C}{H_2O(l)} \rightleftharpoons \underset{100^\circ C}{H_2O(g)}\ ;\ \Delta S_4 = \frac{\Delta H_{m,\,vap}}{373}$

$\underset{100^\circ C}{H_2O(g)} \rightarrow \underset{110^\circ C}{H_2O(g)}\ ;\ \Delta S_5 = \int_{373}^{383} \frac{C_{p,\,m}dT}{T}$

$\Delta S_{total} = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4 + \Delta S_5$

55. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H atom is suitable for this?

Given: Rydberg constant $R_H = 10^5\ cm^{-1}$, $h = 6.6 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m/s)

(1) Lyman series, $\infty \rightarrow 1$

(2) Paschen series, $5 \rightarrow 3$

(3) Paschen series, $\infty \rightarrow 3$

(4) Balmer series, $\infty \rightarrow 2$

Answer (3)

Sol. $\frac{1}{\lambda} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\frac{1}{\lambda} = 10^5 \left(\frac{1}{3^2} - \frac{1}{\infty^2}\right)$

$\frac{1}{\lambda} = 10^5 \times \frac{1}{9}$

$\lambda = 9 \times 10^{-5}$

$\lambda = 900 \times 10^{-7} cm$

$= 900$ Å

56. The major product of the following reaction is

$$CH_3CH_2CH = O \xrightarrow[\text{reflux}]{\text{excess HCHO alkali}} ?$$

(1) $CH_3 - CH_2 - CH_2 - OH$

(2) $CH_3 - \underset{\displaystyle CH_2 - OH}{\overset{\displaystyle |}{\underset{\displaystyle |}{CH}}} - CH = O$

(3) $CH_3 - \underset{\displaystyle CH_2 - OH}{\overset{\displaystyle CH_2 - OH}{\overset{\displaystyle |}{\underset{\displaystyle |}{C}}}} - CH_2 - OH$

(4) $CH_3 - \underset{\displaystyle CH_2}{\overset{\displaystyle O}{\overset{\displaystyle ||}{\underset{\displaystyle |}{C}}}} - CH = O$

Answer (3)

Sol. Propanal undergoes aldol condensation with excess of HCHO in presence of OH^- ions to 2,2-dihydroxy methyl propanal which further reacts with HCHO and undergoes Cannizzaro reaction to give 2,2-hydroxymethylpropan-1-ol.

$$CH_3 - CH_2 - \underset{\displaystyle |}{\overset{\displaystyle O}{\overset{\displaystyle ||}{C}}} - H + 2HCHO \xrightarrow{OH^-} \underset{\displaystyle CH_2 - OH}{\overset{\displaystyle CH_2OH}{\overset{\displaystyle |}{\underset{\displaystyle |}{CH_3 - C - CHO}}}}$$

$$\underset{\displaystyle CH_2 - OH}{\overset{\displaystyle |}{\underset{\displaystyle |}{CH_3 - C - CHO}}} + HCHO \xrightarrow[\text{reflux}]{OH^-} \underset{\displaystyle CH_2 - OH}{\overset{\displaystyle CH_2 - OH}{\overset{\displaystyle |}{\underset{\displaystyle |}{CH_3 - C - CH_2OH}}}} + HCOO^-$$

57. Match the **LIST-I** with **LIST-II**.

	LIST-I (Classification of molecules based on octet rule)		**LIST-II (Example)**
A.	Molecules obeying octet rule	I.	NO, NO_2
B.	Molecules with incomplete octet	II.	BCl_3, $AlCl_3$
C.	Molecules with incomplete octet with odd electron	III.	H_2SO_4, PCl_5
D.	Molecules with expanded octet	IV.	CCl_4, CO_2

Choose the ***correct*** answer from the options given below.

(1) A-IV, B-I, C-III, D-II (2) A-II, B-IV, C-III, D-I

(3) A-IV, B-II, C-I, D-III (4) A-III, B-II, C-I, D-IV

Answer (3)

Sol. $NO = 7e^-$, $NO_2 = 7e^-$ } Incomplete octet with odd electron

$BCl_3 = 6e^-$, $AlCl_3 = 6e^-$ } Incomplete octet

$H_2SO_4 = 12e^-$, $PCl_5 = 10e^- \Rightarrow$ molecules with expanded octet

$CCl_4 = 8e^-$, $CO_2 = 8e^- \Rightarrow$ molecules obeying octet rule

58. Math the **LIST-I** with **LIST-II**.

	LIST-I (Name reaction)		**LIST-II (Product obtainable)**
A.	Swarts reaction	I.	Ethyl benzene
B.	Sandmeyer's reaction	II.	Ethyl iodide
C.	Wurtz-Fittig reaction	III.	Cyanobenzene
D.	Finkelstein reaction	IV.	Ethyl fluoride

Choose the ***correct*** answer from the options given below.

