MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let a circle *C* pass through the points (4, 2) and (0, 2), and its centre lie on $3x + 2y + 2 = 0$ Then the length of the chord, of the circle *C*, whose mid-point is (1, 2), is:

(1) $2\sqrt{3}$

(2) $\sqrt{3}$

(3) $4\sqrt{2}$

(4) $2\sqrt{2}$

Answer (1)

Sol. Let the centre be

$$\left(-2a, \frac{6a-2}{2}\right) \equiv (-2a, 3a-1)$$

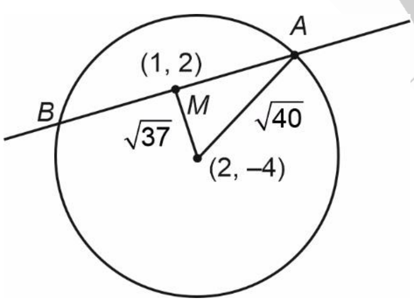
Centre is equal distance from (4, 2) and (0, 2)

$$\Rightarrow \sqrt{(4+2a)^2+(3a-3)^2} = \sqrt{(-2a-0)^2+(3a-3)^2}$$

$$\Rightarrow (2a+4)^2+9(a-1)^2 = 4a^2+9(a-1)^2$$

$$\Rightarrow 4a^2+16+16a = 4a^2 \Rightarrow a = -1$$

$$\Rightarrow \text{centre} \equiv (2,-4) \Rightarrow \text{Radius} = \sqrt{40}$$



$$\Rightarrow AM^2 = (\sqrt{40})^2 - (\sqrt{37})^2$$

$$\Rightarrow 2AM = AB = 2\sqrt{3}$$

2. Let $f(x) = \int_0^x t(t^2 - 9t + 20)dt$, $1 \le x \le 5$. If the range of *f* is [α, β], then 4(α + β) equals

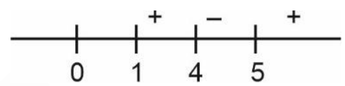
(1) 125 (2) 253

(3) 154 (4) 157

Answer (4)

Sol. $f'(x) = x(x^2 - 9x + 20)$, $x \in (1, 5)$

$= (x - 4)x(x - 5)$



$\Rightarrow f'(x) > 0 \,\forall x \in (1, 4)$

$\Rightarrow f'(x) < 0 \,\forall x \in (4, 5)$

$\Rightarrow$ *f*(*x*) increasing in (1, 4)

f(*x*) decreasing in (4, 5)

$\Rightarrow$ critical points to check:

$x = 1, 4, 5$

$$f(x) = \int_0^x (t^3 - 9t^2 + 20t)dt$$

$$= \frac{t^4}{4} - 3t^3 + 10t^2 \Big|_0^x = \frac{x^4}{4} - 3x^3 + 10x^2$$

$$f(1) = \frac{1}{4} - 3 + 10 = \frac{29}{4}$$

$$f(4) = 4^3 - 3.4^3 + 10.4^2 = -2.4^3 + 10.4^2 = 32$$

$$f(5) = \frac{5^4}{4} - 3.5^3 + 10.25 = \frac{5^4}{4} - 125 = \frac{125}{4}$$

$$\text{Range} \Rightarrow \left[\frac{29}{4}, 32\right] \Rightarrow 4(\alpha+\beta) = 128 + 29$$

$= 157$

3. Let the function $f(x) = (x^2 - 1)|x^2 - ax + 2| + \cos|x|$ be not differentiable at the two points $x = \alpha = 2$ and $x = \beta$. Then the distance of the point (α, β) from the line $12x + 5y + 10 = 0$ is equal to

(1) 3 (2) 2

(3) 5 (4) 4

Answer (1 *Bonus)

Sol. $f(x) = (x^2 - 1)|x^2 - ax + 2| + \cos|x|$

Notice that $\cos(-x) = \cos x = \cos|x|$ which means $\cos|x|$ is differentiable

everywhere in $x \in R$

$\Rightarrow$ $f(x)$ can be non differentiable where $|x^2 - ax + 2| = 0$

$\Rightarrow$ $x^2 - ax + 2 = 0$ $\begin{cases} x = \alpha = 2 \\ x = \beta \end{cases}$

$\Rightarrow$ $4 - 2a + 2 = 0 \Rightarrow a = 3$

$\Rightarrow$ $(x^2 - 3x + 2) = 0$ $\Rightarrow$ $x = 1, 2$

$\beta = 1$

but $f(x)$ is differentiable at $x = 1$.

so it should be bonus.

distance of (α, β) from line

$12x + 5y + 10 = 0$

$$\Rightarrow \frac{|2(12) + 5(1) + 10|}{13} = \frac{39}{13} = 3$$

4. Let $A = [a_{ij}]$ be a matrix of order 3×3, with $a_{ij} = \left(\sqrt{2}\right)^{i+j}$. If the sum of all the elements in the third row of A^2 is $\alpha + \beta\sqrt{2}, \alpha, \beta \in Z$, then $\alpha + \beta$ is equal to

(1) 280 (2) 210

(3) 224 (4) 168

Answer (3)

Sol. $a_{ij} = \left(\sqrt{2}\right)^{i+j}$

$$A^2 = \begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix} \times \begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 28 & 28\sqrt{2} & 56 \\ 28\sqrt{2} & 56 & 56\sqrt{2} \\ 56 & 56\sqrt{2} & 112 \end{bmatrix}$$

Sum of elements of third row = $56 + 112 + 56\sqrt{2}$

$= 168 + 56\sqrt{2}$

$\Rightarrow \alpha = 168$ $\beta = 56$

$\therefore$ $\alpha + \beta = 224$

5. If for the solution curve $y = f(x)$ of the differential equation $\frac{dy}{dx} + (\tan x)y = \frac{2 + \sec x}{(1 + 2\sec x)^2}$, $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right), f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{10}$, then $f\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{\sqrt{3}+1}{10(4+\sqrt{3})}$ (2) $\frac{4-\sqrt{2}}{14}$

(3) $\frac{9\sqrt{3}+3}{10(4+\sqrt{3})}$ (4) $\frac{5-\sqrt{3}}{2\sqrt{2}}$

Answer (2)

Sol. $\frac{dy}{dx} + (\tan x)y = \frac{2 + \sec x}{(1 + 2\sec x)^2}$

$IF = e^{\int \tan x\, dx} = e^{\ln \sec x}$

$= \sec x$

$\therefore$ solution will be

$$y \sec x = \int \frac{(2 + \sec x)}{(1 + 2\sec x)^2} \sec x\, dx$$

$$= \int \frac{\left(2 + \frac{1 + \tan^2 \frac{x}{2}}{1 - \tan^2 \frac{x}{2}}\right)\left(1 + \tan^2 \frac{x}{2}\right)}{\left(1 + \frac{2\left(1 + \tan^2 \frac{x}{2}\right)}{1 - \tan^2 \frac{x}{2}}\right)^2 \left(1 - \tan^2 \frac{x}{2}\right)} dx$$

Let $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

$$= 2\int \frac{(3-t^2)}{(3+t^2)^2}dt$$

$$= \frac{2t}{t^2+3} = \frac{2\tan\frac{x}{2}}{\tan^2\frac{x}{2}+3} + c$$

$$\Rightarrow y\sec x = \frac{2\tan\frac{x}{2}}{\tan^2\frac{x}{2}+3} + c$$

$$y\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{10}$$

$$\Rightarrow 2y = \frac{\frac{2}{\sqrt{3}}}{\frac{1}{3}+3} + c = \frac{\frac{2}{\sqrt{3}}}{\frac{10}{3}} + c = \frac{2\sqrt{3}}{10} + c$$

$$\frac{2\sqrt{3}}{10} = \frac{2\sqrt{3}}{10} + c \Rightarrow c = 0$$

$$\therefore\ y = \frac{2\tan\frac{x}{2}}{\sec x\left[\tan^2\frac{x}{2}+3\right]}$$

Now, $f\left(\frac{\pi}{4}\right) = \frac{2\tan\frac{\pi}{8}}{\sec\frac{\pi}{4}\left[\tan^2\frac{\pi}{8}+3\right]} = \frac{4-2\sqrt{2}}{14}$

6. Let the line $x + y = 1$ meet the axes of x and y at A and B, respectively. A right angled triangle AMN is inscribed in the triangle OAB, where O is the origin and the points M and N lie on the lines OB and AB, respectively. If the area of the triangle AMN is $\frac{4}{9}$ of the area of the triangle OAB and $AN : NB = \lambda : 1$, then the sum of all possible value(s) of is λ :

(1) $\frac{13}{6}$

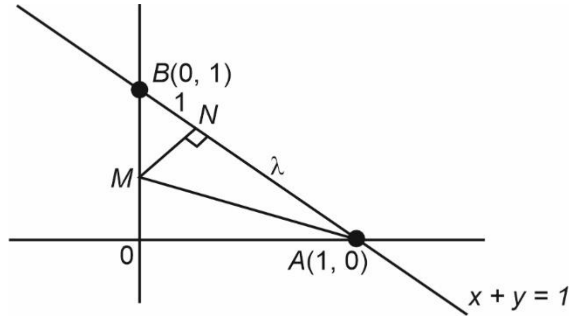
(2) $\frac{5}{2}$

(3) $\frac{1}{2}$

(4) 2

Answer (4)

Sol.



$$\frac{AN}{NB} = \frac{\lambda}{1}$$

Then $N \equiv \left(\frac{1}{1+\lambda}, \frac{\lambda}{1+\lambda}\right)$

$m_{MN} = 1$

$$MN : \left(y - \frac{\lambda}{1+\lambda}\right) = \left(x - \frac{1}{1+\lambda}\right)$$

$$\therefore\ M\left(0, \frac{\lambda-1}{\lambda+1}\right)$$

area (ΔAMN) = $\frac{4}{9}$ ar (ΔOAB)

$$\frac{1}{2}\times|AN\times NM| = \frac{4}{9}\times\frac{1}{2}\times 1\times 1$$

$$|AN\times NM| = \frac{4}{9}$$

$$\left|\frac{\sqrt{2}\lambda}{1+\lambda}\times\frac{\sqrt{2}}{1+\lambda}\right| = \frac{4}{9}$$

$$\left|\frac{2\lambda}{(1+\lambda)^2}\right| = \frac{4}{9}$$

$9|\lambda| = 2(1+\lambda)^2$

$\pm 9\lambda = 2 + 2\lambda^2 + 4\lambda$

$\Rightarrow\ 2\lambda^2 + 13\lambda + 2 = 0$ → α, β

$\Rightarrow\ 2\lambda^2 - 5\lambda + 2 = 0 \quad \begin{cases} \frac{1}{2} \\ 2 \end{cases}$

