MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If the area of the region $\{(x,y): -1 \le x \le 1, 0 \le y \le a + e^{|x|} - e^{-x}, a > 0\}$ is $\frac{e^2+8e+1}{e}$, then the value of a is:

(1) 5 (2) 7

(3) 8 (4) 6

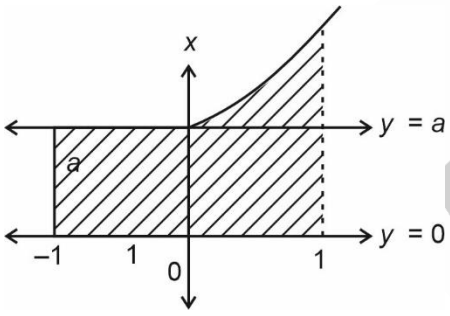
Answer (1)

Sol. $y \in \left[0, a + e^{|x|} - e^{-x}\right]$

(i) If $x \ge 0 \Rightarrow y \in \left(0, a + e^{x} - \frac{1}{e^x}\right)$

if $x < 0 \Rightarrow y \in \left(0, a + e^{-x} - e^{-x}\right)$

$\Rightarrow \quad y \in (0, a)$



$$\text{Area} = (a) + \int_0^1 (a + e^x - e^{-x})\,dx = \frac{e^2+8e+1}{e}$$

$$= a + (ax + e^x + e^{-x})\Big|_0^1 = e + 8 + \frac{1}{e}$$

$$= a + \left(a + e + \frac{1}{e} - 2\right) = e + \frac{1}{e} + 8$$

$\Rightarrow \quad 2a - 2 = 8 \Rightarrow a = 5$

2. If $I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}\,dx$, then

$\int_0^{2I} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$ equals:

(1) $\frac{\pi^2}{12}$ (2) $\frac{\pi^2}{16}$

(3) $\frac{\pi^2}{4}$ (4) $\frac{\pi^2}{8}$

Answer (2)

Sol. $I = \int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\frac{3}{2}}\,dx}{(\sin x)^{\frac{3}{2}} + (\cos x)^{\frac{3}{2}}} = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)dx}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}$

$\Rightarrow$ Adding $2I = \int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\frac{3}{2}} + (\cos x)^{\frac{3}{2}}}{(\sin x)^{\frac{3}{2}} + (\cos x)^{\frac{3}{2}}}\,dx = \frac{\pi}{2}$

$$I_0 = \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right)\sin x \cos x}{(\sin x)^4 + (\cos x)^4}\,dx$$

Adding, $2I_0 = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}(\sin x)\cos x}{(\sin^4 x + \cos^4 x)}\,dx$

$$\Rightarrow I_0 = \frac{\pi}{4}\int_0^{\frac{\pi}{2}} \frac{\tan x (\sec^2 x)\,dx}{1 + \tan^4 x}$$

put $\tan^2 x = t \Rightarrow 2\tan x \sec^2 x\,dx = dt$

$$\Rightarrow I_0 = \frac{\pi}{4}\int_0^{\infty} \frac{\frac{dt}{2}}{(1+t^2)} = \frac{\pi}{8}(\tan^{-1} t)\Big|_0^{\infty} = \frac{\pi}{8}\left(\frac{\pi}{2} - 0\right)$$

$$\Rightarrow I_0 = \frac{\pi^2}{16}$$

3. Let $x = x(y)$ be the solution of the differential equation $y = \left(x - y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right), y > 0$ and $x(1) = \frac{\pi}{2}$.

Then $\cos(x(2))$ is equal to:

(1) $1 - 2(\log_e 2)^2$ (2) $2(\log_e 2) - 1$

(3) $1 - 2(\log_e 2)$ (4) $2(\log_e 2)^2 - 1$

Answer (4)

Sol. $y = \left(x - y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right)$

Let $x = ty \Rightarrow \frac{dx}{dy} = y\frac{dt}{dy} + t$

$y = \left[yt - y\left(\frac{ydt}{dy} + t\right)\right]\sin t$

since $y > 0$

$1 = \left(t - \frac{ydt}{dy} - t\right)\sin t = (\sin t)y\left(\frac{-dt}{dy}\right)$

$\Rightarrow \frac{dy}{y} = -\sin t dt$

Integration $\int\frac{dy}{y} = \int -\sin t dt + c$

$\ln|y| = \cos t + c \Rightarrow \ln y = \cos\left(\frac{x}{y}\right) + c$

$x(1) = \frac{\pi}{2}$

$\Rightarrow \ln(1) = \cos\left(\frac{\frac{\pi}{2}}{1}\right) + c \Rightarrow c = 0$

$\Rightarrow x(y) = y\cos^{-1}(\ln y)$

$x(2) = 2\cos^{-1}(\ln 2) \Rightarrow \cos(x(2))$

$= \cos\left(2\cos^{-1}(\ln 2)\right)$

4. The distance of the line $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ from the point (1, 4, 0) along the line $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ is

(1) $\sqrt{13}$ (2) $\sqrt{15}$

(3) $\sqrt{14}$ (4) $\sqrt{17}$

Answer (3)

Sol. Line passing through (1, 4, 0) and parallel to $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ is $L : \frac{x-1}{1} = \frac{y-4}{2} = \frac{z}{3}$

Any point on $L : (\lambda + 1, 2\lambda + 4, 3\lambda)$

Any point on $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ is $(2\mu + 2, 3\mu + 6, 4\mu + 3)$

$\left.\begin{aligned}\lambda + 1 &= 2\mu + 2\\ 2\lambda + 4 &= 3\mu + 6\\ 3\lambda &= 4\mu + 3\end{aligned}\right\} \lambda = 1\ \mu = 0$

Point: (2, 6, 3)

Distance = $\sqrt{(2-1)^2 + (6-4)^2 + (3-0)^2}$

$= \sqrt{1+4+9} = \sqrt{14}$

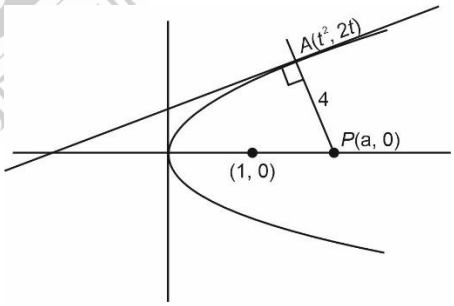
5. Let the shortest distance from $(a, 0)$, $a > 0$, to the parabola $y^2 = 4x$ be 4. Then the equation of the circle passing through the point $(a, 0)$ and the focus of the parabola, and having its centre on the axis of the parabola is

(1) $x^2 + y^2 - 6x + 5 = 0$ (2) $x^2 + y^2 - 10x + 9 = 0$

(3) $x^2 + y^2 - 4x + 3 = 0$ (4) $x^2 + y^2 - 8x + 7 = 0$

Answer (1)

Sol.



Shortest distance lie along the normal

Slope of normal at A is $(-t)$

$\Rightarrow (-t) = \frac{2t - 0}{t^2 - a}$

$\Rightarrow t = 0$ $t^2 - a = -2$

$t^2 = a - 2$

$(t^2 - a)^2 + (2t)^2 = 16$

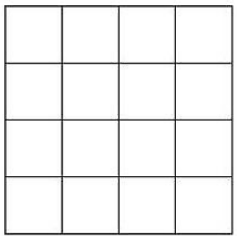
$\Rightarrow \quad t^2 = 3, a = 5$

Equation of circle passing through (1, 0) and (5, 0) and having centre on the axis will be diametric form

$(x - 1)(x - 5) + y.y = 0$

$x^2 + y^2 - 6x + 5 = 0$

6. A board has 16 squares as shown in the figure:



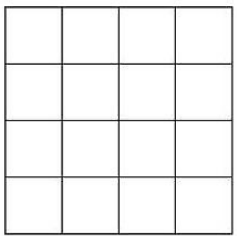
Out of these 16 squares, two squares are chosen at random. The probability that they have no side in common is

(1) $\frac{3}{5}$ (2) $\frac{23}{30}$

(3) $\frac{7}{10}$ (4) $\frac{4}{5}$

Answer (4)

Sol.



Total = $^{16}C_2$

Required ways = Total – (adjacent square)

= $^{16}C_2$ –

[3 pair in vertical & horizontal for each row and column]

$= {}^{16}C_2 - [3 \times 4 + 3 \times 4]$

= 96

Probability = $\frac{96}{120} = \frac{4}{5}$

7. Let $A = [a_{ij}]$ be a 3 × 3 matrix such that $A\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$, $A\begin{bmatrix}4\\1\\3\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$ and $A\begin{bmatrix}2\\1\\2\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$, then a_{23} equals

(1) 1

(2) –1

(3) 2

(4) 0

Answer (2)

Sol. Let $A = \begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$

$\therefore \quad \begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$

$\therefore \quad b = 0, e = 0, h = 1$

and $\begin{bmatrix}a & 0 & c\\ d & 0 & f\\ g & 1 & i\end{bmatrix}\begin{bmatrix}4\\1\\3\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$

$\therefore \quad \left.\begin{aligned}4a + 3c &= 0\\ 4d + 3f &= 1\\ 4g + 1 + 3i &= 0\end{aligned}\right\}$ ...(1)

and $\begin{bmatrix}a & 0 & c\\ d & 0 & f\\ g & 1 & i\end{bmatrix}\begin{bmatrix}2\\1\\2\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$

$\therefore \quad \left.\begin{aligned}2a + 2c &= 1\\ 2d + 2f &= 0\\ 2g + 1 + 2i &= 0\end{aligned}\right\}$ ...(2)

