FIITJEE

Solutions to JEE(Main) -2023

Test Date: 25th January 2023 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (1 – 10)*** *contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. T is the time period of simple pendulum on the earth's surface. Its time period becomes x T when taken to a height R (equal to earth's radius) above the earth's surface. Then, the value of x will be:

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) 4 (D) 2

Q2. A car travels a distance of 'x' with speed v_1 and the same distance 'x' with speed v_2 in the same direction. The average speed of the car is :

(A) $\frac{2x}{v_1+v_2}$ (B) $\frac{2v_1v_2}{v_1+v_2}$

(C) $\frac{v_1v_2}{2(v_1+v_2)}$ (D) $\frac{v_1+v_2}{2}$

Q3. The ratio of the density of oxygen nucleus $\left({}^{16}_{8}O\right)$ and helium nucleus $\left({}^{4}_{2}He\right)$ is

(A) 8 : 1 (B) 1 : 1

(C) 4 : 1 (D) 2 : 1

Q4. An electromagnetic wave is transporting energy in the negative z direction. At a certain point and certain time the direction of electric field of the wave is along positive y direction. What will be the direction of the magnetic field of the wave at that point and instant?

(A) Positive direction of z (B) Negative direction of x

(C) Negative direction of y (D) Positive direction of x

Q5. A uniform metallic wire carries a current 2A, when 3.4V battery is connected across it. The mass of uniform metallic wire is $8.92\times10^{-3}\,kg/m^3$ density is $8.92\times10^{3}\,kg/m^3$ and resistivity is $1.7\times10^{-8}\,\Omega-m$. The length of wire is :

(A) $\ell = 100m$ (B) $\ell = 5m$

(C) $\ell = 10m$ (D) $\ell = 6.8m$

Q6. A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2m long and 0.2m in diameter. The magnetic intensity at the centre of the solenoid when a current of 2 A flows through it is :

(A) $1\,A\,m^{-1}$ (B) $1.2\times10^{3}\,A\,m^{-1}$

(C) $2.4\times10^{3}\,A\,m^{-1}$ (D) $2.4\times10^{-3}\,A\,m^{-1}$

Q7. Electron beam used in an electron microscope, when eccelerated by a voltage of 20kV. Has a de-Broglie wavelength of λ_0. If the voltage is increased to 40kV, then the de-Broglie wavelength associated with the electron beam would be :

(A) $\frac{\lambda_0}{\sqrt{2}}$ (B) $\frac{\lambda_0}{2}$

(C) $9\lambda_0$ (D) $3\lambda_0$

Q8. In Young's double slits experiment, the position of 5th bright fringe from the central maximum is 5 cm. The distance between slits and screen is 1m and wavelength of used monochromatic light is 600nm. The separation between the slits is :

(A) $48\mu m$ (B) $36\mu m$

(C) $12\mu m$ (D) $60\mu m$

Q9. The root mean square velocity of molecules of gas is

(A) Proportional to temperature (T)

(B) Proportional to square of temperature $\left(T^2\right)$

(C) Inversely proportional to square root of temperature $\left(\sqrt{\frac{1}{T}}\right)$

(D) Proportional to square root of temperature $\left(\sqrt{T}\right)$

Q10. Given below are two statements : one is labelled as Assertion A and the other is labeled as Reason R

Assertion A : Photodiodes are used in forward bias usually for measuring the light intensity.

Reason R : For a p-n junction diode, at applied voltage V the current in the forward bias is more than the current in the reverse bias for $\left|V_z\right| > \pm V \geq \left|V_0\right|$ where V_0 is the threshold voltage and V_z is the breakdown voltage.

In the light of the above statements, choose the correct answer from the options given below

(A) A is true but R is false

(B) Both A and R are true and R is correct explanation A

(C) Both A and R are true but R is NOT the correct explanation A.

(D) A is false but R is true

Q11. A message signal of frequency 5 kHz is used to modulate a carrier single of frequency 2 MHz. The bandwidth for amplitude modulation is :

(A) 2.5 kHz (B) 5 kHz

(C) 20 kHz (D) 10 kHz

Q12. In an LC oscillator, if values of inductance and capacitance become twice and eight times, respectively, then the resonant frequency of oscillator becomes x times its initial resonant frequency ω_0. The value of x is :

(A) 1/16 (B) 4

(C) 1/4 (D) 16

Q13. Assume that the earth is a solid sphere of uniform density and a tunnel is dug along its diameter throughout the earth. It is found that when a particle is released in this tunnel. it executes a simple harmonic motion. The mass of the particle is 100g. The time period of the motion of the particle will be (approximately)

(Take $g = 10 ms^{-2}$, radius of earth = 6400 km)

(A) 12 hours (B) 1 hours 24 minutes

(C) 1 hours 40 minutes (D) 24 hours

Q14. A Carnot engine with efficiency 50% takes heat from a source at 600K. In order to increase the efficiency to 70%, keeping the temperature of sink same, the new temperature of the source will be :

(A) 900 K (B) 300 K
(C) 1000 K (D) 360 K

Q15. A bowl filled with very hot soup cools from $98^{\circ}C$ to $86^{\circ}C$ in 2 minutes when the room temperature is $22^{\circ}C$. How long it will take to cool from $75^{\circ}C$ to $69^{\circ}C$?

(A) 2 minutes (B) 1 minutes
(C) 0.5 minutes (D) 1.4 minutes

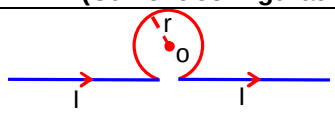
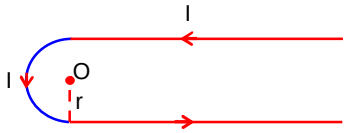
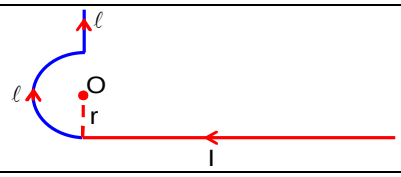
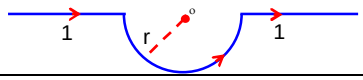
Q16. Match List I with List II

List – I	**List – II**
A. Surface tension	I. $kg\ m^{-1}s^{-1}$
B. Pressure	II. $kg\ ms^{-1}$
C. Viscosity	III. $kg\ m^{-1}\ s^{-2}$
D. Impulse	IV. $Kg\ s^{-2}$

Choose the correct answer from the options given below :

(A) A – II, B – I, C – III, D – IV (B) A – III, B – IV, C – I, D – II
(C) A – IV, B – III, C – II, D – I (D) A – IV, B – III, C – I, D – II

Q17. Match List I with List II

List – I (Current configuration)		**List – II (Magnitude of Magnetic Field at point O)**	
A.		I.	$B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 2]$
B.		II.	$B_0 = \frac{\mu_0}{4}\frac{I}{r}$
C.		III.	$B_0 = \frac{\mu_0 I}{2\pi r}[\pi - 1]$
D.		IV.	$B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 1]$

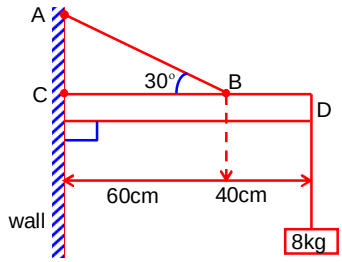
Choose the correct answer from the options below :

(A) A – III, B – IV, C – I, D – II (B) A – II, B – I, C – IV, D – III
(C) A – I, B – III, C – IV, D – II (D) A – III, B – I, C – IV, D – II

Q18. A car is moving with a constant speed of 20 m/s in a circular horizontal track of radius 40 m. A bob is suspended from the roof of the car by a massless string. The angle made by the string with the vertical will be : (Take g = 10 m/s^2)

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$
(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{3}$

Q19. An object of mass 8 kg is hanging from one end of a uniform rod CD of mass 2kg and length 1m pivoted at its end C on a vertical wall as shown in figure. It is supported by a cable AB such that the system is in equilibrium. The tension in the cable is : (Take g = 10 m/s^2)



(A) 300 N
(B) 240 N
(C) 90 N
(D) 30 N

Q20. A parallel plate capacitor has plate area $40\,cm^2$ and plates separation 2mm. The space between the plates is filled with a dielectric medium of a thickness 1mm and dielectric constant 5. The capacitance of the system is :

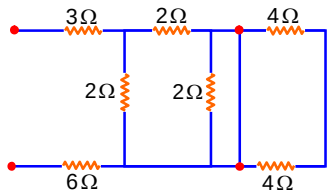
(A) $\frac{3}{10}\varepsilon_0$ F
(B) $10\varepsilon_0$ F
(C) $\frac{10}{3}\varepsilon_0$ F
(D) $24\varepsilon_0$ F

SECTION - B

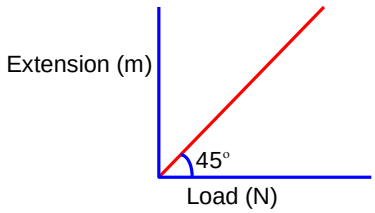
(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

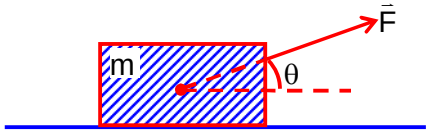
Q1. In the given circuit, the equivalent resistance between the terminal A and B is _____ Ω.



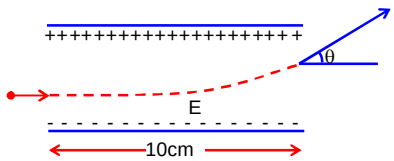
Q2. As shown in the figure, in an experiment to determine Young's modulus of a wire, the extension-load curve is plotted. The curve is a straight line passing through the origin and makes an angle of 45° with the load axis. The length of wire is 62.8 cm and its diameter is 4mm. The Young's modulus is found to be $x \times 10^{4}\,Nm^{-2}$. The value of x is _____.



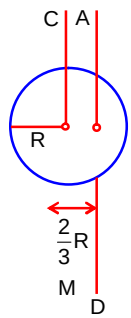
Q3. An object of mass 'm' initially at rest on a smooth horizontal plane starts moving under the action of force $F = 2N$. In the process of its linear motion. The angle θ (as shown in figure) between the direction of force and horizontal varies as $\theta = kx$, where k is a constant and x is the distance covered by the object from its initial position. The expression of kinetic energy of the object will be $E = \frac{n}{k}\sin\theta$, The value of n is _____.



Q4. A uniform electric field of 10 N/C is created between two parallel charge plates (as shown in figure). An electron enters the field symmetrically between the plates with a kinetic energy 0.5 eV. The length of each plate is 10cm. The angle (θ) of deviation of the path of electron as it comes out of the field is_____(in degree).



Q5. I_{CM} is the moment of inertia of a circular disc about an axis (CM) passing through its centre and perpendicular to the plane of disc. I_{AB} is it's moment of inertia about an axis AB perpendicular to plane and parallel to axis CM at a distance $\frac{2}{3}R$ from centre. Where R is the radius of the disc. The ratio of I_{AB} and I_{CM} is x : 9. The value of x is _____.



Q6. A ray of light is incident from air on a glass plate having thickness $\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence of a ray is equal to the critical angle for glass-air interface. The lateral displacement of the ray when it passes through the plate is _____ $\times 10^{-2}$ cm. (given $\sin 15^{\circ} = 0.26$)

Q7. The wavelength of the radiation emitted is λ_0 when an electron jumps from the second excited state to the first excited state of hydrogen atom. If the electron jumps from the third excited state to the second orbit of the hydrogen atom, the wavelength of the radiation emitted will be $\frac{20}{x}\lambda_0$. The value of x is _____.

Q8. An LCR series circuit of capacitance 62.5nF and resistance of 50Ω, is connected to an A.C. source of frequency 2.0 kHz. For maximum value of amplitude of current in circuit, the value of inductance is _____ mH.
(Take $\pi^2 = 10$)

Q9. If $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$ and $\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$ then, The unit vector in the direction of $\vec{P} \times \vec{Q}$ is $\frac{1}{x}\left(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right)$. The value if x is

Q10. The distance between two consecutive points with phase difference of 60° in a wave of frequency 500Hz is 6.0 m. The velocity with which wave is traveling is_____km/s

PART – B (CHEMISTRY)

SECTION - A

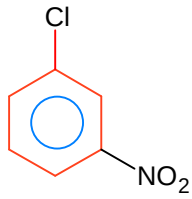
(One Options Correct Type)

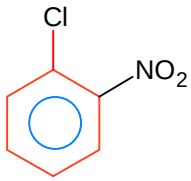
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

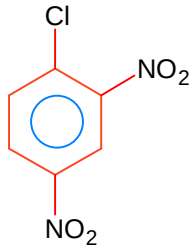
Q1. Which of the following statements is incorrect for antibiotics?
(A) An antibiotic should be effective in low concentrations.
(B) An antibiotic must be a product of metabolism.
(C) An antibiotic should promote the growth or survival of microorganisms
(D) An antibiotic is a synthetic substance produced as a structural analogue of naturally occurring antibiotic.

