MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let $S_n = \frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\cdots$ upon n terms. If the sum of the first six terms of an A.P. with first term $-p$ and common difference p is $\sqrt{2026 S_{2025}}$, then the absolute difference between 20[th] and 15[th] terms of the A.P. is

(1) 25 (2) 90

(3) 20 (4) 45

Answer (1)

Sol. $S_n = \frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots\frac{1}{n\times(n+1)}$

$$=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=1-\frac{1}{n+1}=\frac{n}{n+1}$$

$$\sqrt{S_{2025}\times 2026}=\sqrt{2026\times\frac{2025}{2026}}=45$$

$$S_6=\frac{6}{2}(2(-p)+5(p))=9p=45$$

$\Rightarrow$ $p = 5$

$T_{20} - T_{15} = 5d = 5(p) = 25$

2. For a statistical data $x_1, x_2, \cdots, x_{10}$ of 10 values, a student obtained the mean as 5.5 and $\sum_{i=1}^{10} x_1^2 = 371$.

He later found that he had noted two values in the data incorrectly as 4 and 5, instead of the correct values 6 and 8, respectively. The variance of the corrected data is

(1) 7 (2) 5

(3) 9 (4) 4

Answer (1)

Sol. $\sum x_i = (5.5)10 = 55$

$\sum x_i^2 = 371$

Corrected mean

$$=\frac{\sum x_i'}{10}=\frac{55+6+8-(4+5)}{10}=6$$

Now $\sum (x_i')^2 = 371 + 6^2 + 8^2 - (4^2+5^2)$

= 471 – (41) = 430

New variance

$$=\frac{\sum (x_i')^2}{10}-\left(\frac{\sum x_i'}{10}\right)^2=\frac{430}{10}-36=7$$

3. Let in a ΔABC, the length of the side AC be 6, the vertex B be (1, 2, 3) and the vertices A, C lie on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Then the area (in sq. units) of ΔABC is:

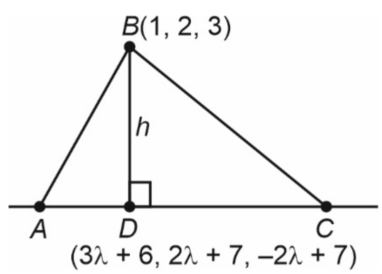
(1) 17 (2) 56

(3) 21 (4) 42

Answer (3)

Sol. Area = $\frac{1}{2}\times h(AC) = 3h$

Where h is minimum distance



$3(3\lambda + 5) + 2(2\lambda + 5) + (-2)(-2\lambda + 4) = 0$

$\Rightarrow$ $BD = 7 = h$

Area = 3(7) = 21

4. Let $f:\mathbb{R}-\{0\}\to\mathbb{R}$ be a function such that $f(x)-6f\left(\frac{1}{x}\right)=\frac{35}{3x}-\frac{5}{2}$. If the $\lim_{x\to 0}\left(\frac{1}{\alpha x}+f(x)\right)=\beta$; $\alpha,\beta\in\mathbb{R}$, then α + 2β is equal to

(1) 6 (2) 3

(3) 4 (4) 5

Answer (3)

Sol. $f(x)-6f\left(\frac{1}{x}\right)=\frac{35}{3x}-\frac{5}{2}$ ...(1)

$$6\left(f\left(\frac{1}{x}\right)-6f(x)=\frac{35x}{3}-\frac{5}{2}\right)$$

$$6f\left(\frac{1}{x}\right)-36f(x)=\frac{210x}{3}-\frac{30}{2} \quad \text{...(2)}$$

(1) + (2)

$$-35f(x)=\frac{35}{3}\left[\frac{1}{x}+6x\right]-\frac{5}{2}(1+6)$$

$$-f(x)=\frac{1}{3}\left(\frac{1}{x}+6x\right)-\frac{1}{2}$$

$$f(x)=-\frac{1}{3x}-2x+\frac{1}{2}$$

$$\lim_{x\to 0}\left[\frac{1}{\alpha x}-\frac{1}{3x}-2x+\frac{1}{2}\right]=\beta$$

$$\Rightarrow \quad \alpha=3 \qquad \beta=\frac{1}{2}$$

$$\alpha+2\beta=3+2\times\frac{1}{2}=4$$

5. Let the line passing through the points (–1, 2, 1) and parallel to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}$ intersect the line $\frac{x+2}{3}=\frac{y-3}{2}=\frac{z-4}{1}$ at the point *P*. Then the distance of *P* from the point *Q*(4, –5, 1) is

(1) 10 (2) $5\sqrt{5}$

(3) 5 (4) $5\sqrt{6}$

Answer (2)

Sol. Line passing through (–1, 2, 1) & parallel to

$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{3}{4}$$

$$L:\frac{x+1}{2}=\frac{y-2}{3}=\frac{z-1}{4}=\lambda$$

$$L_1:\frac{x+2}{3}=\frac{y-3}{2}=\frac{z-4}{1}=\mu$$

If lines are intersecting

$$\left.\begin{matrix}2\lambda-1=3\mu-2\\3\lambda+2=2\mu+3\end{matrix}\right\}\lambda=1 \quad \mu=1$$

4λ + 1 = μ + 4

$\therefore$ Point *P*(1, 5, 5) *Q*(4, –5, 1)

$$PQ=\sqrt{(1-4)^2+(5+5)^2+(5-1)^2}$$

$$=\sqrt{9+100+16}=\sqrt{125}=5\sqrt{5}$$

6. $\lim_{x\to 0}\text{cosec}x\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$ is

(1) $\frac{1}{\sqrt{15}}$ (2) $-\frac{1}{2\sqrt{15}}$

(3) $\frac{1}{2\sqrt{15}}$ (4) 0

Answer (2)

Sol. $\lim_{x\to 0}\text{cosec}\,x\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$

$$\lim_{x\to 0}\frac{\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}{\sin x\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$$

$$=\frac{1}{2\sqrt{5}}\lim_{x\to 0}\left[\frac{\cos^2 x+3\cos x-\sin x-4}{\sin x}\right]$$

Apply L' Hopital's rule

$$=\frac{1}{2\sqrt{5}}\lim_{x\to 0}\left[\frac{-\sin 2x-3\sin x-\cos x}{\cos x}\right]$$

$$=-\frac{1}{2\sqrt{5}}$$

7. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. If the vector $\vec{c}$ is perpendicular to $\vec{b}$ and $\vec{a}\cdot\vec{c} = 5$, then $|\vec{c}|$ is equal to

(1) 18 (2) $\frac{1}{3\sqrt{2}}$

(3) $\sqrt{\frac{11}{6}}$ (4) 16

Answer (3)

Sol. $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$

and $\vec{c} = k(\vec{a} + \lambda\vec{b})$ (for some scalar λ)

$\therefore\ \vec{c} = k\left((1+3\lambda)\hat{i} + (2+\lambda)\hat{j} + (3-\lambda)\hat{k}\right)$

$\because\ \vec{b}\cdot\vec{c} = 0$

$3(1+3\lambda) + (2+\lambda) - (3-\lambda) = 0$

$\therefore\ \lambda = \frac{-2}{11}$

$\therefore\ \vec{c} = k\left(\frac{5}{11}\hat{i} + \frac{20}{11}\hat{j} + \frac{35}{11}\hat{k}\right)$

$\because\ \vec{c}\cdot\vec{a} = 5 \Rightarrow k = \frac{11}{30}$

$\therefore\ \vec{c} = \frac{1}{30}(5\hat{i} + 20\hat{j} + 35\hat{k})$

$= \frac{1}{6}(\hat{i} + 4\hat{j} + 7\hat{k})$

$\therefore\ |\vec{c}| = \sqrt{\frac{11}{6}}$

8. Consider the region

$$R = \left\{(x,y) : x \le y \le 9 - \frac{11}{3}x^2,\ x \ge 0\right\}.$$

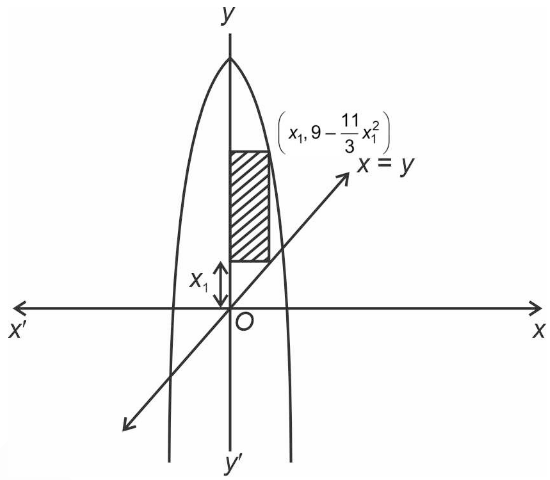
The area, of the largest rectangle of sides parallel to the coordinate axes and inscribed in R, is:

(1) $\frac{625}{111}$ (2) $\frac{821}{123}$

(3) $\frac{567}{121}$ (4) $\frac{730}{119}$

Answer (3)

Sol. $R = \left\{(x,y);\ x \le y \le 9 - \frac{11}{3}x^2,\ x \ge 0\right\}$



Area of rectangle $(A) = \left(9 - \frac{11}{3}x_1^2 - x_1\right)x_1$

$A = 9x_1 - x_1^2 - \frac{11}{3}x_1^3$

$\frac{dA}{dx_1} = 9 - 2x_1 - 11x_1^2$

For maximum area = $x_1 = \frac{9}{11}$

$\therefore$ Required maximum area = $\frac{81}{11} - \frac{81}{121} - \frac{243}{121}$

$= \frac{567}{121}$ sq. units

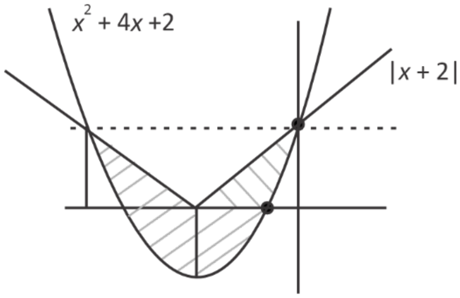
9. The area of the region $\{(x, y) : x^2 + 4x + 2 \le y \le |x + 2|\}$ is equal to