From equation (1) and (2) we get

$d = 1, f = -1$

$\therefore \quad a_{23} = -1$

8. Let the range of the function $f(x) = 6 + 16\cos x \cdot \cos\left(\frac{\pi}{3} - x\right) \cdot \cos\left(\frac{\pi}{3} + x\right) \cdot \sin 3x \cdot \cos 6x$, $x \in R$ be $[\alpha, \beta]$. Then the distance of the point (α, β) from the line $3x + 4y + 12 = 0$ is

(1) 11 (2) 9

(3) 8 (4) 10

Answer (1)

Sol. $f(x) = 6 + 16\cos x \cdot \cos\left(\frac{\pi}{3} - x\right) \cdot \cos\left(\frac{\pi}{3} + x\right) \cdot \sin 3x \cdot \cos 6x$

$f(x) = 6 + 4\cos 3x \cdot \sin 3x \cdot \cos 6x$

$\therefore$ $f(x) = 6 + \sin 12x$

$\therefore$ Range of $f(x) = [5, 7]$

$\therefore$ $[\alpha, \beta] = [5, 7]$

$\therefore$ Distance of point from $3x + 4y + 12 = 0$

$$= \left|\frac{3.5 + 4.7 + 12}{\sqrt{3^2 + 4^2}}\right|$$

= 11 units

9. A spherical chocolate ball has a layer of ice-cream of uniform thickness around it. When the thickness of the ice-cream layer is 1 cm, the ice-cream melts at the rate of 81 cm^3/min and the thickness of the ice-cream layer decreases at the rate of $\frac{1}{4\pi}$ cm/min. The surface area (in cm^2) of the chocolate ball (without the ice-cream layer) is:

(1) 256 π (2) 255 π

(3) 128 π (4) 196 π

Answer (1)

Sol. Let thickness of ice-cream is x cm at any instant and radius of chocolate ball is r cm.

$\therefore$ Volume of ice-cream = $\frac{4}{3}\pi\{(r + x)^3 - r^3\}$

$\therefore$ $V = \frac{4}{3}\pi\{(r + x)^3 - r^3\}$

$V = \frac{4}{3}\pi(3r^2x + 3rx^2 + x^3)$

On Differentiating both sides w.r.t. t we get:

$\frac{dV}{dt} = 4\pi(r^2 + 2rx + x^2) \cdot \frac{dx}{dt}$

$\therefore$ $81 = 4\pi(r^2 + 2r \cdot 1 + 1^2) \cdot \frac{1}{4\pi}$

$\therefore$ $r^2 + 2r - 80 = 0$

$\therefore$ $r = 8$.

$\therefore$ Surface area = $4\pi r^2$

$= 4\pi \cdot 8^2$

$= 256\ \pi\ cm^2$

10. The length of the chord of the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$, whose mid-point is $\left(1, \frac{1}{2}\right)$, is:

(1) $\frac{1}{3}\sqrt{15}$ (2) $\frac{5}{3}\sqrt{15}$

(3) $\frac{2}{3}\sqrt{15}$ (4) $\sqrt{15}$

Answer (3)

Sol. $T = S_1$

$\frac{x \cdot 1}{4} + \frac{y}{4} = \frac{1}{4} + \frac{1}{8}$

$\Rightarrow$ $2x + 2y = 3$

$\frac{x^2}{4} + \frac{\left(\frac{3 - 2x}{2}\right)^2}{2} = 1$

$\Rightarrow$ $x = \frac{12 \pm \sqrt{120}}{12} \Rightarrow y = \frac{1}{2} \mp \frac{\sqrt{120}}{12}$

So length of chord

$= \frac{2\sqrt{15}}{3}$

11. If in the expansion of $(1 + x)^p (1 - x)^q$, the coefficients of x and x^2 are 1 and –2, respectively, then $p^2 + q^2$ is equal to :

(1) 8 (2) 18

(3) 13 (4) 20

Answer (3)

Sol. $(1+x)^p = p_{C_0} + p_{C_1}x + p_{C_2}x^2 + ... + p_{C_p}$

$(1-x)^q = q_{C_0} - q_{C_1}x + q_{C_2}x^2 ... + (-1)^q q_{C_q}$

$-q_{C_1} + p_{C_1} = 1 \Rightarrow p - q = 1$

$q_{C_2} - pq + p_{C_2} = -2$

$\frac{q(q-1)}{2} + \frac{p(p-1)}{2} - pq = -2$

$\Rightarrow \quad p = 3$

$q = 2$

$p_2 + q^2 = 13$

12. If the square of the shortest distance between the lines $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+3}{-3}$ and $\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers, then $m + n$ is equal to :

(1) 21 (2) 9

(3) 6 (4) 14

Answer (2)

Sol. $a_1 = (2, 1, -3), a_2 = (-1, -3, -5)$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$$

$= 2\hat{i} - \hat{j}$

$$(SD)^2 = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|^2$$

$$= \left(\frac{2}{\sqrt{5}} \right)^2 = \frac{4}{5} = \frac{m}{n}$$

$m + n = 9$

13. The number of complex numbers z, satisfying $|z| = 1$ and $\left| \frac{z}{\bar{z}} + \frac{\bar{z}}{z} \right| = 1$, is

(1) 8 (2) 10

(3) 6 (4) 4

Answer (1)

Sol. $|z| = 1$

$$\left| \frac{z}{\bar{z}} + \frac{\bar{z}}{z} \right| = 1$$

$$\Rightarrow \quad \left| z^2 + (\bar{z})^2 \right| = 1$$

Let $z = x + iy$

$\Rightarrow \quad |(x + iy)^2 + (x - iy)^2| = 1$

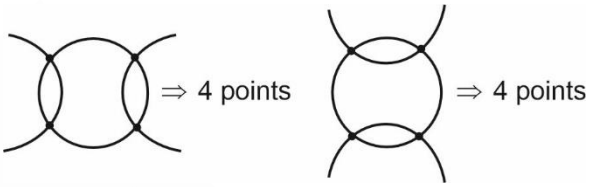
$\Rightarrow \quad |2x^2 - 2y^2| = 1$

$$\Rightarrow \quad \left| x^2 - y^2 \right| = \frac{1}{2}$$

$$\Rightarrow \quad x^2 - y^2 = \frac{\pm 1}{2}$$

and $x^2 + y^2 = 1$

Case I: $x^2 - y^2 = \frac{1}{2}$ Case II: $x^2 - y^2 = -\frac{1}{2}$



Hence, we get 8 complex number

14. A rod of length eight units moves such that its ends A and B always lie on the lines $x - y + 2 = 0$ and $y + 2 = 0$, respectively. If the locus of the point P, that divides the rod AB internally in the ratio 2 : 1 is $9(x^2 + \alpha y^2 + \beta xy + \gamma x + 28y) - 76 = 0$, then $\alpha - \beta - \gamma$ is equal to

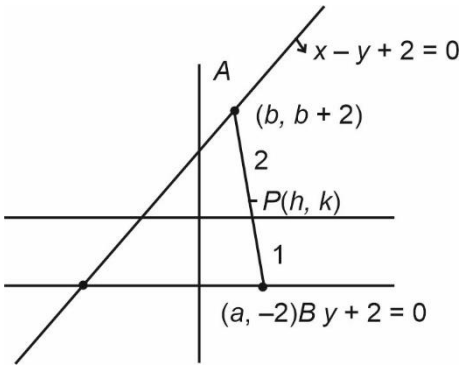
(1) 24 (2) 23

(3) 22 (4) 21

Answer (2)

Sol. $AB = 8$

$AB^2 = 64$



$\Rightarrow (a-b)^2 + (b+4)^2 = 64$ ...(1)

Now P divides AB in the ratio 2 : 1 internally

$\Rightarrow h = \frac{2a+b}{3}$ and $k = \frac{-4+b+2}{3}$

$\Rightarrow 2a + b = 3h$ ...(2) $k = \frac{b-2}{3}$

From equation (2) and (3) $\Rightarrow b = 3k + 2$ ...(3)

$\Rightarrow 2a = 3h - 3k - 2$

$\Rightarrow a = \frac{3h-3k-2}{2}$

Now by putting value of a and b in equation

$\Rightarrow \left(\frac{3h-3k-2}{2} - (3k+2)\right)^2 + (3k+2+4)^2 = 64$

$\Rightarrow \left(\frac{3h-3k-2-6k-4}{2}\right)^2 + (3k+6)^2 = 64$

$\Rightarrow (3h-9k-6)^2 + 4(3k+6)^2 = 4\times 64$

$\Rightarrow 9(h-3k-2)^2 + 36(k+2)^2 = 256$

$\Rightarrow 9(h^2 + 9k^2 + 4 - 6hk - 4h + 12k)$

$+ 36(k^2 + 4 + 4k) = 256$

$\Rightarrow 9(h^2 + 13k^2 + 20 - 6hk - 4h + 28k) = 256$

Replacing h by x and k by y

$\Rightarrow 9(x^2 + 13y^2 - 6xy - 4x + 28y) + 180 - 256 = 0$

$\Rightarrow 9(x^2 + 13y^2 - 6xy - 4x + 28y) - 76 = 0$

By comparing $\alpha = 13$, $\beta = -6$, $\gamma = -4$

$\alpha - \beta - \gamma = 13 + 6 + 4 = 23$

15. Let $A = \{(x, y) \in \mathbf{R} \times \mathbf{R}: |x + y| \geq 3\}$ and $B = \{(x, y) \in \mathbf{R} \times \mathbf{R} : |x| + |y| \leq 3\}$.