(1) A-II, B-I, C-III, D-IV (2) A-IV, B-I, C-III, D-II

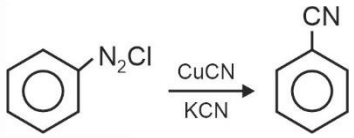
(3) A-IV, B-III, C-I, D-II (4) A-II, B-III, C-I, D-IV

Answer (3)

Sol. Swarts rection : Halogen exchange rxn

$$CH_3 - Br + AgF \xrightarrow{DMF} CH_3F$$

Sandmeyer's reaction :



Wurtz-Fittig reaction :

$$Ph - Br + C_2H_5Br \xrightarrow[\text{Dry ether}]{Na} Ph - C_2H_5$$

Finkelstein reaction : Halogen exchange rxn

$$R - Cl \xrightarrow[\text{Acetone}]{NaI} R - I$$

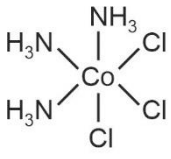
59. The complex that shows Facial - Meridional isomerism is

(1) $[Co(en)_2Cl_2]^+$ (2) $[Co(en)_3]^{3+}$

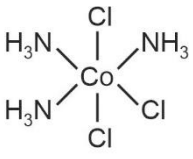
(3) $[Co(NH_3)_4Cl_2]^+$ (4) $[Co(NH_3)_3Cl_3]$

Answer (4)

Sol. Ma_3b_3 type of complex can show Facial-Meridional type isomerism.(where a and b are monodentate ligands)



Facial



Meridional

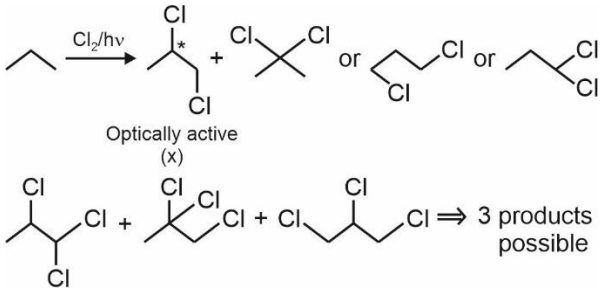
60. Propane molecule on chlorination under photochemical condition gives two di-chloro products, "x" and "y". Amongst "x" and "y", "x" is an optically active molecule. How many tri-chloro products (consider only structural isomers) will be obtained from "x' when it is further treated with chlorine under the photochemical condition?

(1) 4 (2) 3

(3) 2 (4) 5

Answer (2)

Sol.



61. The correct set of ions (aqueous solution) with same colour from the following is:

(1) Ti^{4+}, V^{4+}, Mn^{2+} (2) Zn^{2+}, V^{3+}, Fe^{3+}

(3) V^{2+}, Cr^{3+}, Mn^{3+} (4) Sc^{3+}, Ti^{3+}, Cr^{2+}

Answer (3)

Sol. V^{2+} = Violet

Cr^{3+} = Violet

Mn^{3+} = Violet

62. $CrCl_3.xNH_3$ can exist as a complex. 0.1 molal aqueous solution of this complex shows a depression in freezing point of 0.558°C. Assuming 100% ionisation of this complex and coordination number of Cr is 6, the complex will be

(Given K_f = 1.86 K kg mol^{-1})

(1) $[Cr(NH_3)_4Cl_2]$ Cl (2) $[Cr(NH_3)_3Cl_3]$

(3) $[Cr(NH_3)_6]$ Cl_3 (4) $[Cr(NH_3)_5Cl]$ Cl_2

Answer (4)

Sol. $\Delta T_f = iK_fm$

$0.558 = i \times 1.86 \times 0.1$

$$i = \frac{0.558}{0.186} = 3$$

Number of ions when 100% ionisation takes place = 3

$$[Cr(NH_3)_5Cl]Cl_2 \xrightarrow{\text{aqueous solution}} [Cr(NH_3)_5Cl]^{2+} + 2Cl^-$$

Number of ions = 3

63. What amount of bromine will be required to convert 2 g of phenol into 2,4,6-tribromophenol?

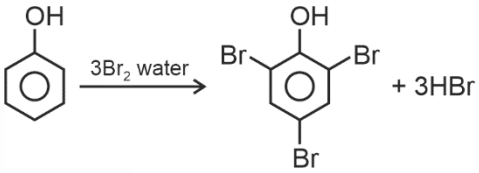
(Given molar mass in g mol^{-1} of C, H, O, Br are 12, 1, 16, 80 respectively)

(1) 4.0 g (2) 6.0 g

(3) 10.22 g (4) 20.44 g

Answer (3)

Sol.



