FIITJEE

Solutions to JEE(Main) -2023

Test Date: 25th January 2023 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (1 – 10)*** *contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Match List I with List II

List – I	**List – II**
A. Young's Modulus (Y)	I. $[ML^{-1}T^{-1}]$
B. Co-efficient of viscosity (η)	II. $[ML^{2}T^{-1}]$
C. Planck's Constant (h)	III. $[ML^{-1}T^{-2}]$
D. Work Function (φ)	IV. $[ML^{2}T^{-2}]$

Choose the correct answer from the options gives below :

(A) A – II, B – III, C – IV, D – I
(B) A – I, B – II, C – III, D – IV
(C) A – III, B – I, C – II, D – IV
(D) A – I, B – III, C – IV, D – II

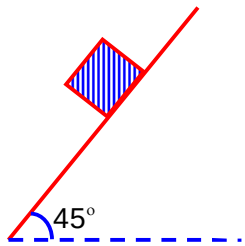
Q2. For a moving coil galvanometer, the deflection in the coil is 0.05rad when a current of 10mA is passed through it, If the torsional constant of suspension wire is 4.0×10^{-5} N m rad^{-1}, the magnetic field is 0.01T and the number of turns in the coil is 200, the area of each turn (in cm^2) is :
(A) 1.0
(B) 0.5
(C) 1.5
(D) 2.0

Q3. Given below are statements :
Statement I : Stopping potential in photoelectric effect does not depend on the power of the light source.
Statement II : For a given metal, the maximum kinetic energy of the photoelectron depends on the wavelength of the incident light.
In the light of above statements, choose the most appropriate answer from the options given below
(A) Both Statement **I** and Statement **II** are incorrect
(B) Both Statement **I** and statement **II** are correct
(C) Statement **I** is incorrect but statement **II** is correct
(D) Statement **I** is correct but statement **II** is incorrect

Q4. The light rays from an object have been reflected towards an observer from a standard flat mirror, the image observed by the observer are :-
A. Real
B. Erect
C. Smaller in size then object
D. Laterally inverted
Choose the most appropriate answer from the options given below:

(A) A,C, and D only
(B) A and D only
(C) B and D only
(D) B and C only

Q5. Consider a block kept on an inclined plane (inclined at 45°) as shown in the figure. If the force required to just push it up the incline is 2 times the force required to just prevent it from sliding down, the coefficient of friction between the block and inclined plane (μ) is equal to :



(A) 0.60 (B) 0.33
(C) 0.50 (D) 0.25

Q6. Two objects are projected with same velocity 'u' however at different angles α and β with the horizontal. If $\alpha + \beta = 90^\circ$, the ratio of horizontal range of the first object to the 2nd object will be :

(A) 1 : 1 (B) 2 : 1
(C) 4 : 1 (D) 1 : 2

Q7. Match List I with List II

List – I		List – II	
A.	Isothermal Process	I.	Work done by the gas decreases internal energy
B.	Adiabatic Process	II.	No change in internal energy
C.	Isochoric Process	III.	The heat absorbed goes partly to increase internal energy and partly to do work
D.	Isobaric Process	IV.	No work is done on or by the gas

Choose the correct answer from the options given below :

(A) A – II, B – I, C – IV, D – III (B) A – II, B – I, C – III, D – IV
(C) A – I, B – II, C – III, D – IV (D) A – I, B – II, C – IV, D – III

Q8. The resistance of a wire is $5\,\Omega$. It's new resistance in ohm if stretched to 5 times of it's original length will be :

(A) 625 (B) 125
(C) 25 (D) 5

Q9. A point charge of $10\,\mu C$ is placed at the origin. At what location on the X-axis should a point charge of $40\,\mu C$ be placed so that the net electric field is zero at x = 2cm on the X-axis ?

(A) x = -4 cm (B) x = 6 cm
(C) x = 4 cm (D) x = 8 cm

Q10. The distance traveled by a particle is related to time t as $x = 4t^2$. The velocity of the particle at $t = 5\,s$ is :-

(A) 40 ms^{-1} (B) 20 ms^{-1}
(C) 25 ms^{-1} (D) 8 ms^{-1}

Q11. A wire of length 1m moving with velocity 8m/s at right angles to a magnetic field of 2T. The magnitude of induced emf, between the ends of wire will be _____

(A) 8 V (B) 12 V
(C) 16 V (D) 20 V

Q12. Match List I with List II

List – I		List – II	
A.	Troposphere	I.	Approximate 65-75km over Earth's surface
B.	E- Part of Stratosphere	II.	Approximate 300km over Earth's surface
C.	F_2- Part of Thermosphere	III.	Approximate 10km over Earth's surface
D.	D- Part of Stratosphere	IV.	Approximate 100km over Earth's surface

Choose the correct answer from the options given below:

(A) A – III, B – IV, C – II, D – I

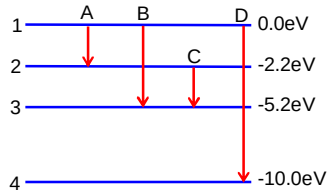
(B) A – III, B – II, C – I, D – IV

(C) A – I, B – II, C – IV, D – III

(D) A – I, B – IV, C – III, D – II

Q13. The energy levels of an atom is shown in figure. Which one of these transitions will result in the emission of a photon of wavelength 124.1nm ?

Given ($h = 6.62 \times 10^{-34}$ Js)



(A) B

(B) D

(C) A

(D) C

Q14. A body of mass m is taken from earth surface to the height h equal to twice the radius of earth (R_e), the increase in potential energy will be:

(g = acceleration due to gravity on the surface of Earth)

(A) $\frac{2}{3}mgR_e$

(B) $\frac{1}{2}mgR_e$

(C) $\frac{1}{3}mgR_e$

(D) $3mgR_e$

Q15. Every planet revolves around the sun in an elliptical orbit :-

A. The force acting on a planet is inversely proportional to square of distance from sun.

B. Force acting on planet is inversely proportional to product of the masses of the planet and the sun.

C. The Centripetal force acting on the planet is directed away from the sun.

D. The square of time period of revolution of planet around sun is directly proportional to cube of semi-major axis of elliptical orbit.

Choose the correct answer from the options given below :

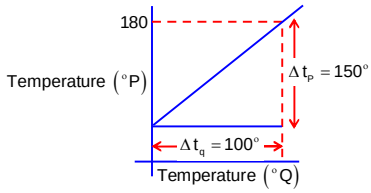
(A) B and C only

(B) A and C only

(C) C and D only

(D) A and D only

Q16. The graph between two temperature scales P and Q is shown in the figure. Between upper fixed point and lower fixed point there are 150 equal divisions of scale P and 100 divisions on scale Q. The relationship for conversion between the two scales is given by :-



(A) $\frac{t_p}{180} - \frac{t_Q - 40}{100}$

(B) $\frac{t_Q}{150} = \frac{t_P - 180}{100}$

(C) $\frac{t_Q}{100} = \frac{t_P - 30}{150}$

(D) $\frac{t_P}{100} = \frac{t_Q - 180}{150}$

Q17. A particle executes simple harmonic motion between x = -A and x = +A. If time taken buy particle to go from x = 0 to $\frac{A}{2}$ is 2s; then time taken by particle in going from x = $\frac{A}{2}$ to A is

(A) 1.5s

(B) 3s

(C) 2s

(D) 4s

Q18. **Match L I with List II**

List – I	**List – II**
A. Gauss's Law in Electrostatics	I. $\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_B}{dt}$
B. Faraday's Law	II. $\oint \vec{B} \cdot d\vec{A} = 0$
C. Gauss's Law in Magnetism	III. $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i_c + \mu_0 \in_0 \frac{d\phi_E}{dt}$
D. Ampere – Maxwell Law	IV. $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\in_0}$

Choose the correct answer from the options given below :

(A) A – II, B – III, C – IV, D – I
(B) A – IV, B – I, C – II, D – III
(C) A – III, B – IV, C – I, D – II
(D) A – I, B – II, C – III, D – IV

Q19. According to law of equipartition of energy the molar specific heat of a diatomic gas at constant volume where the molecule has additional vibrational mode is:-

(A) $\frac{9}{2}R$
(B) $\frac{7}{2}R$
(C) $\frac{3}{2}R$
(D) $\frac{5}{2}R$

Q20. **Statement I :** When a Si sample is doped with Boron, it becomes P type and when doped by Arsenic it becomes N – type semi conductor such that P-type has excess holes and N-type has excess electrons.
Statement II : When such P – type and N – type semi-conductors, are fused to make a junction, a current will automatically flow which can be detected with an externally connected ammeter.

In the light of above statements, choose the most appropriate answer from the options given below

(A) Statement **I** is incorrect but statement **II** is correct
(B) Statement **I** is correct but statement **II** is incorrect
(C) Both Statement **I** and Statement **II** are incorrect
(D) Both Statement **I** and statement **II** are correct

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

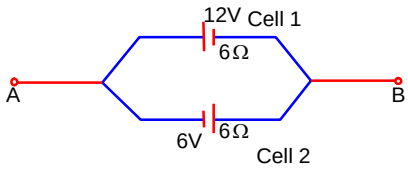
Q1. A train blowing a whistle of frequency 320 Hz approaches an observer standing on the platform at a speed of 66 m/s. The frequency observed by the observer will be (given speed of sound = $330ms^{-1}$)_____Hz.

