FIITJEE

Solutions to JEE(Main) -2023

Test Date: 29th January 2023 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (1 – 10)*** *contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

PART – A (PHYSICS)

SECTION - A

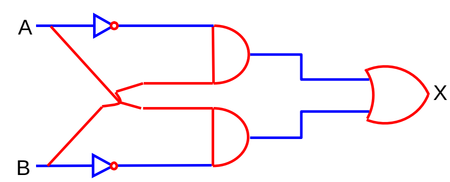
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. The equation of a circle is given by $x^2 + y^2 = a^2$, where a is the radius. If the equation is modified to change the origin other than (0,0), then find out the correct dimensions of A and B in a new equation : $(x - At)^2 + \left(y - \frac{t}{B}\right)^2 = a^2$. The dimensions of t given as $\left[T^{-1}\right]$.

(A) $A = \left[L^{-1}T^{-1}\right], B = \left[LT^{-1}\right]$
(B) $A = \left[L^{-1}T\right], B = \left[LT^{-1}\right]$
(C) $A = \left[LT\right], B = \left[L^{-1}T^{-1}\right]$
(D) $A = \left[L^{-1}T^{-1}\right], B = \left[LT\right]$

Q2. For the given gates combination, the correct truth table will be



(A)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

(B)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

(C)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

(D)

A	B	X
0	0	1
0	1	0
1	0	1
1	1	0

Q3. A fully loaded boeing aircraft has a mass of 5.4×10^5 kg. Its total wing area is $500 m^2$. It is in level flight with a speed of 1080km/h. If the density of air ρ is $1.2 kg\ m^{-3}$, the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be. ($g = 10\ m/s^2$)

(A) 10
(B) 6
(C) 16
(D) 8

Q4. The time taken by an object to slide down 45° rough inclined plane is n times as it takes to slide down a perfectly smooth 45° incline plane. The coefficient of kinetic friction between the object and the incline plane is :

(A) $1 + \frac{1}{n^2}$
(B) $\sqrt{1 - \frac{1}{n^2}}$
(C) $\sqrt{\frac{1}{1 - n^2}}$
(D) $1 - \frac{1}{n^2}$

Q5. The time period of a satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value, then its new time period will become.
(A) 6 hours (B) 3 hours
(C) 4 hours (D) 12 hours

Q6. With the help of potentiometer, we can determine the value of emf of a given cell. The sensitively of the potentiometer is
(A) Directly proportional to the length of the potentiometer wire
(B) Directly proportional to the potential gradient of the wire
(C) Inversely proportional to the potential gradient of the wire
(D) Inversely proportion to the length of the potentiometer wire

Choose the correct option for the above statements :
(A) A only (B) B and D only
(C) C only (D) A and C only

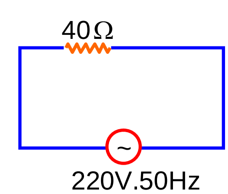
Q7. A force acts for 20s on a body of mass 20 kg, starting from rest, after which the force ceases and then body describes 50m in the next 10s. The value of force will be :
(A) 40 N (B) 20 N
(C) 5 N (D) 10 N

Q8. An object moves at a constant speed along a circular path in a horizontal plane with centre at the origin. When the object is at x = +2 m, its velocity is $-4\hat{j}\,m/s$. The object's velocity (v) and acceleration (a) at $x = -2m$ will be
(A) $v = -4\hat{i}\,m/s,\ a = -8\hat{j}\,m/s^2$ (B) $v = 4\hat{j}\,m/s, a = 8\hat{i}\,m/s^2$
(C) $v = -4\hat{j}\,m/s, a = 8\hat{i}\,m/s^2$ (D) $v = 4\hat{i}\,m/s, a = 8\hat{j}\,m/s^2$

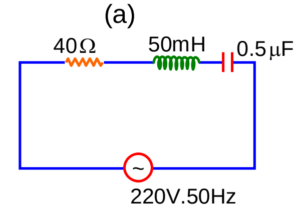
Q9. A scientist is observing a bacteria through a compound microscope. For better analysis and to improve its resolving power he should. (Select the best option)
(A) Decrease the diameter of the objective lens
(B) Increase the refractive index of the medium between the object and objective lens
(C) Increase the wave length of the light
(D) Decrease the focal length of the eye piece.