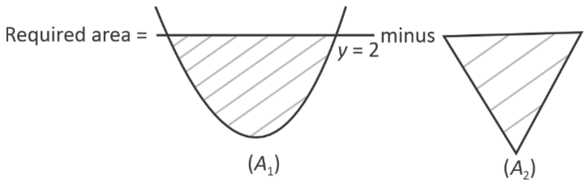
(1) 5 (2) $\frac{24}{5}$

(3) $\frac{20}{3}$ (4) 7

Answer (3)

Sol. $x^2 + 4x + 2 \le y \le |x + 2|$





$$A_1 = \int_{-4}^{0} [2-(x^2+4x+2)]dx - \frac{1}{2}\times 4 \times 2$$

$$= \left(\frac{-x^3}{3} - 2x^2\right)\Bigg|_{-4}^{0} - 4$$

$$= 0 - \left(\frac{64}{3} - 32\right) - 4$$

$$= 32 - \frac{64}{3} - 4 = \frac{20}{3}$$

10. *A* and *B* alternately throw a pair of dice. *A* wins if he throws a sum of 5 before *B* throws a sum of 8, and *B* wins if he throws a sum of 8 before *A* throws a sum of 5. The probability, that *A* wins if *A* makes the first throw, is

(1) $\frac{8}{19}$ (2) $\frac{9}{19}$

(3) $\frac{9}{17}$ (4) $\frac{8}{17}$

Answer (2)

Sol. For sum '5' $\rightarrow$ (1, 4), (2, 3), (3, 2)

$$(4, 1) \Rightarrow P(A) = \frac{4}{36}$$

For sum '8' $\rightarrow$ (2, 6), (3, 5), (4, 4)

$$(5, 3), (6, 2) \Rightarrow P(B) = \frac{5}{36}$$

$$P(\bar{A}) = \frac{32}{36}, P(\bar{B}) = \frac{31}{36}$$

$$P(A\,\text{wins}) = P(A) + P(\bar{A})P(\bar{B})P(A) +$$

$$+P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A) + \ldots$$

$$= \frac{P(A)}{1-P(\bar{A})P(\bar{B})} = \frac{9}{19}$$

11. Let $y = y(x)$ be the solution of the differential equation $\left(xy - 5x^2\sqrt{1+x^2}\right)dx + (1+x^2)dy = 0$, $y(0) = 0$. Then $y\left(\sqrt{3}\right)$ is equal to

(1) $\sqrt{\frac{14}{3}}$ (2) $\frac{5\sqrt{3}}{2}$

(3) $2\sqrt{2}$ (4) $\sqrt{\frac{15}{2}}$

Answer (2)

Sol. $\frac{dy}{dx} + \frac{xy}{1+x^2} = \frac{5x^2}{\sqrt{1+x^2}}$

$$\text{I.F.} = e^{\int \frac{x}{1+x^2}dx} = \sqrt{1+x^2}$$

$$y\cdot\sqrt{1+x^2} = \int \frac{5x^2}{\sqrt{1+x^2}}\cdot\sqrt{1+x^2}\,dx$$

$$y\sqrt{1+x^2} = \frac{5x^3}{3} + c$$

$y(0) = 0 \Rightarrow c = 0$

$$y\sqrt{1+x^2} = \frac{5x^3}{3}$$

$$y\left(\sqrt{3}\right) = \frac{5\sqrt{3}}{2}$$

12. If $I(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx, m, n > 0$, then $I(9, 14) + I(10, 13)$ is

(1) $I(9, 1)$ (2) $I(1, 13)$

(3) $I(9, 13)$ (4) $I(19, 27)$

Answer (3)

Sol. Beta function

$$\beta(P, q) = \int_0^1 x^{P-1}(1-x)^{q-1}dx$$

$$= \frac{(P-1)!(q-1)!}{(P+q-1)!}, P, q \in I$$

$$I(9, 14) = \frac{8!13!}{22!}$$

$$I(10, 13) = \frac{9!12!}{22!}$$

$$I(9, 14) + I(10, 13) = \frac{8!12!}{21!}$$

$$= \frac{(9-1)!(13-1)!}{(9+13-1)!} = I(9, 13)$$

13. The product of all the rational roots of the equation $(x^2 - 9x + 11)^2 - (x - 4)(x - 5) = 3$, is equal to

(1) 7 (2) 14

(3) 21 (4) 28

Answer (2)

Sol. $(x^2 - 9x + 11)^2 - (x-4)(x-5) = 3$

$(x^2 - 9x + 11)^2 - (x^2 - 9x + 20) = 3$

Let $x^2 - 9x + 11 = t$

$t^2 - (t + 9) = 3$

$\Rightarrow t^2 - t - 12 = 0$

$\Rightarrow t^2 - 4t + 3t - 12 = 0$

$\Rightarrow t(t-4) + 3(t-4) = 0$

$\Rightarrow t = 4$ or -3

$x^2 - 9x + 11 = 4$

$x^2 - 9x + 7 = 0$

Here, we will get irrational roots

$x^2 - 9x + 11 = -3$

$x^2 - 9x + 14 = 0$

$x^2 - 7x - 2x + 14 = 0$

$\Rightarrow x = 7, 2$

$\Rightarrow$ Product of all rational roots = 14

14. Let circle C be the image of $x^2 + y^2 - 2x + 4y - 4 = 0$ in the line $2x - 3y + 5 = 0$ and A be the point on C such that OA is parallel to x-axis and A lies on the right-hand side of the centre O of C. If $B(\alpha, \beta)$, with $\beta < 4$, lies on C such that the length of the arc AB is $(1/6)^{th}$ of the perimeter of C, then $\beta - \sqrt{3}\alpha$ is equal to

(1) 4 (2) 3

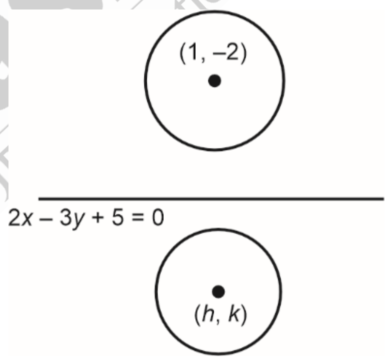
(3) $4 - \sqrt{3}$ (4) $3 + \sqrt{3}$

Answer (1)

Sol. $x^2 + y^2 - 2x + 4y - 4 = 0$

Centre $\equiv (1, -2)$

Radius = $\sqrt{1+4+4} = 3$



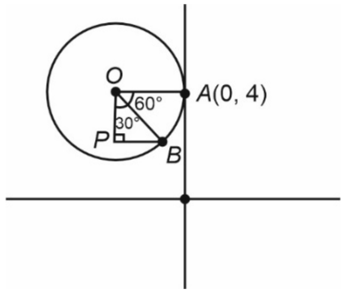
$$\frac{h-1}{2} = \frac{k+2}{-3} = \frac{-2(2+6+5)}{2^2+(-3)^2}$$

$$\Rightarrow \frac{h-1}{2} = \frac{k+2}{-3} = -\frac{26}{13}$$

$\Rightarrow h = -3, k = 4$

Centre of circle C (–3, 4)

$C : (x + 3)^2 + (y - 4)^2 = (3)^2$



Co-ordinate of $A(0, 4)$

As OA is parallel to x-axis length of are

$AB = \frac{1}{6}$ × perimeter

$= \frac{1}{6} \times 2\pi \times 3$

$\theta = \frac{\pi}{180} \times 3 = \pi$

$\Rightarrow \quad \theta = 60°$

In ΔOPB

$\sin 30° = \frac{PB}{OB}$

$\frac{PB}{3} = \frac{1}{2}$

$\Rightarrow \quad PB = \frac{-3}{2}$

$\cos 30° = \frac{OP}{3}$

$\Rightarrow \quad OP = \frac{3\sqrt{3}}{2}$

$B\left(\frac{-3}{2}, 4 - \frac{3\sqrt{3}}{2}\right)$

$\beta - \sqrt{3}\alpha = 4 - \frac{3\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} = 4$

15. If α and β are the roots of the equation $2z^2 - 3z - 2i = 0$, where $i = \sqrt{-1}$, then

$$16 \cdot \text{Re}\left(\frac{\alpha^{19} + \beta^{19} + \alpha^{11} + \beta^{11}}{\alpha^{15} + \beta^{15}}\right) . \text{Im}\left(\frac{\alpha^{19} + \beta^{19} + \alpha^{11} + \beta^{11}}{\alpha^{15} + \beta^{15}}\right)$$

is equal to

(1) 441 (2) 398

(3) 312 (4) 409

Answer (1)

Sol. $2z^2 - 3z - 2i = 0$ ...(i)

$2\left(z - \frac{i}{z}\right) = 3$

As α, β are roots of (i)

$\alpha - \frac{i}{\alpha} = \frac{3}{2}$

$\Rightarrow \alpha^2 - \frac{1}{\alpha^2} - 2i = \frac{9}{4}$

$\Rightarrow \alpha^2 - \frac{1}{\alpha^2} = \frac{9}{4} + 2i$

Squaring both sides

$\Rightarrow \alpha^4 + \frac{1}{\alpha^4} - 2 = \frac{81}{16} - 4 + 9i$

$\Rightarrow \alpha^4 + \frac{1}{\alpha^4} = \frac{49}{16} + 9i$

Similarly, $\beta^4 + \frac{1}{\beta^4} = \frac{49}{16} + 9i$

$\frac{\alpha^{19} + \beta^{19} + \alpha^{11} + \beta^{11}}{\alpha^{15} + \beta^{15}}$

$= \frac{\alpha^{15}\left(\alpha^4 + \frac{1}{\alpha^4}\right) + \beta^{15}\left(\beta^4 + \frac{1}{\beta^4}\right)}{\alpha^{15} + \beta^{15}}$

$= \frac{49}{16} + 9i$

$\text{Re}\left(\frac{\alpha^{19} + \beta^{19} + \alpha^{11} + \beta^{11}}{\alpha^{15} + \beta^{15}}\right) = \frac{49}{16}$

$\text{Im}\left(\frac{\alpha^{19} + \beta^{19} + \alpha^{11} + \beta^{11}}{\alpha^{15} + \beta^{15}}\right) = 9$