Q2. Two long parallel wires carrying currents 8A and 15A in opposite directions are placed at a distance of 7cm from each other. A point P is at equidistant from both the wires such that lines joining the point P to the wire are perpendicular to each other. The magnitude of magnetic field at P is _____$\times 16^{-6}$T.

(Given : $\sqrt{2} = 1.4$)

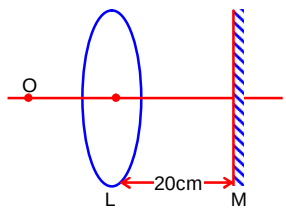
Q3. A series LCR circuit is connected to an AC source of 220V, 50Hz. The circuit contains a resistance $R = 80\Omega$, an inductor of inductive reactance $X_L = 70\Omega$, and a capacitor of capacitive reactance $X_C = 130\Omega$. The power factor of circuit is $\frac{x}{10}$. The value of x is :

Q4. Two cells are connected between points A and B as shown. Cell 1 has emf of 12 V and internal resistance of 3Ω. Cell 2 has emf of 6V and internal resistance of 6Ω. An external resistor R of 4Ω is connected across A and B. The current flowing through R will be _____A.



Q5. A nucleus disintegrates into two smaller parts, which have their velocities in the ratio 3 : 2. The ratio of their nuclear size will be $\left(\frac{x}{3}\right)^{\frac{1}{3}}$. The value of 'x' is :-

Q6. An object is placed on the principal axis of convex lens of focal length 10cm as shown A plane mirror is placed on the other side of lens at a distance of 20cm. The image produced by the plane mirror is 5cm inside the mirror. The distance of the object from the lens is _____cm.



Q7. If a solid sphere of mass 5kg and a disc of mass 4kg have the same radius. Then the ratio of moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be $\frac{x}{7}$. The value of x is _____.

Q8. A capacitor has capacitance $5\mu F$ when it's parallel plates are separated by air medium of thickness d. A slab of material of dielectric constant 1.5 having area equal to that of plates but thickness $\frac{d}{2}$ is inserted between the plates. Capacitance of the capacitor in the presence of slab will be _____μF.

Q9. A spherical drop of liquid splits into 1000 identical spherical drops. If u_i is the surface energy of the original drop and u_f is the total surface energy of the resulting drops, the (ignoring evaporation), $\frac{u_f}{u_i} = \left(\frac{10}{x}\right)$. Then value of x is _____:

Q10. A body of mass 1kg collides head on elastically with a stationary body of mass 3kg. After collision, the smaller body reverses its direction of motion and moves with a speed of 2m/s. The initial speed of the smaller body before collision is _____ ms^{-1}.

PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

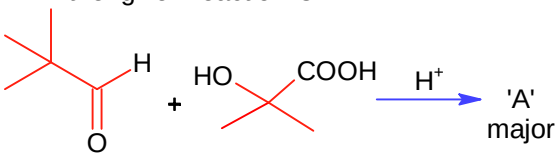
Q1. Which one among the following metals is the weakest reducing agent?
(A) Rb (B) Li
(C) Na (D) K

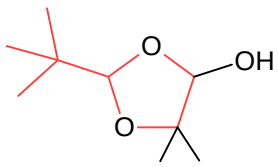
Q2. Match List I with List II

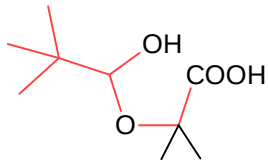
List-I Isomeric pairs		List-II Type of isomers	
A.	Propanamine and N-Methylethanamine	I.	Metamers
B.	Hexan-2-one and Hexan-3-one	II.	Positional isomers
C.	Ethanamide and Hydroxyethanimine	III.	Functional isomers
D.	o-nitrophenol and p-nitrophenol	IV.	Tautomers

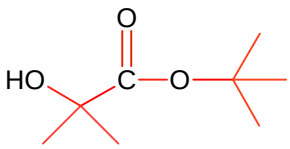
Choose the correct answer from the options given below:
(A) A- II, B-III, C-I, D- IV (B) A-IV, B-III, C-I, D-II
(C) A-III, B-IV, C-I, D-II (D) A-III, B-I, C-IV, D-II

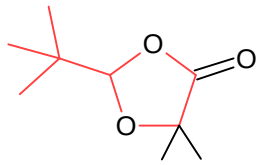
Q3. 'A' in the given reaction is



(A) 

(B) 

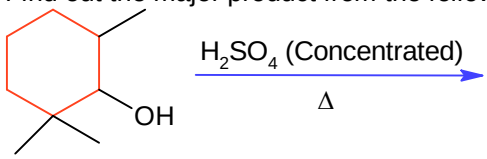
(C) 

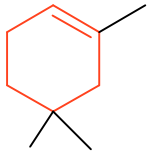
(D) 

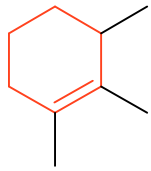
Q4. Which of the following represents the correct order of metallic character of the given elements?
(A) K < Mg < Be < Si (B) Si < Be < Mg < K
(C) Be < Si < Mg < K (D) Be < Si < K < Mg

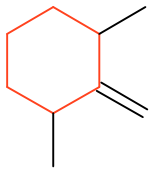
Q5. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**
Assertion A: Carbon forms two important oxides-CO and CO_2, CO is neutral whereas CO_2 is acidic in nature.
Reason R: CO_2 can combine with water in a limited way to form carbonic acid, while CO is sparingly soluble in water.
In the light of the above statements, choose the correct answer from the option given below:
(A) A is not correct but R is correct
(B) Both A and R are correct but R is NOT the correct explanation of A
(C) Both A and R are correct and R is the correct explanation of A
(D) A is correct but R is not correct

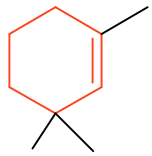
Q6. Find out the major product from the following reaction.



(A) 

(B) 

(C) 

(D) 

Q7. Match List I with List II

List-I		List-II	
A.	Cobalt catalyst	I.	(H_2+Cl_2) production
B.	Syngas	II.	Water gas production
C.	Nickel catalyst	III.	Coal gasification
D.	Brine solution	IV.	Methanol production

Choose the correct answer from the options given below:

(A) A- II, B-III, C-IV, D- I
(B) A-IV, B-III, C-I, D-II
(C) A-IV, B-III, C-II, D-I
(D) A-IV, B-I, C-II, D-III

Q8. A chloride salt solution acidified with dil HNO_3 gives a curdy white precipitate, [A], on addition of $AgNO_3$. [A] on treatment with NH_4OH gives a clear solution B.
A and B are respectively.

(A) AgCl & $(NH_4)[Ag(OH)_2]$
(B) $H[AgCl_3]$ & $[Ag(NH_3)_2]Cl$
(C) $H[AgCl_3]$ & $(NH_4)[Ag(OH)_2]$
(D) AgCl & $[Ag(NH_3)_2]Cl$

Q9. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**
Assertion A: Butylated hydroxy anisole when added to butter increases it shelf life.
Reason R: Butylated hydroxy anisole is more reactive towards oxygen than food.
In the light of the above statements, choose the correct answer from the option given below:
(A) Both A and R are correct but R is NOT the correct explanation of A
(B) A is not correct but R is correct
(C) A is correct but R is not correct
(D) Both A and R are correct and R is the correct explanation of A

Q10. Potassium dichromate acts as a strong oxidizing agent in acidic solution. During this process, the oxidation state changes from

(A) +2 to +1
(B) +6 to +2
(C) +3 to +1
(D) +6 to +3

Q11. The isomeric deuterated bromide with molecular formula C_4H_8DBr having two chiral carbon atoms is.

(A) 2-Bromo-1- deuterobutane
(B) 2- Bromo-3-deuterobutane
(C) 2-Bromo-1-deutero-2- methylpropane
(D) 2- Bromo-2-deuterobutane

Q12. Given below are two statements :
Statement I: In froth floatation method a rotating paddle agitates the mixture to drive air out of it.
Statement II: Iron pyrites are generally avoided for extraction of iron due to environmental reasons.
In the light of the above statements, choose the correct answer from the options given below:
(A) Statement I is true but Statement II is false
(B) Both statement I and Statement II are false
(C) Statement I is false but Statement II is true
(D) Both Statement I and Statement II are true

Q13. Match List I with List II

List-I		List-II	
A.	Aniline	I.	3.25
B.	Ethanamine	II.	3.00
C.	N-Ethylethanamine	III.	9.38
D.	N,N-Diethylethanamine	IV.	3.29

Choose the correct answer from the options given below:
(A) A- III, B-IV, C-II, D- I
(B) A-I, B-IV, C-II, D-III
(C) A-III, B-II, C-IV, D-I
(D) A-III, B-II, C-I, D-IV

Q14. When the hydrogen ion concentration $[H^+]$ changes by a factor of 1000, the value of pH of the solution _____.
(A) Decreases by 2 units
(B) Increases by 1000 units
(C) Decreases by 3 units
(D) Increases by 2 units

Q15. **Statement I:** Dipole moment is a vector quantity and by convention it is depicted by a small arrow with tail on the negative centre and head pointing towards the positive centre.
Statement II: The crossed arrow of the dipole moment symbolizes the direction of the shift of charges in the molecules.
In the light of the above statements, choose the most appropriate answer from the options given below:
(A) Both Statement I and Statement II are incorrect
(B) Statement I is correct but Statement II is incorrect
(C) Both statement I Statement II are correct
(D) Statement I is incorrect but Statement II is correct.

