MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to:

(1) 8575 (2) 8750

(3) 8925 (4) 9100

Answer (3)

Sol.

Group A		Group B		
B	G	B	G	Ways
4	1	0	3	${}^7C_4 \cdot {}^3C_1 \cdot {}^6C_0 \cdot {}^5C_3$
3	2	1	2	${}^7C_3 \cdot {}^3C_2 \cdot {}^6C_1 \cdot {}^5C_2$
2	3	2	1	${}^7C_2 \cdot {}^3C_3 \cdot {}^6C_2 \cdot {}^5C_1$

Total ways = $30 \cdot {}^7C_4 + 180 \cdot {}^7C_3 + 75 \cdot {}^7C_2 = 8925$

2. Let $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = \vec{a} \times (\hat{i} - 2\hat{k})$ and $\vec{c} = \vec{b} \times \hat{k}$. Then the projection of $\vec{c} = 2\hat{j}$ on $\vec{a}$ is:

(1) $\sqrt{14}$ (2) $2\sqrt{7}$

(3) $2\sqrt{14}$ (4) $3\sqrt{7}$

Answer (3)

Sol.

$$\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k} \quad \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 0 & -2 \end{vmatrix} = 2\hat{i} + 8\hat{j} + \hat{k}$$

$$\vec{b} = \vec{a} \times (\hat{i} - 2\hat{k}) \Rightarrow$$

$$\vec{c} = \vec{b} \times \hat{k} = 8\hat{i} - 2\hat{j} \Rightarrow \vec{c} - 2\hat{j} = 8\hat{i} - 4\hat{j}$$

Projection of $(\vec{c} - 2\hat{j})$ on $\vec{a}$

$$= (\vec{c} - 2\hat{j}) \cdot \frac{\vec{a}}{|\vec{a}|} = \left(\frac{24+4}{\sqrt{14}}\right) = 2\sqrt{14}$$

3. Let $f: (0, \infty) \to \mathbf{R}$ be function which is differentiable at all points of its domain and satisfies the condition $x^2 f'(x) = 2xf(x) + 3$, with $f(1) = 4$. Then $2f(2)$ is equal to:

(1) 23 (2) 19

(3) 29 (4) 39

Answer (4)

Sol. $x^2 f'(x) = 2xf(x) + 3$

Let $y = f(x) \Rightarrow \dfrac{dy}{dx} = f'(x)$

$$\frac{dy}{dx} = \frac{2y}{x} + \frac{3}{x^2}$$

$$\Rightarrow \frac{dy}{dx} + y\left(\frac{-2}{x}\right) = \frac{3}{x^2}$$

$$\text{I.F} = e^{\int \frac{-2}{x}dx} = e^{-2\ln x} = \frac{1}{x^2}$$

$$\Rightarrow y\left(\frac{1}{x^2}\right) = \int \frac{3}{x^2} \times \frac{1}{x^2} dx + c$$

$$\frac{y}{x^2} = -x^{-3} + c, \; f(1) = 4 = y$$

$$\frac{4}{(1)^2} = -(1)^{-3} + c \Rightarrow c = 5$$

$$\Rightarrow \frac{y}{x^2} = \frac{-1}{x^3} + 5 \Rightarrow f(x) = \frac{-1}{x} + 5x^2$$

$$2f(2) = 2\left[\frac{-1}{2} + 5(2)^2\right] = 2\left[\frac{-1}{2} + 20\right] = 39$$

4. Let the position vectors of three vertices of a triangle be $4\vec{p}+\vec{q}-3\vec{r}$, $-5\vec{p}+\vec{q}+2\vec{r}$ and $2\vec{p}-\vec{q}+2\vec{r}$. If the position vectors of the orthocentre and the circumcentre of the triangle are $\frac{\vec{p}+\vec{q}+\vec{r}}{4}$ and $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$ respectively, then $\alpha + 2\beta + 5\gamma$ is equal to:

(1) 4 (2) 6

(3) 3 (4) 1

Answer (3)

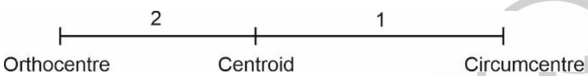
Sol. Orthocentre $=\frac{\vec{p}+\vec{q}+\vec{r}}{4}$

Circumcentre $=\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$

Centroid $=\left(\frac{4\vec{p}+\vec{i}-3\vec{r}-5\vec{p}+\vec{q}+2\vec{r}+2\vec{p}-\vec{q}+2\vec{r}}{3}\right)$

$=\left(\frac{\vec{p}+\vec{q}+\vec{r}}{3}\right)$

Now



$$\frac{2(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r})+\frac{\vec{p}+\vec{q}+\vec{r}}{4}}{2+1}=\left(\frac{\vec{p}+\vec{q}+\vec{r}}{3}\right)$$

$\Rightarrow 8(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r})=3(\vec{p}+\vec{q}+\vec{r})$

$\Rightarrow 8\alpha=3,\ 8\beta=3,\ 8\gamma=3$

$\alpha=\frac{3}{8}=\beta=\gamma$

$\alpha+2\beta+5\gamma=\frac{3}{8}+\frac{6}{8}+\frac{15}{8}=\frac{27}{8}=3$

5. Let (2, 3) be the largest open interval in which the function $f(x) = 2\log_e(x-2) - x^2 + ax + 1$ is strictly increasing and (b, c) be the largest open interval, in which the function $g(x) = (x-1)^3(x+2-a)^2$ is strictly decreasing. Then $100(a+b-c)$ is equal to:

(1) 280 (2) 360

(3) 160 (4) 420

Answer (2)

Sol. $f(x) = 2\ln(x-2) - x^2 + ax + 1$

$f'(x)=\frac{2}{x-2}-2x+a>0$

$f''(x)=\frac{-2}{(x-2)^2}-2<0$

$\Rightarrow$ $f'(x)$ is decreasing

and $f'(3)\geq 0$

$2-6+a\geq 0 \Rightarrow a\geq 4$

$a_{min} = 4$

$\because$ $g(x)$ is strictly decreasing

Now, $g(x) = (x-1)^3(x-2)^2$

$g'(x)=2(x-1)^3(x-2)+3(x-1)^2(x-2)^2<0$

$=(x-1)^2(x-2)[2(x-1)+3(x-2)]<0$

$=(x-1)^2(x-2)(5x-8)<0$

$\Rightarrow x\in\left(\frac{8}{5},2\right) \Rightarrow b=\frac{8}{5}\quad c=2$

Now, $100(a+b-c)=100\left[4+\frac{8}{5}-2\right]$

$=100\times\frac{18}{5}=360$

6. The equation of the chord, of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid-point is (3, 1) is:

(1) $5x + 16y = 31$

(2) $25x + 101y = 176$

(3) $48x + 25y = 169$

(4) $4x + 122y = 134$

Answer (3)

Sol. $\frac{x^2}{25}+\frac{y^2}{16}=1$

Chord with mid-point (3, 1)

$T = S_1$

$$\frac{xx_1}{25}+\frac{yy_1}{16}-1=\frac{x_1^2}{25}+\frac{y_1^2}{16}$$

$$\frac{3x}{25}+\frac{y}{16}=\frac{9}{25}+\frac{1}{16}$$

$48x + 25y = 9\times16 + 25$

$48x + 25y = 169$

7. Let $[x]$ denote the greatest integer function, and let m and n respectively be the numbers of the points, where the function $f(x) = [x] + |x - 2|$, $-2 < x < 3$, is not continuous and not differentiable. Then $m + n$ is equal to :

(1) 8

(2) 6

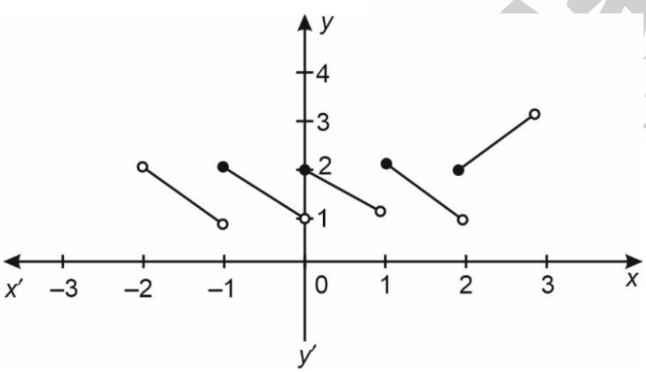
(3) 7

(4) 9

Answer (1)

Sol. $f(x) = [x] + |x - 2|, -2 < x < 3$

$$\therefore\ f(x)=\begin{cases}-x, & -2<x<-1\\ 1-x, & -1\le x<0\\ 2-x, & 0\le x<1\\ 3-x, & 1\le x<2\\ x, & 2\le x<3\end{cases}$$



It is clearly discontinues at 4 points and non-differentiable at 4 points.

$\therefore\ m + n = 8$

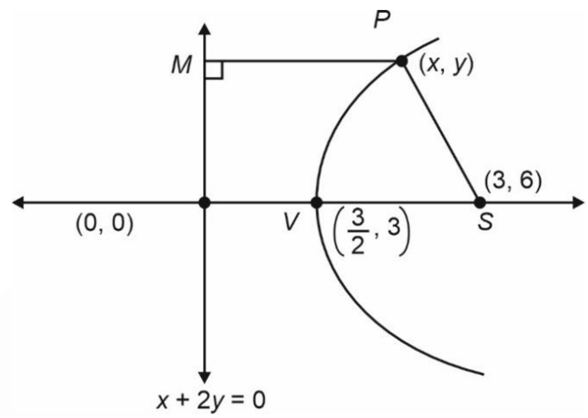
8. If the equation of the parabola with vertex $V\left(\frac{3}{2}, 3\right)$ and the directrix $x + 2y = 0$ is $\alpha x^2 + \beta y^2 - \gamma xy - 30x - 60y + 225 = 0$, then $\alpha + \beta + \gamma$ is equal to:

(1) 7 (2) 6

(3) 9 (4) 8

Answer (3)

Sol.



$\therefore$ Equation of parabola is $PS = (1)\ PM$

$\Rightarrow\ PS^2 = PM^2$

$$(x-3)^2+(y-6)^2=\left(\frac{x+2y}{\sqrt{5}}\right)^2$$

$$5x^2-30x+5y^2-60y+225=x^2+4y^2+4xy$$

$$4x^2+y^2-4xy-30x-60y+225=0$$

We get : $\alpha = 4, \beta = 1, \gamma = 4$

$\therefore\ \alpha + \beta + \gamma = 9$

9. Let $A = [a_{ij}]$ be a square matrix of order 2 with entries either 0 or 1. Let E be the event that A is an invertible matrix. Then the probability $P(E)$ is :

(1) $\frac{3}{16}$ (2) $\frac{3}{8}$

(3) $\frac{1}{8}$ (4) $\frac{5}{8}$

Answer (2)

Sol. $A = [a_{ij}]_{2\times2}$ and entries are 0 or 1.