Q10. The modulation index for an A.M. wave having maximum and minimum peak-to-peak voltages of 14 mV and 6 mV respectively is -
(A) 0.4 (B) 1.4
(C) 0.2 (D) 0.6

Q11. For the given figures, choose the correct options :
(A) The rms current in figure (a) is always equal to that in figure(b)
(B) At resonance, current in (b) is less than that in (a)
(C) The rms current in circuit (b) can never be larger than that in(a)
(D) The rms current in circuit (b) can be larger than that in (a)



(a)



(b)

Q12. Identify the correct statements from the following

A. Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is negative.

B. Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative.

C. Work done by friction on a body sliding down an inclined plane is positive.

D. Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity is zero.

E. Work done by the air resistance on an oscillating pendulum is negative.

Choose the correct answer from the options given below :

(A) B and E only
(B) A and C only
(C) B and D only
(D) B, D and E only

Q13. Heat energy of 184 kJ is given to ice of mass 600 g at $-12^{\circ}C$. Specific heat of ice is 2222.3 J $kg^{-1^{\circ}}C^{-1}$ and latent heat of ice is 336 kJ/kg^{-1}

A. Final temperature of system will be $0^{\circ}C$.

B. Final temperature of the system will be greater than $0^{\circ}C$.

C. The final system will have a mixture of ice and water in the ratio of 5 : 1.

D. The final system will have a mixture of ice and water in the ratio of 1 : 5.

E. The final system will have water only.

Choose the correct answer from the options given below :

(A) B and D only
(B) A and D only
(C) A and E only
(D) A and C only

Q14. Given below are statements :

Statement I : Electromagnetic waves are not deflected by electric and magnetic field.

Statement II : The amplitude of electric field and the magnetic field in Electromagnetic waves are related to each other as $E_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} B_0$.

In the light of the above statements, choose the correct answer from the options given below :

(A) Both Statement I and Statement II are false
(B) Statement I is false but statement II is true
(C) Both Statement I and Statement II are true
(D) Statement I is true but statement II is false

Q15. Substance A has atomic mass number 16 and half life of 1 day. Another substance B has atomic mass number 32 and half life of $\frac{1}{2}$ day. If both A and B simultaneously start undergo radio activity at the same time with initial mass 320 g each, how many total atoms of A and B combined would be left after 2 days.

(A) 6.76×10^{24}
(B) 1.69×10^{24}
(C) 3.38×10^{24}
(D) 6.76×10^{23}

Q16. The electric current in a circular coil of four turns produces a magnetic induction 32 T at its centre. The coil is unwound and is rewound into a circular coil of single turn, the magnetic induction at the centre of the coil by the same current will be :

(A) 16 T
(B) 4T
(C) 2T
(D) 8T

Q17. The ratio of de-Broglie wavelength of an α particle and a proton accelerated from rest by the same potential is $\frac{1}{\sqrt{m}}$, the value of m is -
(A) 2 (B) 8
(C) 4 (D) 16

Q18. A square loop of area 25cm^2 has a resistance of 10Ω. The loop is placed in uniform magnetic field of magnitude 40.0 T. The plane of loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0 sec, will be
(A) 5×10^{-3} J (B) 1.0×10^{-3} J
(C) 2.5×10^{-3} J (D) 1.0×10^{-4} J

Q19. At 300 K, the rms speed of oxygen molecules is $\sqrt{\frac{\alpha+5}{\alpha}}$ times to that of its average speed in the gas. Then, the value of α will be
$\left(\text{used } \pi=\frac{22}{7}\right)$
(A) 27 (B) 28
(C) 32 (D) 24

Q20. A point charge 2×10^{-2}C is moved from P to S in a uniform electric of 30NC^{-1} directed along positive x-axis. If coordinates of P and S are (1,2,0) m and (0,0,0) m respectively, the work done by electric field will be
(A) -600 mJ (B) 1200 mJ
(C) -1200 mJ (D) 600 mJ

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. A particle of mass 250g executes a simple harmonic motion under a periodic force $F = (-25x)$ N. The particle attains a maximum speed of 4m/s during its oscillation. The amplitude of the motion is ________cm.