Q16. Given below are two statements; one is labelled as **Assertion A** and the other is labelled as **Reason R**
Assertion A: The alkali metals and their salts impart characteristic colour to reducing flame.
Reason R: Alkali metals can be detected using flame tests.
In the light of the above statements, choose the most appropriates answer from the options given below:
(A) Both A and R are correct but R is NOT the correct explanation of A
(B) A is correct but R is not correct
(C) Both A and R are correct and R is the correct explanation of A
(D) A is not correct but R is correct.

Q17. Match List I with List II

List-I (Name of polymer)		List-II (Uses)	
A.	Glyptal	I.	Flexible pipes
B.	Neoprene	II.	Synthetic wool
C.	Acrilan	III.	Paints and Lacquers
D.	LDP	IV.	Gaskets

Choose the correct answer from the options given below:
(A) A- III, B-II, C-IV, D- I
(B) A-III, B-IV, C-II, D-I
(C) A-III, B-I, C-IV, D-II
(D) A-III, B-IV, C-I, D-II

Q18. What is the mass ratio of ethylene glycol ($C_2H_6O_2$, molar mass = 62 g/mol) required for making 500 g of 0.25 molal aqueous solution and 250 mL of 0.25 molal aqueous solution?
(A) 3:1
(B) 1:2
(C) 2:1
(D) 1:1

Q19. Match List I with List II

List-I (Coordination entity)		**List-II (Wavelength of light absorbed in nm)**	
A.	$[CoCl(NH_3)_5]^{2+}$	I.	310
B.	$[Co(NH_3)_6]^{3+}$	II.	475
C.	$[Co(NH_3)_6]^{3-}$	III.	535
D.	$[Cu(H_2O)_4]^{2+}$	IV.	600

Choose the correct answer from the options given below:
(A) A- IV, B-I, C-III, D- II
(B) A-III, B-I, C-II, D-IV
(C) A-II, B-III, C-IV, D-I
(D) A-III, B-II, C-I, D-IV

Q20. A. Ammonium salts produce haze in atmosphere.
B. Ozone gets produced when atmospheric oxygen reacts with chlorine radicals.
C. Polychlorinated biphenyls act as cleansing solvents.
D. 'Blue baby' syndrome occurs due to the presence of excess of sulphate ions in water.
Choose the correct answer from the option given below:
(A) A and D only
(B) A and C only
(C) B and C only
(D) A, B and C only

SECTION - B

(Numerical Answer Type)

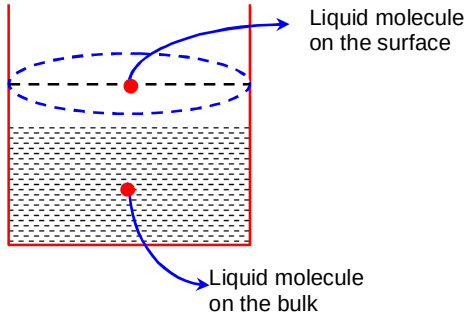
This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. Number of hydrogen atoms per molecule of a hydrocarbon A having 85.8% carbon is____
(Given: Molar mass of A = 84 g mol^{-1})

Q2. Total number of moles of AgCl precipitated on addition of excess of $AgNO_3$ to one mole each of the following complexes $[Co(NH_3)_4Cl_2]Cl$,$[Ni(H_2O)_6]Cl_2$,$[Pt(NH_3)_2Cl_2]$ and $[Pd(NH_3)_4]Cl_2$ is_____.

Q3. Based on the given figure, the number of
(A) Surface tension is the outcome of equal
(B) Surface tension is due to uneven forces
(C) The molecule in the bulk can never come to

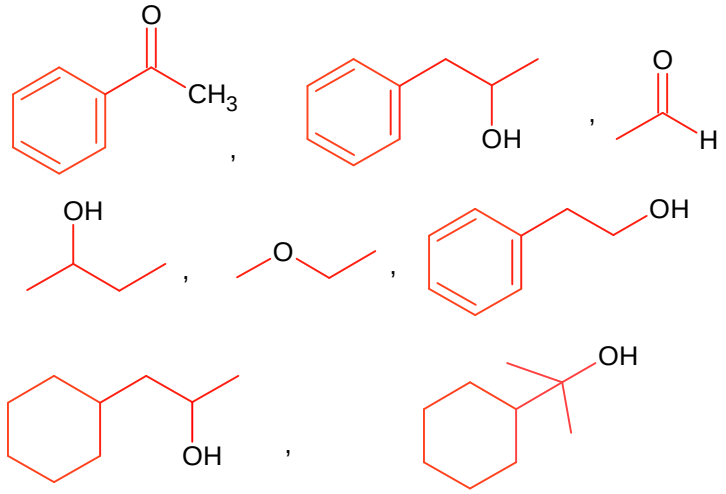
(D) The molecules on the surface are



Q4. The number of pairs of the solutions having the same value of the osmotic pressure from the following is_____.
(Assume 100% ionization)
A. 0.500 M $C_2H_5OH_{(aq)}$ and 0.25 M $KBr_{(aq)}$
B. 0.100 M $K_4[Fe(CN)_6]_{(aq)}$ and 0.100 M $FeSO_4(NH_4)_2SO_{4(aq)}$
C. 0.05 M $K_4[Fe(CN)_6]_{(aq)}$ and 0.25 M $NaCl_{(aq)}$
D. 0.15 M $NaCl_{(aq)}$ and 0.1 M $BaCl_{2(aq)}$
E. 0.02 M $KCl.MgCl_2.6H_2O_{(aq)}$ and 0.05 M $KCl_{(aq)}$

Q5. 28.0L of CO_2 is produced on complete combustion of 16.8 L gaseous mixture of ethene and methane at 25°C and 1 atm. Heat evolved during the combustion process is ____kJ.
Given: $\Delta H_C(CH_4) = -900 kJ\ mol^{-1}$
$\Delta H_C(C_2H_4) = -1400 kJ\ mol^{-1}$

Q6. Number of compound giving (i) red colouration with ceric ammonium nitrate and also (ii) positive iodoform test from the following is_____.



Q7. A first order reaction has the rate constant, $k = 4.6\times10^{-3}\ s^{-1}$. The number of correct statements from the following is / are _____.
Given log 3 = 0.48
A. Reaction completes in 1000s.
B. The reaction has a half-life of 500s
C. The time required for 10% completion is 25 times the time required for 90% completion.
D. The degree of dissociation is equal to $(1-e^{-kt})$
E. The rate and the rate constant have the same unit.

Q8. $Pt(s)\left|H_{2(g)}(1bar)\right|H^{+}_{(aq)}(1M)\left\|M^{3+}_{(aq)}, M^{+}_{(aq)}\right|Pt_{(s)}$

The E_{cell} for the given cell is 0.1115 V at 298 K When $\frac{\left[M^{+}(aq)\right]}{\left[M^{3+}(aq)\right]} = 10^{a}$

The value of a is_____.
Given: $E^{\theta}_{M^{3+}/M^{+}} = 0.2V$
$\frac{2.303RT}{F} = 0.059V$

Q9. The number of given orbitals which have electron density along the axis is_____
$p_x, p_y, p_z, d_{xy}, d_{yz}, d_{xz}, d_{z^2}, d_{x^2-y^2}$

Q10. The number of **incorrect** statement/s from the following is / are_____.
A. Water vapours are adsorbed by anhydrous calcium chloride.
B. There is decrease in surface energy during adsorption.
C. As the adsorption proceeds, ΔH becomes more and more negative.
D. Adsorption is accompanied by decrease in entropy of the system.

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Let $f(x) = 2x^n + \lambda, \lambda \in R, n \in N$ and $f(4) = 133, f(5) = 255$. Then the sum of all the positive integer divisors of $(f(3) - f(2))$ is

(A) 60 (B) 61
(C) 59 (D) 58

Q2. Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i = \sqrt{-1}$.

If $M = A^T B A$, then the inverse of the matrix $AM^{2023}A^T$ is

(A) $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$

Q3. Let A, B, C be 3 x 3 matrices such that A is symmetric and B and C are skew-symmetric. Consider the statements

(S_1) $A^{13}B^{26} - B^{26}A^{13}$ is symmetric

$(S2)$ $A^{26}C^{13} - C^{13}A^{26}$ is symmetric

(A) Both S1 and S2 are false (B) Both S1 and S2 are true
(C) Only S1 is true (D) Only S2 is true

Q4. The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1,3,5,7,9 without repetition, is

(A) 72 (B) 120
(C) 6 (D) 12

Q5. Let $f : R \to R$ be a function defined by

$f(x) = \log_{\sqrt{m}}\left\{\sqrt{2}(\sin x - \cos x) + m - 2\right\}$, for some m, such that the range of f is $[0,2]$. Then the value of m is…….