$\Rightarrow 16\text{Re}\left(\frac{\alpha^{19} + \beta^{19} + \alpha^{11} + \beta^{11}}{\alpha^{15} + \beta^{15}}\right) \cdot \text{Im}\left(\frac{\alpha^{19} + \beta^{19} + \alpha^{11} + \beta^{11}}{\alpha^{15} + \beta^{15}}\right)$

$= 16 \times \frac{49}{16} \times 9$

$= 441$

16. Let $f(x)=\dfrac{2^{x+2}+16}{2^{2x+1}+2^{x+4}+32}$. Then the value of $8\left(f\left(\dfrac{1}{15}\right)+f\left(\dfrac{2}{15}\right)+...+f\left(\dfrac{59}{15}\right)\right)$ is equal to

(1) 118 (2) 92
(3) 102 (4) 108

Answer (1)

Sol. $f(x)=\dfrac{2^{x+2}+16}{2^{2x+1}+2^{x+4}+32}=\dfrac{4(2^x+4)}{2(2^{2x})+16(2^x)+32}$

$=\dfrac{2(2^x+4)}{(2^x+4)^2}=\dfrac{2}{2^x+4}$

Now, $f(x)+f(4-x)=\dfrac{2}{2^x+4}+\dfrac{2}{2^{4-x}+4}$

$=\dfrac{2}{2^x+4}+\dfrac{2\times 2^x}{16+4\times 2^x}=\dfrac{2}{2^x+4}+\dfrac{2x}{2(4+2^x)}$

$=\dfrac{1}{2}$

So, $f\left(\dfrac{1}{15}\right)+f\left(\dfrac{59}{15}\right)=\dfrac{1}{2}$

$f\left(\dfrac{2}{15}\right)+f\left(\dfrac{58}{15}\right)=\dfrac{1}{2}$

and $f\left(\dfrac{30}{15}\right)=\dfrac{2}{4+4}=\dfrac{1}{2}$

$\therefore 8\left(f\left(\dfrac{1}{15}\right)+f\left(\dfrac{2}{15}\right)+...f\left(\dfrac{59}{15}\right)\right)$

$=8\left(\dfrac{29}{2}+\dfrac{1}{4}\right)=118$

17. If the system of equations

$2x-y+z=4$

$5x+\lambda y+3z=12$

$100x-47y+\mu z=212$

has infinitely many solutions, then $\mu-2\lambda$ is equal to

(1) 59 (2) 55
(3) 56 (4) 57

Answer (4)

Sol. $2x-y+z=4$

$5x+\lambda y+3z=12$

$100x-47y+\mu z=212$

$\Delta x=\Delta y=\Delta z=0$

$$\Delta z=\begin{vmatrix}2 & -1 & 4\\ 5 & \lambda & 12\\ 100 & -47 & 212\end{vmatrix}=0$$

$\Rightarrow$ $2(212\lambda+564)+1(1060-1200)$
$+4(-235-100\lambda)=0$

$\Rightarrow$ $424\lambda+1128-140-940-400\lambda=0$

$\Rightarrow$ $\lambda=-2$

$$\Delta y=\begin{vmatrix}2 & -1 & 1\\ 5 & -2 & 3\\ 100 & -47 & \mu\end{vmatrix}=0$$

$\Rightarrow$ $2(-2\mu+141)+1(5\mu-300)+1(-235+200)=0$

$\Rightarrow$ $\mu=53$

$\mu-2\lambda=53-2(-2)=57$

18. For some $n\neq 10$, let the coefficients of the 5th, 6th and 7th terms in the binomial expansion of $(1+x)^{n+4}$ be in A.P. Then the largest coefficient in the expansion of $(1+x)^{n+4}$ is:

(1) 10 (2) 70
(3) 35 (4) 20

Answer (3)

Sol. Binomial coefficient of $T_{r+1}={}^{n+4}C_r$

${}^{n+4}C_4$, ${}^{n+4}C_5$, ${}^{n+4}C_6$ in A.P.

$\Rightarrow$ $\dfrac{(n+4)!}{4!\,n!}+\dfrac{(n+4)!}{6!\,(n-2)!}=2\dfrac{(n+4)!}{(n-1)!\,5!}$

$\Rightarrow$ $n=3$

$\Rightarrow$ Greatest binomial coefficient in the expansion of $(1+x)^{n+4}$

$=(1+x)^7$

$\Rightarrow$ ${}^7C_3={}^7C_4\Rightarrow\dfrac{7!}{3!\,4!}=\dfrac{7\times6\times5}{3\times2}=35$

19. Let the product of the focal distances of the point $\left(\sqrt{3}, \frac{1}{2}\right)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a>b)$, be $\frac{7}{4}$. Then the absolute difference of the eccentricities of two such ellipses is

(1) $\frac{3-2\sqrt{2}}{3\sqrt{2}}$ (2) $\frac{1-2\sqrt{2}}{\sqrt{3}}$

(3) $\frac{1-\sqrt{3}}{\sqrt{2}}$ (4) $\frac{3-2\sqrt{2}}{2\sqrt{3}}$

Answer (4)

Sol. Given $\left(a+e\sqrt{3}\right)\left(a-e\sqrt{3}\right)=\frac{7}{4}$ ...(i)

Also, $\frac{3}{a^2}+\frac{1}{4b^2}=1$ ...(ii)

And $b^2 = a^2(1-e^2)$ ...(iii)

From (i), (ii) and (iii)

$12e^4 - 17e^2 + 6 = 0$

$\Rightarrow (3e^2-2)(4e^2-3)=0 \Rightarrow e=\sqrt{\frac{2}{3}}$ or $\sqrt{\frac{3}{4}}$

20. Let the lines $3x-4y-\alpha=0$, $8x-11y-33=0$, and $2x-3y+\lambda=0$ be concurrent. If the image of the point (1, 2) in the line $2x-3y+\lambda=0$ is $\left(\frac{57}{13}, \frac{-40}{13}\right)$, then $|\alpha\lambda|$ is equal to

(1) 91 (2) 84

(3) 113 (4) 101

Answer (1)

Sol. $\begin{vmatrix} 3 & -4 & -\alpha \\ 8 & -11 & -33 \\ 2 & -3 & \lambda \end{vmatrix}=0$

$\Rightarrow 2\alpha-\lambda=33$ ...(i)

$\frac{x-1}{2}=\frac{y-2}{-3}=\frac{-2.|2-6+\lambda|}{4+9}=\frac{-2\lambda+8}{13}$

Image: $x=\frac{29-4\lambda}{13}=\frac{57}{13}$ (given)

$\Rightarrow \lambda=-7$

From (i) $\alpha = 13$

$|\alpha.\lambda| = 91$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The number of 3-digit numbers, that are divisible by 2 and 3, but not divisible by 4 and 9, is

Answer (125)

Sol. Number of 3 digit number = 900

Number of 3 digit number divisible by 2 and 3

$=\frac{900}{6}=150$

Number of 3 digit number divisible by 4 and 9

$=\frac{900}{36}=25$

Number of 3 digit number divisible by 2 and 3 but not divisible by 4 and 9

= 150 – 25 = 125

22. Let f be a differentiable function such that $2(x+2)^2 f(x) - 3(x+2)^2 = 10\int_0^x (t+2)f(t)dt$, $x \geq 0$. Then $f(2)$ is equal to ______.

Answer (19)

Sol. $2(x+2)^2 f(x)-3(x+2)^2=10\int_0^x (t+2)f(t)dt$

Differentiating both side

$4(x+2)f(x)+2f'(x)(x+2)^2-6(x+2)=10(x+2)f(x)$

$=(x+2)\frac{dy}{dx}-3y=3$

$\frac{1}{3}\int\frac{dy}{y+1}=\int\frac{dx}{x+2}$

$\ln|y+1|=3\ln|x+2|+\ln c$

$y+1=(x+2)^3 c$

$\because y(0)=\frac{3}{2}$

$\Rightarrow \frac{5}{16}=c$

$\therefore y=\frac{5}{16}(x+2)^3-1$

$y(2)=\frac{5}{16}\times 64-1=19$

23. Let A be a 3 × 3 matrix such that $X^T AX = O$ for all nonzero 3 × 1 matrices $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. If $A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$, $A\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$, and det (adj(2(A + I))) = $2^\alpha 3^\beta 5^\gamma$. $\alpha, \beta, \gamma \in \mathbb{N}$, then $\alpha^2 + \beta^2 + \gamma^2$ is______.

Answer (44)

Sol. $X^T AX = O$

$$(xyz)\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$(xyz)\begin{bmatrix} a_1x + a_2y + a_3z \\ b_1x + b_2y + b_3z \\ c_1x + c_2y + c_3z \end{bmatrix} = 0$$

$x(a_1x + a_2y + a_3z) + y(b_1x + b_2y + b_3z) + z(c_1x + c_2y + c_3z)$

$a_1 = 0, b_2 = 0, c_3 = 0$

$a_2 + b_1 = 0, a_3 + c_1 = 0, b_3 + c_2 = 0$

A = skew symmetric matrix

$$A = \begin{bmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{bmatrix}; A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$$

$x + y = 1$

$-x + z = 4$

$y + z = 5$

$$\begin{bmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -8 \end{bmatrix}$$

$2x + y = 0$ $x = -1$

$-x + z = 4$ $y = 2$

$-y - 2z = -8$ $z = 3$

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$$

$$2(A + I) = \begin{bmatrix} 2 & -2 & 4 \\ 2 & 2 & 6 \\ -2 & -6 & 2 \end{bmatrix}$$

$2(A + I) = 120$

$\Rightarrow$ det(adj(2A + I))

$= 120^2 = 2^6 . 3^2 . 5^2$

$\therefore$ $\alpha = 6, \beta = 2, \gamma = 2$

Hence $\alpha^2 + \beta^2 + \gamma^2 = 6^2 + 2^2 + 2^2 = 44$

24. Let $S = \{p_1, p_2 ...., p_{10}\}$ be the set of first ten prime numbers. Let $A = S \cup P$, where P is the set of all possible products of distinct elements of S. Then the number of all ordered pairs (x, y), $x \in S$, $y \in A$, such that x divides y, is ______.