If $C = \{(x, y) \in A \cap B : x = 0 \text{ or } y = 0\}$, then $\sum_{(x,y)\in C} |x + y|$ is

(1) 18 (2) 12

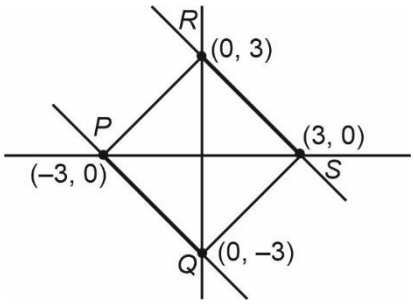
(3) 24 (4) 15

Answer (2)

Sol. $A = \{(x, y) \in \mathbf{R} \times \mathbf{R}: |x + y| \geq 3\}$

and $B = \{(x, y) \in \mathbf{R} \times \mathbf{R} : |x| + |y| \leq 3\}$

$C = \{(x, y) \in A \cap B : x = 0 \text{ or } y = 0\}$



$A \cap B$ will have only common points lying on the line PQ and RS

Now, $C = \{(-3, 0), (3, 0), (0, 3), (0, -3)\}$

$\sum_{(x,y)\in C} |x + y| = 3 + 3 + 3 + 3 = 12$

16. $\lim_{x\to\infty} \frac{(2x^2 - 3x + 5)(3x - 1)^{\frac{x}{2}}}{(3x^2 + 5x + 4)\sqrt{(3x+2)^x}}$ is equal to

(1) $\frac{2}{\sqrt{3e}}$ (2) $\frac{2e}{\sqrt{3}}$

(3) $\frac{2}{3\sqrt{e}}$ (4) $\frac{2e}{3}$

Answer (3)

Sol. $\lim_{x\to\infty} \frac{(2x^2 - 3x + 5)(3x - 1)^{x/2}}{(3x^2 + 5x + 4)\sqrt{(3x+2)^x}}$

$= \lim_{x\to\infty} \frac{x^2\left(2 - \frac{3}{x} + \frac{5}{x^2}\right)(3x)^{\frac{x}{2}}}{x^2\left(3 + \frac{5}{x} + \frac{4}{x^2}\right)(3x)^{\frac{x}{2}}} \cdot \frac{\left(1 - \frac{1}{3x}\right)^{\frac{x}{2}}}{\left(1 + \frac{2}{3x}\right)^{\frac{x}{2}}}$

$\lim_{x\to\infty}\left(1 - \frac{1}{3x}\right)^{\frac{x}{2}} = e^{\lim_{x\to\infty}\left(1-\frac{1}{3x}-1\right)\times\frac{x}{2}} = e^{\frac{-1}{6}}$

$\lim_{x\to\infty}\left(1 + \frac{2}{3x}\right) = e^{\lim_{x\to\infty}\left(1+\frac{2}{3x}-1\right)\times\frac{x}{2}}$

$= e^{\frac{1}{3}}$

So, $\frac{2}{3}\times\frac{e^{\frac{-1}{6}}}{e^{\frac{1}{3}}} = \frac{2}{3}\times\frac{1}{e^{\frac{1}{3}+\frac{1}{6}}} = \frac{2}{3\sqrt{e}}$

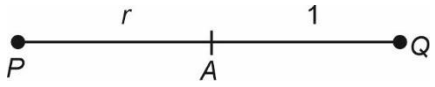
17. Let the point A divide the line segment joining the points $P(-1, -1, 2)$ and $Q(5, 5, 10)$ internally in the ratio $r : 1 (r > 0)$. If O is the origin and $\left(\overrightarrow{OQ}\cdot\overrightarrow{OA}\right)-\frac{1}{5}\left|\overrightarrow{OP}\times\overrightarrow{OA}\right|^2 = 10$, then the value of r is:

(1) $\sqrt{7}$ (2) 7

(3) 3 (4) 14

Answer (2)

Sol.



$$A=\left(\frac{5r-1}{r+1},\frac{5r-1}{r+1},\frac{10r+2}{r+1}\right)$$

$$\left(\overrightarrow{OQ}\cdot\overrightarrow{OA}\right)-\frac{1}{5}\left|\overrightarrow{OP}\times\overrightarrow{OA}\right|^2=10$$

$$\overrightarrow{OQ}=5\hat{i}+5\hat{j}+10\hat{k}$$

$$\overrightarrow{OA}=\frac{5r-1}{r+1}\hat{i}+\frac{5r-1}{r+1}\hat{j}+\frac{10r+2}{r+1}\hat{k}$$

$$\overrightarrow{OP}=-\hat{i}-\hat{j}+2\hat{k}$$

$$\overrightarrow{OP}\times\overrightarrow{OA}=\frac{1}{r+1}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\5r-1&5r-1&10r+2\\-1&-1&2\end{vmatrix}$$

$$=\frac{1}{r+1}(\hat{i}(20r)-\hat{j}(20r))$$

$$=5\left(\frac{5r-1}{r+1}\right)+5\left(\frac{5r-1}{r+1}\right)+10\left(\frac{10r+2}{r+1}\right)$$

$$-\frac{1}{5}\left(\frac{2\times400r^2}{(r+1)^2}\right)=10$$

$$\frac{150r+10}{r+1}-\frac{1}{5}\left(\frac{2\times400r^2}{(r+1)^2}\right)=10$$

$(150r + 10)(r + 1) - 160r^2 = 10(r + 1)^2$

$(15r + 1)(r + 1) - 16r^2 = (r + 1)^2$

$15r^2 + 16r + 1 - 16r^2 = r^2 + 2r + 1$

$-2r^2 + 14r = 0$

$r = 0, 7$

18. The system of equations

$x + y + z = 6$,

$x + 2y + 5z = 9$,

$x + 5y + \lambda z = \mu$

has no solution if

(1) $\lambda \neq 17, \mu \neq 18$ (2) $\lambda = 17, \mu = 18$

(3) $\lambda = 15, \mu \neq 17$ (4) $\lambda = 17, \mu \neq 18$

Answer (4)

Sol. $\left[\begin{array}{ccc|c}1&1&1&6\\1&2&5&9\\1&5&\lambda&\mu\end{array}\right]\sim\left[\begin{array}{ccc|c}1&1&1&6\\0&1&4&3\\0&4&\lambda-1&\mu-6\end{array}\right]\sim\left[\begin{array}{ccc|c}1&1&1&6\\0&1&4&3\\0&0&\lambda-17&\mu-18\end{array}\right]$

$\lambda = 17$ and $\mu \neq 18$

19. Let $\int x^3 \sin x dx = g(x) + C$, where C is the constant of integration. If $8\left(g\left(\frac{\pi}{2}\right)+g'\left(\frac{\pi}{2}\right)\right) = \alpha\pi^3 + \beta\pi^2 + \gamma$, $\alpha, \beta, \gamma \in Z$, then $\alpha + \beta - \gamma$ equals:

(1) 62

(2) 55

(3) 48

(4) 47

Answer (2)

Sol. $g(x) + C = \int x^3 \cdot \sin x dx$

$= -x^3\cos x + 3x^2\sin x$

$+6x\cos x - 6\sin x + C$

Hence, $g(x) = -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x$

$$g\left(\pi/2\right)=\frac{3\pi^2}{4}-6$$

Also, $g'(x) = x^3 \sin x \Rightarrow g'\left(\pi/2\right)=\frac{\pi^3}{8}$

$$8\left(g\left(\pi/2\right)+g'\left(\pi/2\right)\right)=8\left(\frac{3\pi^2}{4}-6+\frac{\pi^3}{8}\right)$$

$= 6\pi^2 + \pi^3 - 48 = \alpha\pi^3 + \beta\pi^2 + \gamma$

$\Rightarrow \alpha = 1, \beta = 6, \gamma = -48$

Hence, $\alpha + \beta - \gamma = 1 + 6 + 48 = 55$

20. Let $X = R \times R$. Define a relation R on X as:

$$(a_1, b_1)\ R\ (a_2, b_2) \Leftrightarrow b_1 = b_2$$

Statement I : R is an equivalence relation.

Statement II: For some $(a, b) \in X$, the set $S = \{(x, y) \in X : (x, y)\ R\ (a, b)\}$ represents a line parallel to $y = x$.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is true but Statement II is false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are false

(4) Both Statement I and Statement II are true

Answer (1)

Sol. $(a_1, b_1)R(a_2, b_2) \Leftrightarrow b_1 = b_2$

Check reflexive

$(a_1, b_1)R(a_1, b_1)$

$\Rightarrow\ b_1 = b_1$

$\Rightarrow$ Reflexive

Check symmetric

$(a_1, b_1)R(a_2, b_2) \Leftrightarrow b_1 = b_2$

$\Rightarrow (a_2, b_2)R(a_1, b_1) \Leftrightarrow b_2 = b_1$

$\Rightarrow\ b_1 = b_2 \Rightarrow$ symmetric

Check transitive

$\Rightarrow (a_1, b_1)R(a_2, b_2) \Leftrightarrow b_1 = b_2$

$(a_2, b_2)R(a_3, b_3) \Leftrightarrow b_2 = b_3$

$\Rightarrow\ b_1 = b_2 = b_3$

$\Rightarrow\ b_1 = b_3$

$\Rightarrow\ (a_1, b_1)\ R(a_3, b_3)$

$\Rightarrow\ R$ is transitive

$\Rightarrow\ R$ is equivalence

II :

$(x, y)R(a, b) \Leftrightarrow y = b \neq y = x$

$\Rightarrow$ Not true.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The roots of the quadratic equation $3x^2 - px + q = 0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88, then $q - 2p$ is equal to _____.