(A) 3 (B) 5
(C) 2 (D) 4

Q6. The number of functions

$f : \{1,2,3,4\} \to \{a \in : Z|a| \le 8\}$ satisfying $f(n) + \frac{1}{n}f(n+1) = 1, \forall n \in \{1,2,3\}$ is

(A) 2 (B) 1
(C) 3 (D) 4

Q7. Let $y = y(t)$ be a solution of the differential equation $\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$ where, $\alpha > 0, \beta > 0$ and $\gamma > 0$. Then $\lim\limits_{t \to \infty} y(t)$

(A) is 0
(B) is -1
(C) is 1
(D) does not exist

Q8. The integral of $16\int\limits_1^2 \frac{dx}{x^3\left(x^2+2\right)^2}$ is equal to

(A) $\frac{11}{12} - \log_e 4$
(B) $\frac{11}{6} - \log_e 4$
(C) $\frac{11}{6} + \log_e 4$
(D) $\frac{11}{12} + \log_e 4$

Q9. If the four points, whose position vectors are $3\hat{i} - 4\hat{j} + 2\hat{k}, \hat{i} + 2\hat{j} - \hat{k}, -2\hat{i} - \hat{j} + 3\hat{k}$ and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then α is equal to

(A) $\frac{73}{17}$
(B) $-\frac{107}{17}$
(C) $-\frac{73}{17}$
(D) $\frac{107}{17}$

Q10. Let T And C respectively by the transverse and conjugate axes of the hyperbola $16x^2 - y^2 + 64x + 4y + 44 = 0$. Then the area of the region above the parabola $x^2 = y + 4$, below the transverse axis T and on the right of the conjugate axis C is :

(A) $4\sqrt{6} - \frac{28}{3}$
(B) $4\sqrt{6} + \frac{44}{3}$
(C) $4\sqrt{6} - \frac{44}{3}$
(D) $4\sqrt{6} + \frac{28}{3}$

Q11. $\sum\limits_{k=0}^{6} {}^{51-k}C_3$ is equal to

(A) ${}^{52}C_4 - {}^{45}C_4$
(B) ${}^{52}C_3 - {}^{45}C_3$
(C) ${}^{51}C_3 - {}^{45}C_3$
(D) ${}^{51}C_4 - {}^{45}C_4$

Q12. Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}, \vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to

(A) $3\left(\hat{i} + \hat{j} + \hat{k}\right)$
(B) $3\left(\hat{i} - \hat{j} + \hat{k}\right)$
(C) $3\left(\hat{i} + \hat{j} - \hat{k}\right)$
(D) $3\left(\hat{i} - \hat{j} - \hat{k}\right)$

Q13. Let the function $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$ have a maxima for some value of $x < 0$ and a minima for some value of x > 0. Then, the set of all values of p is

(A) $\left(-\frac{9}{2}, \frac{9}{2}\right)$
(B) $\left(\frac{9}{2}, \infty\right)$
(C) $\left(-\infty, \frac{9}{2}\right)$
(D) $\left(0, \frac{9}{2}\right)$

Q14. Let z be a complex number such that $\left|\frac{z-2i}{z+i}\right| = 2, z \neq -i$. Then z lies on the circle of radius 2 and centre

(A) $(0, 2)$ (B) $(2, 0)$

(C) $(0, 0)$ (D) $(0, -2)$

Q15. The equations of two sides of a variable triangle are x = 0 and y = 3, and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is :

(A) $4y^2 - 18y - 3x + 18 = 0$ (B) $4y^2 - 18y + 3x + 18 = 0$

(C) $4y^2 + 18y + 3x + 18 = 0$ (D) $4y^2 - 18y - 3x - 18 = 0$

Q16. If the function $f(x) = \begin{cases} \left(1+|\cos x|\right)\frac{\lambda}{|\cos x|} & , \quad 0 < x < \frac{\pi}{2} \\ \mu & , \quad x = \frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}} & , \quad \frac{\pi}{2} < x < \pi \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda}$ is equal to

(A) 10 (B) 11

(C) $2e^4 + 8$ (D) 8

Q17. The shortest distance between the lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ is

(A) 3 (B) 2

(C) $\frac{5}{2}$ (D) $\frac{3}{2}$

Q18. Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N-2, \sqrt{3N}, N+2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is

(A) 4 (B) 8

(C) 2 (D) 16

Q19. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $(p \to q)\Delta(p \nabla q)$ is a tautology. Then

(A) $\Delta = \wedge, \nabla = \vee$ (B) $\Delta = \wedge, \nabla = \wedge$

(C) $\Delta = \vee, \nabla = \wedge$ (D) $\Delta = \vee, \nabla = \vee$

Q20. The foot of perpendicular of the point $(2,0,5)$ on the line $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ). Then, which of the following is NOT correct?

(A) $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$ (B) $\frac{\alpha}{\beta} = -8$

(C) $\frac{\gamma}{\alpha} = \frac{5}{8}$ (D) $\frac{\beta}{\gamma} = -5$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. Let $a \in R$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is……..

Q2. The remainder when $(2023)^{2023}$ is divided by 35 is……..

Q3. 25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer than a non smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is $\frac{k}{10}$. Then the value of k is……..

Q4. A triangle is formed by X-axis, Y-axis and the line $3x + 4y = 60$. Then the number of points $P(a,b)$ which lie strictly inside the triangle, where a is an integer and b is a multiplier of a, is…….

Q5. For the two positive numbers a, b, if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}$,10 and $\frac{1}{b}$ are in an arithmetic progression, then $16a + 12b$ is equal to…….

Q6. If m and n respectively are the numbers of positive and negative values of θ in the interval $[-\pi, \pi]$ that satisfy the equation $\cos 2\theta \cos\frac{\theta}{2} = \cos 3\theta \cos\frac{9\theta}{2}$ m then mn is equal to……..

Q7. If $\int_{\frac{1}{3}}^{3} |\log_e x| dx = \frac{m}{n}\log_e\left(\frac{n^2}{e}\right)$, where m and n are coprime natural numbers, then $m^2 + n^2 - 5$ is equal to…….

Q8. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 oranges, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is…….

Q9. If the shortest distance between the line joining the points $(1,2,3)$ and $(2,3,4)$, and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is α, then $28\alpha^2$ is equal to…….

Q10. Points $P(-3,2), Q(9,10)$ and $R(\alpha, 4)$ lie on a circle C with PR as its diameter. The tangents to C at the points Q and R intersect at the point S. If S lies on the line $2x - ky = 1$, then k is equal to…….

FIITJEE

KEYS to JEE (Main)-2023

PART – A (PHYSICS)

SECTION – A

1.	C	2.	A	3.	B	4.	C
5.	B	6.	A	7.	A	8.	B
9.	B	10.	A	11.	C	12.	A
13.	B	14.	A	15.	D	16.	C
17.	D	18.	B	19.	B	20.	B

SECTION – B

1.	400	2.	68	3.	8	4.	1
5.	2	6.	30	7.	5	8.	6
9.	1	10.	4				

PART – B (CHEMISTRY)

SECTION - A

1.	C	2.	D	3.	D	4.	B
5.	C	6.	B	7.	C	8.	D
9.	D	10.	D	11.	B	12.	C
13.	A	14.	C	15.	B	16.	D
17.	B	18	C	19.	D	20.	B

SECTION - B

1.	12	2.	5	3.	2	4.	4
5.	847	6.	3	7.	1	8.	3
9.	5	10.	2				

PART – C (MATHEMATICS)

SECTION - A

1.	A	2.	B	3.	D	4.	A
5.	B	6.	A	7.	A	8.	B
9.	A	10.	D	11.	A	12.	A
13.	C	14.	D	15.	B	16.	DROP
17.	B	18	A	19.	D	20.	D

SECTION - B

1.	45	2.	7	3.	9	4.	31
5.	3	6.	25	7.	20	8.	6860
9.	18	10.	3				

FIITJEE

Solutions to JEE (Main)-2023

PART – A (PHYSICS)

SECTION – A

Sol1. A $\rightarrow$ Young modulus

$y = \frac{\text{stress}}{\text{strain}} = \left[ML^{-1}T^{-2}\right]$

B $\rightarrow$ coefficient of viscosity

$F = \eta A \frac{dv}{dx}$

$\eta = \left[ML^{-1}T^{-1}\right]$

C $\rightarrow$ Hanck's constant

hr = Energy

h = $[ML^2 T^{-1}]$

D $\rightarrow$ Work function

$[ML^2 T^{-2}]$

Sol2. Formula = $K\phi = NIAB$

Given $K = 4\times10^{-5}\text{Nm rad}^{-1}$

$\phi = 0.05\text{rad}$

N = 200

$I = 10^{-2}A$

B = 0.01T

$4\times10^{-5}\times0.05 = 200\times10^{-2}\times A\times0.01$

$A = 10^{-4}\text{m}^2 = 1\text{cm}^2$

Sol3. $KE_{max} = \frac{hc}{\lambda} - \phi = eV_s$

ϕ is work function

Sol4. Properties of image formed by plane mirror.

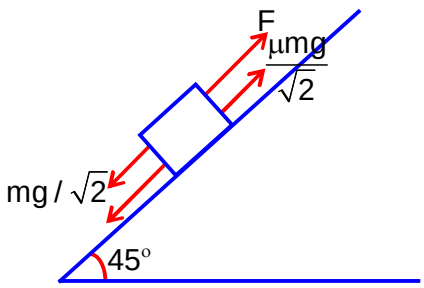
The plane mirror forms virtual , erect , same size and laterally inverted image.