Answer (5120)

Sol. Let $\frac{y}{x} = \lambda$

$y = \lambda x$

$= 10 \times \left({}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + ... + {}^9C_9\right)$

$= 10 \times 2^9$

$= 10 \times 512 = 5120$

25. If for some α, β; $\alpha \le \beta$, $\alpha + \beta = 8$ and $\sec^2(\tan^{-1}\alpha) + \text{cosec}^2(\cot^{-1}\beta) = 36$, then $\alpha^2 + \beta$ is _______.

Answer (14)

Sol. Let $\tan^{-1}\alpha = A \Rightarrow \tan A = \alpha$

$\cot^{-1}\beta = B \Rightarrow \cot B = \beta$

$\sec^2 A + \text{cosec}^2 B = 36$

$\Rightarrow$ $1 + \tan^2 A + 1 + \cot^2 B = 36$

$\Rightarrow$ $\alpha^2 + \beta^2 = 34$

Also $\alpha + \beta = 8$ (Given)

$\therefore$ $(\alpha + \beta)^2 = 34 + 2\alpha\beta = 64$

$\Rightarrow$ $\alpha\beta = 15$

$\Rightarrow$ α, β are roots of equation

$x^2 - 8x + 15 = 0$

$\Rightarrow$ $(x - 3)(x - 5) = 0$

$\Rightarrow$ $x = 3, 5$

$\therefore$ $\alpha = 3, \beta = 5$ $(\alpha < \beta)$

$\therefore$ $\alpha^2 + \beta = 9 + 5 = 14$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. Consider a parallel plate capacitor of area A (of each plate) and separation 'd' between the plates. If E is the electric field and ε_0 is the permittivity of free space between the plates, then potential energy stored in the capacitor is

(1) $\frac{1}{4}\varepsilon_0 E^2 Ad$ (2) $\frac{1}{2}\varepsilon_0 E^2 Ad$

(3) $\varepsilon_0 E^2 Ad$ (4) $\frac{3}{4}\varepsilon_0 E^2 Ad$

Answer (2)

Sol. We know energy density

$$\rho_{av} = \frac{1}{2}\varepsilon_0 E^2$$

So potential energy = $\rho_{av} \times$ Volume

$$\Rightarrow \text{P.E.} = \frac{1}{2}\varepsilon_0 E^2 Ad$$

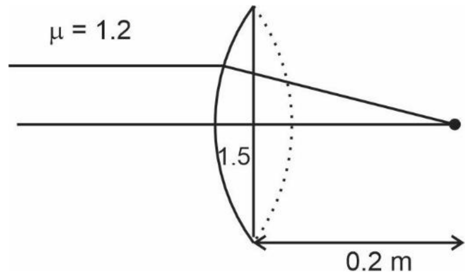
27. A thin plano convex lens made of glass of refractive index 1.5 is immersed in a liquid of refractive index 1.2. When the plane side of the lens is silver coated for complete reflection, the lens immersed in the liquid behaves like a concave mirror of focal length 0.2 m. The radius of curvature of the curved surface of the lens is

(1) 0.20 m (2) 0.15 m

(3) 0.25 m (4) 0.10 m

Answer (4)

Sol.



We have shown extrapolated figure for reflecting side as it went undeviated.

$$\frac{1.5}{v_1} - \frac{1.2}{\infty} = \frac{(1.5-1.2)}{R}$$

$$\frac{1.2}{0.2} - \frac{1.5}{v_1} = \frac{(1.2-1.5)}{-R}$$

$$\frac{1.2}{0.2} = \frac{0.3}{R} + \frac{0.3}{R}$$

$$\Rightarrow 6 = \frac{2\times 3}{10R} \Rightarrow R = \frac{1}{10}\text{m} = 0.10 \text{ m}$$

28. For an experimental expression $y = \frac{32.3\times 1125}{27.4}$, where all the digits are significant. Then to report the value of y we should write

(1) y = 1330

(2) y = 1326.2

(3) y = 1326.19

(4) y = 1326.186

Answer (1)

Sol. $y = \frac{32.3\times 1125}{27.4} = 1326.18$

So we need to report to three significant digit.

So, y = 1330

29. A satellite is launched into a circular orbit of radius 'R' around the earth. A second satellite is launched into an orbit of radius 1.03R. The time period of revolution of the second satellite is larger than the first one approximately by

(1) 9%

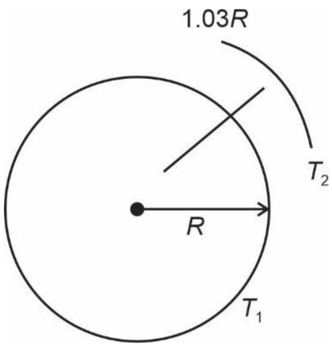
(2) 2.5%

(3) 3%

(4) 4.5%

Answer (4)

Sol.



$T_1 \propto (R)^{3/2}$

and $T_2 \propto (1.03R)^{3/2}$

$\Rightarrow \quad T_2 = (1.03R)^{3/2} \cdot T_1 \approx 1.045T_1$

So T_2 will larger by 4.5% w.r.t. T_1.

30. The amount of work done to break a big water drop of radius 'R' into 27 small drops of equal radius is 10 J. The work done required to break the same big drop into 64 small drops of equal radius will be

(1) 10 J

(2) 20 J

(3) 5 J

(4) 15 J

Answer (4)

Sol. $W_1 = s\,4\pi\left(\frac{R}{3}\right)^2 \times 27 - 4\pi R^2 \cdot s$

$\Rightarrow \quad W_1 = 4\pi R^2 \cdot s(2) = 10 \text{ Joule}$

Now, $W_2 = s\,4\pi R^2(4-1) = 4\pi R^2 s \times 3$

$\Rightarrow \quad W_2 = 3 \times \frac{10}{2} = 15 \text{ Joule}$

31. The Young's double slit interference experiment is performed using light consisting of 480 nm and 600 nm wavelengths to form interference patterns. The least number of the bright fringes of 480 nm light that are required for the first coincidence with the bright fringes formed by 600 nm light is

(1) 6

(2) 8

(3) 5

(4) 4

Answer (3)

Sol. Fringe width of λ_1 = 480 nm

$$\Delta w_1 = \frac{\lambda_1 D}{d} = 480\left(\frac{D}{d}\right)$$

Similarly fringe width for λ_2 = 600 nm

$$\Delta w_2 = \lambda_2\left(\frac{D}{d}\right) = 600\left(\frac{D}{d}\right)$$

Clearly we can see that for minimum value

$$5(\Delta w_1) = 4(\Delta w_2)$$

That mean 5th maxima of λ_1 = 480 nm

Coincides with 4th maxima of λ_2 = 600 nm

32. An ideal gas goes from an initial state to final state. During the process, the pressure of gas increases linearly with temperature.

A. The work done by gas during the process is zero.

B. The heat added to gas is different from change in its internal energy.

C. The volume of the gas is increased.

D. The internal energy of the gas is increased.

E. The process is isochoric (constant volume process)

Choose the **correct** answer from the options given below:

(1) A, D, E only (2) E only

(3) A, C only (4) A, B, C, D only

Answer (1)

Sol. Given for ideal gas.

P = constant T

$\Rightarrow$ PT^{-1} = constant

$\Rightarrow \dfrac{P}{PV} = \text{constant} = V$ is constant

So, isochoric process

Work done by gas will be zero.

A, D, E only correct.

33. A car of mass 'm' moves on a banked road having radius 'r' and banking angle θ. To avoid slipping from banked road, the maximum permissible speed of the car is v_0. The coefficient of friction μ between the wheels of the car and the banked road is

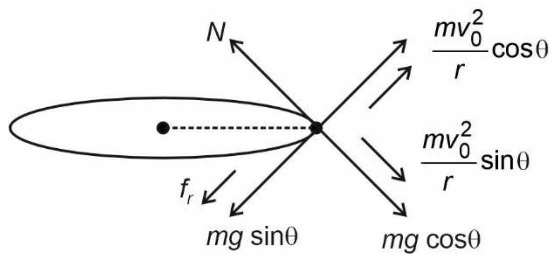
(1) $\mu = \dfrac{v_0^2 + rg\tan\theta}{rg - v_0^2\tan\theta}$ (2) $\mu = \dfrac{v_0^2 + rg\tan\theta}{rg + v_0^2\tan\theta}$

(3) $\mu = \dfrac{v_0^2 - rg\tan\theta}{rg + v_0^2\tan\theta}$ (4) $\mu = \dfrac{v_0^2 - rg\tan\theta}{rg - v_0^2\tan\theta}$

Answer (3)

Sol. Given maximum possible speed = v_0

For this



So, $N = mg\cos\theta + \dfrac{mv_0^2}{r}\sin\theta$

$f_r = \mu mg\cos\theta + \dfrac{\mu mv_0^2}{r}\sin\theta$

And $\dfrac{mv_0^2}{r}\cos\theta = mg\sin\theta + f_r$

$\Rightarrow \dfrac{mv_0^2}{r}\cos\theta - mg\sin\theta = \mu\left(mg\cos\theta + \dfrac{mv_0^2}{r}\sin\theta\right)$

$\Rightarrow \left(v_0^2 - gr\tan\theta\right) = \mu\left(v_0^2\tan\theta + gr\right)$

$\Rightarrow \mu = \dfrac{v_0^2 - gr\tan\theta}{gr + v_0^2\tan\theta}$

34. An air bubble of radius 0.1 cm lies at a depth of 20 cm below the free surface of a liquid of density 1000 kg/m^3. If the pressure inside the bubble is 2100 N/m^2 greater than the atmospheric pressure, then the surface tension of the liquid in SI unit is (use g = 10 m/s^2)

(1) 0.02

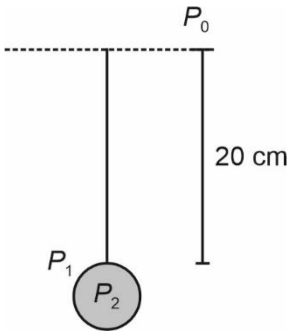
(2) 0.05

(3) 0.1

(4) 0.25

Answer (2)

Sol.



$$P_1 = P_0 + \rho gh = P_0 + 1000 \times 10 \times \frac{20}{100}$$

$\Rightarrow$ $P_1 = P_0 + 2000$

So, $P_2 - P_1 = \frac{2S}{R} = \left(\frac{2S}{1\times10^{-3}}\right)$

$\Rightarrow$ $P_2 = P_0 + 2100$ (given)

So, $P_0 + 2100 - P_0 - 2000 = 2S \times 10^3$

$\Rightarrow$ $100 = 2S \times 10^3$

$\Rightarrow$ $s = \left(\frac{1}{20}\right) = 0.05$

35. A parallel plate capacitor was made with two rectangular plates, each with a length of l = 3 cm and breadth of b = 1 cm. The distance between the plates is 3 μm. Out of the following, which are the ways to increase the capacitance by a factor of 10?