Q2. The compound which will have the lowest rate towards nucleophilic aromatic substitution on treatment with OH^- is

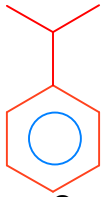
(A) 

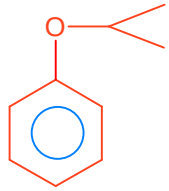
(B) 

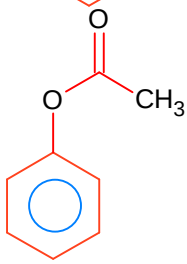
(C) 

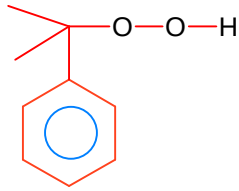
(D) 

Q3. In the cumene to phenol preparation in presence of air, the intermediate is

(A) 

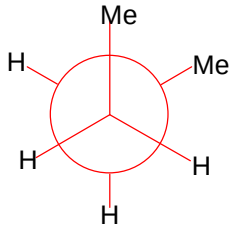
(B) 

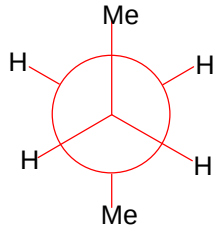
(C) 

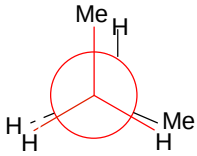
(D) 

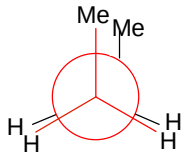
Q4. '25 volume' hydrogen peroxide means
(A) 1 L marketed solution contains 250g of H_2O_2.
(B) 1 L marketed solution contains 25g of H_2O_2.
(C) 100 mL marketed solution contains 25 g of H_2O_2.
(D) 1 L marketed solution contains 75 g of H_2O_2.

Q5. Which of the following conformations will be the most stable?

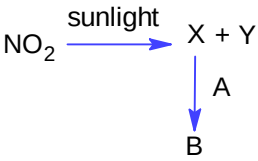
(A) 

(B) 

(C) 

(D) 

Q6. Some reactions of NO_2 relevant to photochemical smog formation are



Identify A, B, X and Y

(A) X = N_2O, Y= [O], A = O_3, B= NO

(B) X = $\frac{1}{2}O_2$, Y = NO_2, A = O_3, B = O_2

(C) X = NO, Y= [O], A = O_2, B=N_2O_3

(D) X= [O], Y= NO, A = O_2, B = O_3

Q7. Match List I with List II

List-I (Elements)		List-II (Colour imparted to the flame)	
A.	K	I.	Brick Red
B.	Ca	II.	Violet
C.	Sr	III.	Apple Green
D.	Ba	IV.	Crimson Red

Choose the correct answer from the options given below:

(A) A- IV, B-III, C-II, D- I

(B) A-II, B-I, C-III, D-IV

(C) A-II, B-IV, C-I, D-III

(D) A-II, B-I, C-IV, D-III

Q8. Match List I with List II

List-I (Cations)		List-II (Group reagents)	
A.	Pb^{2+},Cu^{2+}	I.	H_2S gas in presence of dilute HCl
B.	Al^{3+}, Fe^{3+}	II.	$(NH_4)_2CO_3$ in presence of NH_4OH
C.	Co^{2+}, Ni^{2+}	III.	NH_4OH in presence of NH_4Cl
D.	Ba^{2+}, Ca^{2+}	IV.	H_2S in presence of NH_4OH

Choose the correct answer from the options given below:

(A) A- III, B-I, C-IV, D- II

(B) A-I, B-III, C-II, D-IV

(C) A-I, B-III, C-IV, D-II

(D) A-IV, B-II, C-III, D-I

Q9. The correct order in aqueous medium of basic strength in case of methyl substituted amines is:

(A) $Me_2NH>MeNH_2>Me_3N>NH_3$

(B) $Me_2NH>Me_3N>MeNH_2>NH_3$

(C) $NH_3>Me_3N>MeNH_2>Me_2NH$

(D) $Me_3N>Me_2NH>MeNH_2>NH_3$

Q10. Reaction of thionyl chloride with white phosphorus forms a compound [A], which on hydrolysis gives [B], a diabasic acid. [A] and [B] are respectively.

(A) P_4O_6 and H_3PO_3

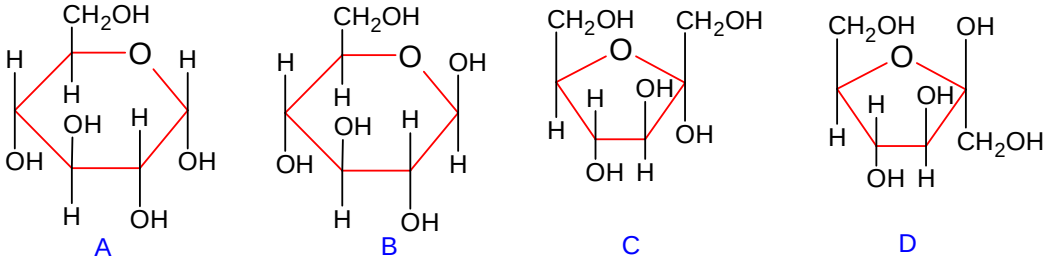
(B) $POCl_3$ and H_3PO_4

(C) PCl_3 and H_3PO_3

(D) PCl_5 and H_3PO_4

Q11. Match items of Row I with those of Row II

Row I:



Row II:

(i) α–D-(–)-Fructofuranose, (ii) β-D-(–)- Fructofuranose

(iii) α–D-(–)- Glucopyranose. (iv) β-D-(–)- Glucopyranose

Correct match is :

(A) A–iii, B–iv, C–ii,D–i (B) A–iv, B-iii, C–i, D–ii

(C) A–iii, B-iv, C–i, D–ii (D) A–i, B–ii, C–iii, D–iv

Q12. The radius of the 2nd orbit of Li^{2+} is x. The expected radius of the 3rd orbit of Be^{3+} is

(A) $\frac{27}{16}x$ (B) $\frac{16}{27}x$

(C) $\frac{4}{9}x$ (D) $\frac{9}{4}x$

Q13. Inert gases have positive electron gain enthalpy. Its correct order is

(A) He < Ne < Kr < Xe (B) He < Xe < Kr < Ne

(C) Xe < Kr < Ne < He (D) He < Kr < Xe < Ne

Q14. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**:

Assertion A: Acetal / Ketal is stable in basic medium.

Reason R: The light leaving tendency of alkoxide ion gives the stability to acetal / ketal in basic medium.

In the high of the above statements, choose the correct answer from the options given below:

(A) A is true but R is false

(B) A is false but R is true

(C) Both A and R are true but R is NOT the correct explanation of A

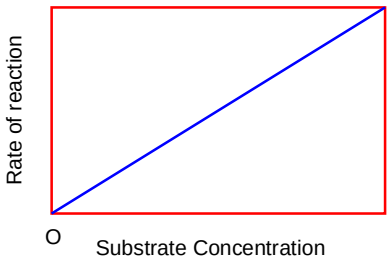
(D) Both A and R are true and R is the correct explanation of A

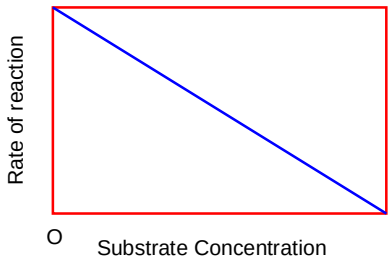
Q15. A cubic solid is made up of two elements X and Y. Atoms of X are present on every alternate corner and one at the center of cube. Y is at $\frac{1}{3}$rd of the total faces. The empirical formula of the compound is.

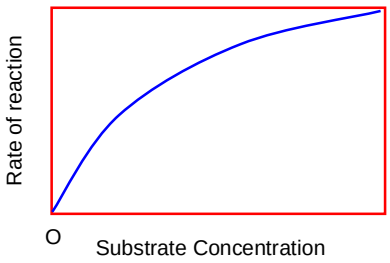
(A) $X_2Y_{1.5}$ (B) $XY_{2.5}$

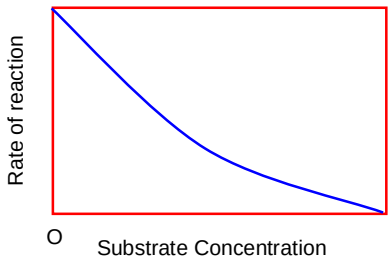
(C) $X_{2.5}Y$ (D) $X_{1.5}Y_2$

Q16. The variation of the rate of an enzyme catalyzed reaction with substrate concentration is correctly represented by graph

A.


B.


C.


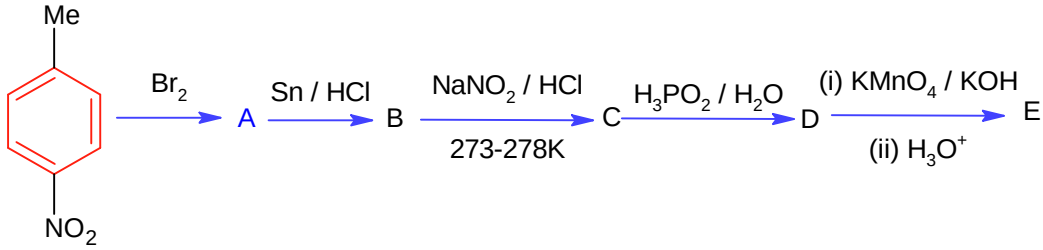
D.


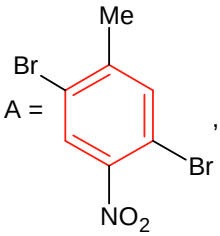
(A) C
(B) D
(C) B
(D) A

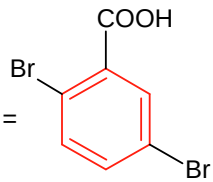
Q17. Compound A reacts with NH_4Cl and forms a compound B. Compound B reacts with H_2O and excess of CO_2 to form compound C which on passing through or reaction with saturated NaCl solution forms sodium hydrogen carbonate. Compound A, B and C, are respectively.

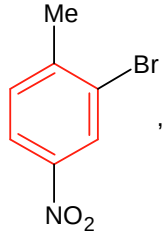
(A) $Ca(OH)_2$, NH_3, NH_4HCO_3
(B) $CaCl_2$, NH_3, NH_4HCO_3
(C) $Ca(OH)_2$, $NH_4^{\oplus}$, $(NH_4)_2CO_3$
(D) $CaCl_2$, $NH_4^{\oplus}$, $(NH_4)_2CO_3$

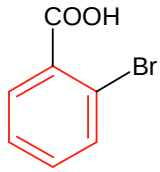
Q18. Identify the product formed (A and E)

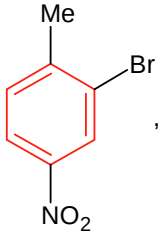


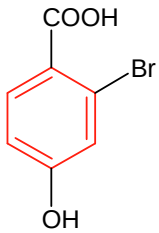
(A) A = 

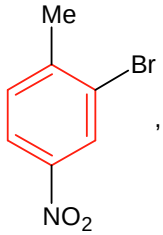
, E = 

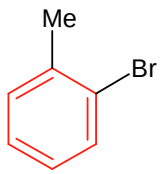
(B) A = 

, E = 

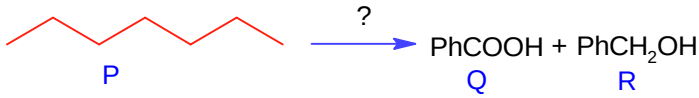
(C) A = 

, E = 

(D) A = 

, E = 

Q19.