Q2. An inductor of inductance, $2\,\mu H$ is connected in series with a resistance, a variable capacitor and an AC source of frequency $7\,kHz$. The value of capacitance for which maximum current is drawn into the circuit is $\frac{1}{x}$ F, where the value of x is ________

(Take $\pi = \frac{22}{7}$)

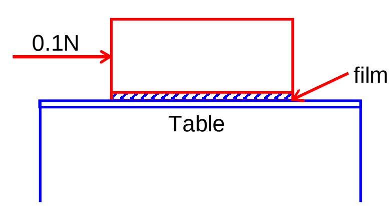
Q3. A null point is found at 200 cm in potentiometer when cell in secondary circuit is shunted by 5Ω. When a resistance of 15Ω. is used for shunting, null point moves to 300cm. The internal resistance of the cell is _________Ω.

Q4. Unpolarised light is incident on the boundary between two dielectric media, whose dielectric constants are 2.8(medium – 1) and 6.8 (medium – 2), respectively. To satisfy the condition, so that the reflected and refracted rays are perpendicular to each other, the angle of incidence should be $\tan^{-1}\left(1+\frac{10}{\theta}\right)^{\frac{1}{2}}$ the value of θ is ________.

(Given for dielectric media, $\mu_r = 1$)

Q5. A car is moving on a circular path of radius 600m such that the magnitude of the tangential acceleration and centripetal acceleration are equal. The time taken by the car to complete first quarter of revolution, if it is moving with an initial speed of 54 km/hr is $t\left(1-e^{-\pi/2}\right)$s. The value of t is ________.

Q6. A metal block of base area 0.20 m^2 is placed on a table. as shown in figure. A liquid film of thickness 0.25 mm is inserted between the block and the table. The block is pushed by a horizontal force of 0.1 N and moves with a constant speed. If the viscosity of the liquid is 5.0 × 10^{-3} Pl, the speed of block is_______$\times 10^{-3}$ m / s.



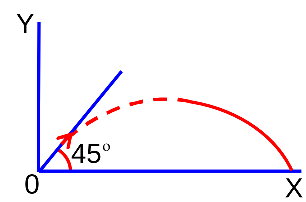
Q7. When two resistance R_1 and R_2 connected in series and introduced into the left gap of a meter bridge and a resistance of 10Ω is introduced into the right gap, a null point is found at 60cm from left side. When R_1 and R_2 are connected in parallel and introduced into the left gap, a resistance of 3Ω is introduced into the right-gap to get null point at 40cm from left end. The product of R_1 and R_2 is ______Ω^2

Q8. For a charged spherical ball, electrostatic potential inside the ball varies with r as V = $2ar^2 + b$. Here, a and b are constant and r is the distance from the centre. The volume charge density inside the ball is $-\lambda a\varepsilon$. the value of λ is ________.

$\varepsilon =$ permittivity of the medium

Q9. In an experiment of measuring the refractive index of a glass slab using traveling microscope in physics lab, a student measures real thickness of the glass slab as 5.25 mm and apparent thickness of the glass slab as 5.00 mm. Travelling microscope has 20 divisions in one cm on main scale and 50 divisions on vernier scale is equal to 49 divisions on main scale. The estimate uncertainly in the measurement of refractive index of the slab is $\frac{x}{10}\times 10^{-3}$, where x is _______.

Q10. A particle of mass 100g is projected at time t = 0 with a speed 20 ms^{-1} at an angle 45° to the horizontal as given in the figure. The magnitude of the angular momentum of the particle about the starting point at time t = 2s is found to be $\sqrt{K}$ kg m^2 / s. The value of K is _______.
(Take g = 10 ms^{-2})



PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Given below are two statements:
Statement I: Nickel is being used as the catalyst for producing syn gas and edible fats.
Statement II: Silicon forms both electron rich and electron deficient hydrides.
In the light of the above statements, choose the most appropriate answer from the options given below:
(A) Statement I is correct but Statement II is incorrect
(B) Statement I is incorrect but Statement II is correct
(C) Both the Statements I and II are correct
(D) Both the statements I and II are incorrect

Q2. The major component of which of the following ore is sulphide based mineral?
(A) Sphalerite
(B) Siderite
(C) Calamine
(D) Malachite

Q3. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement)
(A) 2-Bromopentane
(B) 2-Bromopropane
(C) 2-Bromo-3,3- dimethylpentane
(D) 1-Bromo-2-methylbutane

Q4. The concentration of dissolved oxygen in water for growth of fish should be more than X ppm and Biochemical oxygen demand in clean water should be less than Y ppm X and Y in ppm are, respectively.