$\because\ M\left(0, \dfrac{\lambda-1}{\lambda+1}\right)$

$\because$ M lies between (0, 0) and (0, 1)

$\Rightarrow\ \lambda \neq \dfrac{1}{2}, \alpha, \beta$

$\therefore\ \lambda = 2$

$\therefore$ Only possible value of $\lambda = 2$

7. If the domain of the function $\log_5(18x - x^2 - 77)$ is (α, β) and the domain of the function $\log_{(x-1)}\left(\dfrac{2x^2+3x-2}{x^2-3x-4}\right)$ is (γ, δ), then $\alpha^2+\beta^2+\gamma^2$ is equal to

(1) 174 (2) 195

(3) 179 (4) 186

Answer (4)

Sol. $f_1(x) = \log_5(18x - x^2 - 77)$

$\therefore\ 18x - x^2 - 77 > 0$

$x^2 - 18x + 77 < 0$

$x \in (7, 11)$

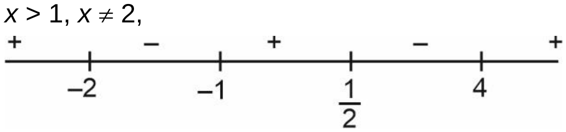
$\alpha = 7, \beta = 11$

$f_2(x) = \log_{(x-1)}\left(\dfrac{2x^2+3x-2}{x^2-3x-4}\right)$

$x > 1,\ x - 1 \neq 1,\ \dfrac{2x^2+3x-2}{x^2-3x-4} > 0$

$x > 1,\ x \neq 2,\ \dfrac{(2x-1)(x+2)}{(x-4)(x+1)} > 0$

$x > 1, x \neq 2,$



$\therefore\ x \in (4, \infty)$

$\therefore\ \gamma = 4$

$\therefore\ \alpha^2 + \beta^2 + \gamma^2 = 49 + 121 + 16$

$= 186$

8. Let a straight line L pass through the point $P(2, -1, 3)$ and be perpendicular to the lines $\dfrac{x-1}{2} = \dfrac{y+1}{1} = \dfrac{z-3}{-2}$ and $\dfrac{x-3}{1} = \dfrac{y-2}{3} = \dfrac{z+2}{4}$. If the line L intersects the yz-plane at the point Q, then the distance between the points P and Q is

(1) $\sqrt{10}$ (2) 3

(3) 2 (4) $2\sqrt{3}$

Answer (2)

Sol. Vector parallel to L

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 3 & 4 \end{vmatrix} = 10\hat{i} - 10\hat{j} + 5\hat{k}$

$= 5\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$

Equation of 'L'

$\dfrac{x-2}{2} = \dfrac{y+1}{-2} = \dfrac{z-3}{1} = \lambda(\text{say})$

Let $Q(2\lambda+2, -2\lambda-1, \lambda+3)$

$\Rightarrow\ 2\lambda + 2 = 0 \Rightarrow \lambda = -1$

$\Rightarrow\ Q(0, 1, 2)$

$d(P, Q) = 3$

9. If all the words with or without meaning made using all the letters of the word "KANPUR" are arranged as in a dictionary, then the word at 440th position in this arrangement, is:

(1) PRNAKU (2) PRNAUK

(3) PRKANU (4) PRKAUN

Answer (4)

Sol. A, K, N, P, R, U

[A] ……………… 5! = 120

[K] ……………… 5! = 120

[N] ……………… 5! = 120

[P] [A] ………… 4! = 24

[P] [K] ………… 4! = 24

$\boxed{P}\ \boxed{N}$ .... 4! = 24

$\boxed{P}\ \boxed{R}\ \boxed{A}$ ....3! = 6

$\boxed{P}\ \boxed{R}\ \boxed{K}\ \boxed{A}\ \boxed{N}\ \boxed{U}$ = 1

$\boxed{P}\ \boxed{R}\ \boxed{K}\ \boxed{A}\ \boxed{U}\ \boxed{N}$ = 1

Total = 440

$\Rightarrow$ 440th word is P R K A U N

10. Let $S = \mathbb{N} \cup \{0\}$. Define a relation R from S to **R** by:

$$R = \left\{(x, y) : \log_e y = x \log_e \left(\frac{2}{5}\right), x \in S, y \in \mathbf{R}\right\}.$$

Then, the sum of all the elements in the range of R is equal to

(1) $\frac{10}{9}$ (2) $\frac{5}{3}$

(3) $\frac{3}{2}$ (4) $\frac{5}{2}$

Answer (2)

Sol. $\log_e y = x \log_e \left(\frac{2}{5}\right)$

$$y = \left(\frac{2}{5}\right)^x$$

$x \in N \cup \{0\}$

$$y = 1, \frac{2}{5}, \left(\frac{2}{5}\right)^2, \ldots$$

$$\sum y = \frac{1}{1 - \frac{2}{5}} = \frac{5}{3}$$

11. Bag 1 contains 4 white balls and 5 black balls, and Bag 2 contains n white balls and 3 black balls. One ball is drawn randomly from Bag 1 and transferred to Bag 2. A ball is then drawn randomly from Bag 2. If the probability, that the ball drawn is white, is 29/45, then n is equal to

(1) 6 (2) 3

(3) 5 (4) 4

Answer (1)

Sol. Bag 1 $\rightarrow$ $4w$, $5B$

Bag 2 $\rightarrow$ nw, $3B$

(I) $\rightarrow$ Transferred ball is white $P(w) = \frac{n+1}{n+4} \cdot \frac{4}{9}$

(II) $\rightarrow$ Transferred ball is black $P(w) = \frac{5}{9} \cdot \frac{n}{n+4}$

$$\frac{4n+4}{9n+36} + \frac{5n}{9n+36} = \frac{29}{45}$$

$$\frac{9n+4}{9n+36} = \frac{29}{45} \Rightarrow n = 6$$

12. If $\alpha x + \beta y = 109$ is the equation of the chord of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, whose mid point is $\left(\frac{5}{2}, \frac{1}{2}\right)$, then $\alpha + \beta$ is equal to

(1) 37 (2) 58

(3) 72 (4) 46

Answer (2)

Sol. Chord with given mid-point

$T = S_1$

$$\frac{5}{18}x + \frac{y}{8} = \frac{25}{36} + \frac{1}{16} = \frac{109}{144}$$

$40x + 18y = 109$

$\alpha + \beta = 40 + 18 = 58$

13. Let $A = [a_{ij}]$ be a 2 × 2 matrix such that $a_{ij} \in \{0, 1\}$ for all i and j. Let the random variable X denote the possible values of the determinant of the matrix A. Then, the variance of X is:

(1) $\frac{3}{8}$ (2) $\frac{5}{8}$

(3) $\frac{1}{4}$ (4) $\frac{3}{4}$

Answer (1)

Sol.

x	0	1	–1
$P(x)$	$\frac{10}{16}$	$\frac{3}{16}$	$\frac{3}{16}$

$\text{Var}(x) = E(x^2) - [E(x)]^2$

$$= \sum_{i=1}^{3} x_i^2 P(x_i) - (\mu)^2$$

$$= 1 \times \frac{3}{16} + 1 \times \frac{3}{16} \quad [\mu = 0]$$

$$= \frac{6}{16} = \frac{3}{8}$$

14. Let P be the foot of the perpendicular from the point (1, 2, 2) on the line $L : \frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2}$. Let the $\vec{r} = \left(-\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$, $\lambda \in \mathbf{R}$, intersect the line L at Q. Then $2(PQ)^2$ is

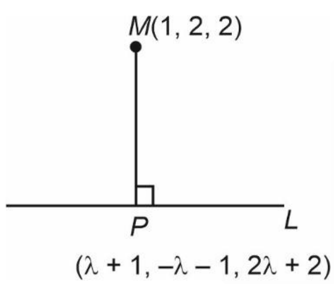
(1) 29 (2) 27

(3) 25 (4) 19

Answer (2)

Sol. General point on line L : $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-2}{2}$

is $(\lambda + 1, -\lambda - 1, 2\lambda + 2)$



DR's of PM are $(\lambda, -\lambda - 3, 2\lambda)$

$PM \perp L$

$\Rightarrow \lambda + (-1)(-\lambda - 3) + 2(2\lambda) = 0$

$\Rightarrow 6\lambda + 3 = 0$

$\Rightarrow \boxed{\lambda = \frac{-1}{2}}$

$P\left(\frac{1}{2}, \frac{-1}{2}, 1\right)$

Let another line $L' : \frac{x+1}{1} = \frac{y-1}{-1} = \frac{z+2}{1}$

General point on line L' is $(\mu - 1, -\mu + 1, \mu - 2)$

Point of intersection of line L and L' is

$\lambda + 1 = \mu - 1$	$2\lambda + 2 = \mu - 2$
$\Rightarrow \mu - \lambda = 2$ .... (1)	$\Rightarrow 2\lambda = \mu - 4$

$\Rightarrow \boxed{\lambda = -2}$ and $\boxed{\mu = 0}$

$Q(-1, 1, -2)$

$$2(PQ)^2 = 2\left(\left(\frac{1}{2}+1\right)^2 + \left(\frac{-1}{2}-1\right)^2 + (1+2)^2\right)$$

$$= 2\left(\frac{9}{4} + \frac{9}{4} + 9\right)$$

$= 27$

15. If $\sin x + \sin^2 x = 1, x \in \left(0, \frac{\pi}{2}\right)$, then $(\cos^{12} x + \tan^{12} x) + 3(\cos^{10} x + \tan^{10} x + \cos^8 x + \tan^8 x) + (\cos^6 x + \tan^6 x)$ is equal to