The correct sequence of reagent for the preparation of Q and R is

(A) (i) $KMnO_4, OH^-$; (ii) Mo_2O_3, Δ ; (iii) NaOH ; (iv) H_3O^+

(B) (i) CrO_2Cl_2, H_3O^+ ; (ii) Cr_2O_3, 770K , 20 atm (iii) NaOH ; (iv) H_3O^+

(C) (i) Cr_2O_3, 770K , 20 atm; (ii) CrO_2Cl_2, H_3O^+ ; (iii) NaOH ; (iv) H_3O^+

(D) (i) Mo_2O_3, Δ ;(ii) CrO_2Cl_2 , H_3O^+ ; (iii) NaOH ; (iv) H_3O^+

Q20. Which one of the following reaction does not occur during extraction of copper?

(A) $FeO + SiO_2 \rightarrow FeSiO_3$

(B) $2FeS + 3O_2 \rightarrow 2FeO + 2SO_2$

(C) $2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$

(D) $CaO + SiO_2 \rightarrow CaSiO_3$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. The number of paramagnetic species from the following is_____.

$\left[Ni(CN)_4\right]^{2-}, \left[Ni(CO)_4\right], [NiCl_4]^{2-}$

$\left[Fe(CN)_6\right]^{4-}, \left[Cu(NH_3)_4\right]^{2+}$

$\left[Fe(CN)_6\right]^{3-}$ and $\left[Fe(H_2O)_6\right]^{2+}$

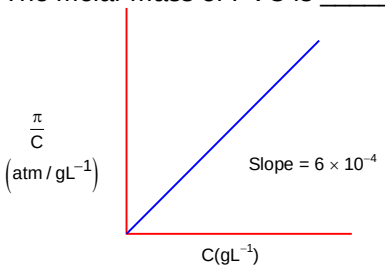
Q2. Consider the cell

$Pt_{(s)} \mid H_{2(g)}(1atm) \mid H^+_{(aq)}.[H^+] = 1) \mid\mid Fe^{3+}_{(aq)}, Fe^{2+}_{(aq)} \mid Pt_{(s)}$

Given $E^o_{Fe^{3+}/Fe^{2+}} = 0.771\,V$ and $E^o_{H^+/\frac{1}{2}H_2} = 0V, T = 298K$

If the potential of the cell is 0.712 V, the ratio of concentration of Fe^{2+} to Fe^{3+} is _____ (Nearest integer)

Q3. The osmotic pressure of solutions of PVC in cyclohexanone at 300K are plotted on the graph. The molar mass of PVC is _____ g mol^{-1}(Nearest integer)



(Given: R = 0.083L atm K^{-1} mol^{-1})

Q4. A litre of buffer solution contains 0.1 mole of each of NH_3 and NH_4Cl. On the addition of 0.02 mole of HCl by dissolving gaseous HCl, the pH of the solution is found to be_____$\times 10^{-3}$(Nearest integer)

[Given: pKb(NH_3) = 4.745
log2 = 0.301
log3 = 0.477
T = 298 KJ

Q5. An athlete is given 100g of glucose ($C_6H_{12}O_6$) for energy. This is equivalent to 1800 kJ of energy. The 50% of this energy gained is utilized by the athlete for sports activities at the event. In order to avoid storage of energy, the weight of extra water be would need to perspire is_____ g (Nearest integer)

Assume that there is no other way of consuming stored energy.

Given: The enthalpy of evaporation of water is 45 kJ mol^{-1}

Molar mass of C, H & O are 12, 1 and 16 g mol^{-1}.

Q6. For the first order reaction A→B, the half life is 30 min. The time taken for 75% completion of the reaction is_____ min. (Nearest integer)
Given: log2 = 0.3010
log 3= 0.4771
log5 = 0.6989

Q7. The density of a monobasic strong acid (Molar mass 24.2 g/mol) is 1.21 gk/L. The volume of its solution required for the complete neutralization of
25mL of 0.24 NaOH is_____ $\times 10^{-2}$mL (Nearest integer)

Q8. The total number of lone pairs of electrons on oxygen atoms of ozone is _____.

Q9. In sulphur estimation, 0.471 g of an organic compound gave 1.4439 g of barium sulphate. The percentage of sulphur in the compound is _____ ((Nearest integer)
(Given: Atomic mass Ba; 137u, S; 32u, O;16u)

Q10. How many of the following metal ions have similar value of spin magnetic moment in gaseous state? _____ .
(Given: Atomic number : V:23; Cr:24; Fe: 26; Ni:28)
V^{3+}, Cr^{3+},Fe^{2+}, Ni^{3+}

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b}\cdot\vec{c}=0$ and $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{\vec{b}-\vec{c}}{2}$. If $\vec{d}$ be a vector such that $\vec{b}\cdot\vec{d}=\vec{a}\cdot\vec{b}$, then $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{c}\times\vec{d}\right)$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) $-\frac{1}{4}$

Q2. The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12. If the new mean of the marks is 10.2, then their new variance is equal to :

(A) 4.08 (B) 4.04

(C) 3.92 (D) 3.96

Q3. The value of $\lim\limits_{n\to\infty}\frac{1+2-3+4+5-6+....+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ is :

(A) $\frac{3}{2}\left(\sqrt{2}+1\right)$ (B) $\frac{3}{2\sqrt{2}}$

(C) $\frac{\sqrt{2}+1}{2}$ (D) $3\left(\sqrt{2}+1\right)$

Q4. Consider the lines L_1 and L_2 given by

$L_1:\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-2}{2}$

$L_2:\frac{x-2}{1}=\frac{y-2}{2}=\frac{z-3}{3}$.

A line L_3 having direction ratios $1,-1,-2$, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is

(A) 4 (B) $2\sqrt{6}$

(C) $3\sqrt{2}$ (D) $4\sqrt{3}$

Q5. The vector $\vec{a}=-\hat{i}+2\hat{j}+\hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a}+\sqrt{2}\vec{b}$ on $\vec{c}=5\hat{i}+4\hat{j}+3\hat{k}$ is :

(A) $2\sqrt{3}$ (B) $\sqrt{6}$

(C) $3\sqrt{2}$ (D) 1

Q6. Let $f:(0,1)\to R$ be a function defined by $f(x)=\frac{1}{1-e^{-x}}$ and $g(x)=\left(f(-x)-f(x)\right)$. Consider two statements

(I) g is an increasing function in $(0,1)$

(II) g is one-one in $(0,1)$

Then,

(A) Neither (I) nor (II) is true
(B) Both (I) and (II) are true
(C) Only (II) is true
(D) Only (I) is true

Q7. Let $z_1=2+3i$ and $z_2=3+4i$. The set $S=\left\{z\in C:|z-z_1|^2-|z-z_2|^2=|z_1-z_2|^2\right\}$ represents a

(A) straight line with the sum of its intercepts on the coordinate axes equals -18
(B) hyperbola with the length of the transverse axis 7
(C) hyperbola with eccentricity 2
(D) straight line with the sum of its intercepts on the coordinate axes equals 14

Q8. Let $x,y,z>1$ and $A=\begin{bmatrix}1 & \log_x y & \log_x z\\ \log_y x & 2 & \log_y z\\ \log_z x & \log_z y & 3\end{bmatrix}$. Then $\left|\text{adj}\left(\text{adj } A^2\right)\right|$ is equal to

(A) 4^8
(B) 2^4
(C) 6^4
(D) 2^8

Q9. Let M be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S=\left\{x\in Z:x(66-x)\ge\frac{5}{9}M\right\}$ and the event $A=\{x\in S:x \text{ is a multiple of } 3\}$. Then $P(A)$ is equal to

(A) $\frac{15}{44}$
(B) $\frac{1}{3}$
(C) $\frac{1}{5}$
(D) $\frac{7}{22}$

Q10. Let $f(x)=\int\frac{2x}{(x^2+1)(x^2+3)}dx$. If $f(3)=\frac{1}{2}(\log_e 5-\log_e 6)$, then $f(4)$ is equal to

(A) $\log_e 19-\log_e 20$
(B) $\frac{1}{2}(\log_e 19-\log_e 17)$
(C) $\log_e 17-\log_e 18$
(D) $\frac{1}{2}(\log_e 17-\log_e 19)$

Q11. Let x = 2 be a local minima of the function $f(x)=2x^4-18x^2+8x+12, x\in(-4,4)$. If M is local maximum value of the function f in $(-4,4)$, then M =

(A) $12\sqrt{6}-\frac{31}{2}$
(B) $18\sqrt{6}-\frac{31}{2}$
(C) $12\sqrt{6}-\frac{33}{2}$
(D) $18\sqrt{6}-\frac{33}{2}$

Q12. Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx} = \frac{y}{x}\left(1 + xy^2\left(1 + \log_e x\right)\right), x > 0, y(1) = 3$. Then $\frac{y^2(x)}{9}$ is equal to :

(A) $\frac{x^2}{2x^3\left(2 + \log_e x^3\right) - 3}$ (B) $\frac{x^2}{7 - 3x^3\left(2 + \log_e x^2\right)}$

(C) $\frac{x^2}{5 - 2x^3\left(2 + \log_e x^3\right)}$ (D) $\frac{x^2}{3x^3\left(1 + \log_e x^2\right) - 2}$

Q13. The points of intersection of the line $ax + by = 0, (a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$. The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is :

(A) $x^2 + y^2 + 3x + 3y + 4 = 0$ (B) $x^2 + y^2 + 3x + 5y + 8 = 0$

(C) $x^2 + y^2 - 5x - 5y + 12 = 0$ (D) $x^2 + y^2 + 5x + 5y + 12 = 0$

Q14. Let S_1 and S_2 be respectively the sets of all $a \in R - \{0\}$ for which the system of linear equations

$ax + 2ay - 3az = 1$

$(2a + 1)x + (2a + 3)y + (a + 1)z = 2$

$(3a + 5)x + (a + 5)y + (a + 2)z = 3$

has unique solution and infinitely many solutions. Then

(A) S_1 is an infinite set and $n(S_2) = 2$ (B) $S_1 = \phi$ and $S_2 = R - \{0\}$

(C) $S_1 = R - \{0\}$ and $S_2 = \phi$ (D) $n(S_1) = 2$ and S_2 is an infinite set

Q15. If a_r is the coefficient of x^{10-r} in the Binomial expansion of $(1 + x)^{10}$, then $\sum_{r=1}^{10} r^3\left(\frac{a_r}{a_{r-1}}\right)^2$ is equal to

(A) 5445 (B) 3025

(C) 4895 (D) 1210

Q16. The distance of the point $P(4, 6, -2)$ from the line passing through the point $(-3, 2, 3)$ and parallel to a line with direction ratios $3, 3, -1$ is equal to :

(A) $2\sqrt{3}$ (B) 3

(C) $\sqrt{14}$ (D) $\sqrt{6}$

Q17. The distance of the point $\left(6, -2\sqrt{2}\right)$ from the common tangent $y = mx + c, m > 0$ of the curve $x = 2y^2$ and $x = 1 + y^2$ is:

(A) $\frac{1}{3}$ (B) 5

(C) $5\sqrt{3}$ (D) $\frac{14}{3}$

Q18. The minimum value of the function $f(x) = \int_0^2 e^{|x-t|} dt$ is :

(A) $2(e - 1)$ (B) $e(e - 1)$

(C) $2e - 1$ (D) 2

Q19. Let $y(x)=(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$. Then $y'-y''$ at $x=-1$ is equal to :

(A) 496
(B) 464
(C) 944
(D) 976

Q20. The statement $(p\wedge(\sim q))\Rightarrow(p\Rightarrow(\sim q))$ is

(A) a contradiction
(B) equivalent to $(\sim p)\vee(\sim q)$
(C) equivalent to $p\vee q$
(D) a tautology

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. If the sum of all the solutions of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}, -1<x<1, x\neq 0$, is $\alpha-\frac{4}{\sqrt{3}}$, then α is equal to……

Q2. Let $S=\{1,2,3,5,7,10,11\}$. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is…….

Q3. Let the equation of the plane passing through the line $x-2y-z-5=0=x+y+3z-5$ and parallel to the line $x+y+2z-7=0=2x+3y+z-2$ be $ax+by+cz=65$. Then the distance of the point (a,b,c) from the plane $2x+2y-z+16=0$ is…….

Q4. Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as $A, A+1, A+2$, respectively. Let a,b,c be the 7th, 9th, 17th terms of A_1, A_2, A_3, respectively such that $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix}+70=0$.

If a = 29, then the sum of first 20 terms of an AP whose first term is c–a–b and common difference is $\frac{d}{12}$, is equal to……..