(A)

X	Y
6	5

(B)

X	Y
4	8

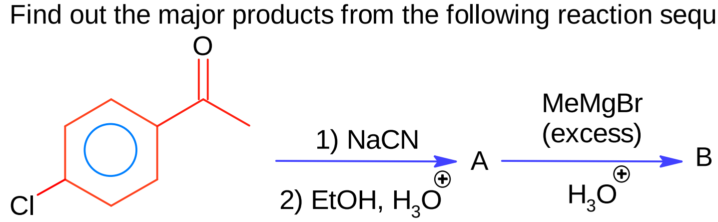
(C)

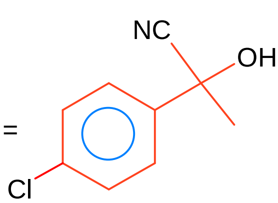
X	Y
6	12

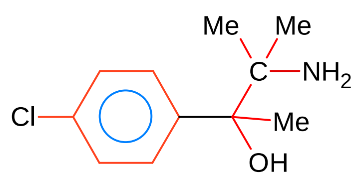
(D)

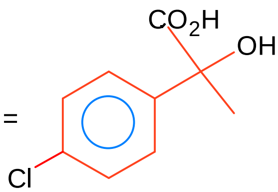
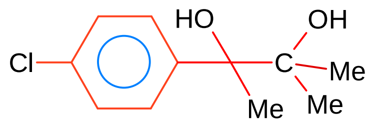
X	Y
4	15

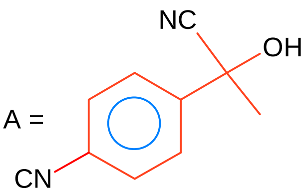
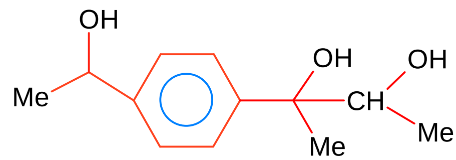
Q5. Find out the major products from the following reaction sequence.

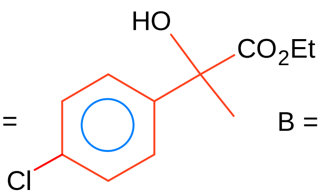
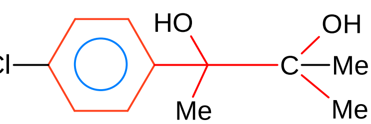


(A) A = 

B = 

(B) A =  B = 

(C) A =  B = 

(D) A =  B = 

Q6. Correct order of spin only magnetic moment of the following complex ions is
(Given At no. Fe:26, Co:27)

(A) $[CoF_6]^{3-} > [FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

(B) $[FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-}$

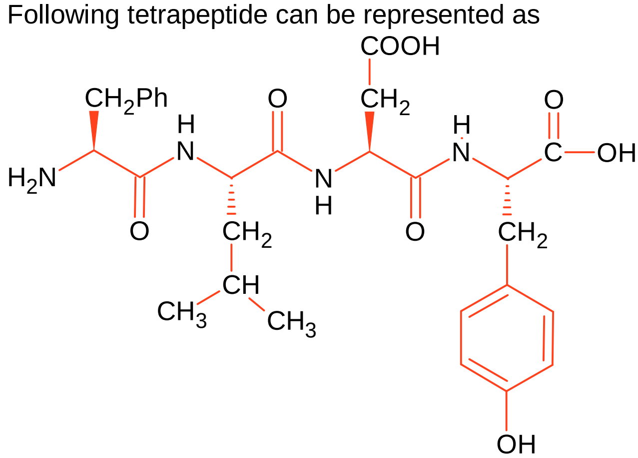
(C) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

(D) $[Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-} > [FeF_6]^{3-}$

Q7. An indicator 'X' is used for studying the effect of variation in concentration of iodide on the rate of reaction of iodide ion with H_2O_2 at room temperature. The indicator 'X' forms blue colored complex with compound 'A' present in the solution. The indicator 'X' and compound 'A' respectively are

(A) Starch and iodine
(B) Methyl orange and H_2O_2
(C) Methyl orange and iodine
(D) Starch and H_2O_2

Q8. Following tetrapeptide can be represented as



(F, L, D, Y, I, Q, P are one letter codes for amino acids)

(A) FLDY
(B) PLDY
(C) YQLF
(D) FIQY

Q9. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease?