$\therefore \begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$

$\Rightarrow$ $ad - bc = 0$

Case I: $ad = bc = 1$

$\therefore$ $a = b = c = d = 1$

Case II: $ad = bc = 0$

$a = 0, d = 0$ $\quad$ $b = 0, c = 0$

$a = 0, d = 1$ $\quad$ $b = 0, c = 1$

$a = 1, d = 0$ $\quad$ $b = 1, c = 0$

$\therefore$ Total 10 cases when matrix is non invertible

Total possible matrix = $2^4 = 16$

Required probability of invertible

$= \dfrac{16-10}{16} = \dfrac{6}{16} = \dfrac{3}{8}$

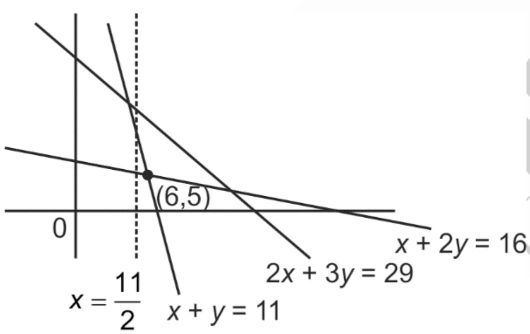
10. Let the points $\left(\dfrac{11}{2}, \alpha\right)$ lie on or inside the triangle with sides $x + y = 11$, $x + 2y = 16$ and $2x + 3y = 29$. Then the product of the smallest and the largest value of α is equal to :

(1) 22 $\quad$ (2) 55

(3) 33 $\quad$ (4) 44

Answer (3)

Sol.



Clearly, $x = \dfrac{11}{2}$ intersect $x + y - 11 = 0$ at $\left(\dfrac{11}{2}, \dfrac{11}{2}\right)$

and $2x + 3y - 29 = 0$ at $\left(\dfrac{11}{2}, 6\right) \Rightarrow \alpha = \left[\dfrac{11}{2}, 6\right]$

$\alpha_{min} \cdot \alpha_{max} = \dfrac{11}{2} \cdot 6 = 33$

11. If $7 = 5 + \dfrac{1}{7}(5+\alpha) + \dfrac{1}{7^2}(5+2\alpha) + \dfrac{1}{7^3}(5+3\alpha) + \ldots \infty$, then the value of α is

(1) 1

(2) 6

(3) $\dfrac{1}{7}$

(4) $\dfrac{6}{7}$

Answer (2)

Sol. $S = a + (a + d)r + (a + 2d)r^2 + \ldots$

Then $S = \dfrac{a}{1-r} + \dfrac{dr}{(1-r)^2}, |r| < 1$

Since, $r = \dfrac{1}{7}$ and $a = 5$, $d = \alpha$

$$7 = \frac{5}{1-\frac{1}{7}} + \frac{\alpha \cdot \frac{1}{7}}{\left(1-\frac{1}{7}\right)^2}$$

$\Rightarrow$ $\alpha = 6$

12. Suppose A and B are the coefficients of 30^{th} and 12^{th} terms respectively in the binomial expansion of $(1 + x)^{2n-1}$. If $2A = 5B$, then n is equal to :

(1) 19 $\quad$ (2) 20

(3) 21 $\quad$ (4) 22

Answer (3)

Sol. $T_{r+1} = {}^{2n-1}C_r\, x^r$

Coefficient of $T_{30} = {}^{2n-1}C_{29} = A$

Coefficient of $T_{12} = {}^{2n-1}C_{11} = B$

$\Rightarrow$ $2\left({}^{2n-1}C_{29}\right) = 5\left({}^{2n-1}C_{11}\right)$

$\dfrac{2n}{30} \cdot {}^{2n-1}C_{29} = \dfrac{2n}{12} \cdot {}^{2n-1}C_{11}$

$\Rightarrow$ $n = 21$

13. Let

$$A = \left\{x \in (0, \pi) - \left(\frac{\pi}{2}\right) : \log_{(2/\pi)} |\sin x| + \log_{(2/\pi)} |\cos x| = 2\right\}$$

and $B = \{x \geq 0 : \sqrt{x}\ (\sqrt{x} - 4) - 3|\sqrt{x} - 2| + 6 = 0\}$.

Then $n(A \cup B)$ is equal to

(1) 2 (2) 6

(3) 4 (4) 8

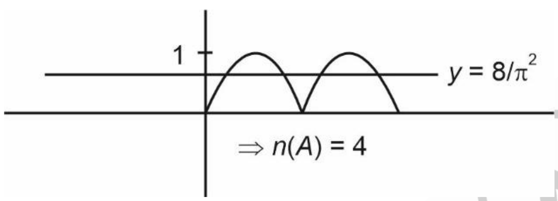
Answer (4)

Sol. $A = \left\{x \in (0, \pi) - \left(\frac{\pi}{2}\right) : \log_{(2/\pi)} |\sin x| + \log_{(2/\pi)} |\cos x| = 2\right\}$

$\log_{2/x} |\sin x \cos x| = 2$

$\frac{|\sin 2x|}{2} = \frac{4}{\pi^2}$

$\Rightarrow \ |\sin 2x| = \frac{8}{\pi^2}$



and $B = \left\{x \geq 0 : \sqrt{x}(\sqrt{x} - 4) - 3|\sqrt{x} - 2| + 6 = 0\right\}$

$x - 4\sqrt{x} - 3\sqrt{x} + 6 + 6 = 0$ when $\sqrt{x} - 2 \geq 0$

$\sqrt{x} \geq 2$

$\Rightarrow \ x - 7\sqrt{x} + 12 = 0$

$\Rightarrow \ x - 4\sqrt{x} - 3\sqrt{x} + 12 = 0$

$\Rightarrow \ \sqrt{x}\sqrt{x} - 4) - 3(\sqrt{x} - 4) = 0$

$\sqrt{x} = 4, \sqrt{x} = 3$

$\Rightarrow \ x = 16, 9$

When $\sqrt{x} - 2 < 0$

$\Rightarrow \ x - 4\sqrt{x} + 3\sqrt{x} - 6 + 6 = 0$

$\Rightarrow \ x - \sqrt{x} = 0$

$\Rightarrow \ \sqrt{x}(\sqrt{x} - 1) = 0$

$\Rightarrow \ \sqrt{x} = 0, 1$

$\Rightarrow \ x = 0, 1$

$\Rightarrow \ n(B) = 4$

Now as A and B are mutually exclusive sets

$N(A \cup B) = 4 + 4 = 8$

14. In an arithmetic progression, if $S_{40} = 1030$ and $S_{12} = 57$, then $S_{30} - S_{10}$ is equal to

(1) 525 (2) 510

(3) 515 (4) 505

Answer (3)

Sol. $S_{40} = 1030 \quad \Rightarrow \frac{40}{2}[2a + 39d] = 1030$

$\Rightarrow \ 2a + 39d = \frac{103}{2} \qquad \ldots(1)$

$S_{12} = 57 \ \Rightarrow \frac{12}{2}[2a + 11d] = 57$

$\Rightarrow \ 2a + 11d = \frac{57}{6} \qquad \ldots(2)$

Equation (1) – equation (2)

$28d = \frac{103}{2} - \frac{57}{6}$

$28d = \frac{309 - 57}{6}$

$d = \frac{3}{2}$

$\Rightarrow \ a = -\frac{7}{2}$

$S_{30} - S_{10} = \frac{30}{2}[2a + 29d] - \frac{10}{2}[2a + 9d]$

$= 15[2a + 29d] - 5[2a + 9d]$

$= 5[6a + 87d - 2a - 9d]$

$= 5[4a + 78d]$

$= 5[-14 + 117]$

$= 515$

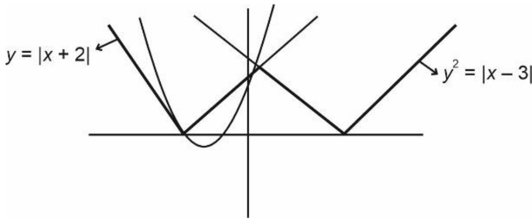
15. The number of real solution(s) of the equation $x^2 + 3x + 2 = \min\{|x-3|, |x+2|\}$ is

(1) 2 (2) 1

(3) 3 (4) 0

Answer (1)

Sol. $x^2 + 3x + 2 = \min\{|x-3|, |x+2|\}$



$y = x^2 + 3x + 2$

$y = x^2 + 2\left(\frac{3}{2}\right)x + \frac{9}{4} - \frac{9}{4} + 2$

$y = \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$

$y + \frac{1}{4} = \left(x + \frac{3}{2}\right)^2$

$\Rightarrow$ Parabola vertex $\left(\frac{-3}{2}, \frac{-1}{4}\right)$

$\Rightarrow$ By graph 2 solution possible

16. For some a, b, let $f(x) = \begin{vmatrix} a + \frac{\sin x}{x} & 1 & b \\ a & 1 + \frac{\sin x}{x} & b \\ a & 1 & b + \frac{\sin x}{x} \end{vmatrix}$, $x \neq 0$, $\lim_{x \to 0} f(x) = \lambda + \mu a + \nu b$. Then $(\lambda + \mu + \nu)^2$ is equal to

(1) 36

(2) 16

(3) 9

(4) 25

Answer (2)

Sol. $\lim_{x \to 0} \begin{vmatrix} a + \frac{\sin x}{x} & 1 & b \\ a & 1 + \frac{\sin x}{x} & b \\ a & 1 & b + \frac{\sin x}{x} \end{vmatrix} = \lambda + \mu a + \nu b$

At lim $x \to 0$,

$f(x) = \begin{vmatrix} a+1 & 1 & b \\ a & 1+1 & b \\ a & 1 & b+1 \end{vmatrix} = \lambda + \mu a + \nu b$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ a & 1 & b+1 \end{vmatrix} = \lambda + \mu a + \nu b$

$C_2 \to C_1 - C_2$

$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ a & a+1 & b+1 \end{vmatrix} = \lambda + \mu a + \nu b$

$a + b + 2 = \lambda + \mu a + \nu b$

$\lambda = 2, \quad \mu = 1, \quad \nu = 1$

$(\lambda + \mu + \nu) = (2 + 1 + 1)^2 = 16$

17. The function $f : (-\infty, \infty) \to (-\infty, 1)$, defined by $f(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$ is:

(1) Onto but not one-one

(2) Both one-one and onto

(3) One-one but not onto

(4) Neither one-one nor onto

Answer (3)

Sol. $f(x)=\dfrac{2^x-2^{-x}}{2^x+2^{-x}}=\dfrac{2^{2x}-1}{2^{2x}+1}=\dfrac{2^{2x}+1-2}{2^{2x}+1}$

$f(x)=1-\dfrac{2}{2^{2x}+1}$

2^{2x} is one-one, so $f(x)$ is one-one

for $x\in(-\infty,\infty)$

$2^x\in(0,\infty)$

$2^{2x}+1\in(1,\infty)$

$\dfrac{1}{2^{2x}+1}\in(1,0)$

$\dfrac{-2}{2^{2x}+1}\in(-2,0)$

$1-\dfrac{2}{2^{2x}+1}\in(-1,1)$

Range of $f(x)$ is $(-1, 1)$ but codomain of $f(x)$ is $(-\infty, 1)$

So $f(x)$ is into

18. The area of the region enclosed by the curves $y=e^x$, $y=|e^x-1|$ and y-axis is:

(1) $1+\log_e 2$

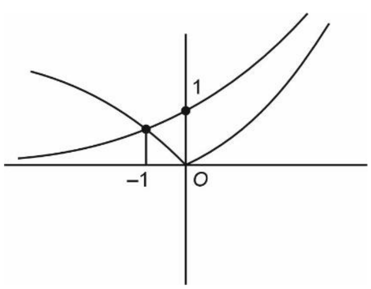
(2) $\log_e 2$

(3) $1-\log_e 2$

(4) $2\log_e 2-1$

Answer (3)

Sol.