Moles of phenol = $\frac{2}{94}$ mol

Moles of $Br_2 = \frac{3 \times 2}{94}$ moles

Mass of $Br_2 = \frac{3 \times 2}{94} \times 160$ g

= 10.22 g

64. Given below are two statements:

Statement I: Fructose does not contain an aldehydic group but still reduces Tollen's reagent.

Statement II: In the presence of base, fructose undergoes rearrangement to give glucose.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Answer (4)

Sol. Fructose has α-hydroxy ketone group which tautomerise to aldehyde group in presence of base, therefore it reduces Tollen's reagent.

In presence of base, fructose undergo rearrangement to give glucose and mannose.

65. The d-electronic configuration of an octahedral Co(II) complex having magnetic moment of 3.95 BM is:

(1) $t_{2g}^5 e_g^2$ (2) $t_{2g}^3 e_g^0$

(3) $t_{2g}^6 e_g^1$ (4) $e^4 t_2^3$

Answer (1)

Sol. Co^{2+} complex having μ = 3.95 BM

Hence number of unpaired electron = 3

$Co^{2+} \Rightarrow 3d^7 = t_{2g}^5 e_g^2$

66. Which of the following happens when NH_4OH is added gradually to the solution containing 1 M A^{2+} and 1 M B^{3+} ions?

Given : $K_{sp}[A(OH)_2] = 9 \times 10^{-10}$ and $K_{sp}[B(OH)_3] = 27 \times 10^{-18}$ at 298 K

(1) $B(OH)_3$ will precipitate before $A(OH)_2$

(2) $A(OH)_2$ will precipitate before $B(OH)_3$

(3) $A(OH)_2$ and $B(OH)_3$ will precipitate together

(4) Both $A(OH)_2$ and $B(OH)_3$ do not show precipitation with NH_4OH

Answer (1)

Sol. Condition for precipitation IP > K_{sp}

For, $[A(OH)_2]$

$[A^{2+}][OH^-]^2 > 9 \times 10^{-10}$

$[A^{2+}] = 1$ M

$[OH^-] > 3 \times 10^{-5}$

For, $[B(OH)_3]$

$[B^{3+}][OH^-]^3 > 27 \times 10^{-18}$

$[B^{3+}] = 1$ M

$[OH^-] > 3 \times 10^{-6}$

So, $B(OH)_3$ will precipitate first

67. The **incorrect** statement among the following is

(1) PF_3 exists but NF_5 does not

(2) PH_3 shows lower proton affinity than NH_3

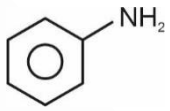
(3) SO_2 can act as an oxidizing agent, but not as a reducing agent

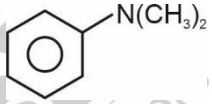
(4) NO_2 can dimerise easily

Answer (3)

Sol. $\overset{+4}{S}O_2$ can acts as both oxidising agent as well reducing agents because due to intermediate oxidation state, it can oxidise and reduce as well.

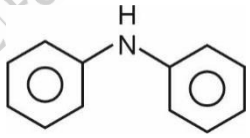
68. Which among the following react with Hinsberg's reagent?

A. 

B. 

C. $CH_3 — NH_2$

D. $N(CH_3)_3$

E. 

Choose the correct answer from the options given below

(1) A, B and E Only

(2) C and D Only

(3) A,C and E Only

(4) B and D Only

Answer (3)

Sol. Primary and secondary amines can reacts with Hinsberg reagent.

69. Given below are two statements

Statement I : In Lassaigne's test, the covalent organic molecules are transformed into ionic compounds.

Statement II: The sodium fusion extract of an organic compound having N and S gives prussian blue colour with $FeSO_4$ and $Na_4[Fe(CN)_6]$

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

(4) Statement I is true but Statement II is false

Answer (4)

Sol. Lassaigne's test is a general test for detection of halogen, nitrogen and sulphur in an organic compound. These elements covalently bonded to the organic compounds, In order to detect them, these have to converted into ionic forms.

$3Na_4[Fe(CN)_6] + 2Fe_2(SO_4)_3 \longrightarrow Fe_4[Fe(CN)_6]_3 + 6\ Na_2SO_4$ (prussian Blue)

70. The element that does not belong to the same period of the remaining elements (modern periodic table) is

(1) Iridium (2) Osmium

(3) Palladium (4) Platinum

Answer (3)

Sol. Ir, Os and Pt belong to 6^{th} period of periodic table while Pd belongs to 5^{th} period.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. For the thermal decomposition of $N_2O_5(g)$ at constant volume, the following table can be formed, for the reaction mentioned below.