Q5. Let $S=\left\{\alpha : \log_2\left(9^{2\alpha-4}+13\right)-\log_2\left(\frac{5}{2}, 3^{2\alpha-4}+1\right)=2\right\}$. Then the maximum value of β for which the equation $x^2-2\left(\sum_{\alpha\in S}\alpha\right)^2 x+\sum_{a\in S}(\alpha+1)^2\beta=0$ has real roots, is…….

Q6. For some $a,b,c\in N$, let $f(x)=ax-3$ and $g(x)=x^b+c, x\in R$. If $(fog)^{-1}(x)=\left(\frac{x-7}{2}\right)^{1/3}$, then $(fog)(ac)+(gof)(b)$ is equal to…..

Q7. The constant term in the expansion of $\left(2x+\frac{1}{x^7}+3x^2\right)^5$ is……

Q8. It the area enclosed by the parabola $P_1: 2y=5x^2$ and $P_2: x^2-y+6=0$ is equal to the area enclosed by P_1 and $y=\alpha x, \alpha>0$, then α^3 is equal to…..

Q9. The vertices of a hyperbola H are $(\pm 6, 0)$ and its eccentricity is $\frac{\sqrt{5}}{2}$. Let N be the normal to H at a point in the first quadrant and parallel to the line $\sqrt{2}x + y = 2\sqrt{2}$. If d is the length of the line segment of N between H and the y-axis then d^2 is equal to……..

Q10. Let x and y be distinct integers where $1 \le x \le 25$ and $1 \le y \le 25$. Then, the number of ways of choosing x and y, such that $x + y$ is divisible by 5, is……..

FIITJEE

KEYS to JEE (Main)-2023

PART – A (PHYSICS)

SECTION - A

1.	D	2.	B	3.	B	4.	D
5.	C	6.	B	7.	A	8.	D
9.	D	10.	D	11.	D	12.	C
13.	B	14.	C	15.	D	16.	D
17.	D	18.	A	19.	A	20.	C

SECTION – B

1.	10	2.	5	3.	2	4.	45
5.	17	6.	52	7.	27	8.	100
9.	4	10.	18				

PART – B (CHEMISTRY)

SECTION - A

1.	C	2.	B	3.	D	4.	D
5.	B	6.	D	7.	D	8.	C
9.	A	10.	C	11.	C	12.	A
13.	B	14.	A	15.	C	16.	A
17.	A	18	B	19.	C	20.	D

SECTION - B

1.	4	2.	10	3.	41500	4.	9079
5.	360	6.	60	7.	12	8.	6
9.	42	10.	2				

PART – C (MATHEMATICS)

SECTION - A

1.	B	2.	D	3.	A	4.	B
5.	C	6.	B	7.	D	8.	D
9.	B	10.	D	11.	C	12.	C
13.	D	14.	C	15.	D	16.	C
17.	B	18	A	19.	A	20.	D

SECTION - B

1.	2	2.	43	3.	9	4.	495
5.	25	6.	2039	7.	1080	8.	600
9.	216	10.	120				

FIITJEE

Solutions to JEE (Main)-2023

PART – A (PHYSICS)

SECTION – A

Sol1. $g\big|_{h=R} = \dfrac{g}{\left(1+\dfrac{R}{R}\right)^2} = \dfrac{g}{4}$

As we know that $T_P = 2\pi\sqrt{\dfrac{\ell}{g_P}}$

$\Rightarrow T_P = 2\pi \times 2\sqrt{\dfrac{\ell}{g}} = 2T$

Sol2. $v_{av} = \dfrac{x+x}{\dfrac{x}{v_1}+\dfrac{x}{v_2}} \Rightarrow v_{av} = \dfrac{2}{\dfrac{1}{v_1}+\dfrac{1}{v_2}} \Rightarrow v_{av} = \dfrac{2v_1v_2}{v_1+v_2}$

Sol3. $\left({}^{16}_{8}O\right)$ & $\left({}^{4}_{2}He\right)$.

Sol4. $\hat{v} = -\hat{k}$ and $\hat{E} = \hat{j}$

$\hat{v} = \hat{E} \times \hat{B} \Rightarrow \hat{j} \times \hat{B} = -\hat{k} \Rightarrow \hat{B} = \hat{i}$

Sol5. Let the length of wire is ℓ, so

$R = \dfrac{\rho\ell}{A} = \dfrac{V}{I} = \dfrac{3.4}{2} = 1.7 \Rightarrow \dfrac{1.7\times10^{-8}\times\ell}{A} = 1.7 \Rightarrow \dfrac{\ell}{A} = 10^8$ .....(1)

$m = A\ell d \Rightarrow 8.92\times10^{-3} = A\times\ell\times8.92\times10^3 \Rightarrow A\times\ell = 10^{-6}$ .....(2)

Multiplying equations (1) and (2) , we have

$\ell^2 = 10^2 \Rightarrow \ell = 10\,m$

Sol6. $H = \dfrac{B}{\mu_0} nI = \dfrac{1200}{2}\times2 = 1.2\times10^3$ A / m

Sol7. $\lambda_1 = \dfrac{h}{\sqrt{2m_e\times20}}$ and $\lambda_2 = \dfrac{h}{\sqrt{2m_e\times40}}$

$\Rightarrow \dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{40}{20}} \Rightarrow \lambda_2 = \dfrac{\lambda_1}{\sqrt{2}}$

Sol8. $x_5 = \dfrac{5\times600\times10^{-9}\,x\,1}{d} = 5\times10^{-2} \Rightarrow d = \dfrac{5\times600\times10^{-9}\,x\,1}{5\times10^{-2}} = 60\ \mu m$

Sol9. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$

Sol10. Basic concept of diode

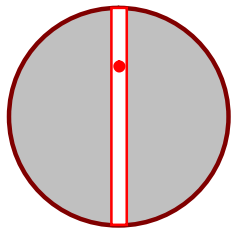
Sol11. Band width = 2 X frequency of message signal = 10 k Hz

Sol12. $\omega = \dfrac{1}{\sqrt{2\times 8LC}} = \dfrac{1}{4\sqrt{LC}} = \dfrac{\omega_0}{4}$

Sol13. When we release the particle in tunnel it will perform SHM whose time period will be

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6400\times 10^3}{10}} = 2\pi\times 800 \text{ sec} = \frac{2\pi\times 800}{60}\text{sec} \approx 84 \text{ min}$$

$$\Rightarrow T \approx 1\text{hour } 24 \text{ min}$$



Sol14. $\dfrac{1-T_2}{T_1} = \dfrac{1}{2} \Rightarrow \dfrac{1}{2} = \dfrac{T_2}{600} \Rightarrow T_2 = 300\text{ K}$

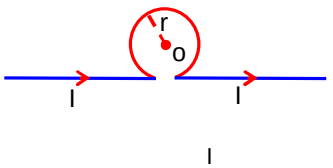
$$1-\frac{T_2}{T_1'} = \frac{7}{10} \Rightarrow \frac{3}{10} = \frac{300}{T_1'} \Rightarrow T_1' = 1000$$

Sol15. $-K(T-S) = \dfrac{dQ}{dt} \Rightarrow \dfrac{12}{2} = -K(92-22) \Rightarrow 6 = -K(70) \Rightarrow \dfrac{6}{70} = -K$

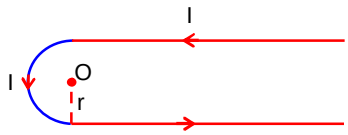
$$\Rightarrow \frac{6}{70}(72-22) = \frac{6}{t}. \Rightarrow t = \frac{70}{50} = 1.4\text{min}$$

Sol16. Basic concept unit and dimension

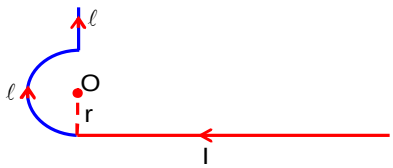
Sol17.



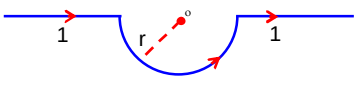
$$B_0 = \frac{\mu_0 I}{2\pi r}[\pi - 1]$$



$$B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 2]$$



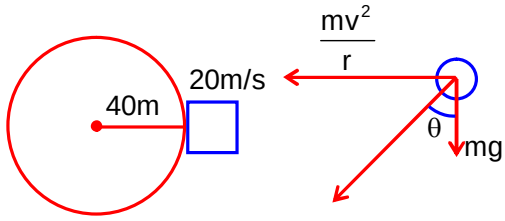
$$B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 1]$$



$$B_0 = \frac{\mu_0}{4}\frac{I}{r}$$

Sol18.

$$\tan\theta = \frac{400}{40\times 10} = 1 \Rightarrow \theta = \frac{\pi}{4}$$



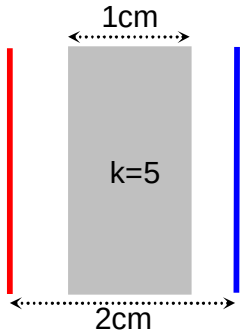
Sol19. $80\times 100 = \frac{T}{2}\times 60 \Rightarrow \frac{80\times 10\times 2}{6}$

$$\Rightarrow 20\times 50 + 80\times 100 = \frac{T}{2}\times 60 \Rightarrow 9000 = \frac{T}{2}\times 60 \Rightarrow T = 300N$$

Sol20.

$$C = \frac{\varepsilon_0 Ak}{d - t\left(1-\frac{1}{k}\right)} = \frac{\varepsilon_0\times 40\times 10^{-4}}{10^{-2}\left[2-1\times\left(1-\frac{1}{5}\right)\right]}$$

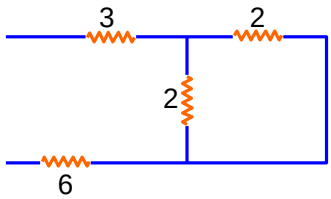
$$\Rightarrow C = \frac{\varepsilon_0\times 40\times 10^{-4}}{10^{-3}\left[2-\frac{4}{5}\right]} = \frac{\varepsilon_0\times 40\times 10^{-4}\times 5}{10^{-2}\times 6} = \frac{10\varepsilon_0}{3}F$$



SECTION – B

Sol1.

$$R_{eq} = 10\Omega$$

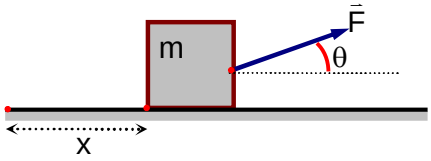


Sol2. $\frac{\Delta\ell}{F} = 1 \Rightarrow Y = \frac{F\ell}{A\Delta\ell} = \frac{\ell}{A} = \frac{62.8\times 10^6}{100\times 3.14\times 2\times 2} = 5\times 10^4\ N/m^2$

Sol3. Using work energy theorem , we can write

$$dK = F\cos\theta dx = 2\cos\theta\times\frac{d\theta}{k}$$

$$\Rightarrow K = F\cos\theta dx = \frac{2}{k}\int_0^{\theta}\cos\theta d\theta = \frac{2}{k}\sin\theta$$

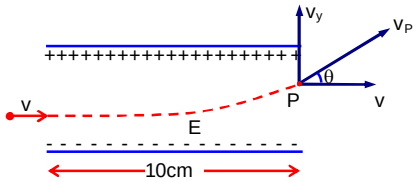


Sol4. $\frac{1}{2}mv^2 = K \Rightarrow v = \sqrt{\frac{2K}{m}} \Rightarrow t = \frac{\ell}{v}$

$$V_y = \frac{eE}{m}t$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{\frac{eEt}{m}}{v} = \frac{\frac{eE}{m}\times\frac{\ell}{v}}{v}$$

$$\Rightarrow \tan\theta = \frac{eE\ell}{mv^2} = \frac{eE\ell}{2k} = \frac{E\ell}{2\times 0.5} = \frac{10\times 0.1}{1} = 1 \Rightarrow \theta = 45^o$$



Sol5. $I_{cm} = \frac{mR^2}{2}$ and $I_{AB} = \frac{mR^3}{2} + m\left(\frac{2}{3}R\right)^2 = mR^2\left(\frac{1}{2}+\frac{4}{9}\right) = \frac{17}{18}mR^2$

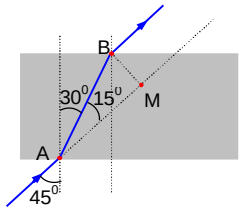
$$\Rightarrow \frac{I_{AB}}{I_{cm}} = \frac{\frac{17}{18}}{\frac{1}{2}} = \frac{17}{9}$$