$e^x=1-e^x\Rightarrow 2e^x=1$

$\Rightarrow e^x=\dfrac{1}{2}$

$\Rightarrow x=\ln\dfrac{1}{2}$

$$\int_{\ln(1/2)}^{0}\left[e^x-(1-e^x)\right]dx$$

$$=\int_{\ln 2}^{0}(2e^x-1)dx=2e^x-x\Big|_{-\ln 2}^{0}$$

$=2-(1+\ln 2)$

$=1-\log_e 2$

19. If $\alpha>\beta>\gamma>0$, then the expression

$$\cot^{-1}\left\{\beta+\frac{(1+\beta^2)}{(\alpha-\beta)}\right\}+\cot^{-1}\left\{\gamma+\frac{(1+\gamma^2)}{(\beta-\gamma)}\right\}+\cot^{-1}\left\{\alpha+\frac{1+\alpha^2}{\gamma-\alpha}\right\}$$ is

equal to:

(1) 0

(2) $\dfrac{\pi}{2}-(\alpha+\beta+\gamma)$

(3) 3π

(4) π

Answer (4)

Sol. $\cot^{-1}\left(\dfrac{\alpha\beta-\beta^2+1+\beta^2}{\alpha-\beta}\right)+\cot^{-1}\left(\dfrac{\beta\gamma-\gamma^2+1+\gamma^2}{\beta-\gamma}\right)+\cot^{-1}\left(\dfrac{\alpha\gamma-\alpha^2+1+\alpha^2}{\gamma-\alpha}\right)$

$=\cot^{-1}\left(\dfrac{1+\alpha\beta}{\alpha-\beta}\right)+\cot^{-1}\left(\dfrac{1+\beta\gamma}{\beta-\gamma}\right)+\cot^{-1}\dfrac{1+\gamma\alpha}{\gamma-\alpha}$

$=\tan^{-1}\left(\dfrac{\alpha-\beta}{1+\alpha\beta}\right)+\tan^{-1}\left(\dfrac{\beta-\gamma}{1+\beta\gamma}\right)+\pi+\tan^{-1}\left(\dfrac{\gamma-\alpha}{1+\gamma\alpha}\right)$

$= \tan^{-1}\alpha - \tan^{-1}\beta + \tan^{-1}\beta - \tan^{-1}\gamma + \pi + \tan^{-1}\gamma - \tan^{-1}\alpha$

$= \pi$

20. If the system of equations

$x + 2y - 3z = 2$

$2x + \lambda y + 5z = 5$

$14x + 3y + \mu z = 33$

has infinitely many solutions, then $\lambda + \mu$ is equal to:

(1) 12

(2) 10

(3) 13

(4) 11

Answer (1)

Sol. $\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & \lambda & 5 \\ 14 & 3 & \mu \end{vmatrix} = 0 \Rightarrow \lambda\mu + 42\lambda - 4\mu + 107 = 0$

$\Delta_1 = \begin{vmatrix} 2 & 2 & -3 \\ 5 & \lambda & 5 \\ 33 & 3 & \mu \end{vmatrix} = 0 \Rightarrow 2\lambda\mu + 99\lambda - 10\mu + 255 = 0$

$\Delta_2 = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 5 & 5 \\ 14 & 33 & \mu \end{vmatrix} = 0 \Rightarrow \mu = 13$

Also, $\lambda = -1$

Hence, $\lambda + \mu = 13 - 1 = 12$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let P be the image of the point $Q(7, -2, 5)$ in the line $L : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ and $R(5, p, q)$ be a point on L. Then the square of the area of ΔPQR is _______.

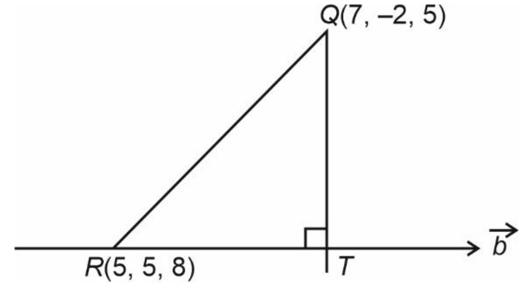
Answer (957)

Sol. Let $R(2\lambda + 1, 3\lambda - 1, 4\lambda)$

$2\lambda + 1 = 5$

$\lambda = 2$

$\Rightarrow R(5, 5, 8)$



Let say $T(2\lambda + 1, 3\lambda - 1, 4\lambda)$

So, $\overrightarrow{QT} = (2\lambda - 6)\hat{i} + (3\lambda + 1)\hat{j} + (4\lambda - 5)\hat{k}$

and $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overrightarrow{QT} \cdot \vec{b} = 0$

$4\lambda - 12 + 9\lambda + 3 + 16\lambda - 20 = 0$

$\lambda = 1$

$T(3, 2, 4)$

$QT = \sqrt{33}, \ RT = \sqrt{29}$

$(\text{ar}(\Delta PQR))^2 = \left(\frac{1}{2}\sqrt{33} \cdot 2 \cdot \sqrt{29}\right)^2$

$= 957$

22. If $\int \frac{2x^2 + 5x + 9}{\sqrt{x^2 + x + 1}} dx = x\sqrt{x^2 + x + 1} + \alpha\sqrt{x^2 + x + 1} + \beta \log_e \left| x + \frac{1}{2} + \sqrt{x^2 + x + 1} \right| + C$, where C is the constant of integration, then $\alpha + 2\beta$ is equal to _____.

Answer (16)

Sol. $I=\int \frac{2x^2+5x+9}{x^2+x+1}dx$

Let $\frac{2x^2+5x+9}{\sqrt{x^2+x+1}}=\frac{A\left(x^2+x+1\right)+B(2x+1)+C}{\sqrt{x^2+x+1}}$

Then, $A=2, B=\frac{3}{2}$ and $C=\frac{11}{2}$

$$\therefore\ I=\int \frac{2\left(x^2+x+1\right)+\frac{3}{2}(2x+1)+\frac{11}{2}}{\sqrt{x^2+x+1}}dx$$

$$=2\int\sqrt{x^2+x+1}\,dx+\frac{3}{2}\cdot 2\sqrt{x^2+x+1}+\frac{11}{2}\int\frac{1}{\sqrt{x^2+x+1}}dx$$

$$=2\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dx+3\sqrt{x^2+x+1}+\frac{11}{2}\int\frac{dx}{\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}}$$

$$\left\{\frac{x+\frac{1}{2}}{2}\sqrt{x^2+x+1}+\frac{3}{8}\ln\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|\right\}+3\sqrt{x^2+x+1}+\frac{11}{2}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

$$=\left(\frac{2x+1}{2}\right)\sqrt{x^2+x+1}+\frac{3}{4}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+3\sqrt{x^2+x+1}+\frac{22}{7}\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

$$=\frac{2x+7}{2}\sqrt{x^2+x+1}+\frac{25}{4}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

$$=x\sqrt{x^2+x+1}+\frac{7}{2}\sqrt{x^2+x+1}+\frac{25}{4}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

$\therefore\ \alpha=\frac{7}{2}, \beta=\frac{25}{4}$

Then $\alpha+2\beta=\frac{7}{2}+\frac{25}{4}=16$

23. Let $H_1:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $H_2:-\frac{x^2}{A^2}+\frac{y^2}{B^2}=1$ be two hyperbolas having length of latus rectums $15\sqrt{2}$ and $12\sqrt{5}$ respectively. Let their eccentricities be $e_1=\sqrt{\frac{5}{2}}$ and e_2 respectively. If the product of the lengths of their transverse axes is $100\sqrt{10}$, then $25e_2^2$ is equal to ______.

Answer (55)

Sol. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{2b^2}{a}=15\sqrt{2}$ ...(i)

$\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{5}{2}}$ ...(ii)

From (i) and (ii)

$a=5\sqrt{2}$ and $b^2=75$

$\frac{x^2}{A^2}-\frac{y^2}{B^2}=-1$

$\frac{2A^2}{B}=12\sqrt{5}$ ...(iii)

Since, product of transverse axis is = $100\sqrt{10}$

$(2A)\cdot(2B) = 100\sqrt{10}$ ...(iv)

From (iii) and (iv)

$A^2 = 150$ and $B = 5\sqrt{5}$

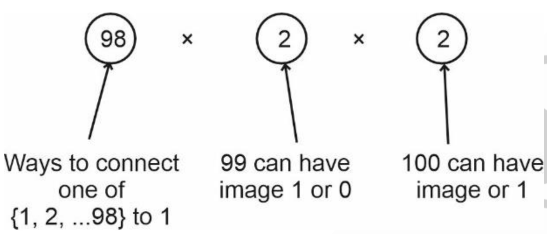
$$e_2 = \sqrt{1+\frac{A^2}{B^2}} = \sqrt{\frac{11}{5}}$$

$$\therefore\ 25e_2^2 = 25\left(\frac{11}{5}\right) = 55$$

24. Number of functions $f : \{1, 2, ..., 100\} \to \{0, 1\}$, that assign 1 to exactly one of the positive integers less than or equal to 98, is equal to _____.

Answer (392)

Sol.



= 98 × 2 × 2 = 392

25. Let $y = y(x)$ be the solution of the differential equation $2\cos x\dfrac{dy}{dx} = \sin 2x - 4y\sin x,\ x \in \left(0, \dfrac{\pi}{2}\right)$. If $y\left(\dfrac{\pi}{3}\right) = 0$, then $y'\left(\dfrac{\pi}{4}\right) + y\left(\dfrac{\pi}{4}\right)$ is equal to_____.