Sol5. Let force required to just prevent it from sliding down = F

$F = \frac{mg}{\sqrt{2}} - \frac{\mu mg}{\sqrt{2}}$ **- (i)**

Force required to push the block = 2F

$2F = \frac{mg}{\sqrt{2}} + \frac{\mu mg}{\sqrt{2}}$ - (ii)



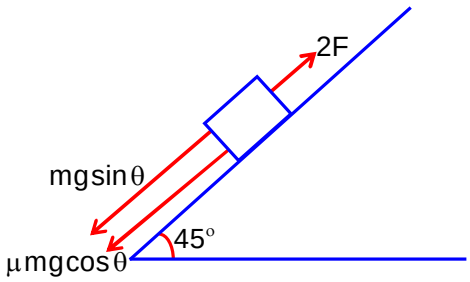
$$\frac{(ii)}{(i)} \Rightarrow 2 = \frac{\frac{mg}{\sqrt{2}} + \frac{\mu mg}{\sqrt{2}}}{\frac{mg - \mu mg}{\sqrt{2}}}$$

$$2 = \frac{1+\mu}{1-\mu}$$

$$2 - 2x = 1 + \mu$$

$$1 = 3\mu$$

$$\frac{1}{3} = \mu$$



Sol6. Ljiven $\alpha + \beta = 90^\circ$

$\therefore \text{Range}_\alpha = \text{Range}_\beta.$

Hence 1 : 1

Sol7. Isothermal process – no change in internal energy Adiabatic work done by gas decreases internal energy

Isochoric – no work done

Sol8. $\frac{\delta L}{A_1} = 5,$

$L \times A_1 = 5L \times A_2$

$\frac{A_1}{5} = A_2$

$\therefore$ **New resistance**

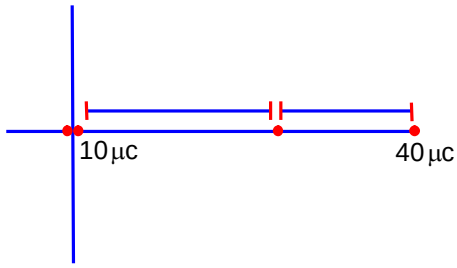
$\frac{\rho \times 5L \times 5}{A_1} = 5 \times 5 \times 5 = 125\Omega$

Sol9. $\frac{10}{4} = \frac{40}{x^2}$

$x^2 = 16$

$x = 4$

$\therefore$ $40\mu c$ **is placed at** $2 + x = 4\text{cm}$ **from origin**



Sol10. $\frac{dx}{dt} = 8t = 8 \times 5 = 40\text{m/s}.$

Sol11. EMF = VBL

$= 1 \times 2 \times 8$

$= 16\text{V}$

Sol12. Troposphere – 10km over Earth's Surface

E – Part of stratosphere - 100 Km over

F_2 of Thermosphere – 300 Km from Earth surface.

Sol13. $\lambda = 124.1\text{nm}$

$\therefore$ Energy $E_2 - E_1 = \dfrac{12\mu_2 \text{ ev nm}}{124.1\text{nm}}$

= 1

D satisfies this case

Sol14. at surface of earth P.E. = mgR

at height 2R P.E. = $\dfrac{-mgR}{3}$

$\therefore$ P.E. gain = $mgR - \dfrac{mgR}{3} = \dfrac{2mgR}{3}$

Sol15. $F = \dfrac{GMm}{r^2}$ and $T^2 \ \alpha \ a^3$

Sol16. Slope of graph $= \dfrac{150}{100}$

$\dfrac{t_P - 30}{t_Q} = \dfrac{150}{100}$

$\dfrac{t_P - 30}{150} = \dfrac{t_Q}{100}$

Sol17. from $x = 0$ to $x = \dfrac{A}{2} \Rightarrow 2\text{sec}$

$\dfrac{T}{12} = 2\text{s}$

$T = 24\,\text{s}$

$\therefore$ Time taken from $x = \dfrac{A}{2}$ to $x = \dfrac{A}{2}$

$\dfrac{T}{4} - \dfrac{T}{12} = \dfrac{T}{6} = 4\,\text{sec}$

Sol18. Basic definition of Law

Sol19. Two additional degree of freedom are added due to vibration at higher temperatures.

$C_V = \dfrac{fR}{2} = \dfrac{7R}{2}$

Sol20. Statement (i) mode is correct but statement (ii) is incorrect

SECTION – B

Sol1. I received = $320 \times \dfrac{330}{330 - 66} = \dfrac{320 \times 330}{264}$ = 400 Hz

Sol2. $x^2 + x^2 = 49$

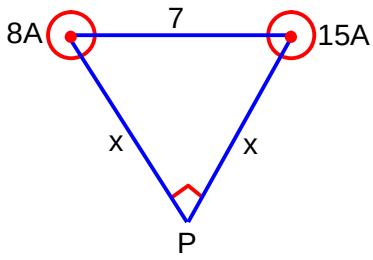
$$x = \frac{7}{\sqrt{2}}$$

$$B_P = \sqrt{\left(\frac{\mu_0 I_1}{2\lambda x}\right)^2 + \left(\frac{\mu_0 I_2}{2\pi x}\right)^2}$$

$$= \frac{\mu_0 \times \sqrt{2}}{2\pi \times 7}\sqrt{15^2 + 8^2}$$

$$= \frac{24\pi \times 10^{-7} \times 1.4}{2\pi \times 7} \times 17$$

$$= 68 \times 10^{-6}\,T$$



Sol3. $R = 80\,\Omega \qquad x_C = 130\,\Omega$

$X_L = 70\Omega$

$$\tan\phi = \frac{x_C - x_L}{R} = \frac{60}{80} = \frac{3}{4}$$

$$\therefore \cos\phi = \frac{4}{5} = \frac{8}{10}$$

Sol4. $$E_{eq} = \frac{\frac{12}{6} + \frac{6}{6}}{\frac{1}{3} + \frac{1}{6}} = \frac{(2+1)2}{6v}$$

$$R_{eq} = 4 + 2 = 6\,\Omega$$

$$\therefore \text{current} = \frac{6}{6} = 1A$$

Sol5. $3m_1 = 2m_2$

$$3 \times \frac{4}{3}\pi r_1^3 \times \rho = 2 \times \frac{4}{3}\pi r_2^3 \times \rho.$$

$$\frac{2}{3} = \left(\frac{\mu_1}{\mu_2}\right)^3$$

$$\Rightarrow \frac{\mu_1}{\mu_2} = \left(\frac{2}{3}\right)^{1/3}$$

Sol6.

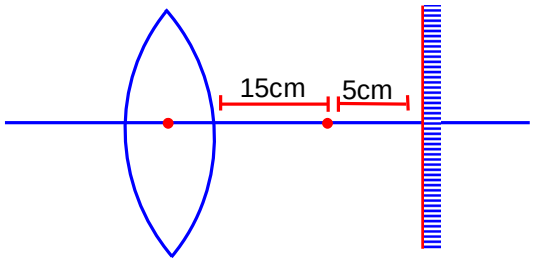
V for convex Q lens = 15cm

$f = 10\,cm$

$$\therefore \frac{1}{15} - \frac{1}{\mu} = \frac{1}{10}$$

$$\frac{1}{15} - \frac{1}{10} = \frac{1}{\mu}$$ **object distance = 30cm**

$$\frac{1}{30} = \frac{1}{\mu} \Rightarrow \mu = 30$$

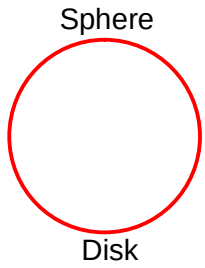


Sol7. $= \frac{2}{5}MR^2 + MR^2$

$= \frac{7}{5}M_1R^2 = 7R^2$

$= \frac{M_2R^2}{4} + M_2R$

$= \frac{5}{4}M_2R^2 = 5R^2$

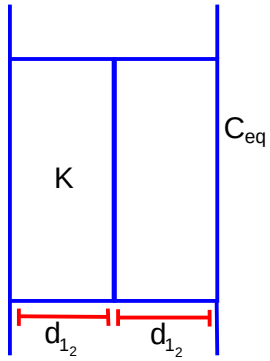


Sol8. $\frac{A\epsilon_0}{d} = 5\mu F$

$C_{eq} = \frac{d}{2A\epsilon_0 K} + \frac{d}{A\epsilon_0 \times 2}$

$= \frac{1}{15} + \frac{1}{10}$

$= \frac{2+3}{30} \Rightarrow 6\mu F$



Sol9. $\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r_1^3$

$R = 10\mu$

$\frac{R}{10} = \mu$

$\mu_i = T \times 4\pi R^2$

$\mu f = 1000 \times T \times \frac{4\pi \times R^2}{100} = 10\mu_i$

$\frac{\mu_f}{\mu_i} = 10$

Sol10. Let initial speed of 1kg = v

$1 \times v = -2 + 3v_1$ -(i)

V_1 = speed of 3kg block after collision

e = 1.