Sol6. $1\times\sin 45^o = \sqrt{2}\sin r \Rightarrow \sin r = \frac{1}{2} \Rightarrow r = 30^0$

$$\frac{\sqrt{3}}{2} = \cos 30^o = \frac{t}{AB} \Rightarrow AB = \frac{2t}{\sqrt{3}} = \frac{2}{\sqrt{3}}\times\sqrt{3} = 2\text{cm}$$

Lateral displacement $= BM = AB\sin 15^o = 2\times 0.26 = 0.52\,\text{cm}$

$\Rightarrow BM = 50\times 10^{-2}\,\text{cm}$



Sol7. $\frac{1}{\lambda_0} = R_H\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = \frac{5R_H}{36}$ .....(1) , and

$\frac{1}{\lambda} = R_H\left[\frac{1}{2^2}-\frac{1}{4^2}\right] = \frac{3R_H}{16}$ .....(2)

Divide equation (1) by (2), we have

$$\frac{\lambda}{\lambda_0} = \frac{5R_H}{36}\times\frac{16}{3R_H} = \frac{20}{27} \Rightarrow \lambda = \frac{20}{27}\lambda_0$$

Sol8. $f = \frac{\omega}{2\pi} \Rightarrow \omega = 2\pi\times 2\times 10^3 = 4\pi\times 10^3\,\text{rad/s}$

$$x_L - x_C = 0 \Rightarrow \omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega L = \frac{1}{\omega C}$$

$$\Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{16\pi^2\times 10^6\times 62.5\times 10^{-9}} = 0.1\text{ H} = 100\text{ mH}$$

Sol9. $\vec{P}\times\vec{Q} = \left(3\hat{i}+\sqrt{3}\hat{j}+2\hat{k}\right)\times\left(4\hat{i}+\sqrt{3}\hat{j}+2.5\hat{k}\right) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & \sqrt{3} & 2\\ 4 & \sqrt{3} & 2.5\end{vmatrix} = \frac{\sqrt{3}}{2}\hat{i}-\frac{1}{2}\hat{j}-\sqrt{3}\hat{k}$

Unit vector along $\vec{P}\times\vec{Q} = \frac{\frac{\sqrt{3}}{2}\hat{i}-\frac{1}{2}\hat{j}-\sqrt{3}\hat{k}}{\sqrt{\frac{3}{4}+\frac{1}{4}+3}} = \frac{\frac{\sqrt{3}}{2}\hat{i}-\frac{1}{2}\hat{j}-\sqrt{3}\hat{k}}{2} = \frac{1}{4}\left(\sqrt{3}\hat{i}-\hat{j}-2\sqrt{3}\hat{k}\right)$

Sol10. $\phi = \frac{2\pi \times \Delta x}{\lambda} \Rightarrow \frac{2\pi \times 6}{\lambda} = \frac{\pi}{3} \Rightarrow \lambda = 36m$

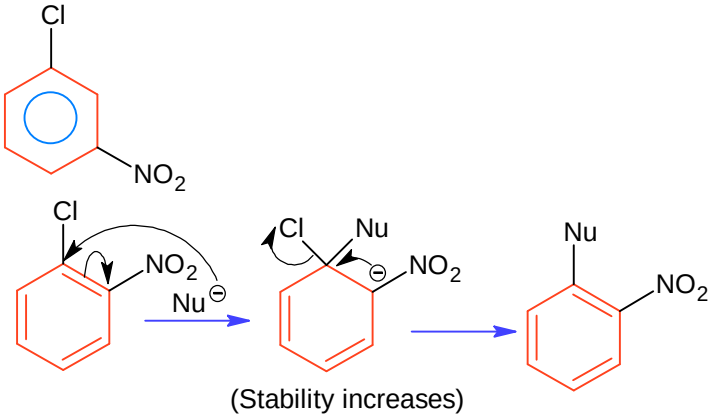
$\Rightarrow v = \lambda \times f = 36 \times 500 \text{ m/s} = 18 \text{ km/s}$

PART – B (CHEMISTRY)

SECTION – A

Sol1. Antibiotics is a antimicrobial substance, it inhibits the growth of microbes. It should not promote growth or survival of microorganism. Low concentration of antibiotics is enough to increase antimicrobial resistance.

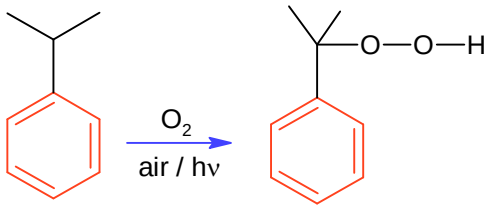
Sol2. When electron withdrawing group present at meta position it will have lowest rate of nuclophilic aromatic substitution.



(Stability increases)

When electron withdrawing group present at ortho & para position it increases the rate of nucleophilic aromatic substitution.

Sol3.



(cummene) (Cummene hydroperoxide)

Cummene hydroperoxide is an intermediate formed during phenol formation from cummene.

Sol4. $2H_2O_2 \longrightarrow H_2O + O_2$

25 'V' means

1 volume of H_2O_2 gives 25 volume H_2O_2

$\therefore M = \frac{V.S}{11.2}$

$= \frac{25}{11.2}$

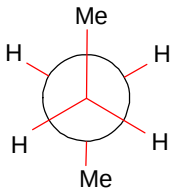
$g/L = \frac{25}{11.2} \times 34 = 75.89 \approx 76$

Or

Strength in g/L $= N \times Eq.wt$

$$= \frac{V.S}{5.6} \times \text{Eq.wt} = \frac{25}{5.6} \times 17 = 75.89$$

Sol5. Anti form is most stable conformer



It has lowest Vanderwaal & torsional strain

Sol6. Photochemical smog formation reactions are:

$$NO_2 \xrightarrow{h\nu} \underset{(Y)}{NO} + \underset{(X)}{[O]}$$

$$[O] \xrightarrow{O_2(A)} \underset{\boxed{B}}{O_3}$$

X = [O] , Y = NO, A = O_2, B = O_3

Sol7.

	Elements	**Colour on Flame**
(A)	K	Violet
(B)	Ca	Brick Red
(C)	Sr	Crimson Red
(D)	Ba	Apple green

Sol8.

Cations	**Group Reagents used**
(A) Pb^{+2}, Cu^{+2}	(i) H_2S gas in presence of dil HCl
(B) Al^{+3}, Fe^{+3}	(ii) NH_4OH in presence of NH_4Cl
(C) Co^{+2}, Ni^{+2}	(iii) H_2S in presence of NH_4OH
(D) Ba^{+2}, Ca^{+2}	(iv) $(NH_4)_2CO_3$ in presence of NH_4OH

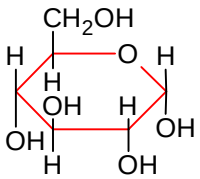
Sol9. Basic strength order of methyl substituted amines are
$2^o > 1^o > 3^o > NH_3$
Basic strength in aqueous solutions decided by (i) I-effect (ii) H-bonding (iii) steric factor.

Sol10.

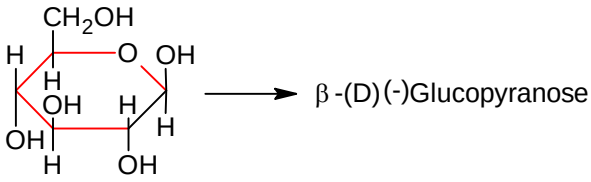
$$\underset{\text{(white P)}}{P_4} + SOCl_2 \longrightarrow \underset{(A)}{PCl_3} + SO_2 + S_2Cl_2$$

$$PCl_3 + H_2O \longrightarrow \underset{(B)}{H_3PO_3} + 3HCl$$

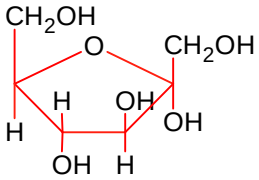
Sol11.



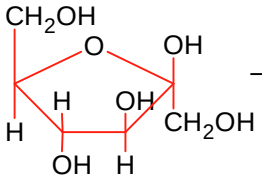
$\longrightarrow$ α -(D) (-)Glucopyranose



β -(D)(-)Glucopyranose



α -(D) (-) Fructofuranose



β -(D) (-) Fructofuranose

Sol12. For Li^{+2}

$$r_2 = x = (r_1)_H \times \frac{x^2}{z^2} = (r_1)_H \times \frac{2^2}{3} = \frac{4(r_1)_H}{3},\ (r_1)_H = \frac{3x}{4}$$

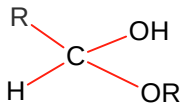
For Be^{+3}

$$r_3 = (r_1)_H \frac{3^2}{4} = (r_1)_H \frac{9}{4}$$

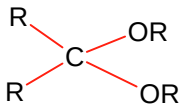
$$= \frac{3x}{4} \times \frac{9}{4} = \frac{27}{16}x$$

Sol13. (+ve) electron gain enthalpy of inert gases because octet is complete. When size increase (+ve) electron gain enthalpy decreases
He (+48 kJ/mole)
Xe (+77 kJ/mole)
Kr(+96 kJ/mole)
Ne(+116 kJ/mole)
He has smallest size therefore highest tendency to accept e^-
Thus correct order is He < Xe < Kr < Ne.

Sol14.



Acetal



Ketal

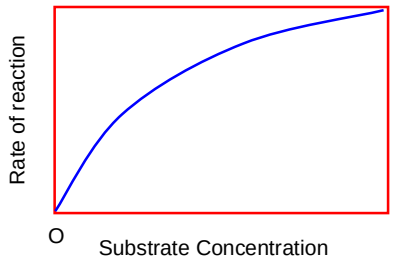
Acetals and ketals are stable in basic medium because R-O group not a good leaving group

Sol15. Atom of $X = \frac{1}{8} \times 4(\text{alternate corner}) + 1 \times 1(\text{centre}) = 3/2$

Atoms of $Y = \left(\frac{1}{3} \times 6\right) \times \frac{1}{2}(\text{At face}) = 1$

$\therefore$ Formula $= X_{3/2}\, Y_1 = X_3\, Y_2$

Sol16. Rate α substrate concentration due to
(i) More & more active sites occupied by substrate
(ii) Higher number of collisions between substrate molecules.
Thus hyperbolic graph.



Sol17.

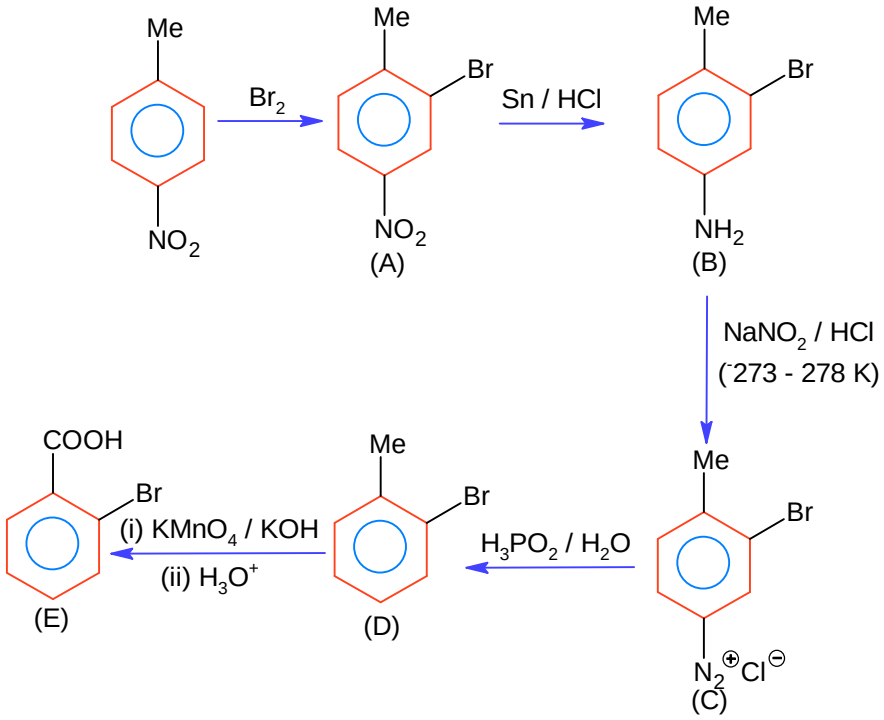
$$A \xrightarrow{NH_4Cl} (B) \xrightarrow[\text{excess } CO_2]{H_2O} (C) \xrightarrow{NaCl} NaHCO_3$$

$$\underset{(A)}{Ca(OH)_2} + NH_4Cl \longrightarrow CaCl_2 + \underset{(B)}{NH_3} + H_2O$$