Answer (01)

Sol. $2\cos x\dfrac{dy}{dx} = \sin 2x - 4y\sin x$

$$\frac{dy}{dx} = \frac{2\sin x\cos x}{2\cos x} - \frac{4y\sin x}{2\cos x}$$

$$\frac{dy}{dx} + 2y\tan x = \sin x$$

$\text{I.F} = e^{\int 2\tan x dx}$

$\text{I.F} = e^{2\ln|\sec x|}$

$\text{I.F} = |\sec x|^2 = \sec^2 x\ \forall\ x \in (0, \pi/2)$

$$y.\sec^2 x = \int \sin x.\sec^2 x dx$$

$$y.\sec^2 x = \int \frac{\sin x}{\cos^2 x}dx$$

Let, $\cos x = t$

$-\sin x dx = dt$

$$y.\sec^2 x = -\int \frac{1}{t^2}dt$$

$$y.\sec^2 x = \frac{1}{t} + C$$

$y.\sec^2 x = \sec x + C$

$y(\pi/3) = 0$

$C = -2$

$\therefore\ y = \cos x - 2\cos^2 x$

$y'(x) = -\sin x + 2\sin 2x$

$$y'\left(\pi/4\right) = \frac{1}{\sqrt{2}} + 2$$

$$y'(\pi/4) = \frac{1}{\sqrt{2}} - 1$$

$$\therefore y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right) = 1$$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. The temperature of a body in air falls from 40°C to 24°C in 4 minutes. The temperature of the air is 16°C. The temperature of the body in the next 4 minutes will be :

(1) $\frac{14}{3}$°C (2) $\frac{28}{3}$°C

(3) $\frac{42}{3}$°C (4) $\frac{56}{3}$°C

Answer (4)

Sol. $\Delta T = T_2 - T_1 = 16°C$

And $T_0 = 16°C$

$$\frac{\Delta T}{t_1} = -k(32-16°) \quad ...(i)$$

$$\frac{(24-T_3)}{4} = -k\left(\frac{24+T_3}{2}-16\right) \quad ...(ii)$$

$$\frac{16}{4} = -k(16)$$

$$\Rightarrow \frac{(24-T_3)}{4} = -k\left(12+\frac{T_3}{2}-16\right)$$

$$\Rightarrow \frac{16}{24-T_3} = \frac{16}{\frac{T_3}{T_2}-4}$$

$$\Rightarrow \frac{T_3}{2}-4 = 24-T_3$$

$$\Rightarrow \frac{3T_3}{2} = 28$$

$$\Rightarrow T_3 = \frac{56}{3}°C$$

27. A solid sphere is rolling without slipping on a horizontal plane. The ratio of the linear kinetic energy of the centre of mass of the sphere and rotational kinetic energy is :

(1) $\frac{5}{2}$ (2) $\frac{4}{3}$

(3) $\frac{3}{4}$ (4) $\frac{2}{5}$

Answer (1)

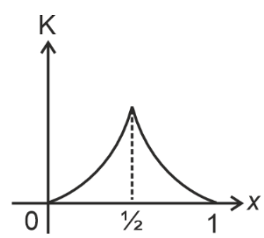
Sol. $KE_{(T)} = \frac{1}{2}mv^2$

$$KE_{(R)} = \frac{1}{2}\cdot\frac{2}{5}mR^2\cdot\frac{v^2}{R^2} = \frac{1}{2}mv^2\left(\frac{2}{5}\right)$$

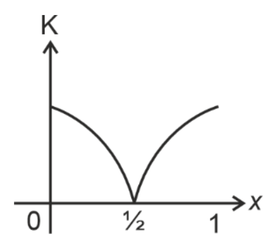
So, $\frac{KE_{(T)}}{KE_{(R)}} = \frac{5}{2}$

28. A particle oscillates along the x-axis according to the law, $x(t) = x_0 \sin^2\left(\frac{t}{2}\right)$ where x_0 = 1 m. The kinetic energy (K) of the particle as a function of x is correctly represented by the graph

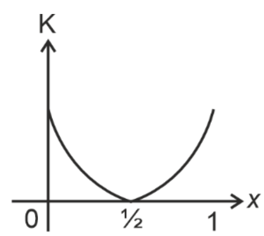
(1)



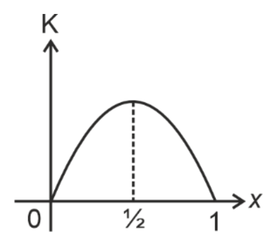
(2)



(3)

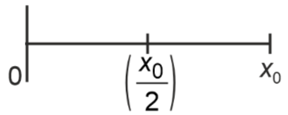


(4)

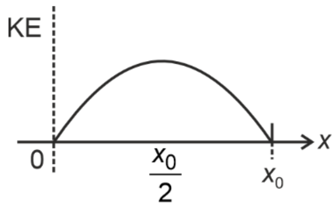


Answer (4)

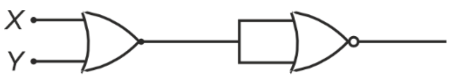
Sol. $x(t) = x_0 \sin^2\left(\frac{t}{2}\right) = \frac{x_0}{2}(1 - \cos t)$



Clearly $\frac{x_0}{2}$ is mean position.



29. The output of the circuit is low (zero) for :



(A) $X = 0, Y = 0$ (B) $X = 0, Y = 1$

(C) $X = 1, Y = 0$ (D) $X = 1, Y = 1$

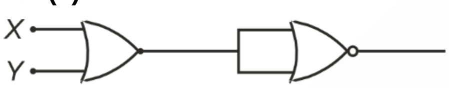
Choose the **correct** answer from the options given below :

(1) (A), (B) and (D) only (2) (A), (C) and (D) only

(3) (A), (B) and (C) only (4) (B), (C) and (D) only

Answer (4)

Sol.



Clearly $\overline{x+y}$ = output

So, when either of x or y is 1 then output will be zero.

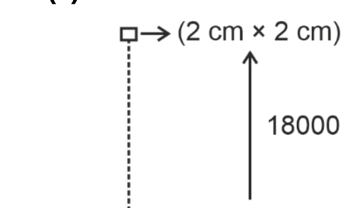
30. A photograph of a landscape is captured by a drone camera at a height of 18 km. The size of the camera film is 2 cm x 2 cm and the area of the landscape photographed is 400 km². The focal length of the lens in the drone camera is :

(1) 0.9 cm (2) 2.5 cm

(3) 1.8 cm (4) 2.8 cm

Answer (3)

Sol.



Linear size of image = 2 cm

Linear size of object = 20×10^5 cm

$$m = \frac{2}{20 \times 10^5} = 10^{-6}$$

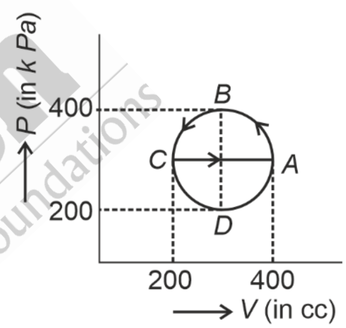
So, $v = \frac{u}{10^6}$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{10^6}{u} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{(1+10^6)}{u} = \frac{1}{f} \Rightarrow f \approx \left(\frac{u}{10^6}\right)$$

$$\Rightarrow f = \frac{18000 \times 100}{10^6}\text{ cm} = 1.8\text{ cm}$$

31. The magnitude of heat exchanged by a system for the given cyclic process $ABCA$ (as shown in figure) is (in SI unit)



(1) 5π

(2) 40π

(3) 10π

(4) Zero

Answer (1)

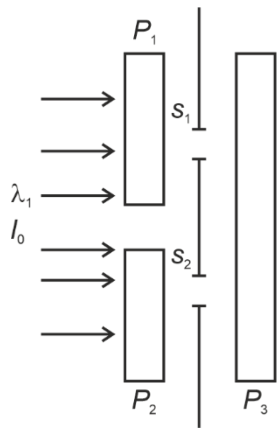
Sol. For cyclic process

$\Delta u = \Delta Q - \Delta\omega = 0$

$$\Rightarrow |\Delta Q| = |\Delta\omega| = \frac{\pi}{2} \times 100 \times 10^{-6} \times 100 \times 10^3$$

$$\Rightarrow |\Delta Q| = 5\pi$$

32. In a Young's double slit experiment, three polarizers are kept as shown in the figure. The transmission axes of P_1 and P_2 are orthogonal to each other. The polarizer P_3 covers both the slits with its transmission axis at 45° to those of P_1 and P_2. An unpolarized light of wavelength λ and intensity I_0, is incident on P_1 and P_2. The intensity at a point after P_3 where the path difference between the light waves from s_1, and s_2 is $\frac{\lambda}{3}$, is



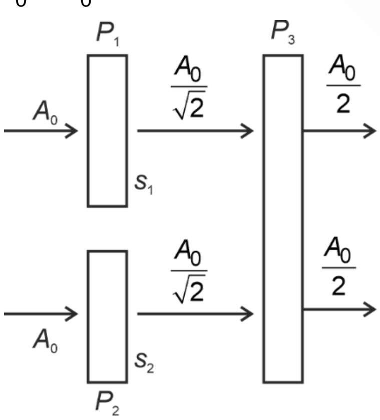
(1) $\frac{I_0}{3}$ (2) $\frac{I_0}{4}$

(3) $\frac{I_0}{2}$ (4) I_0

Answer (2)

Sol. Incident amplitude is A_0 then

$I_0 \propto A_0^2$



Now, path difference is $\frac{\lambda}{3}$ so phase difference is $\frac{2\pi}{3}$

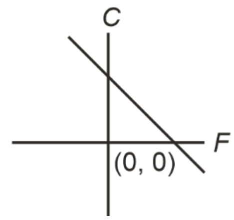
$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\theta$

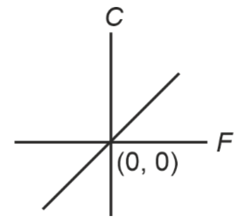
$= \frac{A_0^2}{4} + \frac{A_0^2}{4} + 2\frac{A_0^2}{4}\cdot\cos\left(\frac{2\pi}{3}\right)$

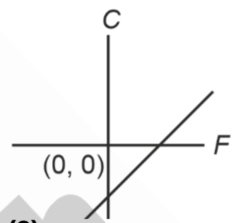
$\Rightarrow \quad A^2 = \frac{A_0^2}{4}$

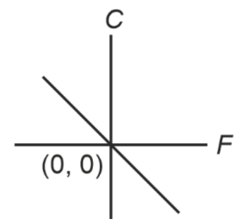
so, $I = \frac{I_0}{4}$

33. Which of the following figure represents the relation between Celsius and Fahrenheit temperatures?

(1)


(2)


(3)


(4)


Answer (3)

Sol. We know $F = 32 + \left(\frac{180}{100}\right)C$

$\Rightarrow \quad \frac{180}{100}C = F - 32$

$\Rightarrow \quad C = \left(\frac{100}{180}\right)F - \frac{160}{9}$

Clearly slope is +ve and intercept is –ve

34. Given below are two statements. One is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : A electron in a certain region of uniform magnetic field is moving with constant velocity in a straight line path.

Reason (R) : The magnetic field in that region is along the direction of velocity of the electron.

In the light of the above statements, choose the correct answer from the options given below

(1) (A) is false but (R) is true

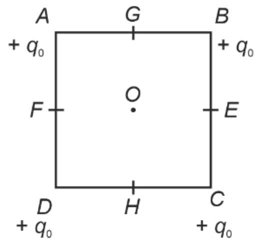
(2) Both (A) and (R) are true and (R) is the correct explanation of (A)

(3) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

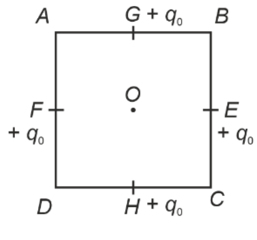
(4) (A) is true but (R) is false

Answer (2)

Sol. If the electron's velocity is along the direction of magnetic field then magnetic force on electron is zero and it will not accelerate.

35. 