Answer (474)

Sol. $S_{11} = \frac{11}{2}[2a + 10d] = 88$

$a + 5d = 8$

$\Rightarrow\ a = \frac{1}{2}$

$T_{10} + T_{11} = \frac{p}{3} = a + 9d + a + 10d$

$\Rightarrow \frac{p}{3} = \frac{59}{2} \Rightarrow p = \frac{177}{2}$

$T_{10} \cdot T_{11} = \frac{q}{3}$

$(a + 9d)(a + 10d) = \frac{q}{3}$

$\left(\frac{1}{2} + 9 \times \frac{3}{2}\right)\left(\frac{1}{2} + 10 \times \frac{3}{2}\right) = \frac{q}{3}$

$\Rightarrow\ q = 651$

$q - 2p = 651 - 177 = 474$

22. The variance of the numbers 8, 21, 34, 47, …, 320 is _____.

Answer (8788)

Sol. 8, 21, 34 ….320

$320 = 8 + (n - 1)\ 13$

$\Rightarrow\ n = 25$

$$\text{Mean} = \frac{\sum x_i}{n} = \frac{8+21+34+...+320}{25} = \frac{\frac{25}{2}[8+320]}{25}$$

$= 164$

$$\text{Variance} = \frac{\sum x_i^2}{n} - [\text{mean}]^2$$

$$= \frac{8^2+21^2+...+320^2}{25} - (164)^2$$

$= 8788$

23. The focus of the parabola $y^2 = 4x + 16$ is the centre of the circle C of radius 5. If the values of λ, for which C passes through the point of intersection of the lines $3x - y = 0$ and $x + \lambda y = 4$, are λ_1 and λ_2, $\lambda_1 < \lambda_2$, then $12\lambda_1 + 29\lambda_2$ is equal to _____.

Answer (15)

Sol. $y^2 = 4(x + 4)$

Equation of circle

$(x + 3)^2 + y^2 = 25$

It passes through the point of intersection of two lines $3x - y = 0$ and $x + \lambda y = 4$

$\left(\frac{4}{3\lambda+1}, \frac{12}{3\lambda+1}\right)$, we get

$\lambda = \frac{-7}{6}, 1; 12\lambda_1 + 29\lambda_2 = -14 + 29 = 15$

24. Let α, β be the roots of the equation $x^2 - ax - b = 0$ with $\text{Im}(\alpha) < \text{Im}(\beta)$. Let $P_n = \alpha^n - \beta^n$. If $P_3 = -5\sqrt{7}i$, $P_4 = -3\sqrt{7}i$, $P_5 = 11\sqrt{7}i$ and $P_6 = 45\sqrt{7}i$, then $|\alpha^4 + \beta^4|$ is equal to _____.

Answer (31)

Sol. $\alpha + \beta = a$, $\alpha\beta = -b$

$P_6 = aP_5 + bP_4$

$45\sqrt{7}i = a \times 11\sqrt{7}i + b(-3\sqrt{7})i$

$45 = 11a - 3b$ …(i)

and $P_5 = aP_4 + bP_3$

$11\sqrt{7}i = a\left(-3\sqrt{7}i\right) + b(-5\sqrt{7}i)$

$11 = -3a - 5b$ …(ii)

$a = 3, b = -4$

$|\alpha^4 + \beta^4| = \sqrt{(\alpha^4 - \beta^4)^2 + 4\alpha^4\beta^4}$

$= \sqrt{-63 + 4.4^4}$

$= \sqrt{-63 + 1024}$

$= \sqrt{961}$

$= 31$

25. The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together, is_____.

Answer (17280)

Sol. Case I, when all boys sit together

$B_1B_2B_3B_4B_5\ G_1G_2G_3G_4$

$= 5!\ 5! = 14400$

Case II, when no two boys sit together

$\uparrow G_1 \uparrow G_2 \uparrow G_3 \uparrow G_4 \uparrow$

$4! \cdot 5! \Rightarrow 2880$

$\therefore$ Total number of ways = 14400 + 2880

= 17280

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. A circular disk of radius R meter and mass M kg is rotating around the axis perpendicular to the disk. An external torque is applied to the disk such that $\theta(t) = 5t^2 - 8t$, where $\theta(t)$ is the angular position of the rotating disc as a function of time t.

How much power is delivered by the applied torque, when $t = 2$ s?

(1) $72MR^2$ (2) $8MR^2$

(3) $108MR^2$ (4) $60MR^2$

Answer (4)

Sol. $I = \frac{MR^2}{2}$

$\omega = 10t - 8 = 12$ for $t = 2$

$\alpha = 10$

$P = \tau\omega = \frac{MR^2}{2} \times 10 \times 12$

$= 60MR^2$

27. A plane electromagnetic wave of frequency 20 MHz travels in free space along the $+x$ direction. At a particular point in space and time, the electric field vector of the wave is $E_y = 9.3\ \text{Vm}^{-1}$. Then, the magnetic field vector of the wave at that point is

(1) $B_Z = 1.55 \times 10^{-8}$ T

(2) $B_Z = 6.2 \times 10^{-8}$ T

(3) $B_Z = 9.3 \times 10^{-8}$ T

(4) $B_Z = 3.1 \times 10^{-8}$ T

Answer (4)

Sol. $E = CB$

$B = \frac{9.3}{3 \times 10^8}$

$= 3.1 \times 10^{-8}$

28. The width of one of the two slits in Young's double slit experiment is d while that of the other slit is xd. If the ratio of the maximum to the minimum intensity in the interference pattern on the screen is 9 : 4 then what is the value of x?

(Assume that the field strength varies according to the slit width.)

(1) 4 (2) 5

(3) 3 (4) 2

Answer (2)

Sol. $A \propto$ width of slit

$\Rightarrow A_1 = A, A_2 = xA$

$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{(x+1)^2}{(x-1)^2} = \frac{9}{4}$

$2x + 2 = 3x - 3$

$x = 5$

29. The energy of a system is given as $E(t) = \alpha^3 e^{-\beta t}$, where t is the time and $\beta = 0.3\ \text{s}^{-1}$. The errors in the measurement of α and t are 1.2% and 1.6%, respectively. At $t = 5$ s, maximum percentage error in the energy is :

(1) 6%

(2) 4%

(3) 8.4%

(4) 11.6%

Answer (1)

Sol. $dE = 3\alpha^2 d\alpha\, e^{-\beta t} + \alpha^3 e^{-\beta t}(-\beta)dt$

$$\frac{dE}{E} = \frac{3d\alpha}{\alpha} + (-\beta)dt$$

$$\frac{\Delta E}{E} = \frac{3\Delta\alpha}{\alpha} + \beta\Delta t$$

(%) $\Delta t = 1.6\% \times 5 = 8\%$

$$\%\left(\frac{3\Delta\alpha}{\alpha}\right) = 3 \times 1.2\% = 3.6\%$$

Putting values we get

$\Rightarrow$ $3.6 + 8 \times 0.3 = 6\%$

30. A massless spring gets elongated by amount x_1 under a tension of 5 N. Its elongation is x_2 under the tension of 7 N. For the elongation of $(5x_1 - 2x_2)$, the tension in the spring will be,

(1) 20 N (2) 11 N

(3) 39 N (4) 15 N

Answer (2)

Sol. As $T = kx$

$$x_1 = \frac{5}{k} \;\&\; x_2 = \frac{7}{k}$$

Then $(5x_1 - 2x_2)k = 5 \times 5 - 2 \times 7 = 11$ N

31. Water flows in a horizontal pipe whose one end is closed with a valve. The reading of the pressure gauge attached to the pipe is P_1. The reading of the pressure gauge falls to P_2 when the valve is opened. The speed of water flowing in the pipe is proportional to

(1) $\sqrt{P_1 - P_2}$ (2) $(P_1 - P_2)^4$

(3) $P_1 - P_2$ (4) $(P_1 - P_2)^2$

Answer (1)

Sol. P_1 | P_1

P_2 $\Rightarrow$ P_1

Using Bernoulli's theorem

$$P_1 - P_2 = \frac{1}{2}\rho v^2$$

$$v \propto \sqrt{P_1 - P_2}$$

32. In photoelectric effect an em-wave is incident on a metal surface and electrons are ejected from the surface. If the work function of the metal is 2.14 eV and stopping potential is 2 V, what is the wavelength of the em-wave?

(Given hc = 1242 eVnm where h is the Planck's constant and c is the speed of light in vaccum.)

(1) 300 nm (2) 400 nm

(3) 600 nm (4) 200 nm

Answer (1)

Sol. $\phi = 2.14$

$V_S = 2$ V

Using photoelectric equation.

$$\frac{hc}{\lambda} = 2.14 + 2 = 4.14 \text{ eV}$$

$$\lambda = \frac{1242}{4.14} = 300 \text{ nm}$$

33. Water of mass m gram is slowly heated to increase the temperature from T_1 to T_2. The change in entropy of the water, given specific heat of water is 1 $Jkg^{-1} K^{-1}$, is

(1) $m(T_2 - T_1)$ (2) $m \ln\left(\frac{T_2}{T_1}\right)$

(3) $m \ln\left(\frac{T_1}{T_2}\right)$ (4) Zero

Answer (2)

Sol. $dQ = msdT$

$$ds = \frac{dQ}{T} = \frac{msdT}{T}$$

$$\Delta s = ms \ln\frac{T_2}{T_1}$$

$$\Delta s = ms \ln\frac{T_2}{T_1} \text{ as } s = 1$$

34. A ball having kinetic energy KE, is projected at an angle of 60° from the horizontal. What will be the kinetic energy of ball at the highest point of its flight?