(1) 2 (2) 1

(3) 3 (4) 4

Answer (1)

Sol. $\sin x + \sin^2 x = 1, x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow \sin x = \cos^2 x \Rightarrow \tan x = \cos x$

$$\left(\cos^{12} + \tan^{12} x\right) + 3\left(\cos^{10} x + \tan^{10} x + \cos^8 x + \tan^8 x\right) + (\cos^6 x + \tan^6 x)$$

$$= (\cos^{12} x + \cos^{12} x) + 3(\cos^{10} x + \cos^{10} x + \cos^8 x + \cos^8 x) + (\cos^6 x + \cos^6 x)$$

$$= 2\cos^{12} x + 6(\cos^{10} x + \cos^8 x) + 2\cos^6 x$$

$$= 2[\cos^{12} x + 3(\cos^{10} x + \cos^8 x) + \cos^6 x]$$

$$= 2[(\sin^2 x)^3 + 3\sin^4 x \cos^2 x + 3\sin^2 x \cos^4 x + (\cos^2 x)^3]$$

$$= 2[(\sin^2 x + \cos^2 x)^3] = 2$$

16. Let $\alpha, \beta\ (\alpha \neq \beta)$ be the values of m, for which the equations $x + y + z = 1$; $x + 2y + 4z = m$ and $x + 4y + 10z = m^2$ have infinitely many solutions. Then the value of $\sum_{n=1}^{10}\left(n^{\alpha} + n^{\beta}\right)$ is equal to

(1) 560 (2) 440

(3) 3080 (4) 3410

Answer (2)

Sol. $\Delta = 0,\ \Delta_x = 0,\ \Delta_y = 0,\ \Delta_z = 0$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & m \\ 1 & 4 & m^2 \end{vmatrix} = 0$$

$1(2m^2 - 4m) - 1(m^2 - m) + 1(4 - 2) = 0$

$m^2 - 3m + 2 = 0$

$(m - 1)(m - 2) = 0$

$m = 1, 2$

$$\Delta_x = \begin{vmatrix} 1 & 1 & 1 \\ m & 2 & 4 \\ m^2 & 4 & 10 \end{vmatrix} = 0 \Rightarrow m = 1, 2$$

$$\Delta_y = \begin{vmatrix} 1 & 1 & 1 \\ 1 & m & 4 \\ 1 & m^2 & 10 \end{vmatrix} = 0 \Rightarrow m = 1, 2$$

$$\sum_{x=1}^{10}(n^{\alpha} + n^{\beta}) = \sum_{n=1}^{10}(n^1 + n^2) = \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6}$$

$= 55 + 385 = 440$

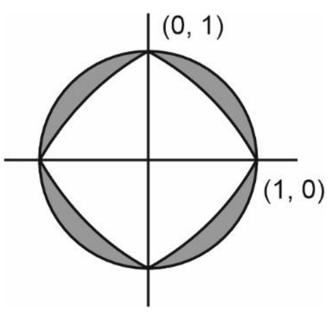
17. Let the area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ be α. If $9\alpha = \beta\pi + \gamma;\ \beta, \gamma$ are integers, then the value of $|\beta - \gamma|$ equals.

(1) 18 (2) 33

(3) 27 (4) 15

Answer (2)

Sol.



Required area $= \pi - 4\int_0^1 (1 - x^2)dx$

$= \pi - 4\left[x - \frac{x^3}{3}\right]_0^1$

$= \pi - 4 \times \frac{2}{3} = \pi - \frac{8}{3}$

$\therefore\ \alpha = \pi - \frac{8}{3}$

$9\alpha = 9\pi - 24 \rightarrow \beta = 9,\ \gamma = -24$

$|\beta - \gamma| = |9 + 24| = 33$

18. Let $\hat{a}$ be a unit vector perpendicular to the vectors $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} - \hat{k}$, and makes an angle of $\cos^{-1}\left(-\frac{1}{3}\right)$ with the vector $\hat{i} + \hat{j} + \hat{k}$. If a makes an angle of $\frac{\pi}{3}$ with the vector $\hat{i} + \alpha\hat{j} + \hat{k}$, then the value of α is

(1) $\sqrt{6}$ (2) $-\sqrt{3}$

(3) $-\sqrt{6}$ (4) $\sqrt{3}$

Answer (3)

Sol. $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(-7) + 7\hat{j} + 7\hat{k}$

$$\hat{a} = \pm\frac{(-7\hat{i} + 7\hat{j} + 7\hat{k})}{\sqrt{7^2 + 7^2 + 7^2}} = \pm\left(\frac{-\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}\right)$$

Now, $\cos\theta = \pm\frac{(-1 + 1 + 1)}{\sqrt{3}\cdot\sqrt{3}} = \pm\frac{1}{3}$

$\Rightarrow \cos^{-1}\left(\frac{-1}{3}\right) \Rightarrow \hat{a} = \frac{-(-\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}}$

$\hat{a} = \frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$

$\cos\frac{\pi}{3} = \frac{1-\alpha-1}{\sqrt{3}.\sqrt{\alpha^2+2}}$

$\frac{1}{2} = \frac{-\alpha}{\sqrt{3}.\sqrt{\alpha^2+2}} \rightarrow \alpha < 0$

$3(\alpha^2 + 2) = 4\alpha^2$

$6 = \alpha^2$

$\alpha = \pm\sqrt{6}$

Clearly, $\alpha = -\sqrt{6}$

19. The remainder, when 7^{103} is divided by 23, is equal to

(1) 9

(2) 6

(3) 17

(4) 14

Answer (4)

Sol. $7,\ 7^2 = 49,\ 7^3 = 343 \equiv (-2)\ (\text{mod}23)$

$\Rightarrow 7^{102} \equiv (7^3)^{34} \equiv (-2)^{34} \equiv 4^{17}\ (\text{mod}23)$

$\Rightarrow 4^6 \equiv 2\ (\text{mod}23)$

$4^{17} \equiv (2)(2)(12) \equiv 2(\text{mod}23)$

$7^{103} \equiv 7.4^{17} \equiv 14\ (\text{mod}23)$

Alter : $7^{\phi(23)} \equiv 1(\text{mod}23)$, gcd(7, 23) = 1

$\phi(23) = (23 - 1) = 22$

$\Rightarrow 7^{22} \equiv 1\ (\text{mod}23) \Rightarrow 7^{11} \equiv (-1)\ \text{mod}(23)$

[as $7^{11} \not\equiv 1$ mod23] $\Rightarrow 7^{99} \equiv -1\ \text{mod}(23)$

$7^{102} \equiv 2\ (\text{mod}23)$

$\Rightarrow 7^{103} \equiv 14\ \text{mod}23$

20. If the set of all $a \in \mathbf{R}$, for which the equation $2x^2 + (a-5)x + 15 = 3a$ has no real root, is the interval (α, β) and $X = \{x \in Z : \alpha < x < \beta\}$, then $\sum_{x \in X} x^2$ is equal to

(1) 2129

(2) 2119

(3) 2139

(4) 2109

Answer (3)

Sol. $(a-5)^2 - 8(15 - 3a) < 0$

$\Rightarrow (a + 19)(a - 5) < 0$

$a \in (-19, 5)$

Hence, $x \in (-19, 5)$

$\sum x_i^2 = (1^2 + 2^2 + ...4^2) + (1^2 + 2^2 + ...18^2)$

= 2139

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $24\int_0^{\frac{\pi}{4}}\left(\sin\left|4x - \frac{\pi}{12}\right| + [2\sin x]\right)dx = 2\pi + \alpha$, where $[\cdot]$ denotes the greatest integer function, then α is equal to______.

Answer (12)

Sol. Let $I = 24\int_0^{\frac{\pi}{2}}\left(\sin\left|4x - \frac{\pi}{2}\right| + [2\sin x]\right)dx$ ...(i)

Now $\left|4x - \frac{\pi}{12}\right| = \begin{cases} -4x + \frac{\pi}{12} & ;\ x < \frac{\pi}{48} \\ 4x - \frac{\pi}{12} & ;\ x \geq \frac{\pi}{48} \end{cases}$

$\therefore$ from (i)

$$I = 24\int_0^{\frac{\pi}{48}} -\sin\left(4x-\frac{\pi}{12}\right)dx + \int_{\frac{\pi}{48}}^{\frac{\pi}{4}} \sin\left(4x-\frac{\pi}{12}\right)$$

$$+\int_0^{\frac{\pi}{6}}[2\sin x]dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{4}}[2\sin x]dx$$

$$I = 24\left[\frac{\left(1-\cos\frac{\pi}{12}\right)}{4} - \frac{\left(-\cos\frac{\pi}{12}-1\right)}{4}\right] + \frac{\pi}{4} - \frac{\pi}{6}$$

$$I = 24\left(\frac{1}{2}\right) + \frac{\pi}{4} - \frac{\pi}{6}$$

$I = 2\pi + 12 = 2\pi + \alpha$ (from above)

$\therefore\ \ \alpha = 12$

22. Let $a_1, a_2, ..., a_{2024}$ be an arithmetic Progression such that

$a_1 + (a_5 + a_{10} + a_{15} + ... + a_{2020}) + a_{2024} = 2233.$

Then $a_1 + a_2 + a_3 + ... + a_{2024}$ is equal to

Answer (11132)

Sol. As $a_1 + a_5 + a_{10} + ... + a_{2020} + a_{2024} = 2233$ ...(1)

We know in arithmetic progression.

Sum of terms equidistant from ends is equal

$\therefore$ from (1)

$$\underbrace{a_1 + a_{2024} = a_5 + a_{2020} = a_{10} + a_{2015} = ...}_{203 \text{ pairs}}$$

$\Rightarrow$ $203\,(a_1 + a_{2024}) = 2233$

$\Rightarrow$ $a_1 + a_{2024} = 11$

Now $\sum_{i=1}^{2024} a_i = S_{2024} = \frac{2024}{2}[a_1 + a_{2024}]$

$= 1012\ (11)$

$= 11132$

23. Let integers $a, b \in [-3, 3]$ be such that $a + b \neq 0$. Then the number of all possible ordered pairs (a, b), for which $\left|\frac{z-a}{z+b}\right| = 1$ and $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 1, z \in C,$ where ω and ω^2 are the roots of $x^2 + x + 1 = 0$, is equal to____.