Configuration (1)



Configuration (2)

In the first configuration (1) as shown in the figure, four identical charges (q_0) are kept at the corners *A*, *B*, *C* and *D* of square of side length '*a*'. In the second configuration (2), the same charges are shifted to mid points G, E, H and F, of the square.

If $K = \frac{1}{4\pi\varepsilon_0}$, the difference between the potential energies of configuration (2) and (1) is given by

(1) $\frac{Kq_0^2}{a}\left(3\sqrt{2}-2\right)$ (2) $\frac{Kq_0^2}{a}\left(4\sqrt{2}-2\right)$

(3) $\frac{Kq_0^2}{a}\left(4-2\sqrt{2}\right)$ (4) $\frac{Kq_0^2}{a}\left(3-\sqrt{2}\right)$

Answer (1)

Sol. $u_{①} = \left(2\frac{Kq_0}{a}+\frac{Kq_0}{\sqrt{2}a}\right)q_0\times 2$

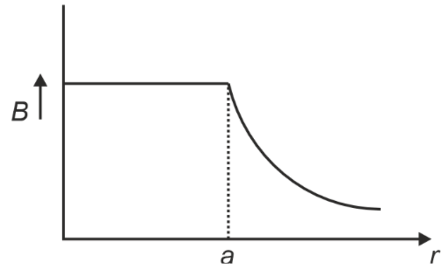
$u_{②} = \left(2\frac{Kq_0\sqrt{2}}{a}+\frac{Kq_0}{a}\right)q_0\times 2$

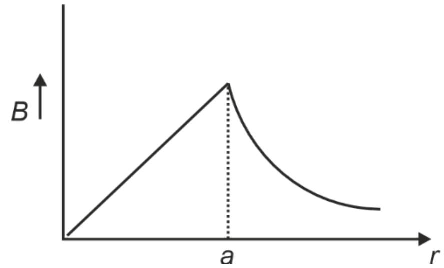
So, $\Delta u = u_2 - u_1 = 2q_0\frac{kq_0}{a}\left[2\sqrt{2}+1-2-\frac{1}{\sqrt{2}}\right]$

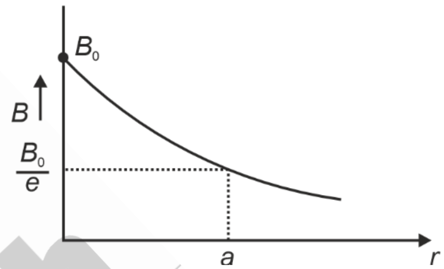
$\Rightarrow \Delta u = \frac{2q_0^2}{4\pi\varepsilon_0 a}\left[\frac{4-\sqrt{2}-1}{\sqrt{2}}\right] = \frac{2q_0^2}{4\pi\varepsilon_0 a}\frac{\left(3-\sqrt{2}\right)}{\sqrt{2}}$

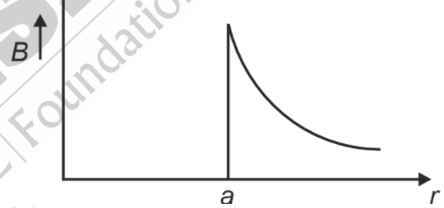
$\Rightarrow \Delta u = \frac{2kq_0^2}{a}\left[\frac{3-\sqrt{2}}{\sqrt{2}}\right] = \frac{kq_0^2}{a}\left(3\sqrt{2}-2\right)$

36. A long straight wire of a circular cross-section with radius '*a*' carries a steady current *I*. The current *I* is uniformly distributed across this cross-section. The plot of magnitude of magnetic field *B* with distance *r* from the centre of the wire is given by

(1) 

(2) 

(3) 

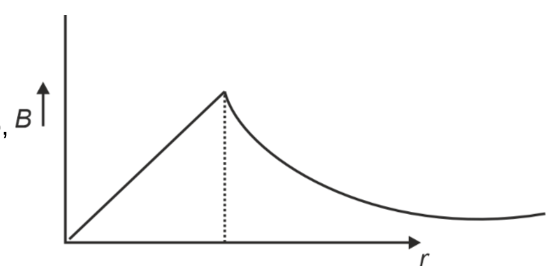
(4) 

Answer (2)

Sol. We know inside the wire

$B = \frac{\mu_0 I}{2\pi R^2}\cdot r$ $(O < r < R)$

And $B = \frac{\mu_0 I}{2\pi r}$ for $(R < r)$

So, 

Option (2) is correct answer.

37 Young's double slit interference apparatus is immersed in a liquid of refractive index 1.44. It has slit separation of 1.5 mm. The slits are illuminated by a parallel beam of light whose wavelength in air is 690 nm. The fringe-width on a screen placed behind the plane of slits at a distance of 0.72 m, will be :

(1) 0.63 mm (2) 0.23 mm

(3) 0.33 mm (4) 0.46 mm

Answer (2)

Sol. $\Delta W = \dfrac{\lambda D}{d \cdot \mu} = \dfrac{690 \times 10^{-9} \times 0.72}{1.5 \times 10^{-3} \times 1.44}$

$\Rightarrow \Delta W = 2.3 \times 10^{-4}$ m = 0.23 mm

Option (2) is correct answer.

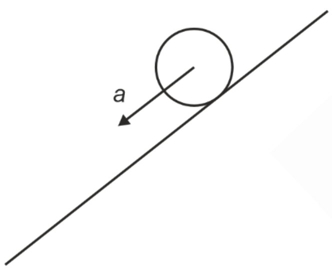
38. A solid sphere and a hollow sphere of the same mass and of same radius are rolled on an inclined plane. Let the time taken to reach the bottom by the solid sphere and the hollow sphere be t_1 and t_2, respectively, then

(1) $t_1 < t_2$ (2) $t_1 = 2t_2$

(3) $t_1 > t_2$ (4) $t_1 = t_2$

Answer (1)

Sol.



$mgR\sin\theta = I\dfrac{a}{R} \quad \Rightarrow \quad a = \dfrac{mgR^2\sin\theta}{I}$

For solid sphere $I_s = \dfrac{7}{5}mR^2$

So, $a_s = \dfrac{mgR^2\sin\theta}{7/5mR^2} = \dfrac{5g}{7}\sin\theta$

For $a_H = \dfrac{mgR^2\sin\theta}{\frac{5}{3}mR^2} = \dfrac{3}{5}g\sin\theta$

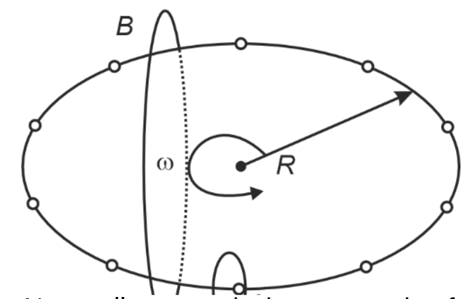
So, $\dfrac{1}{2}a_s t_1^2 = \dfrac{1}{2}a_H t_2^2$

$\Rightarrow \left(\dfrac{t_1}{t_2}\right)^2 = \dfrac{3 \times 7}{5 \times 5} = \dfrac{21}{25}$

Clearly $t_2 > t_1$

Option (1) is correct answer.

39.



N equally spaced charges each of value q, are placed on a circle of radius R. The circle rotates about its axis with an angular velocity ω as shown in the figure. A bigger Amperian loop B encloses the whole circle where as a smaller Amperian loop A encloses a small segment. The different enclosed currents, $I_A - I_B$, for the given Amperian loops is

(1) $\dfrac{N^2}{2\pi}q\omega$ (2) $\dfrac{N}{2\pi}q\omega$

(3) $\dfrac{2\pi}{N}q\omega$ (4) $\dfrac{N}{\pi}q\omega$

Answer (2)

Sol. $I = \dfrac{Nq\omega}{2\pi}$

Now for bigger Amperian loop (loop B)

$I_{B(\text{enclosed})} = 0$

And for smaller Amperian loop (loop A)

$|I_{A(\text{enclosed})}| = I = \dfrac{Nq\omega}{2\pi}$

So, $|I_A - I_B| = \dfrac{Nq\omega}{2\pi}$

Option (2) is correct answer.

40. A small uncharged conducting sphere is placed in contact with an identical sphere but having 4×10^{-8} C charge and then removed to a distance such that the force of repulsion between them is 9×10^{-3} N. The distance between them is (Take $\dfrac{1}{4\pi\varepsilon_0}$ as 9×10^9 in SI units)

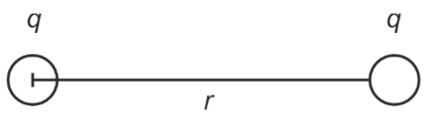
(1) 3 cm (2) 2 cm

(3) 4 cm (4) 1 cm

Answer (2)

Sol. Charges will be shared equally

$$q = \frac{Q}{2}$$



$$F_{(rep)} = \frac{kq^2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = 9\times10^{-3}$$

$$\Rightarrow \frac{9\times10^{9}\times4\times10^{-16}}{r^2} = 9\times10^{-3}$$

$$\Rightarrow r^2 = \frac{4\times10^{-7}}{10^{-3}} = 4\times10^{-4}$$

$\Rightarrow$ $r = 2\times10^{-2}$ m = 2 cm

Option (2) is correct answer.

41. The energy E and momentum p of a moving body of mass m are related by some equation. Given that c represents the speed of light, identify the correct equation

(1) $E^2 = p^2c^2 + m^2c^4$

(2) $E^2 = p^2c^2 + m^2c^2$

(3) $E^2 = pc^2 + m^2c^4$

(4) $E^2 = pc^2 + m^2c^2$

Answer (1)

Sol. We need to check the dimensions only.

With momentum the dimension

$E^2 = p^2c^2$

And with mass $E^2 = m^2c^4$

So $E^2 = p^2c^2 + m^2c^4$ (dimensionally)

42. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): In an insulated container, a gas is adiabatically shrunk to half of its initial volume. The temperature of the gas decreases.

Reason (R): Free expansion of an ideal gas is an irreversible and an adiabatic process.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(2) (A) is true but (R) is false

(3) (A) is false but (R) is true

(4) Both (A) and (R) are true and (R) is the correct explanation of (A)

Answer (3)

Sol. If the container is insulated then temperature is expected to increase in adiabatic compression. So (A) is wrong.

And free expansion is irreversible and adiabatic process. So (R) is correct.

43. In photoelectric effect, the stopping potential (V_0) v/s frequency (ν) curve is plotted. (h is the Planck's constant and ϕ_0 is work function of metal)

(A) V_0 v/s ν is linear

(B) The slope of V_0 v/s v curve = $\frac{\phi_0}{h}$

(C) h constant is related to the slope of V_0 v/s ν line

(D) The value of electric charge of electron is not required to determine h using the V_0 v/s ν curve.

(E) The work function can be estimated without knowing the value of h.

Choose the correct answer from the options given below :

(1) (A), (B) and (C) only

(2) (D) and (E) only

(3) (A), (C) and (E) only

(4) (C) and (E) only

Answer (3)

Sol. We know $h\nu - \phi_0 = eV_0$

$$\Rightarrow \quad V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e}$$

So slope is constant = $\frac{h}{e}$

and intercept is –ve $\left(\frac{\phi}{e}\right)$

$\Rightarrow$ Option (A) is correct, option (B) is wrong.