(1) $\frac{(KE)}{2}$ (2) $\frac{(KE)}{4}$

(3) $\frac{(KE)}{16}$ (4) $\frac{(KE)}{8}$

Answer (2)

Sol. Let speed at projection is v_0

at highest point it would be $v_0\cos\theta$

as $KE \propto v^2$

$KE_{Highest} = KE_{Projection}\cos^2\theta$

$= \frac{1}{4} KE_{Projection}$

35. Match **List - I** with **List - II**.

	List - I		**List - II**
(A)	Permeability of free space	(I)	$[M L^2T^{-2}]$
(B)	Magnetic field	(II)	$[MT^{-2}A^{-1}]$
(C)	Magnetic moment	(III)	$[MLT^{-2}A^{-2}]$
(D)	Torsional constant	(IV)	$[L^2A]$

Choose the **correct** answer from the options given below

(1) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(2) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(3) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(4) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Answer (1)

Sol. (A) $B = \frac{\mu_0 i}{2\pi r}$ & $qvB = F$

$$\mu_0 \equiv \frac{Br}{i} \equiv \frac{Fr}{qvi} \equiv \frac{MLT^{-2}\times L}{A^2TLT^{-1}} \equiv MLT^{-2}A^{-2} \quad \text{(III)}$$

(B) $B \equiv \frac{F}{qv} \equiv \frac{MLT^{-2}}{ATLT^{-1}} = MT^{-2}A^{-1}$ (II)

(C) $M = iA \equiv AL^2$ (IV)

(D) $\tau = C\theta \Rightarrow C \equiv ML^2T^{-2}$ (I)

36. A galvanometer having a coils resistance 30 Ω need 20 mA of current for full-scale deflection. If a maximum current of 3 A is to be measured using this galvanometer, the resistance of the shunt to be added to the galvanometer should be $\frac{30}{X}\Omega$, where X is

(1) 596 (2) 149

(3) 298 (4) 447

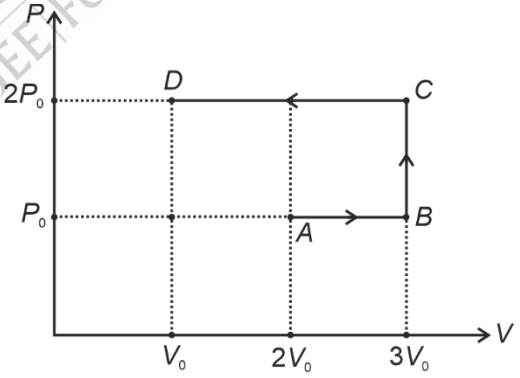
Answer (2)

Sol. $(I - I_g)R = I_gG$

$(3 - 0.02) \times R = 0.02 \times G \Rightarrow R = \frac{30}{149}$

$\Rightarrow$ 149 = Required X

37



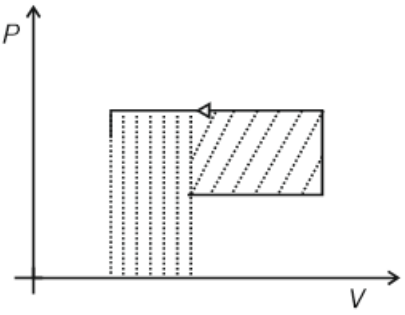
Using the given P-V diagram, the work done by an ideal gas along the path $ABCD$ is

(1) $-3P_0V_0$ (2) $-4P_0V_0$

(3) $4P_0V_0$ (4) $3P_0V_0$

Answer (1)

Sol.

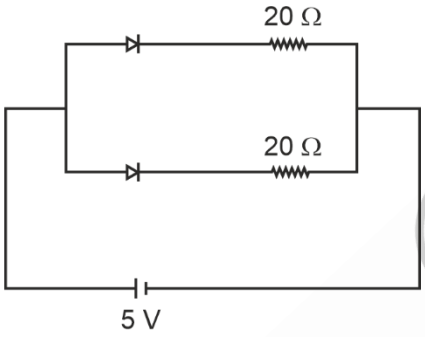


Area under graph will be magnitude of graph and being counterclockwise graph it would be negative

Area = $2P_0 \times V_0 + P_0V_0 = 3P_0V_0$

$W = -3P_0V_0$

38. What is the current through the battery in the circuit shown below?



(1) 1.5 A (2) 1.0 A

(3) 0.5 A (4) 0.25 A

Answer (3)

Sol. Diodes are in forward bias therefore

$$i = \frac{V}{R_{eq}} = \frac{5}{10} = 0.5 \text{ A}$$

39. If a satellite orbiting the Earth is 9 times closer to the Earth than the Moon, what is the time period of rotation of the satellite? Given rotational time period of Moon = 27 days and gravitational attraction between the satellite and the moon is neglected.

(1) 1 day (2) 27 days

(3) 3 days (4) 81 days

Answer (1)

Sol. $T^2 \propto r^3$ from Kepler's 3rd law

$$\frac{27^2}{T^2} = \frac{r^3}{(r/g)^3}$$

$$\frac{27^2}{9^3} = T^2$$

$$\frac{27 \times 27}{9 \times 9 \times 9} = T^2 \Rightarrow T = 1 \text{ day}$$

40. Two point charges –4 μC and 4 μC, constituting an electric dipole, are placed at (–9, 0, 0) cm and (9, 0, 0) cm in a uniform electric field of strength 10^4 NC^{-1}. The work done on the dipole in rotating it from the equilibrium through 180° is

(1) 14.4 mJ (2) 18.4 mJ

(3) 16.4 mJ (4) 12.4 mJ

Answer (1)

Sol. $U = -\vec{P} \cdot \vec{E}$

Therefore $W = 2PE$

$= 2 \times 4 \times 10^{-6} \times 18 \times 10^{-2} \times 10^4$

$= 144 \times 10^{-4}$

$= 14.4$ mJ

41. Given below are two statements. One is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Reason (A): The binding energy per nucleon is found to be practically independent of the atomic number A, for nuclei with mass numbers between 30 and 170.

Reason (R): Nuclear force is long range.

In the light of the above statements, choose the correct answer from the options given below:

(1) (A) is true but (R) is false

(2) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(3) Both (A) and (R) are true and (R) is the correct explanation of (A)

(4) (A) is false but (R) is true

Answer (1)

Sol. From graph between B.E/N & A we can see BE/N is almost constant ⇒ correct

Reason ⇒ incorrect as nuclear forces are short range forces.

42. Two charges 7 μC and –4 μC are placed at (–7 cm, 0, 0) and (7 cm, 0, 0) respectively. Given, $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$, the electrostatic potential energy of the charge configuration is:

(1) – 1.5 J (2) – 1.2 J

(3) – 2.0 J (4) – 1.8 J

Answer (4)

Sol. $U = \dfrac{kq_1q_2}{r}$

$= -\dfrac{9\times10^9\times7\times4\times10^{-12}}{14\times10^{-2}}$

$= -18 \times 10^{-1}$

$= -1.8$ J

43. A concave mirror of focal length f in air is dipped in a liquid of refractive index μ. Its focal length in the liquid will be :

(1) $\dfrac{f}{(\mu-1)}$ (2) $\dfrac{f}{\mu}$

(3) f (4) μf

Answer (3)

Sol. Focal length of mirror is independent of refractive index.

44. The equation of a transverse wave travelling along a string is

$y(x, t) = 4.0 \sin[20 \times 10^{-3}\ x + 600t]$ mm, where x is in mm and t is in second. The velocity of the wave is :

(1) – 60 m/s (2) + 30 m/s

(3) – 30 m/s (4) + 60 m/s

Answer (3)

Sol. $k = 20 \times 10^{-3}\ mm^{-1} = 20\ m^{-1}$

$w = 600\ s^{-1}$

$v = \dfrac{w}{k} = \dfrac{600}{20} = 30$ m/s

and x & t carry same sign

Therefore v = –30 m/s

45. The refractive index of the material of a glass prism is $\sqrt{3}$. The angle of minimum deviation is equal to the angle of the prism. What is the angle of the prism?

(1) 48° (2) 50°

(3) 60° (4) 58°

Answer (3)

Sol. $\mu = \dfrac{\sin(A+\delta m/2)}{\sin A/2}$ and $\delta_m = A$

$\sqrt{3} = \dfrac{\sin A}{\sin A/2} = 2\cos A/2$

$\cos A/2 = \cos 30°$

$A = 60°$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. A time varying potential difference is applied between the plates of a parallel plate capacitor of capacitance 2.5 μF. The dielectric constant of the medium between the capacitor plates is 1. It produces an instantaneous displacement current of 0.25 mA in the intervening space between the capacitor plates, the magnitude of the rate of change of the potential difference will be _____ Vs^{-1}.

Answer (100)

Sol. $q = CV$

Differentiating

$i = \dfrac{CdV}{dt}$

$\Rightarrow \dfrac{dV}{dt} = \dfrac{0.25\times10^{-3}}{2.5\times10^{-6}} = \dfrac{1000}{10} = 100$

47. An air bubble of radius 1.0 mm is observed at a depth of 20 cm below the free surface of a liquid having surface tension 0.095 J/m^2 and density 10^3 kg/m^3. The difference between pressure inside the bubble and atmospheric pressure is _____ N/m^2.

(Take g = 10 m/s^2)

Answer (2190)

Sol. $\Delta p = h\rho g + \frac{2s}{r}$

$= 0.2\times10^3\times10 + \frac{2\times0.095}{10^{-3}}$

= 2000 + 190 = 2190 Pa

48. In a series LCR circuit, a resistor of 300 Ω, a capacitor of 25 μF and an inductor of 100 mH are used. For maximum current in the circuit, the angular frequency of the ac source is _____ × 10^4 radians s^{-1}.