Answer (10)

Sol. $a, b \in I, -3 \le a, b \le 3;\ a + b \neq 0$

$|z - a| = |z + b|$

$$\begin{vmatrix} z+1 & w & w^2 \\ w & z+w^2 & 1 \\ w^2 & 1 & z+w \end{vmatrix} = 1$$

$$\Rightarrow \begin{vmatrix} z & z & z \\ w & z+w^2 & 1 \\ w^2 & 1 & z+w \end{vmatrix} = 1$$

$$\Rightarrow z\begin{vmatrix} 1 & 1 & 1 \\ w & z+w^2 & 1 \\ w^2 & 1 & z+w \end{vmatrix} = 1$$

$$\Rightarrow z\begin{vmatrix} 1 & 0 & 0 \\ w & z+w^2-w & 1-w \\ w^2 & 1-w^2 & z+w-w^2 \end{vmatrix} = 1$$

$\Rightarrow$ $z^3 = 1$

$\Rightarrow$ $z = w, w^2, 1$

Now, $|1 - a| = |1 + b|$

$\Rightarrow$ 10 pairs

24. If $\lim_{t\to0}\left(\int_0^1 (3x+5)^t\,dx\right)^{1/t} = \frac{\alpha}{5e}\left(\frac{8}{5}\right)^{2/3}$, then α is equal to_____.

Answer (64)

Sol. $\frac{\alpha}{5e} = \exp\left(\lim_{t\to0}\frac{1}{t}\left(\int_0^1 (3x+5)^t\,dx - 1\right)\right)$

$$= \exp\left(\lim_{t\to0}\frac{1}{t}\left(\left.\frac{(3x+5)^{t+1}}{3(t+1)}\right|_0^1 - 1\right)\right)$$

$$= \exp\left(\lim_{t\to0}\frac{1}{t}\left(\frac{8^{t+1}-5^{t+1}}{3(t+1)} - 1\right)\right)$$

$$= \exp\left(\lim_{t\to0}\frac{1}{t}\left(\frac{8^{t+1}-5^{t+1}-3t-3}{3(t+1)}\right)\right)$$

$$= \exp\left(\lim_{t\to0}\left(\frac{8^{t+1}\cdot\ln8-5^{t+1}\ln5-3}{3(t+1)}\right)\right)$$

$$= \exp\left(\frac{\ln8^8-\ln5^5-3}{5}\right)$$

$$= \left(\frac{8}{5}\right)^{2/3}\frac{\alpha}{5e} = \exp\left(\frac{\ln\left(\frac{8^8}{5^5}\right)}{5} - 1\right)$$

$$\Rightarrow \left(\frac{8}{5}\right)^{2/3}\frac{\alpha}{5} = \left(\frac{8^8}{5^5}\right)^{1/3} = \left(\frac{8^6\cdot8^2}{5^3\cdot5^2}\right)^{1/3} = \frac{64}{5}\left(\frac{8}{5}\right)^{2/3}$$

$\Rightarrow\ \alpha = 64$

25. Let $y^2 = 12x$ be the parabola and S be its focus. Let PQ be a focal chord of the parabola such that $(SP)(SQ) = \frac{147}{4}$. Let C be the circle described taking PQ as a diameter. If the equation of a circle C is $64x^2 + 64y^2 - \alpha x - 64\sqrt{3}y = \beta$, then $\beta - \alpha$ is equal to_____.

Answer (1328)

Sol. $y^2 = 12x$, $a = 3$, $(SP)(SQ) = \frac{147}{4}$

Let $P(3t^2, 6t)$ and $t_1t_2 = -1$ (ends of focal chord)

$\because Q \equiv \left(\frac{3}{t^2}, \frac{6}{t}\right)$

$(SP)(SQ) = (PM_1)(QM_2)$

$= \left(3+3t^2\right)\left(3+\frac{3}{t^2}\right) = \frac{147}{4}$

$\Rightarrow \frac{\left(1+t^2\right)^2}{t^2} = \frac{49}{12}$

$\Rightarrow t^2 = \frac{3}{4}, \frac{4}{3}$

$\Rightarrow t = \pm\frac{\sqrt{3}}{2}, \frac{\pm2}{\sqrt{3}}$

Considering $t = \frac{-\sqrt{3}}{2}$

$\therefore P\left(\frac{9}{4}, -3\sqrt{3}\right)$ and $Q\left(4, 4\sqrt{3}\right)$

$\therefore$ Equation of circle

$\Rightarrow (x-4)\left(x-\frac{9}{4}\right) + \left(y+3\sqrt{3}\right)\left(y-4\sqrt{3}\right) = 0$

$\Rightarrow x^2 + y^2 - \frac{25}{4}x - \sqrt{3}y - 27 = 0$

$\Rightarrow \alpha = 400$, $\beta = 1728$

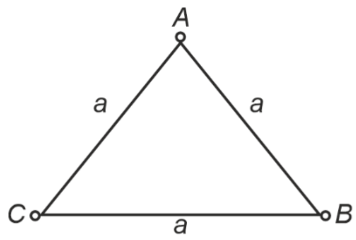
$\therefore \beta - \alpha = 1328$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

26. 

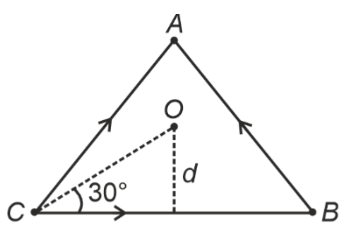
Three equal masses m are kept at vertices (A, B, C) of an equilateral triangle of side a in free space. At t = 0, they are given an initial velocity $\vec{V_A} = V_0\,\vec{AC}$, $\vec{V_B} = V_0\,\vec{BA}$ and $\vec{V_C} = V_0\,\vec{CB}$. Here, $\vec{AC}$, $\vec{CB}$ and $\vec{BA}$ are unit vectors along the edges of the triangle. If the three masses interact gravitationally, then the magnitude of the net angular momentum of the system at the point of collision is :

(1) $\frac{1}{2}amV_0$ (2) $\frac{3}{2}amV_0$

(3) $\frac{\sqrt{3}}{2}amV_0$ (4) $3amV_0$

Answer (3)

Sol.



$$d = \frac{a}{2\sqrt{3}}$$

Angular momentum of one mass about point O

$L = mvd$

$$= mv_0 \cdot \frac{a}{2\sqrt{3}}$$

Net angular momentum about point O

L_{net} = 32

$$= \frac{\sqrt{3}mv_0a}{2}$$

27. Two bodies A and B of equal mass are suspended from two massless springs of spring constant k_1 and k_2, respectively. If the bodies oscillate vertically such that their amplitudes are equal, the ratio of the maximum velocity of A to the maximum velocity of B is

(1) $\sqrt{\frac{k_1}{k_2}}$ (2) $\frac{k_1}{k_2}$

(3) $\frac{k_2}{k_1}$ (4) $\sqrt{\frac{k_2}{k_1}}$

Answer (1)

Sol. Here $\omega = \sqrt{\frac{k}{m}}$

and maximum velocity $V = A\omega = A\sqrt{\frac{k}{m}}$

So, $\frac{V_A}{V_B} = \sqrt{\frac{k_1}{k_2}}$

28. A point charge causes an electric flux of –2 × 10^4 Nm^2C^{-1} to pass through a spherical Gaussian surface of 8.0 cm radius, centred on the charge. The value of the point charge is :

(Given ϵ_0= 8.85 × 10^{-12} $C^2N^{-1}m^{-2}$)

(1) 17.7 × 10^{-8} C (2) –15.7 × 10^{-8} C

(3) 15.7 × 10^{-8} C (4) –17.7 × 10^{-8} C

Answer (4)

Sol. Flux$(\phi) = \dfrac{\theta_{inc}}{\epsilon_0}$

$\theta_{inc} = \epsilon_0 \, \phi$

$= -17.7 \times 10^{-8}$ C

29. In an experiment with photoelectric effect, the stopping potential,

(1) decreases with increase in the intensity of the incident light

(2) is $\left(\dfrac{1}{e}\right)$ times the maximum kinetic energy of the emitted photoelectrons

(3) increases with increase in the wavelength of the incident light

(4) increases with increase in the intensity of the incident light

Answer (2)

Sol. From Einstein photoelectric equation

$\dfrac{hc}{\lambda} = \phi + eV_S$

Maximum K.E. = $(K)_{max} = eVs$

So, $V_S = \dfrac{(K)_{max}}{e}$

30. The number of spectral lines emitted by atomic hydrogen that is in the 4th energy level, is

(1) 6 (2) 0

(3) 3 (4) 1

Answer (1)

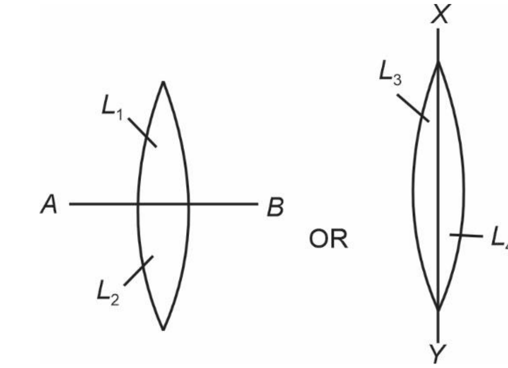
Sol. Number of spectral line $(N) = \dfrac{(n_2 - n_1)(n_2 - n_1 + 1)}{2}$

Here $n_2 = 4$

$n = 1$

So, $N = 6$

31. Two identical symmetric double convex lenses of focal length *f* are cut into two equal parts L_1, L_2 by *AB* plane and L_3, L_4 by *XY* plane as shown in figure respectively. The ratio of focal lengths of lenses L_1 and L_3 is

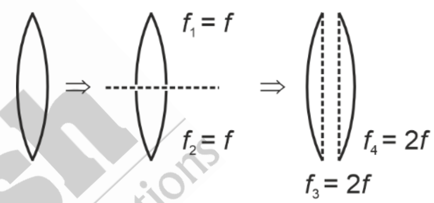


(1) 2 : 1 (2) 1 : 2

(3) 1 : 1 (4) 1 : 4

Answer (2)

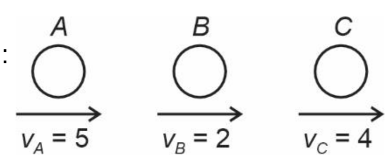
Sol.



so $\dfrac{f_1}{f_3} = 1:2$

32. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) :



Three identical spheres of same mass undergo one dimensional motion as shown in figure with initial velocities v_A = 5 m/s, v_B = 2 m/s, v_C = 4 m/s. If we wait sufficiently long for elastic collision to happen, then v_A = 4 m/s, v_B = 2 m/s, v_C = 5 m/s will be the final velocities.