$\Rightarrow$ Option (C) is correct, option (D) is wrong.

$\Rightarrow$ Option (E) is correct.

(A), (C) & (E) are correct.

44. Arrange the following in the ascending order of wavelength (λ):

(A) Microwaves (λ_1)

(B) Ultraviolet rays (λ_2)

(C) Infrared rays (λ_3)

(D) X-rays (λ_4)

Choose the most appropriate answer from the options given below :

(1) $\lambda_3 < \lambda_4 < \lambda_2 < \lambda_1$ (2) $\lambda_4 < \lambda_3 < \lambda_1 < \lambda_2$

(3) $\lambda_4 < \lambda_3 < \lambda_2 < \lambda_1$ (4) $\lambda_4 < \lambda_2 < \lambda_3 < \lambda_1$

Answer (4)

Sol. X-ray (λ_4) is shortest wavelength

then ultraviolet is (λ_2) is longer than X-ray.

then infrared (λ_3) is longer than ultraviolet

then microwaves (λ_1) is longer than infrared

So $\lambda_4 < \lambda_2 < \lambda_3 < \lambda_1$

45. The position vector of a moving body at any instant of time is given as $\vec{r} = \left(5t^2\hat{i} - 5t\hat{j}\right)$ m. The magnitude and direction of velocity at t = 2 s is,

(1) $5\sqrt{17}$ m/s, making an angle of $\tan^{-1} 4$ with –ve Y axis

(2) $5\sqrt{17}$ m/s, making an angle of $\tan^{-1} 4$ with +ve X axis

(3) $5\sqrt{15}$ m/s, making an angle of $\tan^{-1} 4$ with +ve X axis

(4) $5\sqrt{15}$ m/s, making an angle of $\tan^{-1} 4$ with –ve Y axis

Answer (1)

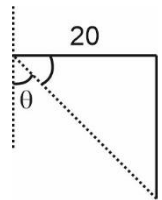
Sol. $\vec{r} = \left(5t^2\hat{i} - 5t\hat{i}\right)$

So $\frac{d\vec{r}}{dt} = \vec{v} = \left(10t\hat{i} - 5\hat{j}\right)$

At (t = 2 second)

$$\vec{v}(t=2) = (20\hat{i} - 5\hat{j})$$

$|\vec{v}| = 5\sqrt{1+16} = 5\sqrt{17}$ m/s

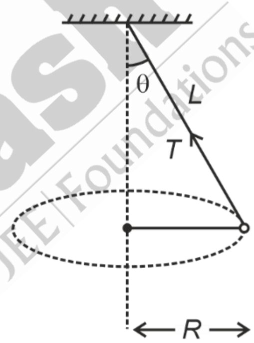


With –ve y-axis $\tan\theta = \frac{20}{5} = 4$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46.

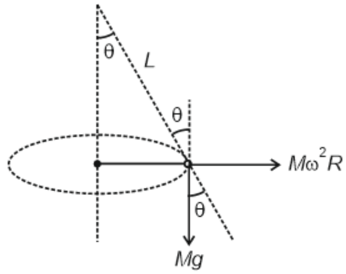


A string of length L is fixed at one end and carries a mass of M at the other end. The mass makes $\left(\frac{3}{\pi}\right)$ rotations per second about the vertical axis passing through end of the string as shown. The tension in the string is _______ ML.

Answer (36)

Sol. $\omega = \frac{3}{\pi} \times 2\pi = 6$ rad/s

$R = L\sin\theta$



and $T = M\sqrt{g^2 + \omega^4 R^2}$

Also, $T\sin\theta = M\omega^2 \cdot L\sin\theta$

$\Rightarrow$ $T = M(36)$ L

$\Rightarrow$ $T = 36$ ML

47. The ratio of the power of a light source S_1 to that the light source S_2 is 2. S_1 is emitting 2×10^{15} photons per second at 600 nm. If the wavelength of the source S_2 is 300 nm, then the number of photons per second emitted by S_2 is _______ $\times 10^{14}$.

Answer (5)

Sol. $p_1 = \dfrac{2\times10^{15}\times E_1}{1\text{ sec}} = \dfrac{2\times10^{15}\times hc}{\lambda_1}$

$p_2 = \dfrac{N\times hc}{\lambda_2}$

$\dfrac{p_1}{p_2} = \dfrac{2\times10^{15}\times hc\times300}{600\times N\times hc} = 2$

$\Rightarrow N = \dfrac{10^{15}}{2} = 5\times10^{14}$ per second

48. The increase in pressure required to decrease the volume of a water sample by 0.2% is P × 10^5 Nm^{-2}. Bulk modulus of water is 2.15 × 10^9 Nm^{-2}. The value of P is _______.

Answer (43)

Sol. We know $\dfrac{\Delta P}{\Delta V/V} = -\beta$

$\Rightarrow \Delta P = -\beta\cdot\dfrac{\Delta V}{V}$

$\Rightarrow |\Delta P| = 2.15\times10^9\times\dfrac{2}{1000} = 4.3\times10^6\ \text{N/m}^2$

$\Rightarrow \Delta P = 43\times10^5\ \text{N/m}^2$

49. Acceleration due to gravity on the surface of earth is 'g'. If the diameter of earth is reduced to one third of its original value and mass remains unchanged, then the acceleration due to gravity on the surface of the earth is _______ g.

Answer (9)

Sol. $g = \dfrac{GM}{R^2}$

Now, $g' = \dfrac{GM}{\left(\dfrac{R}{3}\right)^2} = \dfrac{9GM}{R^2} = 9g$

50. A tightly wound long solenoid carries a current of 1.5 A. An electron is executing uniform circular motion inside the solenoid with a time period of 75 ns. The number of turns per metre in the solenoid is _______.

[Take mass of electron $m_e = 9\times10^{-31}$ kg, charge of electron $|q_e| = 1.6\times10^{-19}$ C, $\mu_0 = 4\pi\times10^{-7}\dfrac{\text{N}}{\text{A}^2}$, 1 ns $= 10^{-9}$ s]

Answer (250)

Sol. We know,

$\beta = \mu_0\eta I$ and $T = \dfrac{2\pi m}{e\beta}$

$\Rightarrow \dfrac{2\pi m}{eT} = \mu_0\eta I$

$\Rightarrow \eta = \dfrac{2\pi m}{eT\mu_0 I}$

$= \dfrac{2\pi\times9\times10^{-31}}{1.6\times10^{-19}\times75\times10^{-9}\times4\pi\times10^{-7}\times1.5}$

$\Rightarrow \eta = \dfrac{9\times10^{-31}}{1.6\times10^{-19}\times75\times10^{-9}\times2\times10^{-7}\times1.5}$

$\Rightarrow \eta = \dfrac{9\times10^4}{360} = 250$ per meter

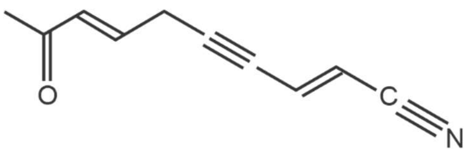
CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

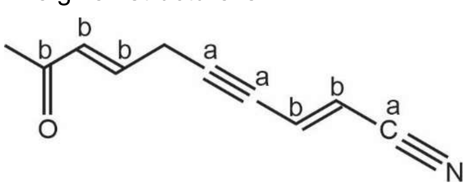
51. In the given structure, number of sp and sp^2 hybridized carbon atoms present respectively are:



(1) 3 and 6
(2) 4 and 6
(3) 4 and 5
(4) 3 and 5

Answer (4)

Sol. The given structure is



Number of sp hybridized C-atoms(a) = 3

Number of sp^2 hybridized C-atoms(b) = 5

52. Identify correct statement/s:

(A) $-OCH_3$ and $-NHCOCH_3$ are activating group.

(B) –CN and –OH are meta directing group.

(C) –CN and $-SO_3H$ are meta directing group.

(D) Activating groups act as ortho- and para directing groups.

(E) Halides are activating groups.

Choose the correct answer from the options given below:

(1) (A), (C) and (D) only
(2) (A) and (C) only
(3) (A) only
(4) (A), (B) and (E) only

Answer (1)

Sol. $-OCH_3$, $NHCOCH_3$ and –OH are activating groups because the atom directly bonded to benzene ring activates the ring by +R effect using its lone pair of electrons. Activating groups are ortho- and para directing groups.

–CN and $-SO_3H$ are deactivating groups because the atom directly bonded to benzene is bonded to more electronegative atom through multiple bonds and they are meta directing groups due to –R effect. Halides are deactivating groups due to –I effect.

Correct statements are (A), (C) and (D) only.

53. The conditions and consequence that favours the $t_{2g}^3 e_g^1$ configuration in a metal complex are:

(1) strong field ligand, high spin complex
(2) weak field ligand, high spin complex
(3) weak field ligand, low spin complex
(4) strong field ligand, low spin complex

Answer (2)

Sol. The conditions and consequence that favour $t_{2g}^3 e_g^1$ configuration in a metal complex are

(i) weak field ligand, and

(ii) high spin complex

For weak field ligands, splitting energy (Δ_0) is lower than pairing energy (P). As a result distribution of electrons for $3d^4$ will be $t_{2g}^3 e_g^1$. It results in high spin complex due to maximum number of unpaired electrons.

54. Given below are two statements:

Statement (I): The first ionization energy of Pb is greater than that of Sn.

Statement (II): The first ionization energy of Ge is greater than that of Si.

In the light of the above statements, choose the correct answer from the options given below:

(1) **Statement I** is false but **Statement II** is true
(2) **Statement I** is true but **Statement II** is false
(3) Both **Statement I** and **Statement II** are false
(4) Both **Statement I** and **Statement II** are true

Answer (2)

Sol. First ionization energy of Lead = 715 kJ mol^{-1}

First ionization energy of Tin = 708 kJ mol^{-1}

(IE_1) of Lead is greater than that of Tin due to ineffective shielding of d- and f-electrons.

Therefore **Statement-I** is true.

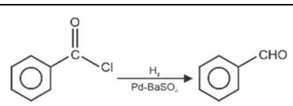
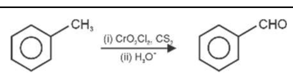
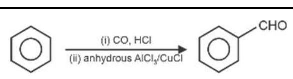
First ionization energy of Germanium = 761 kJ mol^{-1}

First ionization energy of Silicon = 786 kJ mol^{-1}

(IE_1) of Germanium is lower than that of Silicon as the effect of higher atomic radius of Ge outweighs the increase in nuclear charge from Si to Ge and effective shielding of inner electrons.

Therefore **Statement-II** is false.

55. Match **List-I** with **List-II**.

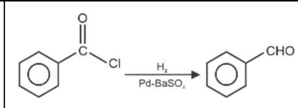
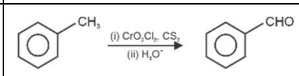
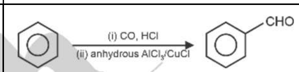
	List-I		List-II
(A)	$RCN \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } SnCl_2,\ HCl} RCHO$	(I)	Etard reaction
(B)		(II)	Gatterman-Koch reaction
(C)		(III)	Rosenmund reduction
(D)		(IV)	Stephen reaction

Choose the **correct** answer from the options given below:

(1) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)
(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(3) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Answer (3)

Sol.