$\frac{V_S}{V_A} = 1.$

$\Rightarrow \frac{v_1 + 2}{v} = 1$ $v_1 + 2 = v$ -(ii)

$v = 3v_1 - 2$

$v = 3(v - 2) - 2$

$v = 3v - 8$

$8 = 2v$

$4 = v$

PART – B (CHEMISTRY)

SECTION – A

Sol1. $E^0_{Li^+/Li} = -3.05V$

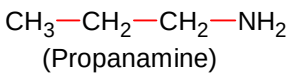
$E^0_{Cs^+/Cs} = -2.92V$

$E^0_{K^+/K} = -2.93V$

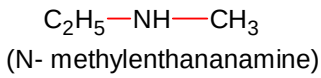
$E^0_{Na^+/Na} = -2.71V$

Higher reduction potential means weaker reducing capability.

Sol2.

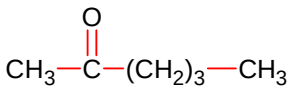


(Propanamine)

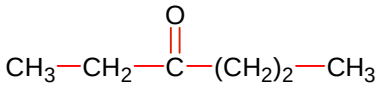


(N- methylenthananamine)

Functional Isomer

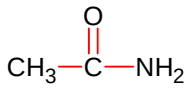


(Hexan-2-one)

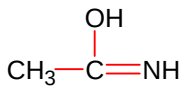


(Hexane-3-one)

Metamer

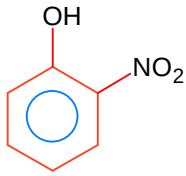


(Ethanamide)



(Hydroxyethanimine)

Tautomer

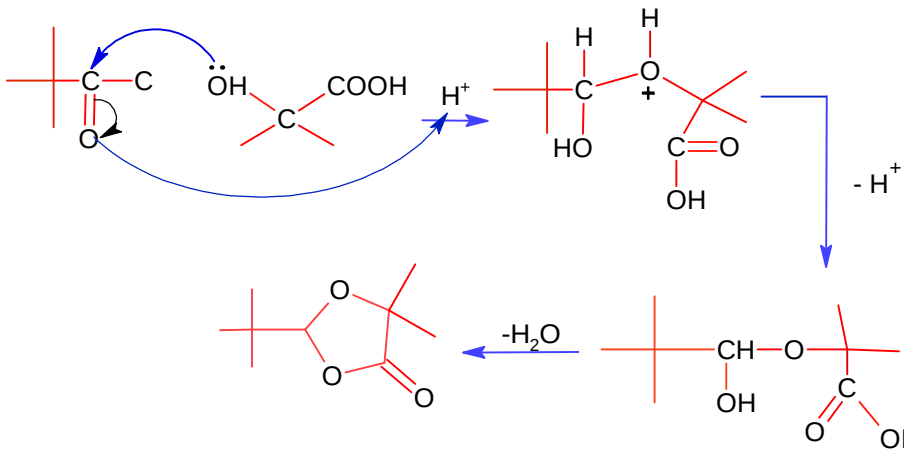


(o- nitrophenol)

(p- nitrophenol)

Positional Isomer

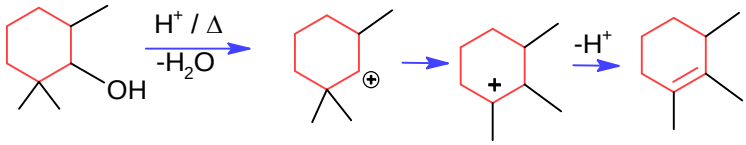
Sol3.



Sol4. Metallic character increases down the group and decreases along the period.

Sol5. Oxide forming acid upon getting dissolved in water is acidic oxide

Sol6.



Sol7. It is fact

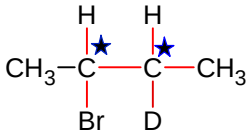
Sol8. $Cl^- + AgNO_3 \longrightarrow \underset{\text{(White)}}{AgCl} \xrightarrow{NH_4OH} \underset{\text{Soluble complex}}{\left[Ag(NH_3)_2\right]Cl}$

Sol9. Butylated hydroxyl anisole is an anti-oxidant

Sol10. $\overset{+6}{Cr_2}O_7 + 14H^+ + 6e \longrightarrow 2Cr^{3+} + 7H_2O$

Change in O.N is from +6 to +3

Sol11.



Sol12. The rotating paddle draws air in and stirs the pulp.

Sol13. Delocalization of lone pair of electron in aniline deceases the basic strength.

Sol14. $\Delta\left[H^+\right] = 1000$

$\Delta(pH) = -\log\left(\Delta\left[H^+\right]\right) = -3$

Sol15. No option is correct.
In chemistry direction of dipole moment is from +ve end to –ve end both statements are false.
Crossed arrow symbolizes direction of shift of electron density in molecule.

Sol16. Alkali metal and their salts impart characteristic colour to oxidisizing flame.

Sol17. It is a fact

Sol18. Assuming mass of solvent = mass of solution

$$0.25 = \frac{w_1}{62} \times \frac{1000}{500}$$

$$0.25 = \frac{w_2}{62} \times \frac{1000}{250}$$

$$\frac{w_1}{w_2} = \frac{2}{1}$$

Sol19. $\Delta(\text{CFSE}) \propto \frac{1}{\lambda_{\text{obserbed}}} \propto$ strength of ligand

Sol20. A & C are correct

B) $Cl^{\bullet} + O_3 \rightarrow O_2 + ClO^{\bullet}$

D) Blue baby syndrome occurs due to the presence of excess of nitrate ions in water.

SECTION – B

Sol1. $C_{\frac{35.12}{12}}H_{\frac{14.2}{1}} \equiv CH_2$ [Empirical formula]

Empirical formula mass = 14 Molecular mass = 84 = 14 ×6

Molecular formula = $(CH_2)_6 = C_6H_{12}$

Sol2. $[Co(NH_3)_4Cl_2]Cl \rightarrow$ Gives 1 mole of AgCl

$[Ni(H_2O)_6]Cl_2 \rightarrow$ Gives 2 moles of AgCl

$[Pt(NH_3)_2Cl_2] \rightarrow$ Gives no ppt

$[Pd(NH_3)_4]Cl_2 \rightarrow$ Gives 2 moles of AgCl

Sol3. B & D options are correct

Sol4. $\pi = i \times CRT$

$\pi \propto i \times C$

A, B, D, E have same value of osmotic pressure.

Sol5. Let volume of C_2H_4 of x lit, and vol of CH_4 = (16.8–x)lit.

Vol. of CO_2 formed = 2x+(16.8 -x)=28

Therefore, x =11.2 lit

$n_{CH_4} = \frac{PV}{RT} = 0.229$ moles

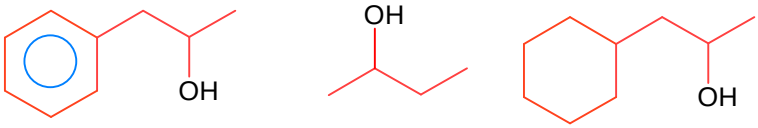
$nC_2H_4 = 0.458$ moles

Heat evolved $= (0.229 \times 900) + (0.458 \times 1400)$

$= 2061. + 641.2$

$= 847.3\,kJ$

Sol6.



Sol7. The degree of dissociation $= \left(1 - e^{-kt}\right)$

Sol8. $H_2 + M^{3+} \longrightarrow M^{+} + 2H^{+}$

$$E_{cell} = E^0_{cell} - \frac{0.059}{2}\log\frac{\left[M^{+}\right] \times (1)^2}{\left[M^{3+}\right] \times 1}$$

$$\log\frac{\left[M^{+}\right]}{\left[M^{3+}\right]} = 3 \Rightarrow a = 3$$

Sol9. P_x, P_y, P_z, d_{z^2}, and $dx^2 - y^2$ are axial orbitals

Sol10. (A) $\rightarrow$ Water vapour are obsorbed by $CaCl_2$
(C) $\rightarrow$ As the adsoption proceeds, ΔH becomes less and less negatiave.

PART – C (MATHEMATICS)

SECTION – A

Sol1. $f(x) = 2x^n + \lambda$
Given that $f(4) = 133$ and $f(5) = 255$
$\Rightarrow 133 = 2 \times 4^n + \lambda$……. (i) and $255 = 2 \times 5^n + \lambda$ …… (ii)
Subtracting (ii) – (i) we get $122 = 2(5^n - 4^n) \Rightarrow 5^n - 4^n = 61 \therefore n = 3$ & $\lambda = 5$
$\therefore f(3) - f(2) = 2(3^3 - 2^3) = 38 \therefore$ Number of Divisors is 1, 2, 19, 38
$\therefore$ Their sum is 60.