Reason (R) : In an elastic collision between identical masses, two objects exchange their velocities.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) **(A)** is true but **(R)** is false

(2) **(A)** is false but **(R)** is true

(3) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**

(4) Both (A) and **(R)** are true and **(R)** is the correct explanation of **(A)**

Answer (2)

Sol. For *A* and *B*.

Before collision

After collision

For *B* and *C*

Before collision

After collision

Final velocity

$V_A = 2$ m/s

$V_B = 4$ m/s

$V_C = 5$ m/s

$\Rightarrow$ Velocity exchange for two identical mass in elastic collision.

33. Given below are two statements : One is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : With the increase in the pressure of an ideal gas, the volume falls off more rapidly in an isothermal process in comparison to the adiabatic process.

Reason (R) : In isothermal process, PV = constant, while in adiabatic process PV^γ = constant. Here γ is the ratio of specific heats, P is the pressure and V is the volume of the ideal gas.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both (A) and **(R)** are true and **(R)** is the correct explanation of **(A)**

(2) **(A)** is false but **(R)** is true

(3) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**

(4) **(A)** is true but **(R)** is false

Answer (1)

Sol. For isothermal process

PV = constant

$\Rightarrow P\Delta V + V\Delta P = 0$

$\Rightarrow \frac{\Delta V}{V} = \frac{-\Delta P}{P}$

$\Rightarrow \Delta V = -\left(\frac{V}{P}\right)\Delta P$

For adiabatic PV^γ = constant

$\Rightarrow P\gamma V^{\gamma-1}dV + V^\gamma dP = 0$

$\Rightarrow dV = \frac{-V}{\gamma P}(dP)$

Magnitude of $\left|\frac{V}{P}\right|$ is greater than $\left|\frac{V}{\gamma P}\right|$

So, volume falls more rapidly in isothermal.

2nd statement is the process description of the isothermal and adiabatic

34. The difference of temperature in a material can convert heat energy into electrical energy. To harvest the heat energy, the material should have

(1) Low thermal conductivity and high electrical conductivity

(2) High thermal conductivity and high electrical conductivity

(3) Low thermal conductivity and low electrical conductivity

(4) High thermal conductivity and low electrical conductivity

Answer (1)

Sol. Material should have low thermal conductivity and high electrical conductivity.

35. A sand dropper drops sand of mass $m(t)$ on a conveyer belt at a rate proportional to the square root of speed (v) of the belt, i.e. $\frac{dm}{dt} \propto \sqrt{v}$. If P is the power delivered to run the belt at constant speed then which of the following relationship is true ?

(1) $P \propto v$ (2) $P \propto \sqrt{v}$

(3) $P^2 \propto v^5$ (4) $P^2 \propto v^3$

Answer (3)

Sol. Power = $\vec{F}.\vec{V}$

and $F = \frac{dp}{dt}$ = Rate of change of linear momentum

$F = V \cdot \frac{dm}{dt} = K_1 V^{\frac{3}{2}}$, K is constant

Power (P) = $\left(KV^{\frac{3}{2}}\right) \cdot (V)$

$= KV^{\frac{5}{2}}$

So, $P^2 \propto V^5$

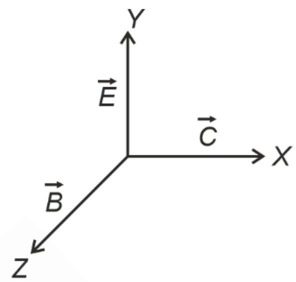
36. A plane electromagnetic wave propagates along the + x direction in free space. The components of the electric field, $\vec{E}$ and magnetic field, $\vec{B}$ vectors associated with the wave in Cartesian frame are

(1) E_x, B_y (2) E_y, B_z

(3) E_y, B_x (4) E_x, B_z

Answer (2)

Sol.



The direction of wave propagation $\rightarrow \vec{E} \times \vec{B}$

37 Match **List** - I with **List - II**.

	List - I		**List - II**
(A)	Magnetic induction	(I)	Ampere meter^2
(B)	Magnetic intensity	(II)	Weber
(C)	Magnetic flux	(III)	Gauss
(D)	Magnetic moment	(IV)	Ampere meter

Choose the **correct** answer from the options given below :

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(2) (A)-(III), (B)-(IV), (C)- (I), (D)-(II)

(3) (A)-(III), (B)-(IV), (C)- (II), (D)-(I)

(4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Answer (3)

Sol. (A) Magnetic induction $\rightarrow$ Gauss (III)

(B) Magnetic intensity $\rightarrow$ Ampere/meter (IV) $\rightarrow$ $\mu = \frac{B}{\mu}$

(C) Magnetic flux $\rightarrow$ weber (II) $\rightarrow$ $\phi = \vec{B}\cdot\vec{A}$

(D) Magnetic moment $\rightarrow$ Ampere-meter^2 (I) $\rightarrow$ $\vec{M} = I\vec{A}$

38. Match **List** - I with **List - II**.

	List - I		**List - II**
(A)	Young's Modulus	(I)	$ML^{-1}T^{-1}$
(B)	Torque	(II)	$ML^{-1}T^{-2}$
(C)	Coefficient of Viscosity	(III)	$M^{-1}L^3T^{-2}$
(D)	Gravitational Constant	(IV)	ML^2T^{-2}

Choose the **correct** answer from the options given below :

(1) (A)-(I), (B)- (III), (C)-(II), (D)-(IV)

(2) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(3) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(4) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Answer (3)

Sol. (A) Young's Modulus (Y) $= Y = \frac{F}{A\left(\frac{\Delta l}{l}\right)}$

[Y] = [$ML^{-1}T^{-2}$] …(II)

(B) Torque (ξ^{-1}) $= \vec{r}\times\vec{F}$

[ξ] = [ML^2T^{-2}] …(IV)

(C) Coefficient of Viscosity (η)

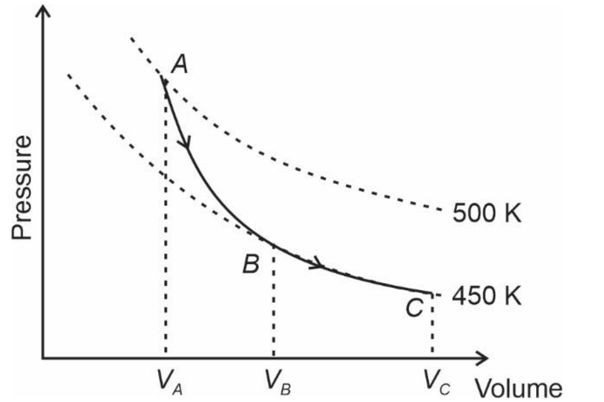
$$F = \eta A\frac{dV}{dt}$$

[η] = [$ML^{-1}T^{-1}$] …(I)

(D) Gravitational Constant $[G] = \frac{F\cdot d^2}{m_1m_2}$

[G] = [$M^{-1}L^3T^{-2}$] …(III)

39.



A poly-atomic molecule ($C_V = 3R$, $C_p = 4R$, where R is gas constant) goes from phase space point $A(P_A = 10^5$ Pa, $V_A = 4\times10^{-6}$ m^3) to point $B(P_B = 5\times10^4$ Pa, $V_B = 6\times10^{-6}$ m^3) to point $C(P_C = 10^4$ Pa, $V_C = 8\times10^{-6}$ m^3). A to B is an adiabatic path and B to C is an isothermal path. The net heat absorbed per unit mole by the system is :

(1) 500R ln2

(2) 500R(ln3 + ln4)

(3) 400R ln4

(4) 450R(ln4 –ln3)

Answer (4)

Sol. For process $A \rightarrow B$

$(\Delta Q)_{AB} = 0$ (adiabatic)

For process $B \rightarrow C$

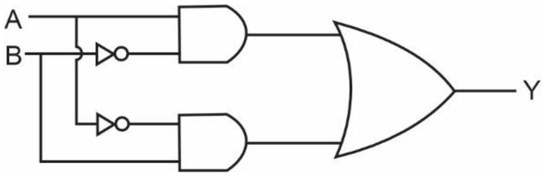
$$(\Delta Q)_{BC} = (\Delta W)_{BC} = nRT\ln\left(\frac{V_C}{V_B}\right)$$

$= 450R$ [ln (4) – ln(3)]

$(\Delta Q)_{net} = (\Delta Q)_{AB} + (\Delta Q)_{BC}$

$= 450R$ [ln (4) – ln(3)]

40. The truth table for the circuit given below is :



(1)

A	B	Y
0	0	0
1	0	1
0	1	0
1	1	0

(2)

A	B	Y
0	0	0
1	1	1
1	0	1
0	1	1

(3)

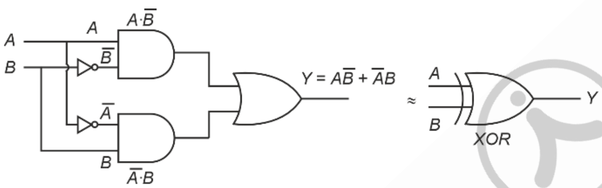
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(4)

A	B	Y
0	0	0
1	0	0
1	1	0
0	1	1

Answer (3)

Sol.