A. l = 30 cm, b = 1 cm, d = 1 μm

B. l = 3 cm, b = 1 cm, d = 30 μm

C. l = 6 cm, b = 5 cm, d = 3 μm

D. l = 1 cm, b = 1 cm, d = 10 μm

E. l = 5 cm, b = 2 cm, d = 1 μm

Choose the correct answer from the options given below:

(1) C and E only

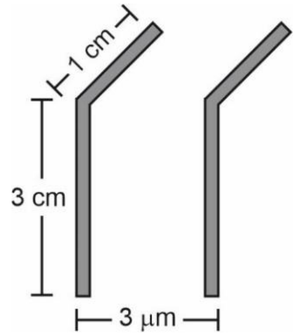
(2) C only

(3) B and D only

(4) A only

Answer (1)

Sol.



We know $C = \frac{A\varepsilon_0}{d}$

$= \frac{b\ell\varepsilon_0}{d}$

So to increase the capacitance by 10 factor $\left(\frac{A}{d}\right)$ has to increase by 10 fator.

For option (A) $C' = \frac{(30\ell)b\varepsilon_0}{\left(\frac{d}{3}\right)} = 30C$

For option (B) $C' = \frac{\ell b\varepsilon_0}{10d} = \frac{C}{10}$

For option (C) $C' = \frac{(2\ell)5b\varepsilon_0}{d} = 10C$

For option (D) $C' = \frac{\left(\frac{\ell}{3}\right)b\varepsilon_0}{\left(\frac{10d}{3}\right)} = \frac{C}{10}$

For option (E) $C' = \frac{\left(\frac{\ell}{3}\right)5(2b)\varepsilon_0}{\left(\frac{d}{3}\right)} = 10C$

Clearly (C) and (E) are the situation for 10C

36. A uniform solid cylinder of mass 'm' and radius 'r' rolls along an inclined rough plane of inclination 45°. If it starts to roll from rest from the top of the plane then the linear acceleration of the cylinder's axis will be

(1) $\frac{1}{3\sqrt{2}}g$

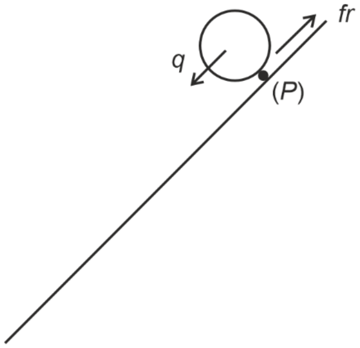
(2) $\frac{\sqrt{2}g}{3}$

(3) $\sqrt{2}\,g$

(4) $\frac{1}{\sqrt{2}}g$

Answer (2)

Sol.



For pure rolling about point '*P*'.

$$\Rightarrow \quad a = \frac{2g}{3}\sin\theta = \frac{2g}{3\sqrt{2}} = \frac{\sqrt{2}}{3}g$$

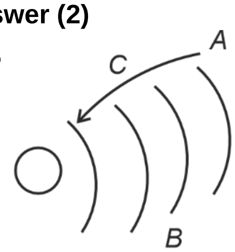
37 During the transition of electron from state *A* to state *C* of a Bohr atom, the wavelength of emitted radiation is 2000 Å and it becomes 6000 Å when the electron jumps from state *B* to state *C*. Then the wavelength of the radiation emitted during the transition of electrons from state *A* to state *B* is

(1) 2000 Å (2) 3000 Å

(3) 6000 Å (4) 4000 Å

Answer (2)

Sol.



For *A* to *C*

$$\frac{hc}{\lambda_1} = E_0 z^2\left(\frac{1}{n_C^2} - \frac{1}{n_A^2}\right) \quad \ldots\text{(i)}$$

And $$\frac{hc}{\lambda_2} = E_0 z^2\left(\frac{1}{n_C^2} - \frac{1}{n_B^2}\right) \quad \ldots\text{(ii)}$$

So for *A* and *B*

$$\frac{hc}{\lambda_3} = E_0 z^2\left(\frac{1}{n_B^2} - \frac{1}{n_A^2}\right)$$

Clearly subtracting equation (ii) from equation (i)

$$hc\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = E_0 z^2\left[\frac{1}{n_B^2} - \frac{1}{n_A^2}\right] = \frac{hc}{\lambda_3}$$

$$\Rightarrow \quad \frac{1}{\lambda_3} = \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \Rightarrow \frac{1}{\lambda_3} = \frac{(6000 - 2000)}{6000 \times 2000} = \frac{1}{3000}$$

λ_3 = 3000Å

38. An object of mass '*m*' is projected from origin in a vertical *xy* plane at an angle 45° with the *x*- axis with an initial velocity v_0. The magnitude and direction of the angular momentum of the object with respect to origin, when it reaches at the maximum height, will be [*g* is acceleration due to gravity]

(1) $\frac{mv_0^3}{4\sqrt{2}g}$ along positive z-axis

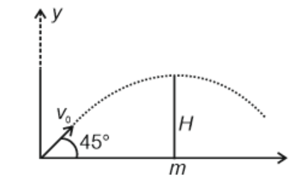
(2) $\frac{mv_0^3}{2\sqrt{2}g}$ along negative z-axis

(3) $\frac{mv_0^3}{2\sqrt{2}g}$ along positive z-axis

(4) $\frac{mv_0^3}{4\sqrt{2}g}$ along negative z-axis

Answer (4)

Sol.



At height point

The speed is $v_x = v\cos 45°$

$$v_x = \frac{v_0}{\sqrt{2}}$$

So angular momentum about origin is

$L = mv_x \cdot H$

$$\Rightarrow \quad L = m\frac{v_0}{\sqrt{2}}\cdot\left(\frac{v_0}{\sqrt{2}}\right)^2 \times \frac{1}{2g}$$

$$\Rightarrow \quad L = \frac{mv_0^3}{4\sqrt{2}g} \ (-\mu\text{- } z\text{-axis})$$

39. Consider the following statements:

A. The junction area of solar cell is made very narrow compared to a photo diode.

B. Solar cells are not connected with any external bias.

C. LED is made of lightly doped p-n junction.

D. Increase of forward current results in continuous increase of LED light intensity.

E. LEDs have to be connected in forward bias for emission of light.

Choose the **correct** answer from the options given below:

(1) B, E Only (2) B, D, E Only

(3) A, C, E Only (4) A, C Only

Answer (1)

Sol. Clearly statement 'A' is wrong as for solar cell junction area are mode wide. LED connected in forward biase and its intensity increases upto certain value of current and further there is no charge as intensity saturates.

Also solar cells are not connected with any external bias. So '*B*' is also correct

40. What is the relative decrease in focal length of a lens for an increase in optical power by 0.1D from 2.5 D? ['D' stands for dioptre]

(1) 0.01 (2) 0.04

(3) 0.1 (4) 0.40

Answer (2)

Sol. $P = \dfrac{1}{f}$

So $P_1 = \dfrac{1}{F_1} \Rightarrow F_1 = \dfrac{1}{2.5}$

Next $F_2 = \dfrac{1}{2.6}$

$$\left|\frac{\Delta F}{F_1}\right| = \frac{|F_2 - F_1|}{F_1} = \left|\frac{F_2}{F_1} - 1\right|$$

$$\Rightarrow \left|\frac{\Delta F}{F_1}\right| = \left|\frac{2.5}{2.6} - 1\right| = \frac{0.1}{2.6} \approx 0.04$$

41. A force $F = \alpha + \beta x^2$ acts on an object in the *x*-direction. The work done by the force is 5 J when the object is displaced by 1 m. If the constant α = 1N then β will be

(1) 8 N/m^2 (2) 15 N/m^2

(3) 12 N/m^2 (4) 10 N/m^2

Answer (3)

Sol. $F = \alpha + \beta x^2$

Work done $\int dw = \int F \cdot dx$

$$\Rightarrow \Delta w = \int F \cdot dx = \int (\alpha + \beta x^2) dx$$

$$\Rightarrow \Delta w = \left| \alpha x + \frac{\beta x^3}{3} \right|_0^1 = \alpha + \frac{\beta}{3} = 5$$

Given α = 1

So, $\dfrac{\beta}{3} = 4$

$\Rightarrow \beta = 12$ N/m^2

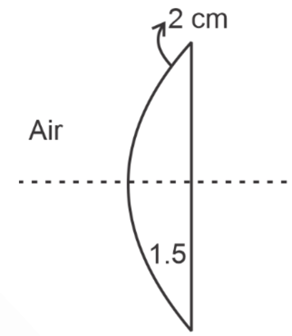
42. A plano-convex lens having radius of curvature of first surface 2 cm exhibits focal length of f_1 in air. Another plano-convex lens with first surface radius of curvature 3 cm has focal length of f_2 when it is immersed in a liquid of refractive index 1.2. If both the lenses are made of same glass of refractive index 1.5, the ratio of f_1 and f_2 will be

(1) 1 : 2 (2) 1 : 3

(3) 2 : 3 (4) 3 : 5

Answer (2)

Sol. For case (1)



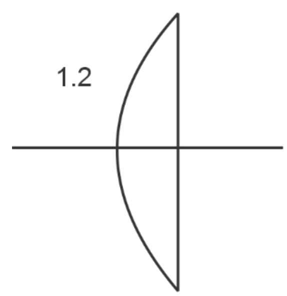
$$\frac{1.5}{v_1} - \frac{1}{\infty} = \frac{(1.5-1)}{R_1}$$

$$\frac{1}{f_1} - \frac{1.5}{v_1} = \frac{1-1.5}{\infty}$$

$$\Rightarrow \frac{1}{f_1} = \frac{(1.5-1)}{R_1} \quad \text{...(1)}$$

$$\Rightarrow \frac{1}{f_1} = \frac{1}{2R_1}$$

For case (2)



$$\frac{1.5}{v_1} - \frac{1.2}{\infty} = \frac{(1.5-1.2)}{R_2}$$

$$\frac{1.2}{f_2} - \frac{1.5}{v_1} = \frac{(1.2-1.5)}{\infty}$$

$$\Rightarrow \frac{1.2}{f_2} = \frac{0.3}{R_2}$$

$$\Rightarrow \frac{1}{f_2} = \frac{1}{4R_2}$$

So clearly, $\dfrac{f_1}{f_2} = \dfrac{2 \times 2}{4 \times 3} = \dfrac{1}{3}$

43. A particle is executing simple harmonic motion with time period 2 s and amplitude 1 cm. If D and d are the total distance and displacement covered by the particle in 12.5 s, then $\frac{D}{d}$ is

(1) 10
(2) $\frac{15}{4}$
(3) $\frac{16}{5}$
(4) 25

Answer (4)

Sol. $T = 2$ s

$A = 1$ cm

In 12.5 s 6 total oscillation and 1 quarter oscillation.