Sol2. $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ also $AA^T = A^TA = I$

$M = A^TBA = IB = B$
$\therefore AM^{2023}A^T = AA^TM^{2023} = IM^{2023} = B^{2023}$

$\Rightarrow BB = B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix} \Rightarrow B^3 = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$

$\Rightarrow B^4 = \begin{bmatrix} 1 & -4i \\ 0 & 1 \end{bmatrix}$ …… $\Rightarrow B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

Inverse of $AM^{2023}A^T$ is $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Sol3. Given, $A^T = A$, $B^T = -B$, $C^T = -C$ Let $M = A^{13}B^{26} - B^{26}A^{13}$ Then, $M^T = (A^{13}B^{26} - B^{26}A^{13})^T = (A^{13}B^{26})^T - (B^{26}A^{13})^T = (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26} = B^{26}A^{13} - A^{13}B^{26} = -M$.
Hence, M is skew symmetric
Let, $N = A^{26}C^{13} - C^{13}A^{26}$ then, $N^T = (A^{26}C^{13})^T - (C^{13}A^{26})T = -C^{13}A^{26} + A^{26}C^{13} = N$
Hence, N is symmetric.
$\therefore$ Only S2 is true

Sol4. Numbers between 5000 & 10000 Using digits 1, 3, 5, 7, 9

5,7or 9			
3 ways	4ways	3 ways	2 ways

Total Numbers = $3 \times 4 \times 3 \times 2 = 72$

Sol5. Since, $-2 \le \sin x - \cos x \le 2 \Rightarrow -2 \le \sqrt{2}(\sin x - \cos x) \le 2 \Rightarrow -2 \le k \le 2$ …….(i)
$f(x) = \log_{\sqrt{m}}(k + m - 2)$ Given, $0 \le f(x) \le 2 \Rightarrow 0 \le \log_{\sqrt{m}}(k + m - 2) \le 2$
$\Rightarrow 1 \le k + m - 2 \le m$
$\Rightarrow -m + 3 \le k \le 2$ …… (ii)
From eq. (i) & (ii), we get, $-m + 3 = -2 \Rightarrow m = 5$

Sol6. $f(1) \Rightarrow -8, \ldots\ldots\ldots\ldots, 8$

$n = 1, f(1) + f(2) = 1 \Rightarrow f(2) = 1 - f(1) \Rightarrow f(2) \Rightarrow -8, \ldots\ldots\ldots\ldots, 8$

$n = 2, f(2) + \frac{1}{2}f(3) = 1 \Rightarrow f(3) = 2f(1) \Rightarrow f(3) \Rightarrow -8, -6, -4, -2, 0, 2, 4, 6, 8$

$n = 3, f(4) = 3 - 6f(1) \Rightarrow f(4) \Rightarrow -6, -3, 0, 3, 6$

f (4) should be odd i.e., - 3 and 3 since

f (3) should be even therefore 2 solutions are possible i.e., 0 and 2

f (2) should be even therefore 2 solutions are possible i.e., 1 and 0

f (1) should be even therefore 2 solutions are possible i.e., 1 and 0

Sol7. $\frac{dy}{dx} + \alpha y = \gamma e^{-\beta t}$ Now I.F = $e^{\alpha t}$

$$\Rightarrow y e^{\alpha t} = \int \gamma e^{(\alpha-\beta)t} dt = \frac{\gamma e^{(\alpha-\beta)t}}{\alpha-\beta} + c \therefore y = \frac{\gamma}{\alpha-\beta} e^{-\beta t} + e^{-\alpha t}$$

$$\text{Now, } \lim_{t\to\infty} y(t) = \lim_{t\to\infty}\left(\frac{\gamma}{\alpha-\beta} e^{-\beta t} + e^{-\alpha t}\right) = 0$$

Sol8. $16\int_1^2 \frac{dx}{x^3(x^2+2)^2} = 16\int_1^2 \frac{dx}{x^3.x^4(1+\frac{2}{x^2})^2}$ put $1+\frac{2}{x^2} = t \Rightarrow -\frac{1}{x^3}dx = dt$

$$\Rightarrow -\int_3^{3/2} \frac{(t-1)^2 dt}{t^2} = -\int_3^{3/2} \frac{(t^2-2t+1)dt}{t^2} = -\int_3^{3/2} (1-\frac{2}{t}+\frac{1}{t^2})dt = \frac{11}{6} - \log_e 4$$

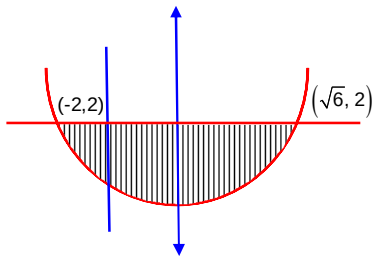
Sol9. Let $\vec{a} = 3\hat{i} - 4\hat{j} + 2\hat{k}$ $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ $\vec{c} = -2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{d} = 5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$

$\overrightarrow{AB} = -2\hat{i} + 6\hat{j} - 3\hat{k}$, $\overrightarrow{AC} = -5\hat{i} + 3\hat{j} + \hat{k}$ and $\overrightarrow{AD} = 2\hat{i} - (2\alpha-4)\hat{j} + 2\hat{k}$

A, B, C, D are coplanar points, then $\begin{vmatrix} -2 & 6 & -3 \\ -5 & 3 & 1 \\ 2 & -(2\alpha-4) & 2 \end{vmatrix} = 0 \Rightarrow \alpha = \frac{73}{17}$

Sol10. $\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16} = 1$

$$A = \int_{-2}^{\sqrt{6}} (2-(x^2-4))dx = \int_{-2}^{\sqrt{6}} (6-x^2)dx = 4\sqrt{6} + \frac{28}{3}$$



Sol11. Using the results $\left({}^{n}C_{r-1} + {}^{n}C_{r} = {}^{n+1}C_{r}\right)$

$$\sum_{k=0}^{6} {}^{51-k}C_3$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + \left({}^{45}C_3 + {}^{45}C_4\right) - {}^{45}C_4$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + \left({}^{46}C_3 + {}^{46}C_4\right) - {}^{45}C_4$$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + \left({}^{47}C_3 + {}^{47}C_4\right) - {}^{45}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + \left({}^{48}C_3 + {}^{48}C_4\right) - {}^{45}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + \left({}^{49}C_3 + {}^{49}C_4\right) - {}^{45}C_4$

$= {}^{51}C_3 + \left({}^{50}C_3 + {}^{50}C_4\right) - {}^{45}C_4$

$= {}^{51}C_3 + {}^{51}C_4 - {}^{45}C_4$

$= {}^{52}C_4 - {}^{45}C_4$

Sol12. Let $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$,Taking cross product with $\vec{a}$ then $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 1 \\ x & y & z \end{vmatrix} = \hat{i} - \hat{j}$

$\Rightarrow \hat{i}(-z-y) + \hat{j}(x+z) + \hat{k}(-y+x) = \hat{i} - \hat{j} \Rightarrow$ - z – y = 1, x + z = - 1 x – y = 0 and also $\vec{a}.\vec{b} = 1 \Rightarrow$ - x – y + z = 1 $\Rightarrow x = -\frac{2}{3}, y = -\frac{2}{3}, z = -\frac{1}{3} \Rightarrow \vec{b} = -\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k} \Rightarrow \vec{a} - 6\vec{b} = 3(\hat{i} + \hat{j} + \hat{k})$

Sol13. $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$

$f'(x) = 6x^2 + 2(2p-7)x + 3(2p-9)$

$f'(0) < 0$

$\therefore 3(2p-9) < 0 \Rightarrow p < \frac{9}{2} \Rightarrow p \in \left(-\infty, \frac{9}{2}\right)$

Sol14. $(z-2i)(\bar{z}+2i) = 4(z+i)(\bar{z}-i) \Rightarrow \bar{z}z + 4 + 2i(z-\bar{z}) = 4(z\bar{z} + 1 + i(\bar{z} - z)) = 0$

$\Rightarrow 3z\bar{z} - 6i(z - \bar{z}) = 0 \Rightarrow x^2 + y^2 - 2i(2iy) = 0 \Rightarrow x^2 + y^2 + 4y = 0.$

Sol15. $y^2 = 6x$ & $y^2 = 4ax \Rightarrow 4a = 6$

$\Rightarrow a = \frac{3}{2}$

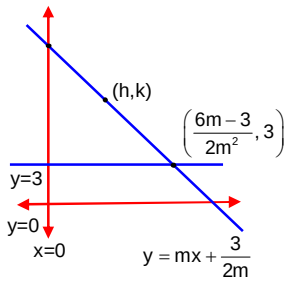
$y = mx + \frac{3}{2m}$; $m \neq 0$

$h = \frac{6m-3}{4m^2}, k = \frac{6m+3}{4m}$

Now eliminating m and we get

$4y^2 - 18y + 3x + 18 = 0$

$\left(0, \frac{3}{2m}\right)$



Sol16. For continuous function $e^{\lim\limits_{x\to\frac{\pi}{2}} \frac{(1+\cos x-1)\lambda}{\cos x}}$ $e^{\lambda} = \mu = e^{\frac{2}{3}} \Rightarrow \lambda = \frac{2}{3}, \ln\mu = \frac{2}{3}$

$\therefore 9\lambda + 6\log_e\mu + \mu^6 - e^{6\lambda} = 10.$

Sol17. $\frac{x+1}{2} = \frac{y}{1} = \frac{z}{\frac{-1}{6}}$ and $\frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1}$

Let A(-1 , 0 , 0) and B(0 , – 2 , 1) $\Rightarrow \vec{b} - \vec{a} = \hat{i} - 2\hat{j} + \hat{k}$

Also let $\vec{p} = 2\hat{i} + \hat{j} - \frac{1}{6}\hat{k}$ and $\vec{q} = 6\hat{i} + 6\hat{j} + \hat{k}$

$$\Rightarrow \vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -\frac{1}{6} \\ 6 & 6 & 1 \end{vmatrix} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$\Rightarrow \text{S.D.} = \left| \frac{(\vec{b} - \vec{a}).\vec{p} \times \vec{q}}{|\vec{p} \times \vec{q}|} \right| = \left| \frac{2 + 2 + 6}{\sqrt{4 + 9 + 36}} \right| = 2$$

Sol18. n(s) = 36 Given : N – 2 , $\sqrt{3N}$,N + 2 are in G.P.