41. A cup of coffee cools from 90°C to 80°C in t minutes when the room temperature is 20°C. The time taken by the similar cup of coffee to cool from 80°C to 60°C at the same room temperature is :

(1) $\frac{13}{5}t$ (2) $\frac{10}{13}t$

(3) $\frac{13}{10}t$ (4) $\frac{5}{13}t$

Answer (1)

Sol. From Newton law

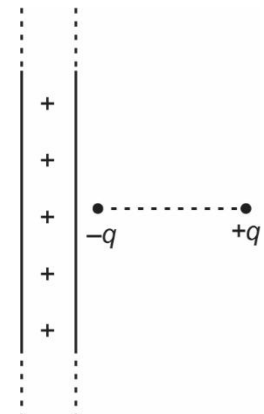
$$\frac{90-80}{t} = c\left(\frac{90+80}{2} - 20\right) \quad \ldots\text{(i)}$$

and $$\frac{80-60}{t_1} = c\left(\frac{80+60}{2} - 20\right) \quad \ldots\text{(ii)}$$

From (i) and (ii)

$$t_1 = \frac{13}{5}t$$

42. An electric dipole is placed at a distance of 2 cm from an infinite plane sheet having positive charge density σ_0. Choose the correct option from the following.



(1) Torque on dipole is zero and net force acts towards the sheet.

(2) Potential energy of dipole is minimum and torque is zero.

(3) Potential energy and torque both are maximum.

(4) Torque on dipole is zero and net force is directed away from the sheet.

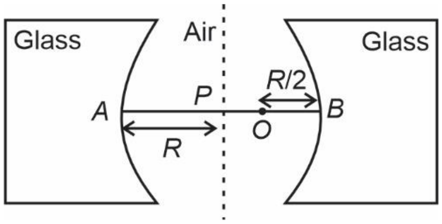
Answer (2)

Sol. Electric field due to sheet $E = \frac{\sigma}{2\varepsilon_0}$

and torque on dipole $\vec{\tau} = \vec{P} \times \vec{E}$

here $\vec{\tau} = 0$

and $U = -\vec{P} \cdot \vec{E}$ → should be minimum

43. 

Two concave refracting surfaces of equal radii of curvature and refractive index 1.5 face each other in air as shown in figure. A point object *O* is placed midway, between *P* and *B*. The separation between the images of *O*, formed by each refracting surface is :

(1) 0.114 *R* (2) 0.411 *R*

(3) 0.214 *R* (4) 0.124 *R*

Answer (1)

Sol. For glass *B*

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\frac{1.5}{V_B}+\frac{1}{\left(\frac{R}{2}\right)}=\frac{0.5}{-R}$$

$V_B = -0.6\ R$

For glass *A*

$$\frac{1.5}{V_A}+\frac{2}{3R}=\frac{0.5}{-R}$$

$$V_A=-\frac{9}{7}R$$

Distance between images

$= 2R-\left(\frac{9}{7}R+0.6R\right)$

$= 0.114\ R$

44. A convex lens made of glass (refractive index = 1.5) has focal length 24 cm in air. When it is totally immersed in water (refractive index = 1.33), its focal length changes to

(1) 24 cm (2) 48 cm

(3) 72 cm (4) 96 cm

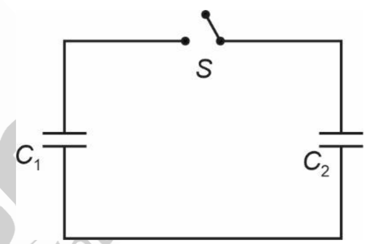
Answer (4)

Sol. $\frac{1}{24}=(1.5-1)\left(\frac{1}{R}+\frac{1}{R}\right)=\frac{1}{R}$, $R = 24$ cm

Again $\frac{1}{f'}=\left(\frac{1.5}{1.33}-1\right)\left(\frac{2}{R}\right)$

On solving, $f' = 96$ cm

45. A capacitor, $C_1 = 6\ \mu F$ is charged to a potential difference of $V_0 = 5V$ using a 5V battery. The battery is removed and another capacitor, $C_2 = 12\ \mu F$ is inserted in place of the battery. When the switch 'S' is closed, the charge flows between the capacitors for some time until equilibrium condition is reached. What are the charges (q_1 and q_2) on the capacitors C_1 and C_2 when equilibrium condition is reached.



(1) $q_1 = 15\ \mu C$, $q_2 = 30\ \mu C$

(2) $q_1 = 10\ \mu C$, $q_2 = 20\ \mu C$

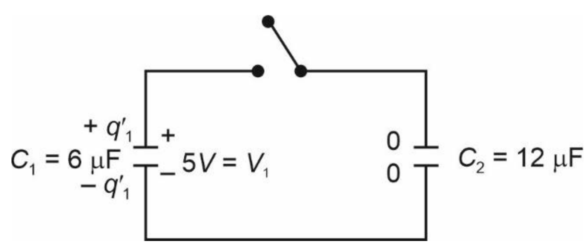
(3) $q_1 = 20\ \mu C$, $q_2 = 10\ \mu C$

(4) $q_1 = 30\ \mu C$, $q_2 = 15\ \mu C$

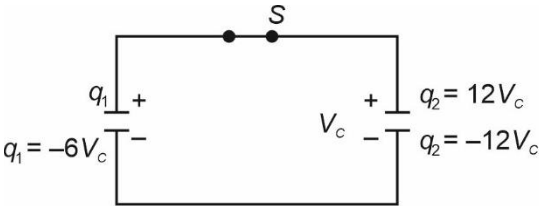
Answer (2)

Sol. at $t = 0$

$q'_1 = C_V = 30\ \mu C$



at $t = t$



$6V_C + 12\ V_C = 30 + 0$

$V_C = \frac{5}{3}V$

$q_1 = \frac{6\times 5}{3} = 10\ \mu C$

$q_2 = \frac{12\times 5}{3} = 20\ \mu C$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

46. A physical quantity Q is related to four observables a, b, c, d as follows :

$$Q = \frac{ab^4}{cd}$$

where, $a = (60 \pm 3)$ Pa; $b = (20 \pm 0.1)$ m; $c = (40 \pm 0.2)$ Nsm^{-2} and $d = (50 \pm 0.1)$ m, then the percentage error in Q is $\frac{x}{1000}$, where x = _______.

Answer (7700)

Sol. Here, $\frac{\Delta Q}{Q}\times 100 = \left[\frac{\Delta a}{a} + 4\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{\Delta d}{d}\right]\times 100$

$$= \left[\frac{3}{60} + \frac{0.4}{20} + \frac{0.2}{40} + \frac{0.1}{50}\right]$$

$$\frac{x}{1000} = \left[\frac{3}{60} + \frac{0.4}{20} + \frac{0.2}{40} + \frac{0.1}{50}\right]\times 100$$

$x = 7700$

47. Two planets, A and B are orbiting a common star in circular orbits of radii R_A and R_B, respectively, with $R_B = 2R_A$. The planet B is $4\sqrt{2}$ times more massive than planet A. The ratio $\left(\frac{L_B}{L_A}\right)$ of angular momentum (L_B) of planet B to that of planet A(L_A) is closest to integer _______.

Answer (8)

Sol. Angular momentum, $L = mV_0R = m\sqrt{GMR}$

where M is the mass of star

$$\frac{L_B}{L_A} = \frac{m_B}{m_A}\sqrt{\frac{R_B}{R_A}} = 8$$

48. Two cars P and Q are moving on a road in the same direction. Acceleration of car P increases linearly with time whereas car Q moves with a constant acceleration. Both cars cross each other at time $t = 0$, for the first time. The maximum possible number of crossing(s) (including the crossing at $t = 0$) is _______.

Answer (3)

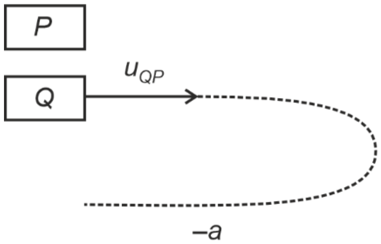
Sol. For first car $P \rightarrow$ acceleration (a_P) = ct, c is constant

For second car $Q \rightarrow$ acceleration (a_Q) = a, a is constant

Let analyze the problem in two case

Case-I

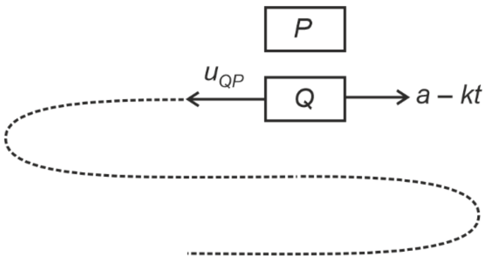
$\Rightarrow$ U_{QP} and a_{QP} in same direction



They cross 2 times in this case

Case-II

U_{PQ} and a_{QP} in opposite directions



They cross 3 time in this case

49. The magnetic field inside a 200 turns solenoid of radius 10 cm is 2.9 × 10^{-4} Tesla. If the solenoid carries a current of 0.29 A, then the length of the solenoid is _______ π cm.

Answer (8)

Sol. Magnetic field inside a solenoid

$$B = \mu_0 i\left(\frac{N}{L}\right)$$

$$L = \frac{\mu_0 N i}{B} = \frac{4\pi\times10^{-7}\times200\times0.29}{2.9\times10^{-4}}$$

$$= 8\pi \text{ cm}$$

50. A parallel plate capacitor consisting of two circular plates of radius 10 cm is being charged by a constant current of 0.15 A. If the rate of change of potential difference between the plates is 7 × 10^{8} V/s then the integer value of the distance between the parallel plates is _______ μm.

$$\left(\text{Take, } \varepsilon_0 = 9\times10^{-12}\,\frac{\text{F}}{\text{m}},\ \pi = \frac{22}{7}\right)$$

Answer (1320)

Sol. $Q = cV$

$$V = \frac{Q}{c} = \frac{it}{\left(\frac{\varepsilon_0 A}{d}\right)}$$

$$d = \frac{\varepsilon_0 \pi r^2}{i}\left(\frac{v}{t}\right)$$

Putting values

$$d = \frac{9\times10^{-12}\times\frac{22}{7}\times(0.1)^2}{0.15}\times(7\times10^{8})$$

$$= 1320\ \mu\text{m}$$

CHEMISTRY

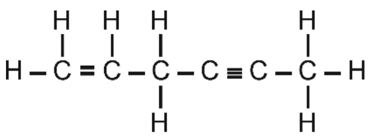
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

51. Total number of sigma (σ) ______ and pi(π) _____ bonds respectively present in hex-1-en-4-yne are :
    (1) 3 and 13
    (2) 11 and 3
    (3) 14 and 3
    (4) 13 and 3

Answer (4)

Sol. 