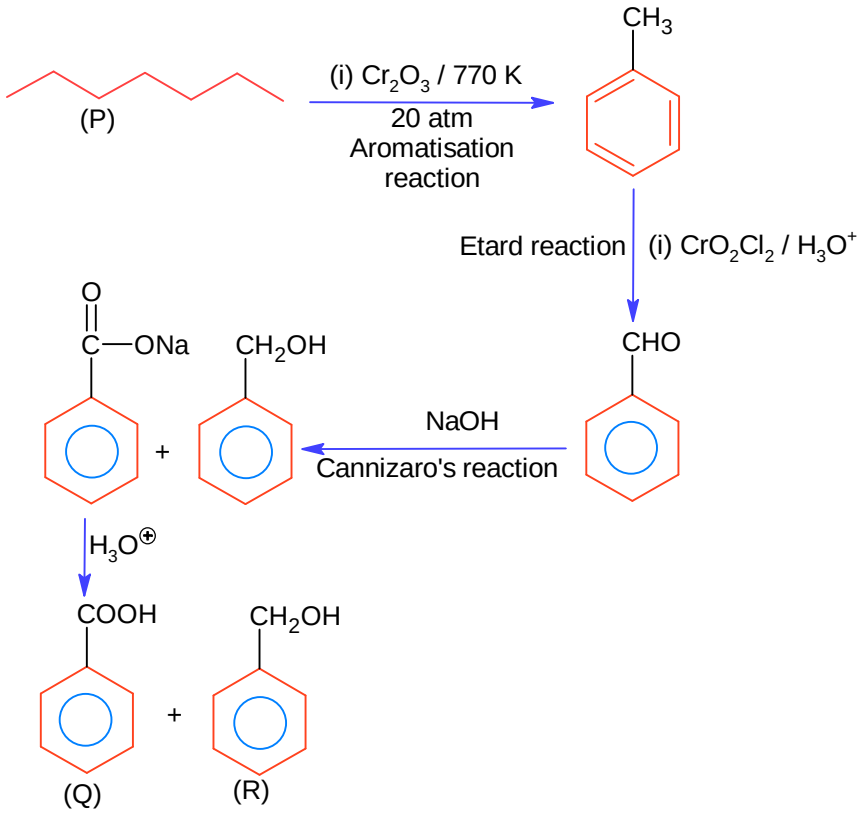
$$\underset{(B)}{NH_3} + H_2O + \underset{(excess)}{CO_2} \longrightarrow \underset{(C)}{NH_4HCO_3}$$

$$\underset{(C)}{NH_4HCO_3} + NaCl \longrightarrow NaHCO_3 \downarrow + NH_4Cl$$

Sol18.



Sol19.



Sol20. $\underset{\text{(Copper Pyrite)}}{CuFeS_2} + O_2 \longrightarrow Cu_2S + FeO + SO_2 + FeS(\text{Little amt})$

$Cu_2S + O_2 \longrightarrow Cu_2O + SO_2$

$FeS + O_2 \longrightarrow FeO + SO_2$

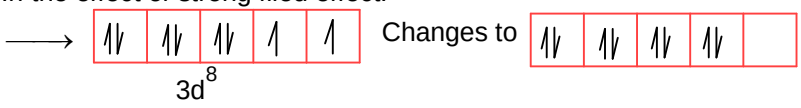
$FeO + SiO_2 \longrightarrow FeSiO_3$

$CaO + SiO_2 \longrightarrow CaSiO_3$ (Not found)

SECTION – B

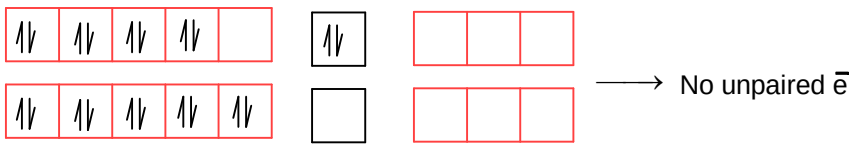
Sol1. (a) $\left[Ni(CN)_4\right]^{-2} \longrightarrow Ni^{+2} = ---- 4s^0 3d^8$

In the effect of strong filed effect.



$\longrightarrow$ No unpaired e^-

(b) $\left[Ni(CO)_4\right] \longrightarrow Ni^o = -4s^2 ---- 3d^8$ In the effect of strong filed effect.



(c) $\left[NiCl_4\right]^{-2} \Rightarrow Ni^{+2} = ---- 4s^0 3d^8$ In the effect of weak filed ligand.



$\longrightarrow$ Unpaired $\bar{e}$ present

(d) $\left[Fe(CN)_6\right]^{-4} \Rightarrow Fe^{+2} = ----4s^0 3d^6$ In the effect of strong filed ligand.

changes to

$\longrightarrow$ No unpaired e^-

(e) $\left[Cu(NH_3)_4\right]^{+2} \longrightarrow Cu^{+2} = ----4s^0 3d^9$ In the effect of strong filed ligand.



$\longrightarrow$ one unpaired e-

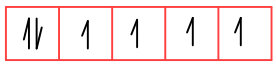
(f) $\left[Fe(CN)_6\right]^{-3} \longrightarrow Fe^{+3} = ----4s^0 3d^5$ In the effect of strong filed ligand.

$\longrightarrow$

changes to

$\longrightarrow$ one unpaired $\bar{e}$

(g) $\left[Fe(H_2O)_6\right]^{+2} \longrightarrow Fe^{+2} = ----4s^0 8d^6$ In the effect of weak filed ligand.



$\longrightarrow$ Unpaired $\bar{e}$ present

Sol2. $\frac{1}{2}H_2(g) + Fe^{+3}_{(aq)} \longrightarrow H^+_{aq} + Fe^{+2}_{(aq)}$

$$E = E^0 - \frac{0.059}{n}\log\frac{\left[Fe^{+2}\right]}{\left[Fe^{+3}\right]\left(P_{H_2}\right)}$$

$$0.712 = (0.771-0) - \frac{0.059}{1}\log\frac{\left[Fe^{+2}\right]}{\left[Fe^{+3}\right](1)}$$

$$0.059\log\frac{\left[Fe^{+2}\right]}{\left[Fe^{+3}\right]} = (0.771-0.712) = 0.059$$

$$\log\frac{\left[Fe^{+2}\right]}{\left[Fe^{+3}\right]} = \frac{0.059}{0.059} = 1 = \log(10)$$

$$\therefore \frac{\left[Fe^{+2}\right]}{\left[Fe^{+3}\right]} = 10$$

Sol3. $\pi = MRT$

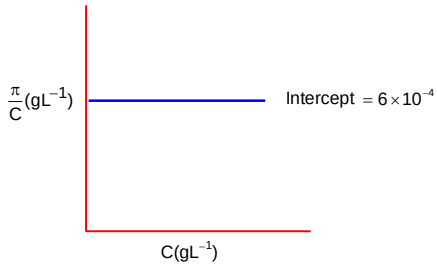
$$= \left(\frac{n}{v}\right)RT = \frac{w}{M\times v}RT$$

$$\pi\times\frac{v}{w} = \frac{RT}{M}$$

$$\left(\frac{\pi}{c}\right) = \frac{RT}{M}$$

Slope = 0

Given graph is incorrect and correct graph is given like above.



Intercept = $\frac{RT}{M} = 6\times10^{-4}$

$M = \frac{RT}{6\times10^{-4}} = \frac{0.083\times300}{6\times10^{-4}} = 41500$

Sol4.

NH_3	+	NH_4Cl		
0.1mole		0.1mole		
		NH_3 +	HCl ⟶	NH_4Cl
		0.1	0.02	0.1
After reaction:-		$(0.1-0.02)$	0	$(0.02+0.1)$
		$=0.08$		$=0.12$

$$\therefore p^{OH} = p^{k_b} + \log\frac{\left[NH_4^+\right]}{\left[NH_3\right]}$$

$$= 4.74 + \log\left(\frac{0.12}{0.08}\right)$$

$$= 4.74 + \log\left(\frac{12}{8}\right)$$

$$= 4.74 + \log\left(\frac{3}{2}\right)$$

$$= 4.74 + (\log3 - \log2)$$

$$= 4.744 + 0.477 - 0.301$$

$$= 4.74 + 0.176 = 4.92$$

$$pH = 14 - 4.92 = 9.08$$

$$= 9080\times10^{-3}$$

Sol5. $C_6H_{12}O_6(s) + 6O_2(g) \longrightarrow 6CO_2(g) + 6H_2O(\ell)$

100g

100g glucose $\equiv 1800\,kJ$ Energy

50% of this energy $= \frac{1800}{2} = 900kJ$

$H_2O(\ell) \longrightarrow H_2O(g)$; $\Delta H_{evaporation} = 45kJ/mole$

$\because$ 45kJ required for 1 mole H_2O evaporation.

$\therefore$ 900kJ required for $\frac{1}{45}\times900$ = 20 mole evaporation.

$n_{H_2O} = 20$

$\therefore \frac{mass}{molar\,mass} = 20$

Mass = $20\times18 = 360\,g$

Sol6. $t_{1/2} = 30\,min$

$K = \frac{0.693}{30}min^{-1}$

$\therefore t_{75\%} = \frac{2.303}{K}\log\left(\frac{Co}{Ct}\right)$

$$= \frac{2.303 \times 30}{0.693} \log\left(\frac{100}{25}\right)$$

$$= \frac{2.303 \times 30}{2.303 \times \log 2} \times \log(4)$$

$$= \frac{30}{\log 2} \times 2\log 2$$

$$= 30 \times 2 = 60\,\text{min}$$

Sol7. Monobasic acid

$d = 1.21\,kg/L = 1.21\,g/m\ell$

Molar mass = 24.2 g / mole

Millimole of acid = millimole of base

$$= 25 \times 0.24 = 6$$

$$\text{Mass of acid} = \frac{\text{m.m}}{1000} \times \text{molar mass}$$

$$= \frac{6 \times 24.2}{1000} g$$

$$\therefore \text{Volume} = \frac{\text{mass}}{\text{density}}$$

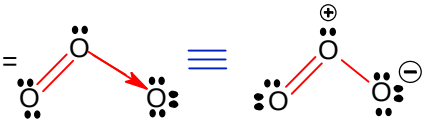
$$= \frac{6 \times 24.2}{1.21} \times 10^{-3} = \frac{145.2}{1.21} \times 10^{-3}$$

$$= 120 \times 10^{-3}\, m\ell$$

$$= 12 \times 10^{-2} m\ell$$

Sol8. Structure of ozone

=



Total number of lone-pairs = 6

Sol9. $$\%\text{of S} = \frac{32}{233} \times \frac{\text{wt. of } BaSO_4}{\text{wt. of organic compound}} \times 100$$

$$= \frac{32}{233} \times \frac{1.4439}{0.471} \times 100$$

$$= \frac{32}{233} \times \frac{144.39}{0.471}$$

$$= \frac{4620.48}{109.743} = 42.10$$

$$\approx 42$$

Sol10. $\mu_s = \sqrt{n(n+2)}\,B.M$

Here n = number of unpaired electrons.

$V^{+3} = [Ar]4s^0 3d^2$ number of u.e = 2

$Cr^{+3} = [Ar]4s^0 3d^3$ number of u.e = 3

$Fe^{+2} = [Ar]4s^0 3d^6$ number of u.e = 4

$Ni^{+3} = [Ar]4s^0 3d^7$ number of u.e = 3

Cr^{+3} & Ni^{+3} have same spin only magnetic moment.