$\Rightarrow 3N = (N - 2)(N + 2) \Rightarrow 3N = N^2 - 4$

$\Rightarrow N^2 - 3N - 4 = 0 \Rightarrow (N - 4)(N + 1) = 0$

$\Rightarrow$ N = 4 or N = –1 rejected (N = 4) $\equiv$ {(1, 3), (3,1), (2,2)} $\Rightarrow$ n(A) = 3

$\Rightarrow$ required probability = $\frac{3}{36} = \frac{1}{12} = \frac{k}{48}$

$\Rightarrow k = 4$

Sol19. Given $(p \to q) \Delta (p \nabla q)$

(A) $\Delta = \wedge$, $\nabla = \vee$

p	q	$(p \to q)$	$(p \vee q)$	$(p \to q) \wedge (p \vee q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	T	F

(B) $\Delta = \vee$, $\nabla = \wedge$

p	q	$(p \to q)$	$(p \wedge q)$	$(p \to q) \vee (p \wedge q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	F

(C) $\Delta = \vee$, $\nabla = \vee$

p	q	$(p \to q)$	$(p \vee q)$	$(p \to q) \vee (p \wedge q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Hence, it is tautology.

(D) $\Delta = \wedge$, $\nabla = \wedge$

p	q	$(p \to q)$	$(p \wedge q)$	$(p \to q) \wedge (p \wedge q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

Sol20. L : $\frac{x + 1}{2} = \frac{y - 1}{5} = \frac{z + 1}{-1} = r \Rightarrow$

P (2,0,5) Let foot of perpendicular is Q(- 1 + 2r,1 + 5r , -1 - r)

Direction ratio of line PQ : 3 – 2r , - 1 – 5r , 6 + r

and also Direction ratio of line L : $2(3 - 2r) + 5(-1 - 5r) - 1(6 + r) = 0 \Rightarrow r = -\frac{1}{6}$

$Q\left(-\frac{4}{3},\frac{4}{3},-\frac{5}{6}\right) = Q(\alpha, \beta, \gamma)$.

$\Rightarrow \alpha = -\frac{4}{3}, \beta = \frac{4}{3}, \gamma = -\frac{5}{6}$

$\Rightarrow \frac{\alpha\beta}{\gamma} = -\frac{32}{15}, \frac{\alpha}{\beta} = -1, \frac{\gamma}{\alpha} = \frac{5}{8}, \frac{\beta}{\gamma} = -\frac{8}{5}$

SECTION – B

Sol1. $\alpha + \beta = -60^{\frac{1}{4}}$ and $\alpha\beta = a$

$\Rightarrow \alpha^4 + \beta^4 = -30$

$\Rightarrow (\alpha^2 + \beta^2)^2 - \alpha^2\beta^2 = -30$

$\Rightarrow \{(\alpha + \beta)^2 - \alpha\beta\}^2 - 2a^2 = -30$

$\Rightarrow 2a^2 - 4.a\,60^{\frac{1}{2}} + 90 = 0$

$\therefore$ product of roots = 45

Sol2. $(2023)^{2023} = (2030 - 7)^{2023} = (35\ K - 7)^{2023} = {}^{2023}C_0(35\ K)^{2023}(-7)^0 + {}^{2023}C_1\ (35\ K)^{2022}\ (-7) + \ldots\ldots + \ldots\ldots + {}^{2023}C_{2023}(-7)^{2023} = 35\ N - 7^{2023}$.

$-7^{2023} = -7 \times 7^{2022} = -7\ (7^2)^{1011} = -7\ (50 - 1)^{1011} = -7({}^{1011}C_0\ 50^{1011} - {}^{1011}C_1(50)^{1010} + \ldots\ldots\ {}^{1011}C_{1011}) = -7\ (5\lambda - 1) = -35\lambda + 7$

Hence remainder is 7

Sol3. E_1 : Smokers $\therefore\ P(E_1) = \frac{1}{4}$ and E_2 : non-smokers $\therefore\ P(E_2) = \frac{3}{4}$

E : diagnosed with lung cancer

$P(E / E_1) = \frac{27}{28}$

$\Rightarrow P(E / E_2) = \frac{1}{28}$

$\Rightarrow P(E_1 / E) = \frac{P(E_1)P(E / E_1)}{P(E)} = \frac{\frac{1}{4} \times \frac{27}{28}}{\frac{1}{4} \times \frac{27}{28} + \frac{3}{4} \times \frac{1}{28}} = \frac{9}{10} = \frac{k}{10}$

$\Rightarrow k = 9$

Sol4. When x = 1 $\Rightarrow$ (1, 1) (1, 2)---------- (1, 14) $\Rightarrow$ 14 pts.
When x = 2 $\Rightarrow$ (2, 2) (2, 4)-------- (2, 12) $\Rightarrow$ 6 pts.
When x = 3 $\Rightarrow$ (3, 3), (3, 6) -------- (3, 12) $\Rightarrow$ 4 pts.
When x = 4 $\Rightarrow$ (4, 4) (4, 8) $\Rightarrow$ 2 pts.
When x = 5 $\Rightarrow$ (5, 5), (5, 10) $\Rightarrow$ 2 pts.
When x = 6, (6, 6) $\Rightarrow$ 1
When x = 7, (7, 7) $\Rightarrow$ 1 pt.
When x = 8 $\Rightarrow$ (8, 8) $\Rightarrow$ 1 pt.
When x = 9 $\Rightarrow$ no pt

Sol5. a, b, $\frac{1}{18}$ are in G.P.

$\therefore \frac{a}{18} = b^2$ ----------(i)

$\frac{1}{a}, 10, \frac{1}{b}$ are in A.P.

$\Rightarrow \frac{1}{a}+\frac{1}{b}=20$

$\Rightarrow a + b = 20ab$

$\Rightarrow 18b^2 + b = 360b^3$

$\Rightarrow 360b^2 - 18b - 1 = 0$

$\Rightarrow b = \frac{1}{12}$

$\Rightarrow a = \frac{1}{8} \therefore 16a + 12b = 3$

Sol6. $\cos 2\theta \cos\frac{\theta}{2} = \cos 3\theta \cos\frac{9\theta}{2} \Rightarrow 2\cos 2\theta \cos\frac{\theta}{2} = 2\cos 3\theta \cos\frac{9\theta}{2}$

$\Rightarrow \cos\frac{15\theta}{2} = \cos\frac{5\theta}{2}$ after solving we get, $\frac{15\theta}{2} = 2n\pi \pm \frac{5\theta}{2} \Rightarrow \theta = \frac{2n\pi}{5}$ or $\theta = \frac{n\pi}{5}$

$\Rightarrow \theta = \left\{-\pi, \frac{-4\pi}{5}, \frac{-3\pi}{5}, \frac{-2\pi}{5}, \frac{-\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi\right\} \Rightarrow m = 5, n = 5$

$\therefore mn = 25$

Sol7. $\int_{\frac{1}{3}}^{3} |\log_e x| dx = \int_{\frac{1}{3}}^{1} -\log_e x dx + \int_{1}^{3} \log_e x dx = \frac{4}{3}(2\log_e 3 - 1) = = \frac{m}{n}\log_e\left(\frac{n^2}{e}\right)$

$\therefore$ m = 4 and n = 3 $\Rightarrow m^2 + n^2 - 5 = 16 + 9 - 5 = 20$.

Sol8. 7 Red apples , 5 white apples,8 oranges

Required ways = ${}^8C_2\ {}^7C_1\ {}^5C_2 + {}^8C_2\ {}^7C_2\ {}^5C_1 + {}^8C_3\ {}^7C_1\ {}^5C_1 = 6860$

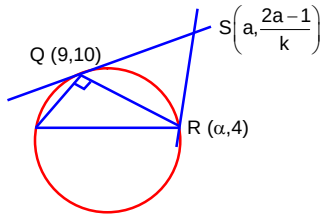
Sol9. $\vec{r} = \vec{a} + \lambda\vec{p}$ where $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{p} = \hat{i} + \hat{j} + \hat{k}$

and $\vec{r} = \vec{b} + \mu\vec{q}$ where $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{q} = 2\hat{i} - \hat{j}$

$\therefore \vec{p} \times \vec{q} = \hat{i} + 2\hat{j} - 3\hat{k}$

S.D.$= \left|\frac{(\vec{b} - \vec{a}).(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right| \Rightarrow \alpha = \frac{3}{\sqrt{14}} \Rightarrow \alpha^2 = \frac{9}{14} \Rightarrow 28\alpha^2 = 18$

Sol10. Product of slopes = $\frac{10-2}{9+3} \times \frac{10-4}{9-\alpha} = -1$



$\Rightarrow \alpha = 13$

$\therefore$ equation of circle is (x+3)(x-13)+ (y-2)(y-4)=0

$x^2 + y^2 - 10x - 6y - 31 = 0$

Now equation of tangent at Q (9,10) is

4x + 7y – 106 = 0 ……..(i)

equation of tangent at R (13,4) is

8x + y – 108 = 0…….(ii)

Solving (i) and (ii) we get $x = \frac{25}{2}$ and y = 8

$\therefore x = \frac{25}{2} = a$ and $\frac{2a-1}{k} = 8 \Rightarrow k = 3$