	List-I		List-II
(A)	$RCN \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } SnCl_2,\ HCl} RCHO$	(IV)	Stephen reaction
(B)		(III)	Rosenmund reduction
(C)		(I)	Etard reaction
(D)		(II)	Gatterman-Koch reaction

Correct match: (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

56. The successive 5 ionisation energies of an element are 800, 2427, 3658, 25024 and 32824 kJ/mol, respectively. By using the above values predict the group in which the above element is present:

(1) Group 14
(2) Group 4
(3) Group 13
(4) Group 2

Answer (3)

Sol. The successive 5 ionisation energies of an element are 800, 2427, 3658, 25024 and 32824 kJ/mol, respectively.

$$\frac{IE_2}{IE_1} = \frac{2427}{800} = 3.03$$

$$\frac{IE_3}{IE_2} = \frac{3658}{2427} = 1.51$$

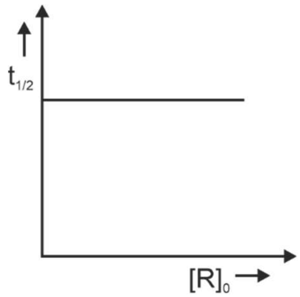
$$\frac{IE_4}{IE_3} = \frac{25024}{3658} = 6.84$$

$$\frac{IE_5}{IE_4} = \frac{32824}{25024} = 1.31$$

Since (IE_4 / IE_3) value is maximum, the element belongs to group 13.

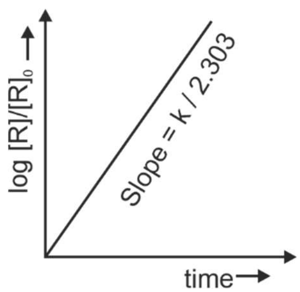
57. Given below are two statements:

Statement (I):



is valid for first order reaction.

Statement (II):



is valid for first order reaction.

In the light of the above statements, choose the correct answer from the options given below:

(1) **Statement I** is true but **Statement II** is false

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is false but **Statement II** is true

(4) Both **Statement I** and **Statement II** are true

Answer (1)

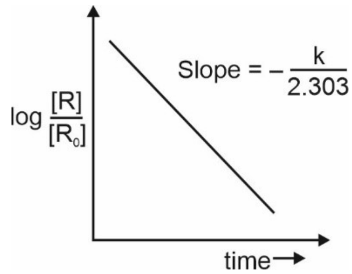
Sol. Half life ($t_{1/2}$) of a first order reaction does not depend on initial concentration of the reactant [R_0]. Therefore **Statement I** is true. Rate constant (k) of a first order reaction is given by

$$k = \frac{2.303}{t}\log\frac{[R_0]}{[R]}$$

$$\log\frac{[R]}{[R_0]} = -\left(\frac{k}{2.303}\right)t$$

Plot of $\log\frac{[R]}{[R_0]}$ versus time is linear having

$$\text{slope} = -\frac{k}{2.303}$$



∴ **Statement II** is false.

58. Given below are two statements:

Statement (I): Experimentally determined oxygen-oxygen bond lengths in the O_3 are found to be same and the bond length is greater than that of a O = O (double bond) but less than that of a single (O – O) bond.

Statement (II): The strong lone pair-lone pair repulsion between oxygen atoms is solely responsible for the fact that the bond length in ozone is smaller than that of a double bond (O = O) but more than that of a single bond (O – O).

In the light of the above statements, choose the correct answer from the options given below :

(1) **Statement I** is false but **Statement II** is true

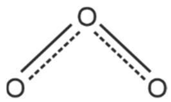
(2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is true but **Statement II** is false

(4) Both **Statement I** and **Statement II** are false

Answer (3)

Sol.

Bond	(O – O) Bond order
(O – O)	1.0
	1.5
O = O	2.0

Bond length $\propto \dfrac{1}{\text{Bond order}}$

Order of O – O bond length

O = O < O_3 < O – O

∴ **Statement I** is true.

Lone pair-lone pair repulsion between O-atoms is not solely responsible for the correct order of O–O bond length. Bond order also should be considered.

∴ **Statement II** is false.

59. When Ethane-1,2-diamine is added progressively to an aqueous solution of Nickel (II) chloride, the sequence of colour change observed will be:

(1) Green → Pale blue → Blue → Violet

(2) Pale blue → Blue → Violet → Green

(3) Pale blue → Blue → Green → Violet

(4) Violet → Blue → Pale blue → Green

Answer (1)

Sol. If ethane-1,2-diamine (en) is added progressively to an aqueous solution of $NiCl_2$ in the molar ratios en : Ni 1 : 1, 2 : 1 and 3 : 1, the following series of reactions and their associated colour changes occur. [Ref : NCERT (XII) Part-1, p 260]

$$\underset{\text{Green}}{[Ni(H_2O)_6]^{2+}}(aq) + en(aq) \longrightarrow \underset{\text{Pale blue}}{[Ni(H_2O)_4en]^{2+}}(aq) + 2H_2O$$

$$[Ni(H_2O)_4en]^{2+}(aq) + en(aq) \longrightarrow \underset{\text{Blue/Purple}}{[Ni(H_2O)_2(en)_2]^{2+}}(aq) + 2H_2O$$

$$[Ni(H_2O)_2(en)_2]^{2+}(aq) + en(aq) \longrightarrow \underset{\text{Violet}}{[Ni(en)_3]^{2+}}(aq) + 2H_2O$$

60. Which of the following mixing of 1 M base and 1 M acid leads to the largest increase in temperature?

(1) 30 mL HCl and 30 mL NaOH

(2) 30 mL CH_3COOH and 30 mL NaOH

(3) 45 mL CH_3COOH and 25 mL NaOH

(4) 50 mL HCl and 20 mL NaOH

Answer (1)

Sol. The rise in temperature of neutralization reaction will be maximum for maximum number of moles of strong acid and strong base neutralized and lower volume of final solution.

(1) HCl + NaOH → NaCl + H_2O

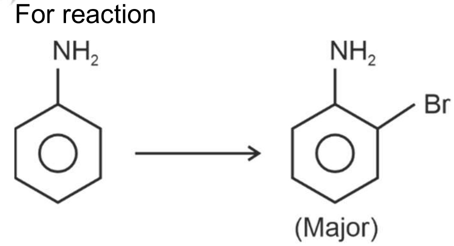
mmol 30 30

- -

Final volume of solution = 60 mL

Option (2) and (3) have weak acids and in option (4) only 20 mmol of HCl will be neutralized with 70 mL final volume.

61. For reaction



The correct order of set of reagents for the above conversion is :

(1) Br_2 | $FeBr_3$, $H_2O(\Delta)$, NaOH

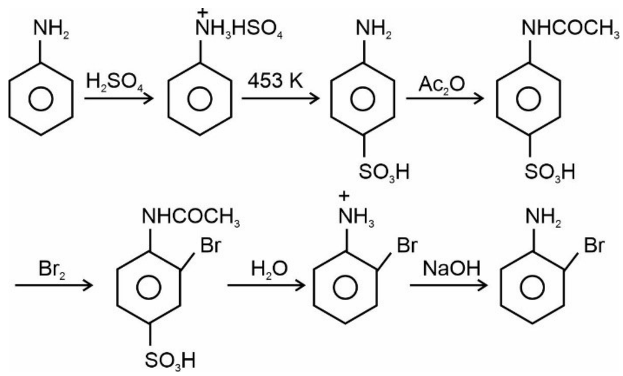
(2) Ac_2O, Br_2, $H_2O(\Delta)$, NaOH

(3) H_2SO_4, Ac_2O, Br_2, $H_2O(\Delta)$, NaOH

(4) Ac_2O, H_2SO_4, Br_2, NaOH

Answer (3)

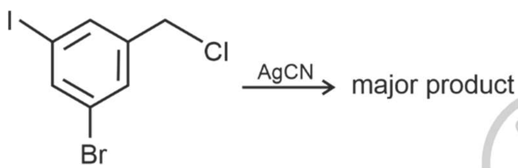
Sol.

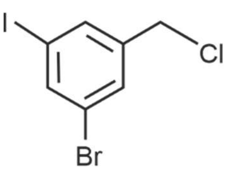


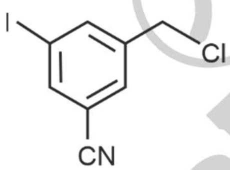
Correct sequence of reagents

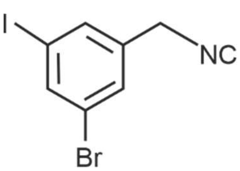
H_2SO_4, Ac_2O, Br_2, $H_2O(\Delta)$, NaOH

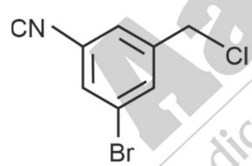
62. The structure of the major product formed in the following reaction is :



(1) 

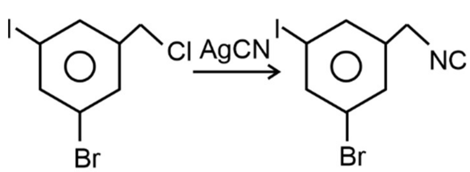
(2) 

(3) 

(4) 

Answer (3)

Sol. Haloalkanes react with AgCN to give isocyanide as major product and haloarenes do not react with AgCN.



(major)

63. Find the compound 'A' from the following reaction sequences.

$$A \xrightarrow{\text{aqua-regia}} B \xrightarrow[\text{(2) AcOH}]{\text{(1) } KNO_2[NH_4OH]} \text{yellow ppt}$$

(1) CoS (2) MnS

(3) NiS (4) ZnS

Answer (1)

Sol. Compound (A) in the given reaction sequence is likely to be CoS.

$$CoS + HNO_3 + 3HCl \rightarrow Co^{2+} + S\downarrow + NOCl\uparrow + 2Cl^- + 2H_2O$$

The above solution is neutralised with NH_4OH.

To a neutral solution of Co^{2+}, acetic acid and saturated solution of KNO_2 are added which results in the formation of yellow precipitate of $K_3[Co(NO_2)_6]$

$$Co^{2+} + 7NO_2^- + 2H^+ + 3K^+ \rightarrow K_3[Co(NO_2)_6] + NO\uparrow + H_2O$$

64. The elemental composition of a compound is 54.2% C, 9.2% H and 36.6% O. If the molar mass of the compound is 132 g mol^{-1}, the molecular formula of the compound is:

[Given: The relative atomic mass of C : H : O = 12 : 1 : 16]

(1) $C_6H_{12}O_6$ (2) $C_4H_8O_2$

(3) $C_4H_9O_3$ (4) $C_6H_{12}O_3$

Answer (4)

Sol.