PART – C (MATHEMATICS)

SECTION – A

Sol1. Given $\vec{b}\cdot\vec{c}=0$ and $\vec{a}\times\vec{b}\times\vec{c}=\dfrac{\vec{b}-\vec{c}}{2}$

$$\Rightarrow \vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\cdot\vec{c}\right)\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{c}=\frac{\vec{b}-\vec{c}}{2}=\frac{1}{2}\vec{b}-\frac{1}{2}\vec{c}$$

$$\therefore \vec{a}\cdot\vec{c}=\frac{1}{2} \text{ and } \vec{a}\cdot\vec{b}=\frac{1}{2}$$

$$\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{c}\times\vec{d}\right)=\left(\vec{a}\cdot\vec{c}\right)\left(\vec{b}\cdot\vec{d}\right)-\left(\vec{a}\cdot\vec{d}\right)\left(\vec{b}\cdot\vec{c}\right)$$

$$=\frac{1}{2}\times\frac{1}{2}-\left(\vec{a}\cdot\vec{d}\right)\cdot 0 \qquad \left[\because \vec{b}\cdot\vec{d}=\vec{a}\cdot\vec{b}\right]$$

$$=\frac{1}{4}$$

Sol2. Mean = 10, variance = 4

$$\therefore \frac{\Sigma x_i}{n}=10 \text{ and } \frac{\Sigma x_i^2}{n}=\left(\frac{\Sigma x_i}{n}\right)^2=4$$

$$\Rightarrow \frac{\Sigma x_i^2}{n}=104$$

$$\therefore \frac{\Sigma x_i}{n}+\frac{4}{n}=10\cdot 2 \Rightarrow \frac{4}{n}=0\cdot 2=\frac{1}{5}\Rightarrow n=20$$

New variance = $\dfrac{\Sigma x_i^2-\left(8^2-12^2\right)}{n}-\left(10\cdot 2\right)^2$

$$=104+\frac{20\times 4}{20}-\left(10\cdot 2\right)^2$$

$$=108-\left(10\cdot 2\right)^2=108-104\cdot 04$$

$$=3\cdot 96$$

Sol3. $\displaystyle\lim_{n\to\infty}\frac{(1+2-3)(4+5-6)+.....+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$

$$=\lim_{n\to\infty}\frac{3\sum_{r=1}^{n}(r-1)\left(\sqrt{2n^4+4n+3}+\sqrt{n^4+5n+4}\right)}{\left(n^4-n-1\right)}$$

$$=\operatorname*{Lt}_{n\to\infty}\frac{3\dfrac{n(n-1)}{2}\times n^2\times\left(\sqrt{2}+1\right)}{n^4\left(1-\dfrac{1}{n^3}-\dfrac{1}{n^4}\right)}$$

$$= \frac{3\left(\sqrt{2}+1\right)}{2}$$

Sol4. $P(1+2r, 3+r, 2+2r)$ and $Q(2+\lambda, 2+2\lambda, 3+3\lambda)$

A/q, $\frac{2r-\lambda-1}{1} = \frac{r-2\lambda+1}{-1} = \frac{2r-3\lambda-1}{-2}$

$\Rightarrow r = \lambda = 3$

$\therefore P(7,6,8)$ and $Q(5,8,12)$

$\therefore PQ = \sqrt{4+4+16} = \sqrt{24} = 2\sqrt{6}$

Sol5. Let $\vec{b} = \lambda\vec{a} + \mu\hat{j}$ [$\because$ it passes through y-axis]

$\vec{a}\cdot\vec{b} = \lambda|\vec{a}|^2 + \mu\,\vec{a}\cdot\hat{j}$

$0 = 6\lambda + 2\mu \Rightarrow \mu = -3\lambda$

$\therefore \vec{b} = \lambda\vec{a} - 3\lambda\hat{j}$

$\vec{b} = \lambda\left(-\hat{i} - \hat{j} + \hat{k}\right)$

$|\vec{b}| = \lambda\sqrt{3}$

$\because |\vec{a}| = |\vec{b}|$

$\sqrt{6} = \lambda\sqrt{3}$

$\therefore \lambda = \pm\sqrt{2}$

$\therefore \vec{b} = -5\hat{i} + 4\hat{j} + 5\hat{k}$

$\therefore \left(3\vec{a} + \sqrt{2}\vec{b}\right)\cdot\frac{\vec{c}}{|\vec{c}|} = \frac{-25+16+15}{\sqrt{50}} = \frac{6}{5\sqrt{2}} = \frac{3\sqrt{2}}{5}$

or $\vec{b} = -\hat{i} + 8\hat{j} + \hat{k}$

$\therefore \left(3\vec{a} + \sqrt{2}\vec{b}\right)\cdot\frac{\vec{c}}{|\vec{c}|} = \frac{-5+32+3}{\sqrt{50}} = \frac{30}{5\sqrt{2}} = 3\sqrt{2}$

Sol6. $f(x) = \frac{1}{1-e^{-x}} = \frac{e^x}{e^x-1}$

$f'(x) = \frac{e^x\left(e^x-1\right) - e^x\cdot e^x}{\left(e^x-1\right)^2} = \frac{e^{2x} - e^x - e^{2x}}{\left(e^x-1\right)^2}$

$= \frac{-e^x}{\left(e^x-1\right)} < 0$

$f'(x) < 0$. Hence $f(x)$ decreasing in $(0,1)$.

$g(x) = \frac{1}{1-e^x} - \frac{e^x}{e^x-1} = \frac{1}{1-e^x} + \frac{e^x}{1-e^x}$

$g'(x) = -f'(-x) - f'(x) = \frac{2e^x}{\left(e^x-1\right)^2} > 0$

$g'(x) > 0$. Hence $g(x)$ is increasing function. Hence it is also one-one function.

Sol7. $(z-z_1)(\bar{z}-\bar{z}_1)-(z-z_2)(\bar{z}-\bar{z}_2)=|2+3i-3-4i|^2$

$\Rightarrow (1-i)z+(1+i)\bar{z}=14$

$\Rightarrow x+y=7$

$\therefore$ sum of intercepts on axes = 14

Sol8. $|A|=\begin{vmatrix} \frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & \frac{2\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{3\log z}{\log z} \end{vmatrix}$

$$\frac{\log x \log y \log z}{\log x \log y \log z}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{vmatrix}=2$$

$$\left|\text{adj}\left(\text{adj}A^2\right)\right|=\left|\text{adj}\left(A^2\right)\right|^2=\left(\left|A^2\right|^2\right)^2=|A|^8=2^8$$

Sol9. $a+b=66$

$p=ab=a(66-a)=66a-a^2$

$\frac{dp}{da}=66-2a$

$\frac{dp}{da}=0, a=33$

$\therefore M=a(66-a)=6 \qquad 33\times 33=33^2$

$S=\left\{x^2-66x+605\le 0\right\}$

$\Rightarrow S=\left\{x\in[11,55]\right\}\Rightarrow n(S)=45$

$A=\left\{x: x \text{ is multiple of } 3\right\}$

$n(A)=15$

$\therefore P(A)=\frac{15}{45}=\frac{1}{3}$

Sol10. $f(x)=\int\frac{2x\,dx}{(x^2+1)(x^2+3)}$

$=\int\frac{dt}{(t+1)(t+3)}$, where $x^2=t\Rightarrow 2xdx=dt$

$=\frac{1}{2}\int\left(\frac{1}{t+1}-\frac{1}{t+3}\right)dt$

$f(x)=\frac{1}{2}\left[\ell n(t+1)-\ell n(t+3)\right]+\frac{c}{2}$

$f(x)=\frac{1}{2}\left(\ell n(x^2+1)-\ell n(x^2+3)\right)+\frac{c}{2}$

$f(3)=\frac{1}{2}(\ell n10-\ell n12)+\frac{c}{2}$

$$\frac{1}{2}(\ell n5-\ell n6)=\frac{1}{2}(\ell n10-\ell n12)+\frac{c}{2}$$

$$=\ell n\left(\frac{10}{12}\right)+c$$

$$c=0$$

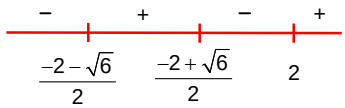
$$f(x)=\frac{1}{2}\left(\ell n\left(x^2+1\right)-\ell n\left(x^2+3\right)\right)$$

$$f(4)=\frac{1}{2}(\ell n17-\ell n19)$$

Sol11. $f(x)=2x^4-18x^2+8x+12$

$$f'(x)=8x^3-36x+8$$

For maximum/minimum, $f'(x)=0, \therefore x=2, \frac{-2+\sqrt{6}}{2}, \frac{-2-\sqrt{6}}{2}$



$$\therefore \ f_{max}=12\sqrt{6}-\frac{33}{2} \text{ at } x=\frac{-2+\sqrt{6}}{2}$$

Sol12. $\frac{dy}{dx}=\frac{y}{x}\left(1+xy^2\left(1+\ell nx\right)\right), x>0$

$$\frac{1}{y^3}\frac{dy}{dx}=\frac{1}{xy^2}+1+\ell nx$$

put $\frac{1}{y^2}=t \Rightarrow -\frac{2}{y^3}\frac{dy}{dx}=\frac{dt}{dx}$

$$\frac{1}{y^3}\frac{dy}{dx}-\frac{1}{xy^2}=1+\ell nx, \frac{-1}{2}\frac{dt}{dx}-\frac{t}{x}=1+\ell nx$$

I.F = x^2

$$tx^2=-2\int x^2\left(1+\ell nx\right)dx$$

$$\frac{x^2}{y^2}=-\frac{2x^3}{3}-\frac{2x^3}{3}\ell nx+\frac{2}{9}x^3+c$$

Put x = 1, y = 1, we get $c=\frac{5}{3}$

$$\therefore \frac{y^2}{9}=\frac{x^2}{5-2x^3\left(2+\ell nx^3\right)}$$

Sol13. $x^2+y^2-2x=0$

$$(x-1)^2+y^2=1$$

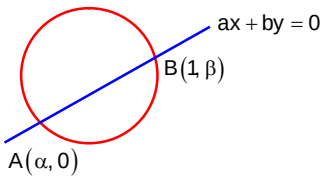
A lies on circle,

$\therefore \alpha^2-2\alpha=0$

$\alpha=0, 2$

B lies on the circle, $1+\beta^2-2=0$

$\beta=\pm1$



$\therefore A(0,0)$ and $B(1,1)$

$\therefore$ Equation of circle as AB diameter is $x^2+y^2-x-y=0$ .....(i)

$\therefore$ centre of (i) $\left(\frac{1}{2},\frac{1}{2}\right)$ and radius = $\frac{1}{\sqrt{2}}$, image of centre of this circle is $\left(-\frac{5}{2},\frac{-5}{2}\right)$.

$\therefore$ equation of required circle is $x^2+y^2+5x+5y+12=0$

Sol14. for $\Delta=\begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$

$\Delta = a\left(15a^2+31a+37\right)$ for infinite solution, $\Delta=0$

$a=0$,

$\therefore S_1 = R-\{0\}$

Hence system has infinite solution but $a \in R-\{0\}$

$\therefore S_2=\phi$

Sol15. $T_{r+1} = {}^{10}C_r x^r = {}^{10}C_{10-r}\ x^{10-r}$

$\therefore a_r = {}^{10}C_{10-r} = {}^{10}C_r$

$a_{r-1} = {}^{10}C_{r-1}$

$$\sum_{r=1}^{10} r^3\left(\frac{a_r}{a_{r-1}}\right)^2 = \sum_{r=1}^{10} r^3\left(\frac{{}^{10}C_r}{{}^{10}C_{r-1}}\right)^2$$

$$= \sum_{r=1}^{10} r^3\left(\frac{11-r}{r}\right)^2 = \sum_{r=1}^{10} r(1-r)^2$$

$$= \sum_{r=1}^{10} r\left(r^2-22r+121\right) = \sum_{r=1}^{10}\left(r^3-22r^2+121r\right)$$

$=1210$

Sol16. Equation of required line is

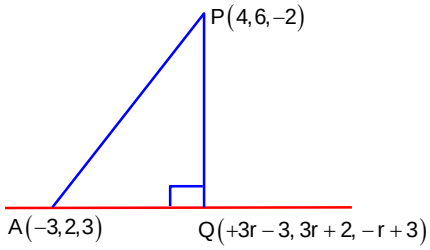
$$\frac{x+3}{3}=\frac{y-2}{3}=\frac{y-3}{-1}=r$$

D. ratios of PQ :

$3r-7, 3r-4, -r+5$

A/q, $3(3r-7)+3(3r-4)-1(-r+5)=0$

$\Rightarrow r=2$ and $PQ=\sqrt{14}$



IInd Method

$\overrightarrow{AP} = 7\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{b} = 3\hat{i} + 3\hat{j} - \hat{k}$

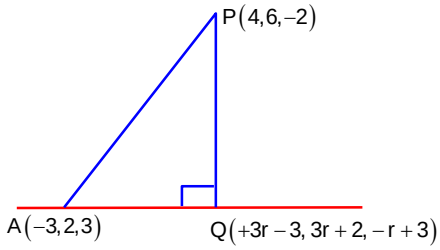
Projection $\overrightarrow{AP}$ on $\vec{b} = \overrightarrow{AP} \cdot \vec{b}$