(A) Depression and hypertension
(B) Stomach ulcers
(C) Anxiety and stress
(D) Hyperacidity

Q10. Given below are two statements:

Statement I: The decrease in first ionization enthalpy from B to Al is much larger than that from Al to Ga.

Statement II: The d orbitals in Ga are completely filled.

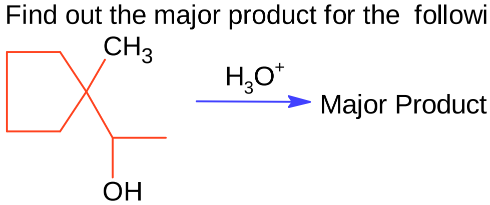
In the light of the above statements, choose the most appropriate answer from the options given below:

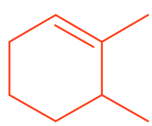
(A) Both the statement I and II are incorrect
(B) Both statement I and II are correct
(C) Statement I is correct but statement II is incorrect
(D) Statement I is incorrect but statement II is correct

Q11. When a hydrocarbon A undergoes combustion in the presence of air, it requires 9.5 equivalents of oxygen and produces 3 equivalents of water. What is the molecular formula of A?

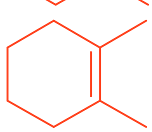
(A) C_6H_6
(B) C_8H_6
(C) C_9H_6
(D) C_9H_9

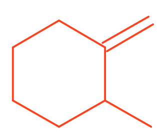
Q12. Find out the major product for the following reaction.



(A) 

(B) 

(C) 

(D) 

Q13. Match **List I** with **List II**

	List-I		**List-II**
A.	Elastomeric polymer	I.	Urea formaldehyde resin
B.	Fibre polymer	II.	Polystyrene
C.	Thermosetting Polymer	III.	Polyester
D.	Thermoplastic Polymer	IV.	Neoprene

Choose the correct answer from the options given below:

(A) A- II, B-I, C-IV, D- III
(B) A-IV, B-I, C-III, D-II
(C) A-IV, B-III, C-I, D-II
(D) A-II, B-III, C-I, D-IV

Q14. Which of the following relations are correct?

(A) $\Delta U = q + p\Delta V$

(B) $\Delta G = \Delta H - T\Delta S$

(C) $\Delta S = \dfrac{q_{rev}}{T}$

(D) $\Delta H = \Delta U - \Delta nRT$

Choose the most appropriate answer from the options given below:

(A) B and C only
(B) B and D only
(C) C and D only
(D) A and B only

Q15. The set of correct statements is:
(i) Maganese exhibits +7 oxidation state in its oxide.
(ii) Ruthenium and Osmium exhibit +8 oxidation in their oxides.
(iii) Sc shows +4 oxidation state which is oxidizing in nature.
(iv) Cr shows oxidizing nature in +6 oxidation state.
(A) (i) and (iii)
(B) (i), (ii) and (iv)
(C) (ii) and (iii)
(D) (ii), (iii) and (iv)

Q16. Match **List I** with **List II**

List-I		**List-II**	
A.	Osmosis	I.	Solvent molecules pass through semi permeable membrane towards solvent side
B.	Reverse osmosis	II.	Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes.
C.	Electro osmosis	III.	Solvent molecules pass through semi permeable membrane towards solution side.
D.	Electrophoresis	IV.	Dispersion medium moves in an electric field.