$2N_2O_5(g) \rightarrow 2N_2O_4(g) + O_2(g)$

Sr. No.	Time/s	Total pressure/(atm
1	0	0.6
2	100	'x'

$x = \_\_\_ \times 10^{-3}$ atm [nearest integer]

Given : Rate constant for the reaction is $4.606 \times 10^{-2}\ s^{-1}$.

Answer (897)

Sol. $2N_2O_5(g) \rightarrow 2N_2O_4(g) + O_2(g)$

$$k = \frac{2.303}{t}\log\frac{0.9-0.6}{(0.9-x)}$$

$$2 \times 10^{-2} \times 100 = \log\frac{0.3}{(0.9-x)}$$

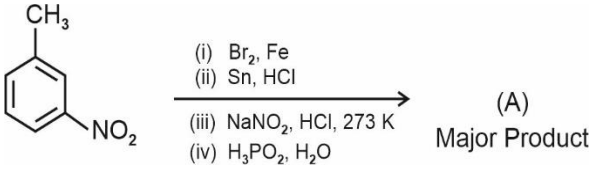
$$100 = \frac{0.3}{(0.9-x)}$$

$$= \frac{0.9-x}{0.3} = 0.01$$

$$0.9 - X = 0.003$$

$$= 897 \times 10^{-3}$$

72. Consider the following sequence of reactions to produce major product (A)

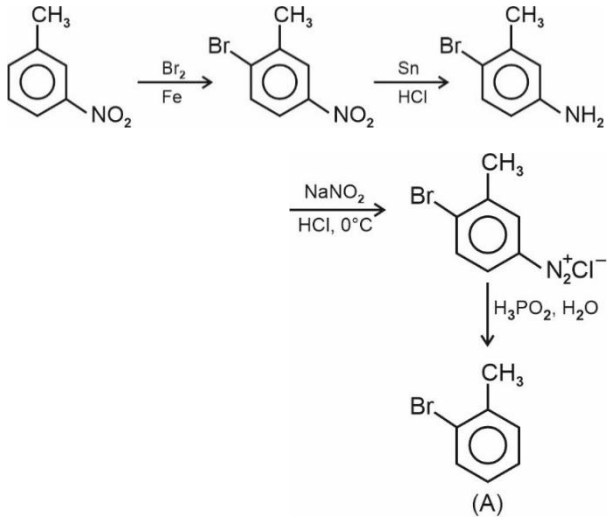


Molar mass of product (A) is ____g mol^{-1}.

(Given molar mass in g mol^{-1} of C : 12, H : 1, O : 16, Br : 80, N : 14, P : 31)

Answer (171)

Sol.



Molar mass of A = 171 g/mol

73. During "S" estimation, 160 mg of an organic compound gives 466 mg of barium sulphate. The percentage of Sulphur in the given compound is ____%.

(Given molar mass in g mol^{-1} of Ba : 137, S : 32, O : 16)

Answer (40)

Sol. m mole of $BaSO_4$ = mmoles of $S = \frac{466}{233}$

Mass of $S = \frac{466}{233} \times 32$ mg

= 64 mg

$\% \ S = \frac{64}{160} \times 100 = 40\%$

74. The standard enthalpy and standard entropy of decomposition of N_2O_4 to NO_2 are 55.0 kJ mol^{-1} and 175.0 J/K/mol respectively. The standard free energy change for this reaction at 25°C in J mol^{-1} is _____ (Nearest integer)

Answer (2850)

Sol. $\Delta H^\circ_{rxn} = 55.0 \text{ kJ/mol}$ T = 298 K

$\Delta S^\circ_{rxn} = 175 \text{ J/mol}$

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

= 55000 – 298 × 175

= 2850 J/mol

75. If 1 mM solution of ethylamine produces pH = 9, then the ionization constant (K_b) of ethylamine is 10^{-x}. The value of x is _____ (nearest integer).

[The degree of ionization of ethylamine can be neglected with respect to unity.]

Answer (7)

Sol. pOH + pH = 14

pOH = 14–9

pOH = 5

$[OH^-] = 10^{-5}$ m

$[OH^-] = \sqrt{K_b \cdot C}$

C = concentration of weak base = 1 mM = 10^{-3} M

$10^{-5} = \sqrt{K_b \times 10^{-3}}$

$10^{-10} = K_b \times 10^{-3}$

$K_b = 10^{-7}$

x = 7

❑ ❑ ❑