σ bond = 13, π bond = 3

52. Drug X becomes ineffective after 50% decomposition. The original concentration of drug in a bottle was 16 mg/mL, which becomes 4 mg/mL, in 12 months. The expiry time of the drug in months is _______

    Assume that decomposition of drug follows first order kinetics
    (1) 6
    (2) 12
    (3) 3
    (4) 2

Answer (1)

Sol. Drug X $\xrightarrow{\text{1st order}}$ Product

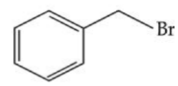
Initial concentration of drug = 16 mg/mL

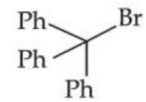
Concentration of drug after 12 months = 4 mg/mL

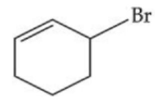
Half life of drug = 6 months

Drug become ineffective after 50% decomposition. The expiry time of drug = 6 months

53. Which among the following halides will generate the most stable carbocation in the nucleophilic substitution reaction?

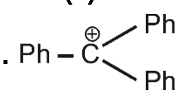
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol.  is most stable carbocation due to extensive resonance stabilization from the three phenyl groups.

54. 0.1 M solution of KI reacts with excess of H_2SO_4 and KIO_3 solutions. According to equation

$5I^- + IO_3^- + 6H^+ \rightarrow 3I_2 + 3H_2O$

Identify the **correct** statements :

(A) 200 mL of KI solution reacts with 0.004 mol of KIO_3

(B) 200 mL of KI solution reacts with 0.006 mol of H_2SO_4

(C) 0.5 L of KI solution produced 0.005 mol of I_2

(D) Equivalent weight of KIO_3 is equal to $\left(\frac{\text{Molecular weight}}{5}\right)$

Choose the **correct** answer from the options given below :

(1) (A) and (D) only
(2) (C) and (D) only
(3) (A) and (B) only
(4) (B) and (C) only

Answer (1)

Sol. $E_{KIO_3} = \frac{\text{Molecular weight}}{n_f}$

$n_f = 5$

$E_{KIO_3} = \frac{\text{Molecular weight}}{5}$

(D) is correct

meq of KI = 0.1 × 200 = 20

meq of KIO_3 = 4 × 5 = 20

(A) is correct

55. Identify the homoleptic complexes with odd number of d electrons in the central metal :

(A) $[FeO_4]^{2-}$

(B) $[Fe(CN)_6]^{3-}$

(C) $[Fe(CN)_5NO]^{2-}$

(D) $[CoCl_4]^{2-}$

(E) $[Co(H_2O)_3F_3]$

Choose the **correct** answer from the options given below :

(1) (A), (B) and (D) only

(2) (A), (C) and (E) only

(3) (C) and (E) only

(4) (B) and (D) only

Answer (4)

Sol. (A) $[FeO_4]^{2-} \Rightarrow Fe^{6+} = 3d^2$

(B) $[Fe(CN)_6]^{3-} \Rightarrow Fe^{3+} = 3d^5$

(C) $[Fe(CN)_5NO]^{2-} \Rightarrow Fe^{2+} = 3d^6$

(D) $[CoCl_4]^{2-} \Rightarrow Co^{2+} = 3d^7$

(E) $[Co(H_2O)_3F_3] \Rightarrow Co^{3+} = 3d^6$

(B) and (D) are homoleptic complex having odd no. of d electrons.

56. First ionisation enthalpy values of first four group 15 elements are given below. Choose the correct value for the element that is a main component of apatite family :

(1) 1012 kJ mol^{-1}

(2) 834 kJ mol^{-1}

(3) 1402 kJ mol^{-1}

(4) 947 kJ mol^{-1}

Answer (1)

Sol. The main component of apatite family is phosphorus.

Order of IE_1 of group 15 elements N > P > As > Sb

IE of phosphorus = 1012 kJ mol^{-1}.

57. If C(diamond) $\rightarrow$ C(graphite) + X kJ mol^{-1}

C(diamond) + O_2(g) $\rightarrow$ CO_2(g) + Y kJ mol^{-1}

C(graphite) + O_2(g) $\rightarrow$ CO_2(g) + Z kJ mol^{-1}

at constant temperature. Then

(1) X = –Y + Z (2) X = Y – Z

(3) X = Y + Z (4) –X = Y + Z

Answer (2)

Sol. C(diamond) + O_2(g) $\rightarrow$ CO_2(g) ; $\Delta H_1 = -Y$ kJ mol^{-1}

CO_2(g) $\rightarrow$ C(graphite) + O_2(g) ; $\Delta H_2 = Z$ kJ mol^{-1}

C(diamond) $\rightarrow$ C(graphite) ; $\Delta H_3 = -Y + Z$

$-X = -Y + Z$

$X = Y - Z$

58. Given below are two statements :

Statement-I : On nitration of m-xylene with HNO_3, H_2SO_4 followed by oxidation, 4-nitrobenzene-1,3-dicarboxylic acid is obtained as the major product.

Statement-II : $-CH_3$ group is o/p-directing while $-NO_2$ group is m-directing group.

In the light of the above statement, choose the correct answer from the options given below:

(1) Both **Statement-I** and **Statement-II** are false

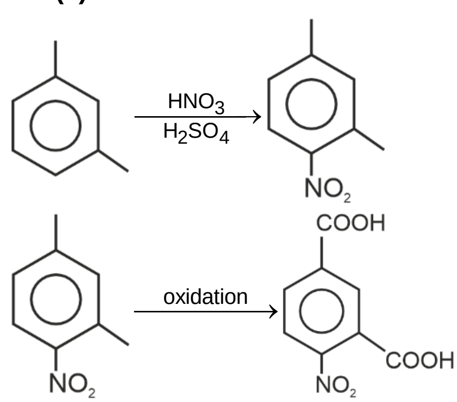
(2) Both **Statement-I** and **Statement-II** are true

(3) **Statement-I** is false but **Statement-II** is true

(4) **Statement-I** is true but **Statement-II** is false

Answer (2)

Sol.



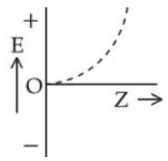
$-CH_3$ group is o/p directing group and

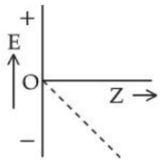
$-NO_2$ is m-directing group.

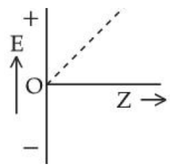
59. For hydrogen like species, which of the following graphs provides the most appropriate representation of E vs Z plot for a constant n ?

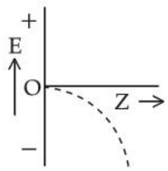
[E : Energy of the stationary state,

Z : atomic number, n = principal quantum number]

(1) 

(2) 

(3) 

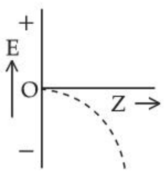
(4) 

Answer (4)

Sol. $E_n = -13.6\frac{z^2}{n^2}$

$E_n \propto -z^2$

$y = kx^2$



60. Match **List-I** with **List-II :**

	List-I Applications		List-II Batteries/Cell
(A)	Transistors	(I)	Anode – Zn/Hg; Cathode – HgO + C
(B)	Hearing aids	(II)	Hydrogen fuel cell
(C)	Invertors	(III)	Anode – Zn; Cathode - Carbon
(D)	Apollo space ship	(IV)	Anode – Pb; Cathode – Pb | PbO_2

Choose the **correct** answer from the options given below :

(1) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(2) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(3) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(4) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Answer (4)

Sol. In transistor, anode is Zn and cathode is carbon.

In hearing, aids mercury battery are used.

In invertors, lead storage battery is used.

In apollo space ship, hydrogen fuel cell was used.

61. Given below are two statements :

Statement (I) : It is impossible to specify simultaneously with arbitrary precision, both the linear momentum and the position of a particle.

Statement (II) : If the uncertainty in the measurement of position and uncertainty in measurement of momentum are equal for an electron, then the uncertainty in the measurement of velocity is $\geq \sqrt{\frac{h}{\pi}} \times \frac{1}{2m}$.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement-I** is false but **Statement-II** is true

(2) Both **Statement-I** and **Statement-II** are true

(3) Both **Statement-I** and **Statement-II** are false

(4) **Statement-I** is true but **Statement-II** is false

Answer (2)

Sol. According to Heisenberg's uncertainty principle, it is impossible to determine simultaneously the exact position and momentum of particle like electron

If $\Delta p = \Delta x$

$\Delta p.\Delta x \geq \frac{h}{4\pi}$

$(\Delta p)^2 \geq \frac{h}{4\pi}$

$\Delta p \geq \sqrt{\frac{h}{\pi}} \times \frac{1}{2}$

$m\Delta v \geq \sqrt{\frac{h}{\pi}} \times \frac{1}{2}$

$\Delta v \geq \sqrt{\frac{h}{\pi}} \times \frac{1}{2m}$

62. Given below are two statements :

Statement (I) : In partition chromatography, stationary phase is thin film of liquid present in the inert support.

Statement (II) : In paper chromatography, the material of paper acts as a stationary phase.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement-I** is true but **Statement-II** is false

(2) Both **Statement-I** and **Statement-II** are false

(3) **Statement-I** is false but **Statement-II** is true

(4) Both **Statement-I** and **Statement-II** are true

Answer (1)

Sol. Paper chromatography is a type of position chromatography in which a special quality paper is used known as chromatography paper, chromatography paper contains water trapped in it, which acts as stationary phase.

63. Given below are two statements :

Statement (I) : NaCl is added to the ice at 0°C, present in the ice cream box to prevent the melting of ice cream.