Choose the correct answer from the options given below:
(A) A- I, B-III, C-IV, D- II
(B) A-I, B-III, C-II, D-IV
(C) A-III, B-I, C-II, D-IV
(D) A-III, B-I, C-IV, D-II

Q17. According to MO theory the bond orders for O_2^{2-}, CO and NO^+ respectively, are
(A) 1,3 and 3
(B) 1,2 and 3
(C) 1,3 and 2
(D) 2, 3 and 3

Q18. A solution of CrO_5 in amyl alcohol has a _______ colour.
(A) Green
(B) Yellow
(C) Blue
(D) Orange-Red

Q19. Reaction of propanamide with Br_2/KOH(aq) produces:
(A) Ethylnitrile
(B) Ethylamine
(C) Propylamine
(D) Propanenitrile

Q20. Match **List I** with **List II**

List-I		**List-II**	
A.	Van't Hoff factor, i	I.	Cryoscopic constant
B.	K_f	II.	Isotonic solutions
C.	Solutions with same osmotic pressure	III.	$\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$
D.	Azeotropes	IV.	Solutions with same composition of vapour above it

Choose the correct answer from the options given below:
(A) A-III, B-I, C-IV, D- II
(B) A-III, B-I, C-II, D-IV
(C) A-III, B-II, C-I, D-IV
(D) A-I, B-III, C-II, D-IV

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. The denticity of the ligand present in the Fehling's reagent is_____________.

Q2. For conversion of compound A→B, the rate constant of the reaction was found to be 4.6×10^{-5} $Lmol^{-1}s^{-1}$. The order of the reaction is__________.

Q3. The volume of HCl, containing $73gL^{-1}$, required to completely neutralize NaOH obtained by reacting 0.69 g of metallic sodium with water, is_________ mL. (Nearest Integer) (Given: molar Masses of Na, Cl, O, H, are 23, 35.5, 16 and 1 g mol^{-1} respectively)

Q4. On heating $LiNO_3$ gives how many compounds among the following?________
Li_2O, N_2,O_2, $LiNO_2$, NO_2

Q5. At 298 K

$$N_2(g)+3H_2(g)\rightleftharpoons 2NH_3(g), K_1=4\times10^5$$

$$N_2(g)+O_2(g)\rightleftharpoons 2NO(g), K_2=1.6\times10^{12}$$

$$H_2(g)+\frac{1}{2}O_2(g)\rightleftharpoons H_2O(g), K_3=1.0\times10^{-13}$$

Based on above equilibria, the equilibrium constant of the reaction ,

$2NH_3(g)+\frac{5}{2}O_2(g)\rightleftharpoons 2NO(g)+3H_2O(g)$ is ____________$\times10^{-33}$ (Nearest integer)

Q6. Assume that the radius of the first Bohr orbit of hydrogen atom is 0.6 $\overset{o}{A}$. The radius of the third Bohr orbit of He^+ is ________ picometer. (Nearest integer)

Q7. A metal M forms hexagonal close- packed structure. The total number of voids in 0.02 mol of it is____________ $\times10^{21}$ (Nearest integer).
(Given $N_A = 6.02\times10^{23}$)

Q8. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of CO_2 was produced. The molar mass of compound is __ g mol^{-1}(Nearest integer)

Q9. Total number of acidic oxides among
N_2O_3, NO_2,N_2O,Cl_2O_7,SO_2,CO,CaO,Na_2O and NO is_______.

Q10. The equilibrium constant for the reaction

$Zn(s)+Sn^{2+}(aq)\rightleftharpoons Zn^{2+}(aq)+Sn(s)$ is 1×10^{20} at 298 K. The magnitude of standard electrode potential of Sn/Sn^{2+} if $E^{\theta}_{zn^{2+}/Zn}=-0.76V$ is__________$\times10^{-2}V$

(Nearest integer)

Given: $\frac{2.303RT}{F}=0.059V$

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Consider a function $f : IN \to IR$, satisfying
$f(1) + 2f(2) + 3f(3) + ... + xf(x) = x(x+1)f(x); x \geq 2$ with $f(1) = 1$.
Then $\dfrac{1}{f(2022)} + \dfrac{1}{f(2028)}$ is equal to

(A) 8200
(B) 8100
(C) 8400
(D) 8000

Q2. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2 + \dfrac{1}{\sqrt{2}}\right)^{200}$. If $\dfrac{{}^{200}C_{99}K}{a} = \dfrac{2^{\ell}m}{n}$, where m and n are odd numbers, then the ordered pair (ℓ, n) is equal to