Element	Mass %	Mole %	Molar ratio
C	54.2	4.52	2
H	9.2	9.2	4
O	36.6	2.28	1

Empirical formula of compound is C_2H_4O

Molecular mass of compound = 132 g mol^{-1}

Molecular formula of compound is $(C_2H_4O)_n$

$$n = \frac{\text{Molecular mass}}{\text{EF mass}} = \frac{132}{44} = 3$$

∴ Molecular formula of compound is $C_6H_{12}O_3$

65. For hydrogen atom, the orbital/s with lowest energy is/are:

(A) 4s

(B) $3p_x$

(C) $3d_{x^2-y^2}$

(D) $3d_{z^2}$

(E) $4p_z$

Choose the **correct** answer from the options given below:

(1) (A) and (E) only

(2) (A) only

(3) (B) only

(4) (B), (C) and (D) only

Answer (4)

Sol. For hydrogen atom and one electron species, the energy of orbitals is decided by the value of principal quantum number. Higher the value of principal quantum number, higher will be the energy of orbital.

(A) 4s	n = 4
(B) $3p_x$	n = 3
(C) $3d_{x^2-y^2}$	n = 3
(D) $3d_{z^2}$	n = 3
(E) $4p_z$	n = 4

∴ (B), (C) and (D) have orbitals with the lowest energy.

66. Based on the data given below:

$$E^{o}_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33V \quad E^{o}_{Cl_2/Cl^-} = 1.36V$$

$$E^{o}_{MnO_4^-/Mn^{2+}} = 1.51V \quad E^{o}_{Cr^{3+}/Cr} = -0.74V$$

The strongest reducing agent is:

(1) Cl^-

(2) Cr

(3) MnO_4^-

(4) Mn^{2+}

Answer (2)

Sol. $E^{o}_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33V \quad E^{o}_{Cl_2/Cl^-} = 1.36V$

$E^{o}_{MnO_4^-/Mn^{2+}} = 1.51V \quad E^{o}_{Cr^{3+}/Cr} = -0.74V$

The species which has the most negative value of standard reduction potential will be the strongest reducing agent. Since Cr^{3+}/Cr has SRP value of –0.74V, Cr is the strongest reducing agent.

67. Match List – I with List – II

List – I (Transition metal ion)		List – II (Spin only magnetic moment (B.M.))	
(A)	Ti^{3+}	(I)	3.87
(B)	V^{2+}	(II)	0.00
(C)	Ni^{2+}	(III)	1.73
(D)	Sc^{3+}	(IV)	2.84

Choose the correct answer from the options given below:

(1) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(2) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(3) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

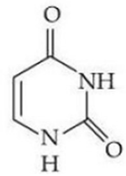
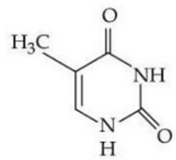
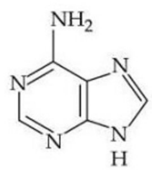
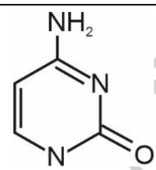
(4) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Answer (3)

Sol.

	Transition Metal ion	Electronic configuration	No. of unpaired electrons	Magnetic moment (BM)
(A)	Ti^{3+}	$3d^1$	1	1.73
(B)	V^{2+}	$3d^3$	3	3.87
(C)	Ni^{2+}	$3d^8$	2	2.84
(D)	Sc^{3+}	$3d^0$	0	0.00

68. Match List-I with List-II

List-I		List-II	
(A)	Adenine	(I)	
(B)	Cytosine	(II)	
(C)	Thymine	(III)	
(D)	Uracil	(IV)	

Choose the correct answer from the options given below:

(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

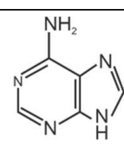
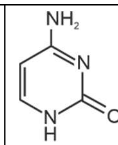
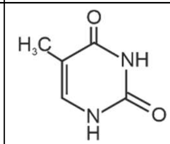
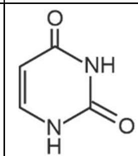
(2) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(3) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Answer (3)

Sol.

Compound		Structure	
(A)	Adenine	III.	
(B)	Cytosine	IV.	
(C)	Thymine	II.	
(D)	Uracil	I.	

69. $S(g)+\frac{3}{2}O_2(g) \rightarrow SO_3(g)+2x\text{ kcal}$

$SO_2(g)+\frac{1}{2}O_2(g) \rightarrow SO_3(g)+y\text{ kcal}$

The heat of formation of $SO_2(g)$ is given by:

(1) $\frac{2x}{y}$ kcal

(2) $y - 2x$ kcal

(3) $x + y$ kcal

(4) $2x + y$ kcal

Answer (2)

Sol.

(i) $S(g)+\frac{3}{2}O_2(g) \rightarrow SO_3(g)+2x\text{ kcal}\ \ \Delta H_1$

(ii) $SO_2(g)+\frac{1}{2}O_2(g) \rightarrow SO_3(g)+y\text{ kcal}\ \ \Delta H_2$

(i) – (ii)

$S(g)+O_2(g) \rightarrow SO_2(g)\ \ \Delta H$

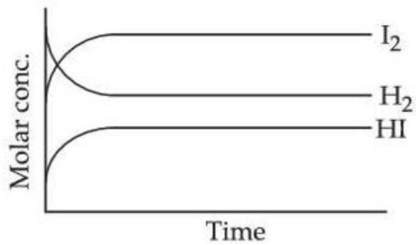
$\Delta H = \Delta H_1 - \Delta H_2$

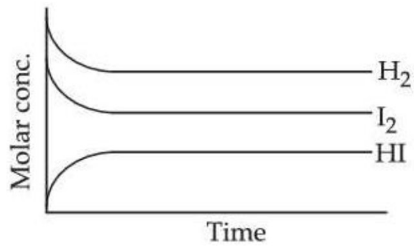
$= -2x - (-y) = (y - 2x)$ kcal

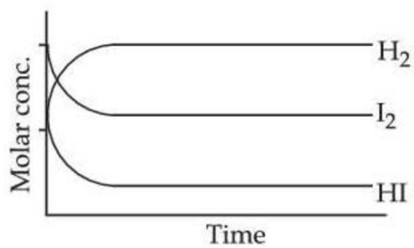
70. For the reaction,

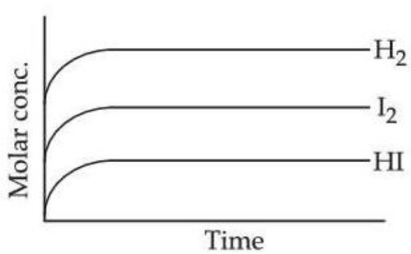
$$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$$

Attainment of equilibrium is predicted correctly by:

(1) 

(2) 

(3) 

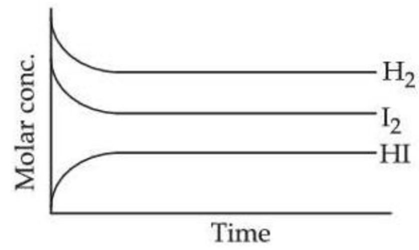
(4) 

Answer (2)

Sol.

$$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$$

Concentrations of $H_2(g)$ and $I_2(g)$ decrease with time while concentration of HI(g) increases with time. At equilibrium $H_2(g)$, $I_2(g)$ and HI(g) attain constant values. Correct plot of molar concentration with time is



71. Consider a complex reaction taking place in three steps with rate constants k_1, k_2 and k_3 respectively. The overall rate constant k is given by the expression $k = \sqrt{\frac{k_1k_3}{k_2}}$. If the activation energies of the three steps are 60, 30 and 10 kJ mol^{-1} respectively, then the overall energy of activation in kJ mol^{-1} is ________. (Nearest integer)

Answer (20)

Sol. k_1, k_2 and k_3 are given as rate constants of three steps of a complex reaction. Rate constant (k) of the overall reaction is given as

$$k = \sqrt{\frac{k_1k_3}{k_2}}$$

Activation energies of the three steps are given as

E_{a_1} = 60 kJ mol^{-1}, E_{a_2} = 30 kJ mol^{-1},

E_{a_3} = 10 kJ mol^{-1}

From Arrhenius equation, we know that

$$k = Ae^{-Ea/RT}$$

If E_a is the activation energy of the overall reaction, then

$$E_a = \frac{1}{2}\left[E_{a_1} + E_{a_3} - E_{a_2}\right]$$

$$= \frac{1}{2}[60 + 10 - 30] = 20 \text{ kJ mol}^{-1}$$

72. In Carius method of estimation of halogen, 0.25 g of an organic compound gave 0.15 g of silver bromide (AgBr). The percentage of Bromine in the organic compound is ____________ × 10^{-1} % (Nearest integer).

(Given: Molar mass of Ag is 108 and Br is 80 g mol^{-1})

Answer (255)

Sol. Mass of organic compound = 0.25 g

Mass of AgBr = 0.15 g

No. of moles of Br = No. of moles of AgBr = $\frac{0.15}{188}$

Mass of Br = $\frac{0.15 \times 80}{188}$ g

% of Br = $\frac{0.15 \times 80 \times 100}{188 \times 0.25}$

= 25.5 %

= 255 × 10^{-1} %

73. The observed and normal molar masses of compound MX_2 are 65.6 and 164 respectively. The percent degree of ionisation of MX_2 is _______ %. (Nearest integer)

Answer (75)

Sol. Normal molar mass of MX_2 = 164.0 g mol^{-1}

Observed molar mass of MX_2 = 65.6 g mol^{-1}

Van't Hoff factor (i) = $\frac{\text{Normal molar mass}}{\text{Observed molar mass}}$

$= \frac{164}{65.6} = 2.5$

If α is the degree of ionisation, then

$$\underset{1-\alpha}{MX_2} \rightleftharpoons \underset{\alpha}{M^{2+}} + \underset{2\alpha}{2X^-}$$

i = 1 – α + α + 2α = 1 + 2α

1 + 2α = 2.5

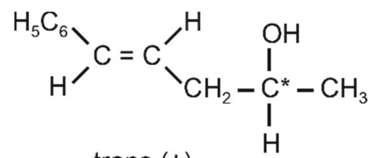
α = 0.75

∴ Percent degree of ionisation = 75%

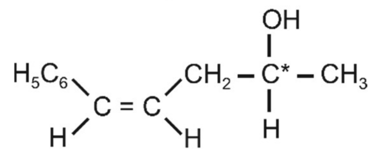
74. The possible number of stereoisomers for 5-phenylpent-4-en-2-ol is ______________.

Answer (4)

Sol. The given compound is 5-phenylpent-4-en-2-ol. Possible stereoisomers are



trans (±)



cis (±)

∴ 4 possible stereoisomers

75. The hydrocarbon (X) with molar mass 80 g mol^{-1} and 90% carbon has _____________ degree of unsaturation.

Answer (3)

Sol.

Element	Mass %	Mole %	Molar ratio
C	90	7.5	1
H	10	10	1.33

∴ Empirical formula is C_3H_4

Molecular mass = 80 g mol^{-1}.

Molecular formula = $(C_3H_4)_n$

$n = \frac{\text{Molecular mass}}{\text{EF mass}} = \frac{80}{40} = 2$

∴ Molecular formula is C_6H_8

Degree of unsaturation = $\frac{2 \times 6 + 2 - 8}{2} = 3$

❑ ❑ ❑