Statement (II) : On addition of NaCl to ice at 0°C, there is a depression in freezing point.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement-I** is true but **Statement-II** is false

(2) Both **Statement-I** and **Statement-II** are true

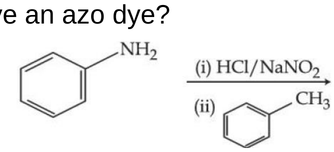
(3) **Statement-I** is false but **Statement-II** is true

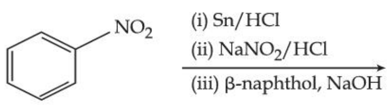
(4) Both **Statement-I** and **Statement-II** are false

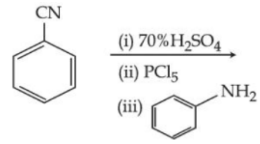
Answer (2)

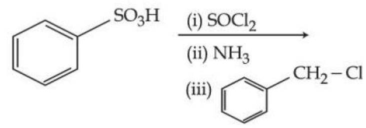
Sol. A mixture of salt and ice is known as freezing mixture. Freezing mixture decreases freezing point of ice. Both statements are true.

64. Which one of the following reaction sequences will give an azo dye?

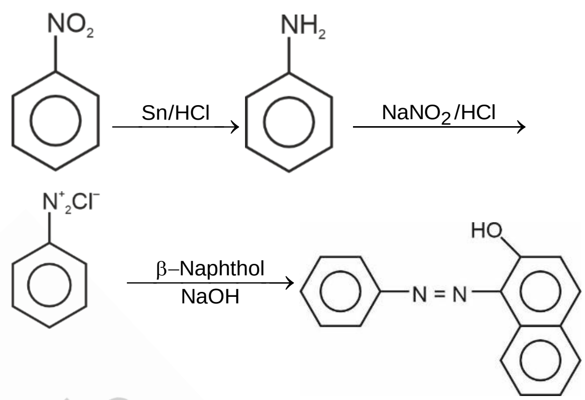
(1) 

(2) 

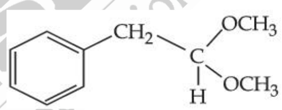
(3) 

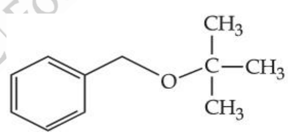
(4) 

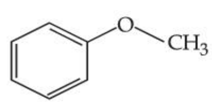
Answer (2)

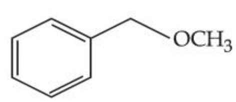
Sol. 

65. Which one of the following, with HBr will give a phenol?

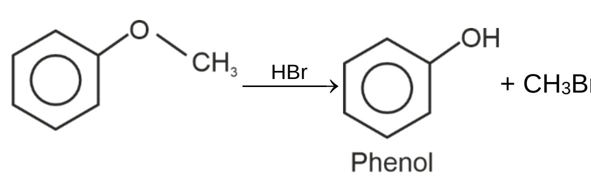
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

66. The calculated spin-only magnetic moments of $K_3[Fe(OH)_6]$ and $K_4[Fe(OH)_6]$ respectively are :

(1) 3.87 and 4.90 B.M.

(2) 5.92 and 4.90 B.M.

(3) 4.90 and 4.90 B.M.

(4) 4.90 and 5.92 B.M.

Answer (2)

Sol.

$K_3\left[Fe(OH)_6\right]$

$Fe^{3+} \Rightarrow 3d^5$

Fe^{3+} with OH^- (WFL)

$= t_{2g}^3 e_g^2$

Number of unpaired electron (n) = 5

$\mu_{\text{spin only}} = 5.92$ BM

$K_4\left[Fe(OH)_6\right]$

$Fe^{2+} \Rightarrow OH^-$ WFL

$Fe^{2+} \Rightarrow 3d^6 = t_{2g}^4 e_g^2$

n = 4

$\mu_{\text{spin only}} = 4.90$ BM

67. Identify the essential amino acids from below :

(A) Valine

(B) Proline

(C) Lysine

(D) Threonine

(E) Tyrosine

Choose the **correct** answer from the options given below :

(1) (B), (C) and (E) only

(2) (C), (D) and (E) only

(3) (A), (C) and (E) only

(4) (A), (C) and (D) only

Answer (4)

Sol. Valine, Lysine and Threonine are example of essential amino acid.

68. Consider the equilibrium

$$CO(g) + 3H_2(g) \rightleftharpoons CH_4(g) + H_2O(g)$$

If the pressure applied over the system increases by two fold at constant temperature then

(A) Concentration of reactants and products increases.

(B) Equilibrium will shift in forward direction.

(C) Equilibrium constant increases since concentration of products increases.

(D) Equilibrium constant remains unchanged as concentration of reactants and products remain same.

Choose the **correct** answer from the options given below :

(1) (B) and (C) only

(2) (A), (B) and (C) only

(3) (A) and (B) only

(4) (A), (B) and (D) only

Answer (3)

Sol. $CO(g) + 3H_2 \rightleftharpoons CH_4(g) + H_2O(g)$

$\Delta n_g = -2$

If pressure of system increases then according to Le-Chatelier's principle reaction will move in forward direction.

Concentration of reactant and products both increases but concentration of product increases more.

69. O_2 gas will be evolved as a product of electrolysis of :

(A) An aqueous solution of $AgNO_3$ using silver electrodes.

(B) An aqueous solution of $AgNO_3$ using platinum electrodes.

(C) A dilute solution of H_2SO_4 using platinum electrodes.

(D) A high concentration solution of H_2SO_4 using platinum electrodes.

Choose the **correct** answer from the options given below :

(1) (A) and (D) only (2) (B) and (D) only

(3) (B) and (C) only (4) (A) and (C) only

Answer (3)

Sol. When an aqueous solution of $AgNO_3$ is electrolysed using Pt electrodes

Cathode : $Ag^{+}(aq) + e^{-} \rightarrow Ag(s)$

Anode : $2H_2O(l) \rightleftharpoons 4H^{+}(aq) + O_2(g) + 4e^{-}$

$\Rightarrow$ When dilute H_2SO_4 is electrolysed using Pt electrodes

Anode : $2H_2O(l) \rightarrow O_2(g) + 4H^{+}(aq) + 4e^{-}$

Cathode : $2H^{+}(aq) + 2e^{-} \rightarrow H_2(g)$

70. The type of oxide formed by the element among Li, Na, Be, Mg, B and Al that has the least atomic radius is :

(1) A_2O (2) AO

(3) A_2O_3 (4) AO_2

Answer (3)

Sol. Among given atoms, Boron has least atomic radius oxide of Boron = B_2O_3

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

71. Isomeric hydrocarbons $\rightarrow$ negative Baeyer's test

(Molecular formula C_9H_{12})

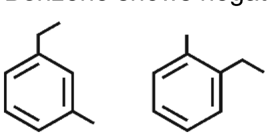
The total number of isomers from above with four different non-aliphatic substitution site is –

Answer (2)

Sol. Degree of unsaturation = C + 1 – $\frac{H}{2}$

= 9 +1 –6 = 4

Benzene shows negative Baeyer's test



Both compounds have four different non-aliphatic substitution sites.

72. Total number of non-bonded electrons present in NO_2^- ion based on Lewis theory is ________.

Answer (12)

Sol. $\ddot{O} = \ddot{N} - \ddot{O}:^{-}$

Number of non-bonding electrons

= 12

73. In the sulphur estimation, 0.20 g of a pure organic compound gave 0.40 g of barium sulphate. The percentage of sulphur in the compound is _____ $\times 10^{-1}$%.

[Molar mass : O = 16, S = 32, Ba = 137 in g mol^{-1}]

Answer (275)

Sol. Moles of $BaSO_4$ = Moles of S = $\frac{0.40}{233}$ mol

Mass of S = $\frac{0.40}{233} \times 32$ g

= 0.055 g

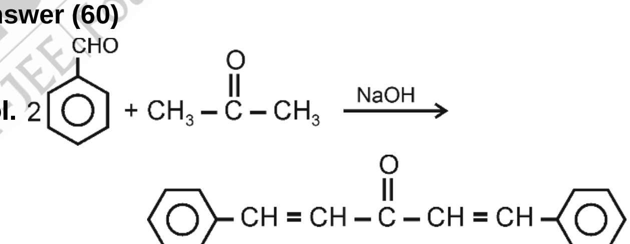
%S = $\frac{0.055}{0.20} \times 100 = 27.5\%$

= 275 × 10^{-1} %

74. In the Claisen-Schmidt reaction to prepare, dibenzalacetone from 5.3 g of benzaldehyde, a total of 3.51 g of product was obtained. The percentage yield in this reaction was _______ %.

Answer (60)

Sol.



dibenzal acetone

Mole = $\frac{5.3}{106} = 0.05$ mol

2 mol benzaldehyde produces 1 mol dibenzalacetone

moles of dibenzalacetone = 0.025 mol

Mass = 0.025 × 234 g = 5.85 g

% yield = $\frac{3.51}{5.85} \times 100$

= 60 %

75. Consider the following low-spin complexes

$K_3[Co(NO_2)_6]$, $K_4[Fe(CN)_6]$, $K_3[Fe(CN)_6]$, $Cu_2[Fe(CN)_6]$ and $Zn_2[Fe(CN)_6]$.

The sum of the spin-only magnetic moment values of complexes having yellow colour is _______ B.M. (answer in nearest integer)

Answer (0)

Sol. $K_3[Co(NO_2)_6]$ = Yellow $\Rightarrow$ $Co^{3+} = 3d^6$ $\Rightarrow$ $t_{2g}^6 e_g^0$

$K_4[Fe(CN)_6]$ = Yellow $\Rightarrow$ $Fe^{2+} = 3d^6$ $\Rightarrow$ $t_{2g}^6 e_g^0$

$K_3[Fe(CN)_6]$ = Bright Red $\Rightarrow$ $Fe^{3+} = 3d^5$ $\Rightarrow$ $t_{2g}^5 e_g^0$

$Cu_2[Fe(CN)_6]$ = Chocolate brown

$Zn_2[Fe(CN)_6]$ = White

Spin only magnetic moment of complex having Yellow colour is zero.

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