So, $D = (6 \times 4A) + A = 25A$

$d = A$

$\frac{D}{d} = 25$

44. An alternating current is given by $I = I_A \sin\omega t + I_B \cos\omega t$. The r.m.s current will be

(1) $\sqrt{\frac{I_A^2 + I_B^2}{2}}$
(2) $\frac{\sqrt{I_A^2 + I_B^2}}{2}$
(3) $\sqrt{I_A^2 + I_B^2}$
(4) $\frac{|I_A + I_B|}{\sqrt{2}}$

Answer (1)

Sol. $I = I_A \sin\omega t + I_B \cos\omega t$

$I_{rms} = \sqrt{I_A^2 + I_B^2}$

So, $I_{RMS} = \sqrt{\frac{I_A^2 + I_B^2}{2}}$

45. An electron of mass 'm' with an initial velocity $\vec{v} = v_0\hat{i}\ (v_0 > 0)$ enters an electric field $\vec{E} = -E_0\hat{k}$. If the initial de Broglie wavelength is λ_0, the value after t would be

(1) $\frac{\lambda_0}{\sqrt{1 + \frac{e^2 E_0{}^2 t^2}{m^2 v_0^2}}}$
(2) $\lambda_0\sqrt{1 + \frac{e^2 E_0{}^2 t^2}{m^2 v_0^2}}$
(3) λ_0
(4) $\frac{\lambda_0}{\sqrt{1 - \frac{e^2 E_0{}^2 t^2}{m^2 v_0{}^2}}}$

Answer (1)

Sol. $mv_0 \longrightarrow \otimes$

$E = -E_0\hat{k}$

$$\lambda_0 = \frac{h}{mv_0}$$

After time 't' $v_{(t)} = \left(\frac{E_0 e}{m}\right) \cdot t$

So, $v = \sqrt{v_0^2 + \left(\frac{eE_0 t}{m}\right)^2}$

Now, $\lambda = \frac{h}{m\sqrt{v_0^2 + \left(\frac{eE_0 t}{m}\right)^2}}$

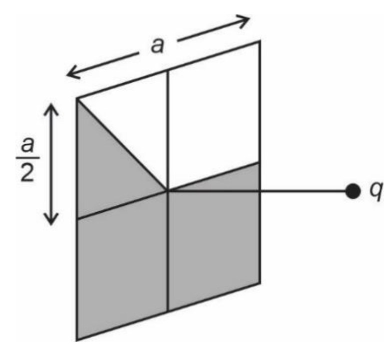
$$\lambda_0 = \frac{h}{mv_0}$$

$$\Rightarrow \frac{\lambda}{\lambda_0} = \frac{1}{\sqrt{1 + \left(\frac{eE_0 t}{mv_0}\right)^2}}$$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. A square loop of side $a = 1$ m is held normally in front of a point charge $q = 1$C. The flux of the electric field through the shaded region is $\frac{5}{p} \times \frac{1}{\varepsilon_0} \frac{\text{Nm}^2}{\text{C}}$, where the value of p is _______.



Answer (48)

Sol. Flux through complete surface

$$\phi_1 = \frac{q}{\varepsilon_0 \times 6}$$

So flux through shaded region is

$$\phi_2 = \phi_1 \frac{5}{8} = \frac{q}{6\varepsilon_0} \times \frac{5}{8} = \frac{5q}{48\varepsilon_0}$$

So, $p = 48$

47. The least count of a screw gauge is 0.01 mm. If the pitch is increased by 75% and number of divisions on the circular scale is reduced by 50%, the new least count will be ________ $\times 10^{-3}$ mm.

Answer (35)

Sol. Least count $= 0.01 = \left(\frac{L}{N}\right)$

Now $\frac{1.75L}{N/2}$ = New least count

$$\Rightarrow 3.5 \times \frac{L}{N} = 3.5 \times 0.01 \text{ mm}$$

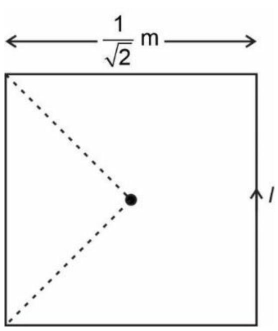
$$= 35 \times 10^{-3} \text{ mm}$$

48. A current of 5 A exists in a square loop of side $\frac{1}{\sqrt{2}}$ m. Then the magnitude of the magnetic field B at the centre of the square loop will be $p \times 10^{-6}$ T. Where, value of p is ________.

[Take $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}].

Answer (8)

Sol.



$$B = \frac{\mu_0 I \times 2}{4\pi\ell} \times 2\cos 45° \times 4$$

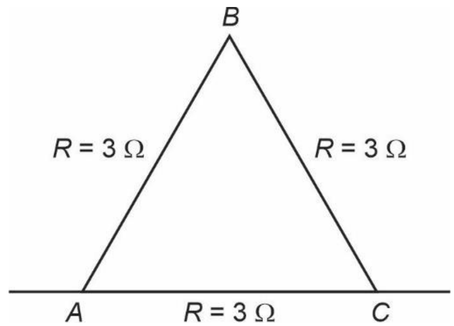
$$\Rightarrow B = \frac{4\pi \times 10^{-7} \times \sqrt{2}}{4\pi \times 1} \times 4 \times 4 \times \frac{1}{\sqrt{2}} \times 5$$

$$\Rightarrow B = 8 \times 10^{-6} \text{ T}$$

49. A wire of resistance 9 Ω is bent to form an equilateral triangle. Then the equivalent resistance across any two vertices will be ________ ohm.

Answer (2)

Sol.



Equivalent resistance across AC is

$$R_1 = 2R \,||\, R = \frac{2R \times R}{3R} = \frac{2R}{3}$$

$\Rightarrow$ $R_1 = 2\ \Omega$

50. The temperature of 1 mole of an ideal monoatomic gas is increased by 50°C at constant pressure. The total heat added the change in internal energy are E_1 and E_2, respectively. If $\frac{E_1}{E_2} = \frac{x}{9}$ then the value of x is ________.

Answer (15)

Sol. ΔT = 50°C = 50 K

$N = 1$ mole ideal gas

$\Delta P = 0$

$$\Delta Q_P = NC_P\Delta T = E_1 = N \cdot \frac{5}{2} R\Delta T$$

$$\Delta Q_V = NC_V\Delta T = E_2 = N\frac{3}{2}R\Delta T$$

$$\frac{E_1}{E_2} = \frac{x}{9} = \frac{C_P}{C_V}$$

For monoatomic gas = $\frac{C_P}{C_V} = \frac{5}{3}$

$$\Rightarrow \frac{x}{9} = \frac{5}{3}$$

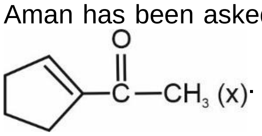
$\Rightarrow$ $x = 15$

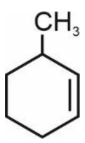
CHEMISTRY

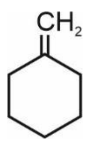
SECTION - A

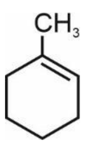
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

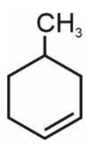
Choose the correct answer :

51. Aman has been asked to synthesise the molecule  (x). He thought of preparing the molecule using an aldol condensation reaction. He found a few cyclic alkenes in his laboratory. He thought of performing ozonolysis reaction on alkene to produce a dicarbonyl compound followed by aldol reaction to prepare "x". Predict the suitable alkene that can lead to the formation of "x".

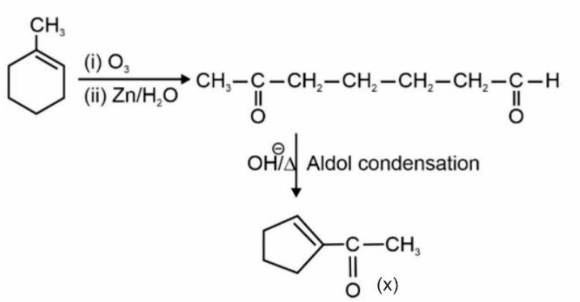
(1) 

(2) 

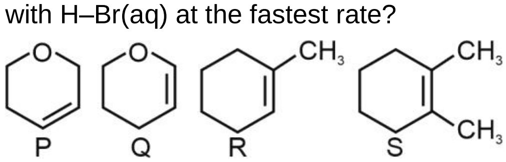
(3) 

(4) 

Answer (3)

Sol. 

52. Following are the four molecules "P", "Q", "R" and "S". Which one among the four molecules will react with H–Br(aq) at the fastest rate?

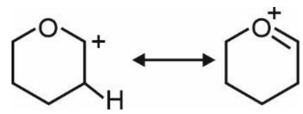


(1) P (2) R

(3) Q (4) S

Answer (3)

Sol. The reaction with H — Br will be fastest for the specie forming most stable carbocation



Due to resonance, it will have most stable carbocation.