Answer (2)

Sol. $\omega = \frac{1}{\sqrt{LC}}$ is resonance condition

$= \frac{1}{\sqrt{100\times10^{-3}\times25\times10^{-9}}}$

$= \frac{10^6}{5\times10}$

= 2 × 10^4

49. A satellite of mass $\frac{M}{2}$ is revolving around earth in a circular orbit at a height of $\frac{R}{3}$ from earth surface. The angular momentum of the satellite is $M\sqrt{\frac{GMR}{x}}$. The value of x is _____, where M and R are the mass and radius of earth, respectively. (G is the gravitational constant)

Answer (3)

Sol. $L = mvr$

$L = m\sqrt{\frac{GM}{R}}R$

$= m\sqrt{GMR}$

$= \frac{M}{2}\sqrt{GM\frac{4R}{3}}$

$= M\sqrt{\frac{GMR}{3}}$

x = 3 (But for comparable mass, this solution is not applicable)

Alternate solution:

$m_1r_1 = m_2r_2 \Rightarrow r_1 = \frac{8R}{9}$

$\frac{GM\frac{M}{2}}{\left(\frac{4R}{3}\right)^2} = \frac{M}{2}\omega^2\frac{8R}{9}$

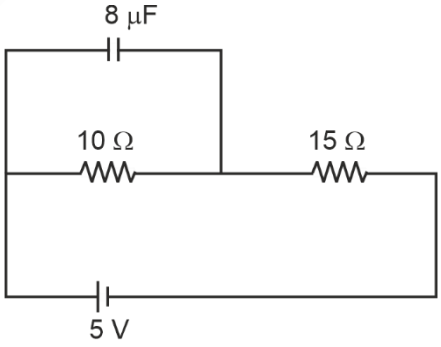
$\omega = \sqrt{\frac{81GM}{128R^3}}$

$L = I\omega = \frac{M}{2}\left(\frac{8R}{9}\right)^2\omega$

$= M\sqrt{GMR\frac{8}{81}}$

$\Rightarrow x \approx 10$

50. At steady state the charge on the capacitor, as shown in the circuit below, is _____ μC.



Answer (16)

Sol. $V_{10\,\Omega} = V_C = \frac{5}{25}\times10 = 2$ V

Q = 8 × 2 = 16 μC

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. Consider a binary solution of two volatile liquid components 1 and 2. x_1 and y_1 are the mole fractions of component 1 in liquid and vapour phase, respectively. The slope and intercept of the linear plot of $\frac{1}{x_1}$ vs $\frac{1}{y_1}$ are given respectively as:

(1) $\frac{P_1^0}{P_2^0}, \frac{P_1^0 - P_2^0}{P_2^0}$ (2) $\frac{P_1^0}{P_2^0}, \frac{P_2^0 - P_1^0}{P_2^0}$

(3) $\frac{P_2^0}{P_1^0}, \frac{P_2^0 - P_1^0}{P_2^0}$ (4) $\frac{P_2^0}{P_1^0}, \frac{P_1^0 - P_2^0}{P_2^0}$

Answer (2)

Sol. Given mole fraction of liquid 1 in liquid and vapour phase is x_1 and y_1 respectively.

$P_1 = P_T y_1$ (DLPP)

$P_1 = P_1^0 x_1$ (Raoult's law)

$P_1 = P_T y_1 = P_1^0 x_1$

$$\frac{P_T}{x_1} = \frac{P_1^0}{y_1}$$

$$\frac{P_2^0 + x_1(P_1^0 + P_2^0)}{x_1} = \frac{P_1^0}{y_1}$$

$$\frac{P_2^0}{x_1} + (P_1^0 + P_2^0) = \frac{P_1^0}{y_1}$$

$$\frac{P_2^0}{x_1} = \frac{P_1^0}{y_1} - (P_1^0 + P_2^0)$$

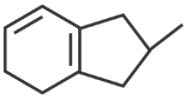
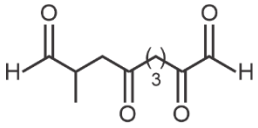
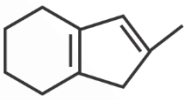
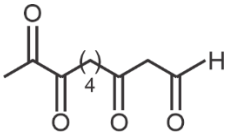
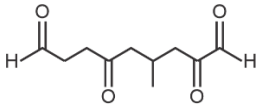
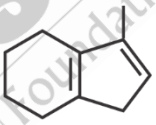
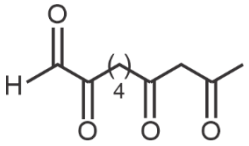
$$\frac{1}{x_1} = \frac{1}{y_2}\left(\frac{P_1^0}{P_2^0}\right) - \frac{(P_1^0 + P_2^0)}{P_2^0}$$

$$\frac{1}{x_1} = \frac{1}{y_2}\left(\frac{P_1^0}{P_2^0}\right) + \frac{(P_2^0 - P_1^0)}{P_2^0}$$

$y = mx + c$

Slope = $\frac{P_1^0}{P_2^0}$; Intercept = $\frac{P_2^0 - P_1^0}{P_2^0}$

52. Match List-I with List-II

List-I (Isomers of $C_{10}H_{14}$)	List-II (Ozonolysis product)
(A) 	(I) 
(B) 	(II) 
(C) 	(III) 
(D) 	(IV) 

Choose the correct answer from the options given below:

(1) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

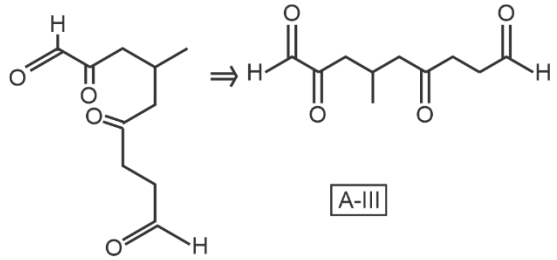
(2) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

(3) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

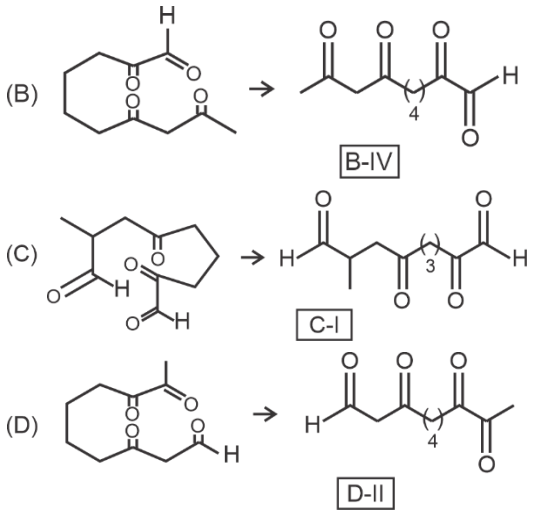
(4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Answer (1)

Sol.

(A) 

A-III



53. Match List-I with List-II

List-I	List-II
(A) Bronze	(I) Cu, Ni
(B) Brass	(II) Fe, Cr, Ni, C
(C) UK silver coin	(III) Cu, Zn
(D) Stainless steel	(IV) Cu, Sn

Choose the **correct** answer from the options given below:

(1) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(3) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(4) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Answer (1)

Sol.
- Bronze is an alloy of copper and Tin. (A-IV)
- Brass is an alloy of copper and Zinc. (B-III)
- UK Silver coin is an alloy of copper and Nickel. (C-I)
- Stainless steel is an alloy of Fe, Cr, Ni, C. (D-II)

54. Given below are two statements:

Statement (I): For a given shell, the total number of allowed orbitals is given by n^2.

Statement (II): For any subshell, the spatial orientation of the orbitals is given by $-l$ to $+l$ values including zero.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **Statement I** and **Statement II** are true

(2) **Statement I** is true but **Statement II** is false

(3) **Statement I** is false but **Statement II** is true

(4) Both **Statement I** and **Statement II** are false

Answer (1)

Sol. Statement I is true and statement II is also true.

55. The effect of temperature on spontaneity of reactions are represented as:

	ΔH	ΔS	Temperature	Spontaneity
(A)	+	–	any T	Non-spontaneous
(B)	+	+	low T	Spontaneous
(C)	–	–	low T	Non-spontaneous
(D)	–	+	any T	Spontaneous

The incorrect combinations are:

(1) (A) and (C) only

(2) (B) and (C) only

(3) (A) and (D) only

(4) (B) and (D) only

Answer (2)

Sol. (B) If, ΔH = +, ΔS = +, ΔG is negative at high temperature

Reaction is spontaneous at high temperature

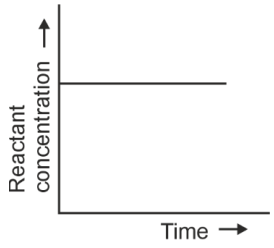
(C) If ΔH = –ve, ΔS = –ve, reaction is non spontaneous at high temperature

ΔG = –ve

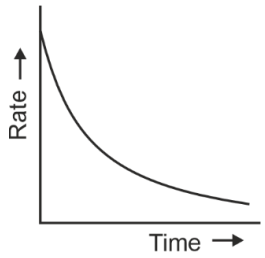
Reaction is spontaneous at low temperature.

56. Which of the following graphs most appropriately represents a zero-order reaction?

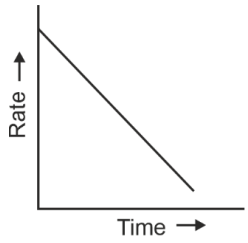
(1)



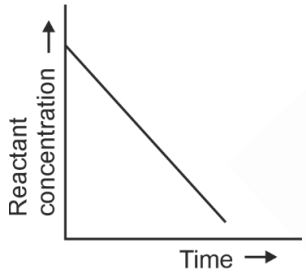
(2)



(3)



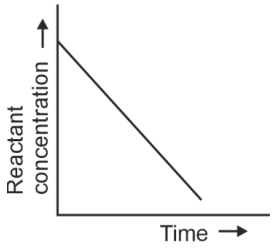
(4)



Answer (4)

Sol. $[A]_t = [A]_0 - kt$

Straight line with negative slope



Graph of reactant concentration versus time for zero order reaction.