(A) $(50, 51)$
(B) $(51, 99)$
(C) $(50, 101)$
(D) $(51, 101)$

Q3. The value of the integral $\int_1^2 \left(\dfrac{t^4+1}{t^6+1}\right)dt$ is

(A) $\tan^{-1}\dfrac{1}{2} + \dfrac{1}{3}\tan^{-1}8 - \dfrac{\pi}{3}$
(B) $\tan^{-1}\dfrac{1}{2} - \dfrac{1}{3}\tan^{-1}8 + \dfrac{\pi}{3}$
(C) $\tan^{-1}2 - \dfrac{1}{3}\tan^{-1}8 + \dfrac{\pi}{3}$
(D) $\tan^{-1}2 + \dfrac{1}{3}\tan^{-1}8 - \dfrac{\pi}{3}$

Q4. The set of all values of $t \in R$, for which the matrix
$\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$ is invertible, is

(A) R
(B) $\left\{(2k+1)\dfrac{\pi}{2}, k \in Z\right\}$
(C) $\left\{k\pi + \dfrac{\pi}{4}, k \in Z\right\}$
(D) $\{k\pi, k \in Z\}$

Q5. If the tangent at a point P on the parabola $y^2 = 3x$ is a parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is :

(A) $\dfrac{9}{\sqrt{5}}$
(B) $\dfrac{3}{2}\sqrt{5}$
(C) $3\sqrt{5}$
(D) $5\sqrt{3}$

Q6. Let R be a relation defined on N as a R b if $2a+3b$ is a multiple of $5, a, b \in N$. Then R is

(A) not reflexive
(B) an equivalence relation
(C) symmetric but not transitive
(D) transitive but not symmetric

Q7. The shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ is

(A) $3\sqrt{3}$
(B) $4\sqrt{3}$
(C) $2\sqrt{3}$
(D) $5\sqrt{3}$

Q8. The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ has a real solution x, is

(A) $\left[-2, -\frac{3}{2}\right]$
(B) $\left[-1, -\frac{1}{2}\right]$
(C) $[-2, -1]$
(D) $\left[-\frac{3}{2}, -1\right]$

Q9. The value of the integral $\int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{x} dx$ is equal to

(A) $\pi \log_e 2$
(B) $\frac{\pi}{2}\log_e 2$
(C) $\frac{\pi}{4}\log_e 2$
(D) $\frac{1}{2}\log_e 2$

Q10. Let $S=\{w_1, w_2, ...\}$ be the sample space associated to a random experiment. Let $P(w_n)=\frac{P(w_{n-1})}{2}, n \geq 2$. Let $A=\{2k+3\ell : k, \ell \in N\}$ and $B=\{w_n : n \in A\}$. Then $P(B)$ is equal to

(A) $\frac{1}{32}$
(B) $\frac{3}{64}$
(C) $\frac{1}{16}$
(D) $\frac{3}{32}$

Q11. Let $y=y(x)$ be the solution of the differential equation $x \log_e x \frac{dy}{dx}+y=x^2 \log_e x, (x>1)$. If $y(2)=2$, then $y(e)$ is equal to

(A) $\frac{1+e^2}{4}$
(B) $\frac{1+e^2}{2}$
(C) $\frac{2+e^2}{2}$
(D) $\frac{4+e^2}{4}$

Q12. Let f and g be twice differentiable functions on R such that

$f''(x)=g''(x)+6x$

$f'(1)=4g'(1)-3=9$

$f(2)=3g(2)=12$

Then which of the following is NOT true?

(A) There exists $x_0 \in (1, 3/2)$ such that $f(x_0) = g(x_0)$
(B) $g(-2) - f(-2) = 20$
(C) $|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$
(D) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

Q13. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is
(A) 86 (B) 89
(C) 79 (D) 84

Q14. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to :
(A) $B \Rightarrow ((\sim A) \Rightarrow B)$ (B) $A \Rightarrow ((\sim A) \Rightarrow B)$
(C) $A \Rightarrow (A \Leftrightarrow B)$ (D) $B \Rightarrow (A \Rightarrow B)$

Q15. The plane $2x - y + z = 4$ intersects the line segment joining the points $A(a, -2, 4)$ and $B(2, b, -3)$ at the point C in the ratio 2 : 1 and the distance of the point C from the origin is $\sqrt{5}$. If $ab < 0$ and P is the point $(a - b, b, 2b - a)$ then CP^2 is equal to
(A) $\frac{17}{3}$ (B) $\frac{97}{3}$
(C) $\frac{16}{3}$ (D) $\frac{73}{3}$