53. One mole of the octahedral complex compound $Co(NH_3)_5Cl_3$ gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with excess of $AgNO_3$ solution to yield two moles of AgCl(s). The structure of the complex is

(1) $[Co(NH_3)_4Cl].Cl_2.NH_3$

(2) $[Co(NH_3)_4Cl_2].Cl.NH_3$

(3) $[Co(NH_3)_3Cl_3].2NH_3$

(4) $[Co(NH_3)_5Cl]Cl_2$

Answer (4)

Sol. $[Co(NH_3)_5Cl]Cl_2 \xrightarrow{H_2O} \underbrace{[Co(NH_3)_5Cl]^{2+} + 2Cl^-}_{\text{3 ions}}$

$\downarrow + AgNO_3$

2 mole AgCl

54. The carbohydrate "Ribose" present in DNA, is

A. A pentose sugar

B. present in pyranose form

C. in "D" configuration

D. a reducing sugar, when free

E. in α – anomeric form

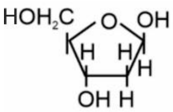
Choose the correct answer from the options given below

(1) A, B and E only (2) B, D and E only

(3) A, C and D only (4) A, D and E only

Answer (3)

Sol. Ribose present in DNA is β-(D)-deoxyribose



It is a pentose sugar present in furanose form having D configuration.

It is a reducing sugar having β-anomeric form

A, C & D are correct

55. Which of the following statement is true with respect to H_2O, NH_3 and CH_4?

A. The central atoms of all the molecules are sp^3 hybridized.

B. The H – O – H, H – N – H and H – C – H angles in the above molecules are 104.5º, 107.5º and 109.5º, respectively.

C. The increasing order of dipole moment is $CH_4 < NH_3 < H_2O$.

D. Both H_2O and NH_3 are Lewis acids and CH_4 is a Lewis base.

E. A solution of NH_3 in H_2O is basic. In this solution NH_3 and H_2O act as Lowry-Bronsted acid and base respectively

Choose the correct answer from the options given below

(1) A, B and C only
(2) A, B, C and E only
(3) C, D and E only
(4) A, D and E only

Answer (1)

Sol.

	H_2O	NH_3	CH_4
Hybridization	sp^3	sp^3	sp^3
Bond angle	104.5º	107.5º	109.5º
Dipole moment	1.85	1.47	0

Both H_2O & NH_3 are Lewis bases. NH_3 acts as base in H_2O, NH_3 & H_2O act as Lowry – Bronsted base & acid respectively.

A, B & C are correct

56. Which of the following statements are NOT true about the periodic table?

A. The properties of elements are function of atomic weights.

B. The properties of elements are function of atomic numbers.

C. Elements having similar outer electronic configuration are arranged in same period.

D. An element's location reflects the quantum numbers of the last filled orbital.

E. The number of elements in a period is same as the number of atomic orbitals available in energy level that is being filled.

Choose the correct answer from the options given below.

(1) D and E only
(2) B, C and E only
(3) A and E only
(4) A, C and E only

Answer (4)

Sol. Properties of elements are function of atomic numbers.

Elements having similar outer electronic configuration are arranged in same group.

Number of elements in a period is double of number of atomic orbitals available in energy level that is being filled.

So, (A), (C) and (E) are incorrect.

57. The large difference between the melting and boiling points of oxygen and sulphur may be explained on the basis of

(1) Atomicity
(2) Electronegativity
(3) Atomic size
(4) Electron gain enthalpy

Answer (1)

Sol. The large difference in the melting and boiling points of oxygen and sulphur is due to atomicity as oxygen exists as O_2 and sulphur exists as S_8.

58. For a reaction, $N_2O_5(g) \rightarrow 2NO_2(g) + \frac{1}{2}O_2(g)$ in a constant volume container, no products were present initially. The final pressure of the system when 50% of reaction gets completed is

(1) 5 times of initial pressure
(2) 5/2 times of initial pressure
(3) 7/4 times of initial pressure
(4) 7/2 times of initial pressure

Answer (3)

Sol. $N_2O_5(g) \rightarrow 2NO_2(g) + \frac{1}{2}O_2(g)$

$t = 0 \quad P_0$

$t = t_{50\%} \quad P_0(1-\alpha) \quad 2P_0\alpha \quad \frac{P_0\alpha}{2}$

Final pressure $= P_0 + P_0\alpha + \frac{P_0\alpha}{2} \quad (\alpha = 0.5)$

$= P_0 + \frac{P_0}{2} + \frac{P_0}{4}$

$= P_0\left(1+\frac{1}{2}+\frac{1}{4}\right)$

$= \frac{7P_0}{4}$

59. Given below are two statements I and II.

Statement I : Dumas method is used for estimation of "Nitrogen" in an organic compound.

Statement II : Dumas method involves the formation of ammonium sulphate by heating the organic compound with conc. H_2SO_4.

In the light of the above statements, choose the ***correct*** answer from the options given below.

(1) Both Statement I and Statement II are true

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Answer (3)

Sol. Dumas method is used to estimate nitrogen which involves heating of compound with copper oxide.

60. Let us consider an endothermic reaction which is non-spontaneous at the freezing point of water. However, the reaction is spontaneous at boiling point of water. Choose the correct option.

(1) Both ΔH and ΔS are (–ve)

(2) Both ΔH and ΔS are (+ve)

(3) ΔH is (+ve) but ΔS is (–ve)

(4) ΔH is (–ve) but ΔS is (+ve)

Answer (2)

Sol. On increasing temperature, the given endothermic reaction becomes spontaneous.

$\Delta G = \Delta H - T\Delta S$

For spontaneity,

$\Delta G < 0$

Given,

$\Delta H > 0$

So, ΔS should be +ve.

61. Which of the following linear combination of atomic orbitals will lead to formation of molecular orbitals in homonuclear diatomic molecules [internuclear axis in z-direction] ?

A. $2p_z$ and $2p_x$

B. $2s$ and $2p_x$

C. $3d_{xy}$ and $3d_{x^2-y^2}$

D. $2s$ and $2p_z$

E. $2p_z$ and $3d_{x^2-y^2}$

Choose the correct answer from the options given below:

(1) A and B Only

(2) D Only

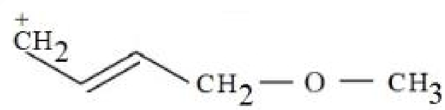
(3) E Only

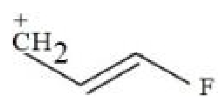
(4) C and D Only

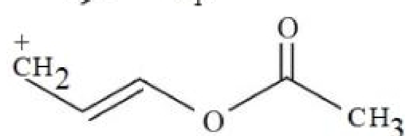
Answer (2)

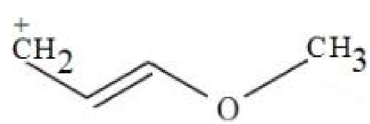
Sol. $2s$ and $2p_z$ will lead to formation of molecular orbital as they have very small difference in energies.

62. Which one of the carbocations from the following is most stable?

(1) 

(2) 

(3) 

(4) 

Answer (4)

Sol. $\overset{\oplus}{C}H_2-CH=CH-O-CH_3$ will be most stable due to extended conjugation

(3) will be less stable due to –M of $-\underset{\underset{O}{\|}}{C}-$

63. Given below are two statements:

Statement I: The conversion proceeds well in the less polar medium.

$$CH_3-CH_2-CH_2-CH_2-Cl \xrightarrow{HO^-} CH_3-CH_2-CH_2-CH_2-OH + Cl^{(-)}$$

Statement II: The conversion proceeds well in the more polar medium.

$$CH_3-CH_2-CH_2-CH_2-Cl \xrightarrow{R_3\ddot{N}} CH_3-CH_2-CH_2-CH_2-\overset{+}{N}R_3\ Cl^{(-)}$$

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) Statement I is true but Statement II is false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Answer (3)

Sol. Both the reactions will proceed via SN^2 mechanism but OH^- will proceed in less polar medium while $R_3\ddot{N}$ will occur well in more polar medium. Both statements are true.

64. The product (A) formed in the following reaction sequence is

$$CH_3-C\equiv CH \xrightarrow[\text{iii) } H_2/Ni]{\text{i) } Hg^{2+}, H_2SO_4 \text{ ii) HCN}} \underset{\text{Product}}{(A)}$$

(1) $CH_3-CH_2-CH(OH)-CH_2-NH_2$

(2) $CH_3-C(OH)(CH_3)-CH_2-NH_2$

(3) $CH_3-CH_2-CH(NH_2)-CH_2-OH$

(4) $CH_3-C(NH_2)(CH_3)-CH_2-OH$

Answer (2)

Sol. $CH_3-C\equiv CH \xrightarrow{Hg^{2+}, H_2SO_4} CH_3-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow{HCN} CH_3-C(OH)(CN)-CH_3 \xrightarrow{H_2/Ni} CH_3-C(OH)(CH_3)-CH_2-NH_2$ (A)

65. K_{sp} for $Cr(OH)_3$ is 1.6×10^{-30}. What is the molar solubility of this salt in water?

(1) $\sqrt[5]{1.8\times10^{-30}}$

(2) $\sqrt[4]{\frac{1.6\times10^{-30}}{27}}$

(3) $\frac{1.8\times10^{-30}}{27}$

(4) $\sqrt[2]{1.6\times10^{-30}}$

Answer (2)

Sol. $\underset{s}{Cr(OH)_3} \rightleftharpoons \underset{s}{Cr^{3+}(aq)} + \underset{3s}{3OH^-(aq)}$

$K_{sp} = s(3s)^3$

$1.6 \times 10^{-30} = 27s^4$

$\sqrt[4]{\frac{1.6\times10^{-30}}{27}} = s$

66. Which of the following ions is the strongest oxidizing agent?

(Atomic Number of Ce = 58, Eu = 63, Tb = 65, Lu = 71)

(1) Ce^{3+}

(2) Eu^{2+}

(3) Tb^{4+}

(4) Lu^{3+}

Answer (3)

Sol. Tb^{4+} is strongest oxidising agent as it will reduce to Tb^{3+} (common O.S. of Ln)