57. Given below are two statements:

Statement (I): The boiling points of alcohols and phenols increase with increase in the number of C-atoms.

Statement (II): The boiling points of alcohols and phenols are higher in comparison to other class of compounds such as ethers, haloalkanes.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Answer (1)

Sol. **Statement I** is correct as boiling point of alcohol phenols increase with increase in the number of C-atoms due to increase in van der Waals forces.

Statement II is correct, since alcohols phenols have intermolecular H-bonding therefore their boiling points are higher in comparison to other class of compounds such as ethers, haloalkanes.

58. Standard electrode potentials for a few half cells are mentioned below:

$E^o_{Cu^{2+}/Cu} = 0.34\ V$, $E^o_{Zn^{2+}/Zn} = -0.76\ V$

$E^o_{Ag^{2+}/Ag} = 0.80\ V$, $E^o_{Mg^{2+}/Mg} = -2.37\ V$

Which one of the following cells gives the most negative value of $\Delta G°$?

(1) $Zn\,|\,Zn^{2+}(1M)\,||\,Ag^{+}(1M)\,|\,Ag$

(2) $Ag\,|\,Ag^{+}(1M)\,||\,Mg^{2+}(1M)\,|\,Mg$

(3) $Zn\,|\,Zn^{2+}(1M)\,||\,Mg^{2+}(1M)\,|\,Mg$

(4) $Cu\,|\,Cu^{2+}(1M)\,||\,Ag^{+}(1M)\,|\,Ag$

Answer (1)

Sol. $E^o_{Cu^{2+}/Cu} = 0.34\ V$, $E^o_{Zn^{2+}/Zn} = -0.76\ V$

$E^o_{Ag^{2+}/Ag} = 0.80\ V$, $E^o_{Mg^{2+}/Mg} = -2.37\ V$

Most negative ΔG° value among the given cells in options is of (1)

$E^o = 0.8 + 0.76 = 1.56$

$\therefore \Delta G^\circ = -2 \times F \times 1.56$

$\boxed{\Delta G^\circ = -3.12\ F\ V}$

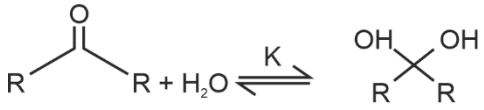
In case of option (2) and (3), E^o = –ve

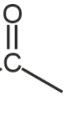
$\therefore$ ΔG° =+ve value,

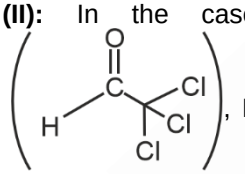
And ΔG° for option (4) will come out to be –0.92 F V, which is less negative than in option (1).

59. Given below are two statements:

Consider the following reaction



Statement (I): In the case of formaldehyde (), K is about 2280, due to small substituents, hydration is faster.

Statement (II): In the case of trichloro acetaldehyde  , K is about 2000 due to –I effect of –Cl.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **Statement I** and **Statement II** are true

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** are false

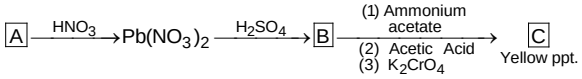
(4) **Statement I** is true but **Statement II** is false

Answer (1)

Sol. Both **Statement I** and **Statement II** are true.

$\boxed{K_{eq} > 1}$, because HCHO and chloral are more electrophilic and equilibrium more lies towards products [Refer: cleyden 1st edition PNo-144 - 145]

60. Identify A, B and C in the given below reaction sequence



(1) $PbCl_2$, $Pb(SO_4)_2$, $PbCrO_4$

(2) PbS, $PbSO_4$, $Pb(CH_3COO)_2$

(3) $PbCl_2$, $PbSO_4$, $PbCrO_4$

(4) PbS, $PbSO_4$, $PbCrO_4$

Answer (4)

Sol. $PbS \xrightarrow{HNO_3} Pb(NO_3)_2 \xrightarrow{H_2SO_4} \underset{(B)}{PbSO_4}$

$\xrightarrow[(2)\ Acetic\ Acid\ (3)\ K_2CrO_4]{(1)\ Ammonium\ acetate} \underset{(C)}{PbCrO_4}$

61. Consider the reaction

$$X_2Y(g) \rightarrow X_2(g) + \frac{1}{2}Y_2(g)$$

The equation representing correct relationship between the degree of dissociation (x) of $X_2Y(g)$ with its equilibrium constant Kp is _____.

Assume x to be very very small.

(1) $x = \sqrt[3]{\frac{2Kp^2}{p}}$ (2) $x = \sqrt[3]{\frac{2\,Kp}{p}}$

(3) $x = \sqrt[3]{\frac{Kp}{2p}}$ (4) $x = \sqrt[3]{\frac{Kp}{p}}$

Answer (1)

Sol. 1 mol

$$\underset{1-x}{X_2Y(g)} \rightarrow \underset{x}{X_2(g)} + \frac{1}{2}\underset{\frac{x}{2}}{Y_2(g)}$$

$$\therefore \quad P_{x_2y} = \frac{1-x}{1+\frac{x}{2}} \times p$$

$$P_{x_2} = \frac{x}{1+\frac{x}{2}} \times p$$

$$P_{y_2} = \frac{x/2}{1+\frac{x}{2}} \times p$$

$$\therefore \quad K_p = \frac{\left(\frac{x}{1+\frac{x}{2}}p\right)\left(\frac{x}{2\left(1+\frac{x}{2}\right)}p\right)^{1/2}}{\left(\frac{1-x}{1+\frac{x}{2}}\right)\times p}$$

$$\therefore \quad K_p = \left(\frac{x}{1-x}\right)\left(\frac{x}{2\left(1+\frac{x}{2}\right)}\right)^{1/2} \times p^{1/2}$$

$\therefore$ x to be very small

$$\therefore \quad K_p = \frac{x^{3/2}}{2^{(1/2)}} \times p^{1/2}$$

$$\therefore \quad x^{3/2} = \frac{K_P \times 2^{1/2}}{p^{1/2}}$$

$$x^3 = \frac{K_p^2 \times 2}{p}$$

$$x = \left(\frac{K_P^2 \times 2}{p}\right)^{1/3}$$

62. Given below are the atomic numbers of some group 14 elements. The atomic number of the element with lowest melting point is:

(1) 82 (2) 6

(3) 50 (4) 14

Answer (3)

Sol. Atomic no. 82 = Lead (Pb)

Atomic no. 6 = Carbon

Atomic no. 50 = Tin (Sn)

Atomic no. 14 = Silicon

The correct order is $\Rightarrow$ C > Si > Pb > Sn

Sn have least melting point

63. pH of water is 7 at 25°C. If water is heated to 80°C., it's pH will :

(1) Increase

(2) Decrease

(3) Remains the same

(4) H^+ concentration increases, OH^- concentration decreases

Answer (2)

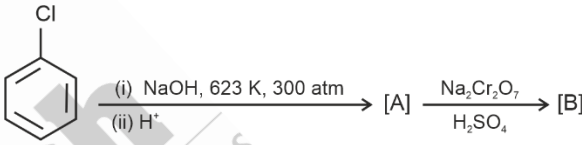
Sol. At 25°C, pure water has pH = 7

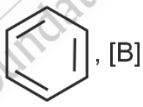
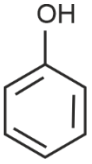
As temperature increased, water molecules dissociate more into hydrogen ions (H^+) and hydroxide ions (OH^-).

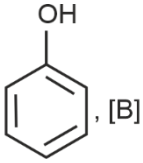
This increased dissociation leads to slightly decrease in pH.

At 80°C, pH ≈ 6.93

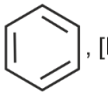
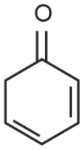
64. Identify the products [A] and [B], respectively in the following reaction :

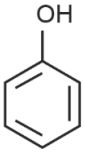


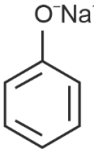
(1) [A]  , [B] 

(2) [A] 

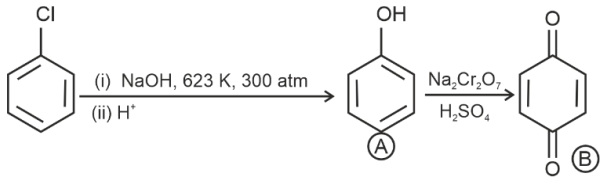
, [B] 

(3) [A]  , [B] 

(4) [A] 

, [B] 

Answer (2)

Sol. 

65. The α-Helix and β- Pleated sheet structures of protein are associated with its:

(1) Secondary structure (2) Primary structure

(3) Quaternary structure (4) Tertiary structure

Answer (1)

Sol. α-Helix and β-pleated sheets are secondary structures of protein.

66. Given below are two statements about X-ray spectra of elements:

Statement (I) : A plot of $\sqrt{\nu}$ (ν = frequency of X-rays emitted) vs atomic mass is a straight line

Statement (II): A plot of ν (ν = frequency of X-rays emitted) vs atomic number is a straight line

In the light of the above statements, choose the correct answer from the options given below:

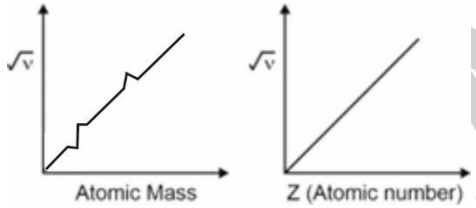
(1) Statement I is true but Statement II is false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are false

(4) Both Statement I and Statement II are true

Answer (3)

Sol. 