$= \left(7i + 4\hat{j} - 5\hat{k}\right) \cdot \dfrac{\left(3i + 3\hat{j} - \hat{k}\right)}{\sqrt{19}}$

$\dfrac{38}{\sqrt{19}} = 2\sqrt{19}$

$\therefore PQ^2 = AP^2 - AQ^2 = 90 - 76 = 14$

$\therefore PQ = \sqrt{14}$



Sol17. Equation of line is $y = mx + c$

solving with $y^2 = \dfrac{1}{2}x$ and $y^2 = x - 1$

we get $m = \dfrac{1}{2\sqrt{2}}$ and $c = \dfrac{\sqrt{2}}{4}$

$\therefore$ line is $\sqrt{2}x - 4y + \sqrt{2} = 10$

Length of perpendicular from $\left(6, -2\sqrt{2}\right)$ is $\dfrac{\left|6\sqrt{2} + 8\sqrt{2} + \sqrt{2}\right|}{\sqrt{18}} = \dfrac{15\sqrt{2}}{3\sqrt{2}} = 5$

Sol18. $f(x) \int_0^2 e^{|x-t|} dt$

Case I :

$x < 0,$

$f(x) = \int_0^2 e^{t-x} dt = e^{-x} \int_0^2 e^t dt = e^{-x}\left(e^2 - 1\right)$

$\therefore f_{min} = e^2 - 1$

Case II :

$0 \le x \le 2,$

$f(x) = \int_0^2 e^{x-t} dt = \int_0^x e^{x-t} dt + \int_x^2 e^{t-x} dt$

$= e^x + e^{2-x} - 2$

$f_{min} = 2e + e - 2 = 2(e - 1)$

Case III :

$x > 2$

$f(x) = \int_0^2 e^{x-t} dt = -e^x\left(e^{-t}\right)_0^2 = -e^x\left(e^{-2} - 1\right)$

$\therefore f_{min} = e^2 - 1$

Sol19. $f(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)\left(1+x^{16}\right)$

$f(x) = \dfrac{1 - x^{32}}{1 - x}$

$$f'(x)=\frac{-32x^{31}(1-x)+\left(1-x^{32}\right)}{(1-x)^2}=\frac{-32x^{31}}{1-x}+\frac{1-x^{32}}{(1-x)^2}$$

$$f'(-1)=\frac{+32\times 1}{2}+0=16$$

$$f''(x)=-32\left[\frac{31x^{30}(1-x)+x^{31}\times 1}{(1-x)t^2}\right]-\frac{(1-x)^2\,32x^{31}-\left(1-x^{32}\right)2(1-x)}{(1-x)^4}$$

$$f''(-1)=-32\frac{\left[31\times 2-1\times 1\right]}{4}+\frac{4\times 32+0}{16}$$

$$=\frac{-32\times 61\times 30}{4}+8$$

$$=-488+8=-480$$

$$f'(-1)-f''(-1)=16+480=496$$

Sol20.

p	q	$\sim p$	$\sim q$	$p\wedge\sim q$	$p\Rightarrow\sim q$	$(p\wedge(\sim q))\Rightarrow(p\Rightarrow(\sim q))$
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Hence it is tautology.

SECTION – B

Sol1. Case-1

$-1<x<0$

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3},\ -1<x<1,\ x\neq 0$$

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\pi+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{3}$$

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)=-\frac{2\pi}{3}$$

$$2\tan^{-1}x=-\frac{\pi}{3}$$

$$x=\tan\left(-\frac{\pi}{6}\right)=\frac{-1}{\sqrt{3}}$$

Case-2

$0<x<1$

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{3}$$

$$\tan^{-1}x=\frac{\pi}{12}$$

$$x=2-\sqrt{3}$$

$$\text{Sum}=-\frac{1}{\sqrt{3}}+2-\sqrt{3}=2-\frac{4}{\sqrt{3}}$$

$\therefore\ \alpha=2$

Sol2. $S = \{1, 2, 3, 5, 7, 10, 11\}$

No. of element of 3k type = 1
No. of elements of 3k + 1 = 3
No. of elements of 3k + 2 type = 3
No. of subset of 1 element = 1
No. of subset of 2 elements = 1 of (3k + 1) type and 1 of (3k + 2) type = 9
No. of subset of 3 elements = 9 + 1 +1 = 11
No. of subset of 4 elements = 1 + 1 + 9 = 11
No. of subset of 5 elements = 9
No. of subset of 6 elements = 1
No. of subset of itself = 1

Sol3. Total = 1 + 9 + 11 + 11 + 9 + 1 + 1 =43

$x - 2y - z - 5 = 0$ ……..(i)

$x + y + 3z - 5 = 0$ ……(ii)

$\left.\begin{aligned} x + y + 2z - 7 = 0 \\ 2x + 3y + z - 2 = 0 \end{aligned}\right\}$ ……(iii)

Let the equation of plane is $x - 2y - z - 5 + \lambda(x + y + 3z - 5) = 0$

$\Rightarrow (1+\lambda)x + (-2+\lambda)y + (-1+3\lambda) - 5 - 5\lambda = 0$ ……(iv)

It's parallel to the line is (iii)

$\therefore$ vector along the line

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = \hat{i}(-5) + \hat{j}(3) + \hat{k}(1)$$

$= -5\hat{i} + 3\hat{j} + \hat{k}$

$\therefore -5(1+\lambda) + 3(-2+\lambda) + 1(-1+3\lambda) = 0$

$\Rightarrow \lambda = 12$

$\therefore$ From (iv) $13x + 10y + 35z = 65$

Distance = $\dfrac{26 + 20 - 35 + 16}{\sqrt{9}} = 9$

Sol4. $A_1 = A + (n-1)d$ $\quad \therefore a = A + 6d$ $\quad b - a = 2d + 1$

$A_2 = A + 1 + (m-1)d$ $\quad b = A + 1 + 8d$ $\quad c - b = 8d + 1$

$A_3 = A + 2 + (1-1)d$ $\quad c = A + 2 + 16d$ $\quad c - a = 10d + 2$

$a(0) + 7(c - 2b) + 1(34b - 17c) + 70 = 0$

$20b - 10c + 70 = 0$

$2b - c + 7 = 0 \Rightarrow c = 2b + 7$

$2b - a - c = -6d$

$2b + 6d = a + c$

$c - 7 + 6d = a + c$

$a = 6d - 7 \Rightarrow 29 + 7 = 6d \Rightarrow d = 6$

$\therefore a = 29, b = 42, c = 91$

$$s = \frac{20}{2}\left[2 \times 20 + 19 \times \frac{1}{2}\right] = 495$$

Sol5. $\dfrac{\log_2\left(9^{2\alpha-4}+13\right)}{\frac{5}{2}3^{2\alpha-4}+1}=2\Rightarrow\dfrac{9^{2\alpha-4}+13}{5\cdot3^{2\alpha-4}+2}=\alpha$

$9^{2\alpha-4}+13=5\cdot3^{2\alpha-4}+2$

Put $3^{2\alpha-4}=t$

$t^2+13=10t+4$

$t^2-10t+9=0\Rightarrow t=1,9$

$\therefore t=1\Rightarrow 3^{2\alpha-4}=3^0\Rightarrow\alpha=2$

$t=9\Rightarrow 3^{2\alpha-4}=3^2\Rightarrow\alpha=3$

$\therefore x^2-50x+25\beta=0\Rightarrow 2500-4\cdot25\beta\geq0$

$\Rightarrow\beta\leq25\Rightarrow\beta=25$

Sol6. $f(x)=ax-3$ and $g(x)=x^b+c,\ x\in R$

$(fog)^{-1}(x)=\left(\dfrac{x-7}{2}\right)^{1/3}$

$f(g(x))=a(x^b+c)-3$

$f^{-1}(g(x))=\left(\dfrac{x+3-ac}{a}\right)^{1/b}=\left(\dfrac{x-7}{2}\right)^{1/3}$

$\Rightarrow a=2,b=3,c=5$

$\therefore f(x)=2x-3,g(x)=x^3+5$

$fog(ac)+gof(b)=2007+32=2039$

Sol7. $\left(2x+\dfrac{1}{x^7}+3x^2\right)^5$, general term $=T_{r+1}={}^5C_r\left(2x+\dfrac{1}{x^7}\right)^{5-r}\left(3x^2\right)^r$

$={}^5C_r\cdot{}^{5-r}C_m(2x)^{5-r-m}\left(\dfrac{1}{x^7}\right)^m\cdot3^r x^{2r}$

$={}^5C_r\cdot{}^{5-r}C_m 2^{5-r-m}\cdot3^r\cdot x^{5-r-m+2r-7m}$

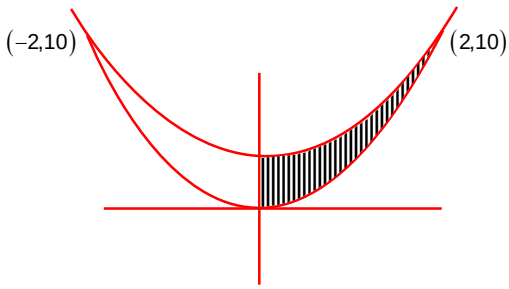
$5+r-8m=0\Rightarrow m=\dfrac{5+r}{8}$

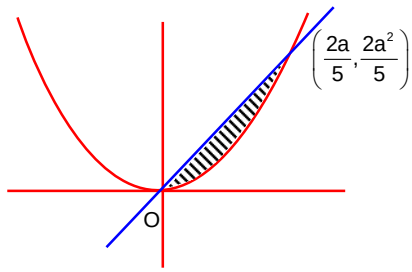
$r=3,m=1$

$\therefore$ constant term $={}^5C_3\cdot{}^2C_1\cdot2\cdot3^3=10\times2\times2\times27=1080$

Sol8. $2y=5x^2\Rightarrow x^2=\dfrac{2}{5}y$ ……..(i)

$x^2-y+6=0\quad x^2=y-6$ …….(ii)





$\therefore$ Area $= \int_{-2}^{2}\left[\left(x^2+6\right)-\frac{5}{2}x^2\right]dx$

$= 2\int_{0}^{2}\left(6-\frac{3}{2}x^2\right)dx = 2\left(6x-\frac{3}{2}\cdot\frac{x^3}{3}\right)_0^2$

$= 2\left(12-\frac{8}{2}\right) = 2\times 8 = 16$

Area $= \int_{0}^{2a/5}\left(ax-\frac{5}{2}x^2\right)dx = \left[\frac{ax^2}{2}-\frac{5}{2}\cdot\frac{x^3}{3}\right]_0^{2a/5}$

$= \frac{2a^3}{75}$

A/q, $\frac{2a^3}{75} = 16 \Rightarrow a^3 = 600$

Sol9. Let $H: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$e = \frac{\sqrt{5}}{2}$

Equation of tangent at P is $\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta = 1$

Equation of normal at P is $\frac{x\tan\theta}{b}+\frac{y\sec\theta}{a} = \frac{a\sec\theta\tan\theta}{b}+\frac{b\tan\theta\sec\theta}{a}$

$= \frac{\left(a^2+b^2\right)\sec\theta\tan\theta}{ab}$

$ax\cos\theta+by\cot\theta = a^2+b^2$

$+\frac{a\cos\theta}{b\cot\theta} = +\sqrt{2} \Rightarrow \sin\theta = \sqrt{2}\frac{b}{a} = \sqrt{2}\times\frac{1}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}$

$\therefore$ equation of hyperbola is $\frac{x^2}{36}-\frac{y^2}{9}=1$

$\therefore$ equation of normal is $\sqrt{2}x + y = 15$

$Q(0,15)$ and $P\left(6\sqrt{2}, 3\right)$

$d^2 = 72 + 144 = 216$

Sol10. $x \in [1,2,3,....25], y \in [1,2,3,.....25]$

types	5k	5k + 1	5k + 2	5k + 3	5k + 4
	0	1	2	3	4
	10	6	7	8	9
	15	11	12	13	14
	20	16	17	18	19
	25	21	22	23	24
	5	5	5	5	5

Total number = $2\left({}^5C_1 \times {}^5C_1 + {}^5C_1 \times {}^5C_1\right) + {}^5C_1 \times {}^4C_1 = 100 + 20 = 120$