67. Which of the following arrangements with respect to their reactivity in nucleophilic addition reaction is correct?

(1) p-nitrobenzaldehyde < benzaldehyde < p-tolualdehyde < acetophenone

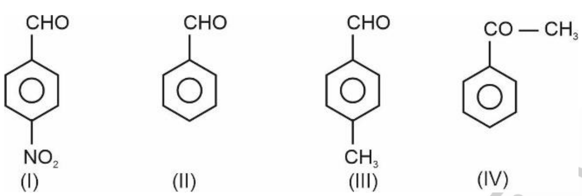
(2) acetophenone < p-tolualdehyde < benzaldehyde < p-nitrobenzaldehyde

(3) benzaldehyde < acetophenone < p-nitrobenzaldehyde < p-tolualdehyde

(4) acetophenone < benzaldehyde < p-tolualdehyde < p-nitrobenzaldehyde

Answer (2)

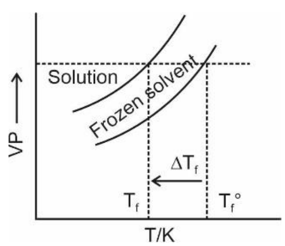
Sol. Reactivity $\propto$ +ve charge on electrophilic carbon



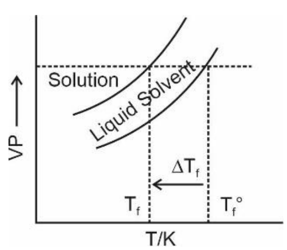
Correct order I > II >III > IV

68. Consider the given plots of vapour pressure (VP) vs temperature (T/K). Which amongst the following options is correct graphical representation showing ΔT_f. depression in the freezing point of a solvent in as solution?

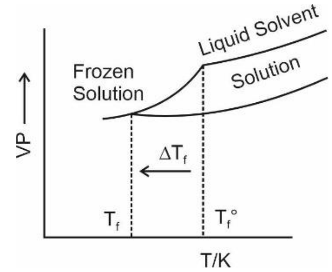
(1)



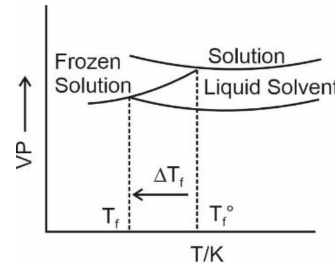
(2)



(3)

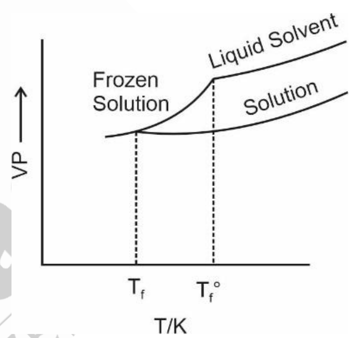


(4)



Answer (3)

Sol. Correct variation



69. For the given cell

$Fe^{2+}(aq) + Ag^{+}(aq) \rightarrow Fe^{3+}(aq) + Ag(s)$

The standard cell potential of the above reaction is

Given:

$Ag^{+} + e^{-} \rightarrow Ag$	$E^{\theta} = x$ V
$Fe^{2+} + 2e^{-} \rightarrow Fe$	$E^{\theta} = y$ V
$Fe^{3+} + 3e^{-} \rightarrow Fe$	$E^{\theta} = z$ V

(1) x + y – z (2) x + 2y

(3) x + 2y – 3z (4) y – 2x

Answer (3)

Sol.

$$Ag^+ + e^- \longrightarrow Ag \qquad E° = x\ V \qquad \Delta G°_1 = -1 \times (x) \times F$$

$$Fe^{2+} + 2e^- \longrightarrow Fe \qquad E° = y\ V \qquad \Delta G°_2 = -2 \times (y) \times F$$

$$Fe \longrightarrow Fe^{3+} + 3e^- \qquad E° = z\ V \qquad \Delta G°_3 = -2 \times (-z) \times F$$

$$Ag^+ + Fe^{2+} \longrightarrow Fe^{3+} + Ag$$

$$\Delta G° = \Delta G°_1 + \Delta G°_2 + \Delta G°_3$$

$-1 \times E° \times F = -1\ (x)\ F - 2(y)\ F + 3(z)F$

$E° = x + 2y - 3z$

70. Preparation of potassium permanganate from MnO_2 involves two step process in which the 1st step is a reaction with KOH and KNO_3 to produce

(1) K_2MnO_4 (2) $K_4(Mn(OH)_6]$

(3) $KMnO_4$ (4) K_3MnO_4

Answer (1)

Sol. Preparation of $KMnO_4$

$$MnO_2 \xrightarrow[KNO_3]{KOH} K_2MnO_4 \xrightarrow[\text{Oxidation in Alkaline solution}]{\text{Electrolytic}} KMnO_4$$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. Among the following cations, the number of cations which will give characteristic precipitate in their identification tests with $K_4[Fe(CN)_6]$ is _____.

$Cu^{2+}, Fe^{3+}, Ba^{2+}, Ca^{2+}, NH_4^+, Mg^{2+}, Zn^{2+}$

Answer (3)

Sol. Cu^{2+}, Ca^{2+} & Zn^{2+} will form precipitate with $K_4[Fe(CN)_6]$. Fe^{3+} forms coloured solution

72. 37.8 g N_2O_5 was taken in a 1 L reaction vessel and allowed to undergo the following reaction at 500 K

$$2N_2O_5(g) \rightleftharpoons 2N_2O_4(g) + O_2(g)$$

The total pressure at equilibrium was found to be 18.65 bar.

Then, Kp = ________ × 10^{-2} [nearest integer]

Assume N_2O_5 to behave ideally under these conditions.

Given : R = 0.082 bar L mol^{-1} K^{-1}

Answer (962)

Sol. $2N_2O_5(g) \rightleftharpoons 2NO_2(g) + O_2(g)$

$$P_i = \frac{n_i R \times T}{V_i}$$

$$= \frac{\frac{37.8}{108} \times 0.082 \times 500}{1}$$

= 14.35 atm

	$2N_2O_5(g)$	$\rightleftharpoons$	$2NO_2$	+	$O_2(g)$
$t = 0$	14.35				
$t = t_{eq}$	14.35 – 2P		2P		P

P_f = 14.35 + P = 18.65

P = 4.3 atm

$$Kp = \frac{(8.6)^2 \times (4.3)}{(5.75)^2}$$

= 9.62 bar

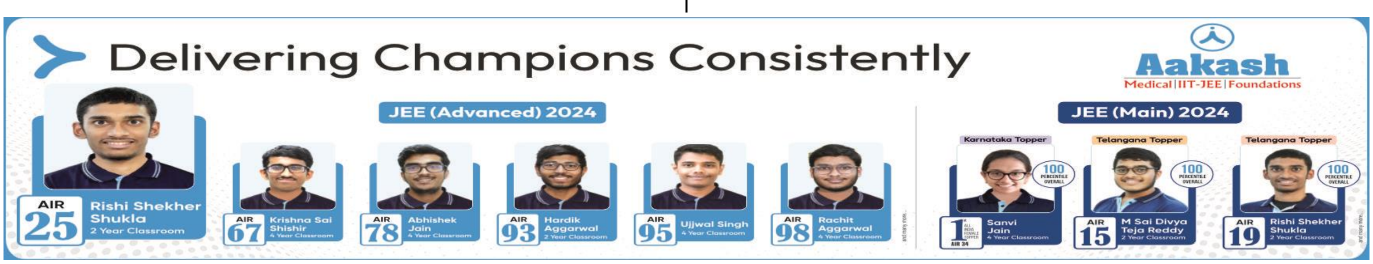
= 962 × 10^{-2} bar

73. Standard entropies of X_2, Y_2 and XY_5 are 70, 50 and 110 J K^{-1} mol^{-1} respectively. The temperature in Kelvin at which the reaction

$$\frac{1}{2}X_2 + \frac{5}{2}Y_2 \rightleftharpoons XY_5 \quad \Delta H° = -35\,kJmol^{-1}$$

will be at equilibrium is ______. (Nearest integer)

Answer (700)



Sol. $\frac{1}{2}X_2 + \frac{5}{2}Y_2 \rightleftharpoons XY_5$

For equilibrium

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$$0 = -35 \times 10^3 - T\left(110 - \left(\frac{5}{2} \times 50\right) - \frac{1}{2} \times 70\right)$$

$35 \times 10^3 = 50 \times T$

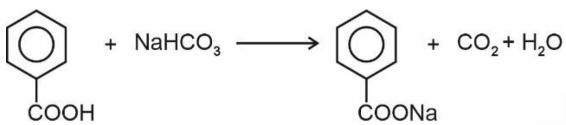
$700K = T$

74. X g of benzoic acid on reaction with aq. $NaHCO_3$ released CO_2 that occupied 11.2 L volume at STP.

X is ______ g.

Answer (61)

Sol.



1 mole benzoic acid will release 1 mole CO_2

11.2 L CO_2 at STP is $\frac{1}{2}$ mole CO_2 which will be released by reaction with $\frac{1}{2}$ mole benzoic acid

Mass of benzoic acid = $\frac{1}{2} \times 122$

= 61 gm

75. Consider the following reaction occurring in the blast furnace :

$Fe_3O_4(s) + 4CO(g) \rightarrow 3Fe(l) + 4CO_2(g)$

'x' kg of iron is produced when 2.32×10^3 kg Fe_3O_4 and 2.8×10^2 kg CO are brought together in the furnace. The value of 'x' is ______. (Nearest integer)

[Given :

molar mass of Fe_3O_4 = 232 g mol^{-1}

molar mass of CO = 28 g mol^{-1}

molar mass of Fe = 56 g mol^{-1}]

Answer (420)

Sol.

	$Fe_3O_4(s)$	+	$4CO(g)$	$\rightarrow$	$3Fe(l) + 4CO_2(g)$
moles taken	$\frac{2.32 \times 10^6}{232}$		$\frac{2.8 \times 10^5}{28}$		

CO is Limiting reagent

Iron produced (in kg) = $\frac{3}{4} \times \frac{2.8}{28} \times 10^5 \times 56 \times 10^{-3}$

= 420 kg

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