[Graphs plotted by Henry Moseley]

67. Consider the following reactions

$$K_2Cr_2O_7 \xrightarrow[-H_2O]{KOH} [A] \xrightarrow[-H_2O]{H_2SO_4} [B] + K_2SO_4$$

The products [A] and [B] respectively are:

(1) $K_2Cr(OH)_6$ and Cr_2O_3

(2) K_2CrO_4 and $K_2Cr_2O_7$

(3) K_2CrO_4 and Cr_2O_3

(4) K_2CrO_4 and CrO

Answer (2)

Sol. $K_2Cr_2O_7 \xrightarrow[-H_2O]{KOH} \underset{(A)}{K_2CrO_4} \xrightarrow{H_2SO_4}$

$$\underset{(B)}{K_2Cr_2O_7} + H_2SO_4$$

68. Identify the coordination complexes in which the central metal ion has d^4 configuration

(A) $[FeO_4]^{2-}$

(B) $[Mn(CN)_6]^{3-}$

(C) $[Fe(CN)_6]^{3-}$

(D) $Cr_2(O-\overset{\overset{O}{\|}}{C}-Me)_4(H_2O)_2$

(E) $[NiF_6]^{2-}$

Choose the correct answer from the options given below:

(1) (B) and (D) only (2) (B), (C) and (D) only

(3) (C) and (E) only (4) (A), (B) and (E) only

Answer (1)

Sol. (A) Fe^{6+} = [Ar] $3d^2$

(B) Mn^{3+} = [Ar] $3d^4$

(C) Fe^{3+} = [Ar] $3d^5$

(D) Cr^{2+} = [Ar] $3d^4$

(E) Ni^{4+} = [Ar] $3d^6$

69. When a non-volatile solute is added to the solvent, the vapour pressure of the solvent decreases by 10 mm of Hg. The mole fraction of the solute in the solution is 0.2. What would be the mole fraction of the solvent if decrease in vapour pressure is 20 mm of Hg?

(1) 0.6 (2) 0.2

(3) 0.4 (4) 0.8

Answer (1)

Sol. $\because P^o - P \propto x_{solute}$ and $\because 10 \propto 0.2$

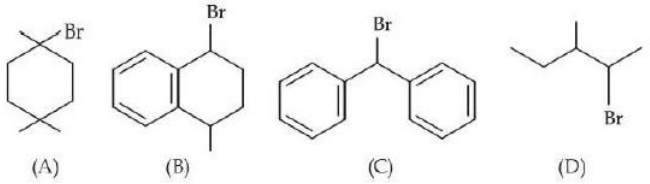
$\therefore 20 \propto 0.4$

$\therefore x_{solvent} = 1\text{-}x_{solute}$

$= 1 - 0.4$

$x_{solvent} = 0.6$

70. The ascending order of relative rate of solvolysis of following compounds is:



(1) (D) < (B) < (A) < (C)

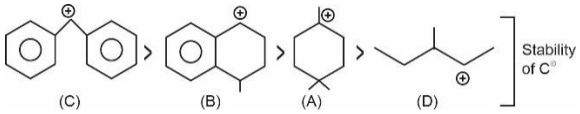
(2) (C) < (D) < (B) < (A)

(3) (C) < (B) < (A) < (D)

(4) (D) < (A) < (B) < (C)

Answer (4)

Sol. Solvolysis or S_N1 order $\propto$ stability of carbocation



SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. A compound 'X' absorbs 2 moles of hydrogen and 'X' upon oxidation with $KMnO_4|H^+$ gives

$CH_3-\overset{\overset{O}{\|}}{C}-CH_3$, $CH_3-\overset{\overset{O}{\|}}{C}-OH$ and

$CH_3-\overset{\overset{O}{\|}}{C}-CH_2CH_2-\overset{\overset{O}{\|}}{C}-OH$.

The total number of σ bonds present in the compound 'X' is _____.

Answer (27)

Sol. $CH_3-\overset{\overset{CH_3}{|}}{\underset{\underset{CH_3}{|}}{C}}-CH_2-CH_2-CH=CH-CH_3$ (X) or $CH_3-\overset{\overset{CH_3}{|}}{\underset{\underset{CH}{|}}{C}}-CH_2-CH_2-CH=\underset{CH_3}{\overset{CH_3}{C}}$ (X)

$\xrightarrow{KMnO_4/H^+}$

$CH_3-\overset{\overset{O}{\|}}{C}-CH_3 + CH_3-\overset{\overset{O}{\|}}{C}-OH + CH_3-\overset{\overset{O}{\|}}{C}-CH_2CH_2-\overset{\overset{O}{\|}}{C}-OH$.

$\therefore$ 27 σ bonds are present in X

72. 0.01 Mole of an organic compound (X) containing 10% hydrogen, on complete combustion produced 0.9 g H_2O. Molar mass of (X) is _____ g mol^{-1}.

Answer (100)

Sol. Organic compound $\xrightarrow{\text{combustion}}$ $\underset{(0.9\text{ g})}{H_2O}$

$\therefore$ Mole of $H_2O = \frac{0.9}{18} = 0.05$ mol

$\therefore$ Mole of H in $H_2O = 0.05 \times 2 = 0.1$ mole

= Mole of H in 0.01 mol organic compound

$\therefore$ Wt. of H-atom in 0.01 mole of compound

$= 0.1 \times 1$

$= 0.1$ g

$\therefore$ Wt. of H atom in 1 mole compound $= \frac{0.1}{0.01}$

$= 10$ g

$\therefore$ Wt.% of H $= \frac{\text{Wt. of H in one mole compound}}{\text{Molar mass of compound}} \times 100$

$10 = \frac{10}{M} \times 100$

$M = 100$ g/mol

73. The bond dissociation enthalpy of X_2 ΔH°_{bond} calculated from the given data is _____ kJ mol^{-1}. (Nearest integer)

$M^+X^-(s) \rightarrow M^+(g) + X^-(g)$ $\Delta H^\circ_{lattice} = 800$ kJ mol^{-1}

$M(s) \rightarrow M(g)$ $\Delta H^\circ_{sub} = 100$ kJ mol^{-1}

$M(g) \rightarrow M^+(g) + e^-(g)$ $\Delta H^\circ_i = 500$ kJ mol^{-1}

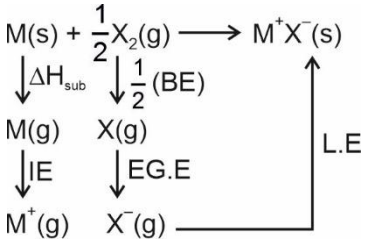
$X(g) + e^-(g) \rightarrow X^-(g)$ $\Delta H^\circ_{eg} = -300$ kJ mol^{-1}

$M(s) + \frac{1}{2}X_2(g) \rightarrow M^+X^-(s)$ $\Delta H^\circ_f = -400$ kJ mol^{-1}

[Given M^+X^- is a pure ionic compound and X forms a diatomic molecule X_2 in gaseous state]

Answer (200)

Sol.



$\therefore \quad \Delta H_f(MX) = \Delta H_{Sub}(M) + I.E.(M) + \frac{1}{2}[B.E.(X-X)] + E.GE(X) + L.E.(M)$

$-400 = 100 + 500 + \frac{1}{2}(B.E.) + (-300) + (-800)$

B.E. = 200 kJ/mol

74. When 81.0 g of aluminium is allowed to react with 128.0 g of oxygen gas, the mass of aluminium oxide produced in grams is _____. (Nearest integer)

Given:

Molar mass of Al is 27.0 g mol^{-1}

Molar mass of O is 16.0 g mol^{-1}

Answer (153)

Sol. $4Al + 3O_2 \rightarrow 2Al_2O_3$

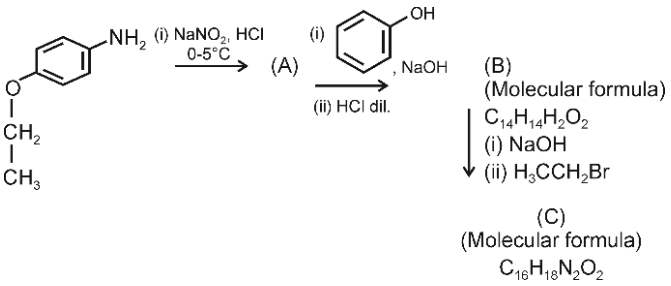
$\frac{81}{27} = 3$ mol $\quad \frac{128}{32} = 4$ mol

Al is Limiting reagent

$\therefore$ Mole of Al_2O_3 formed = $\frac{1}{2} \times 3$ mol

$\therefore$ Wt. of Al_2O_3 formed = $\frac{3}{2} \times 102 = 153$ grams

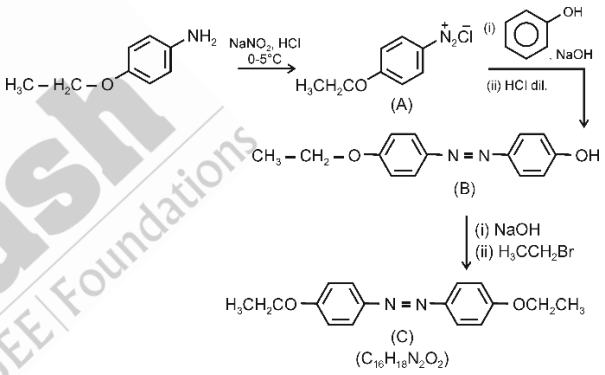
75. Consider the following sequence of reactions.



The total number of sp^3 hybridised carbon atoms in the major product C formed is _____.

Answer (4)

Sol.



Total (4) sp^3 hybridised carbon atoms are present.

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