Q16. If $\vec{a} = \hat{i} + 2\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = 7\hat{i} - 3\hat{j} + 4\hat{k}, \vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$. Then $\vec{r} \cdot \vec{c}$ is equal to
(A) 30 (B) 32
(C) 36 (D) 34

Q17. If the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{1}$ and $\frac{x-a}{2} = \frac{y+2}{3} = \frac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane z = a is :
(A) 16 (B) 22
(C) 10 (D) 28

Q18. Let $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c} \cdot (\vec{a} \times \vec{b}) + 25 = 0, \vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$ and projection of $\vec{c}$ on $\vec{a}$ is 1, then the projection of $\vec{c}$ on $\vec{b}$ equals
(A) $\frac{5}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$
(C) $\frac{3}{\sqrt{2}}$ (D) $\frac{1}{5}$

Q19. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is
(A) 400 (B) 472
(C) 432 (D) 507

Q20. The area of the region $A = \left\{(x, y) : |\cos x - \sin x| \le y \le \sin x, 0 \le x \le \frac{\pi}{2}\right\}$ is

(A) $\frac{3}{\sqrt{5}} - \frac{3}{\sqrt{2}} + 1$

(B) $1 - \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{5}}$

(C) $\sqrt{5} + 2\sqrt{2} - 4.5$

(D) $\sqrt{5} - 2\sqrt{2} + 1$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. Let $\alpha = 8-14i, A = \left\{z \in c : \dfrac{\alpha z - \bar{\alpha}\bar{z}}{z^2 - (\bar{z})^2 - 112i} = 1\right\}$ and $B = \{z \in C : |z+3i| = 4\}$. Then $\sum\limits_{z \in A \cap B} (\text{Re}\,z - \text{Im}\,z)$ is equal to……….

Q2. A triangle is formed by the tangents at the point $(2, 2)$ on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$ and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to……..

Q3. Let A be a symmetric matrix such that $|A| = 2$ and $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$. If the sum of the diagonal elements of A is s, then $\dfrac{\beta s}{\alpha^2}$ is equal to……

Q4. Let $\{a_k\}$ and $\{b_k\}, k \in N$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k, k \in N$. If $c_2 = 5$ and $c_3 = \dfrac{13}{4}$ then $\sum\limits_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to…….

Q5. The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is…….

Q6. A circle with centre $(2, 3)$ and radius 4 intersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to……..

Q7. Let $X = \{11, 12, 13, ..., 40, 41\}$ and $Y = \{61, 62, 63, ..., 90, 91\}$ be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $|\bar{x} + \bar{y} - \sigma^2|$ is equal to……….

Q8. If the equation of the normal to the curve $y = \dfrac{x-a}{(x+b)(x-2)}$ at the point $(1, -3)$ is $x - 4y = 13$, then the value of a + b is equal to…….

Q9. Let $\alpha_1, \alpha_2, ..., \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq .... \geq |\alpha_7|$. Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to………

Q10. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1), b_n = b_{n-1} + a_{n-1}, \forall n \geq 2$. If $S = \sum\limits_{n=1}^{10} \dfrac{b_n}{2^n}$ and $T = \sum\limits_{n=1}^{8} \dfrac{n}{2^{n-1}}$, these $2^7(2S - T)$ is equal to……..

FIITJEE
KEYS to JEE (Main)-2023
PART – A (PHYSICS)
SECTION - A

1.	C	2.	B	3.	A	4.	D
5.	B	6.	D	7.	C	8.	B
9.	B	10.	A	11.	C	12.	A
13.	B	14.	D	15.	C	16.	C
17.	B	18.	B	19.	B	20.	A

SECTION – B

1.	40	2.	3872	3.	5	4.	7
5.	40	6.	25	7.	30	8.	12
9.	41	10.	800				

PART – B (CHEMISTRY)
SECTION - A

1.	A	2.	A	3.	A	4.	A
5.	D	6.	C	7.	A	8.	A
9.	A	10.	B	11.	B	12.	C
13.	C	14.	A	15.	B	16.	D
17.	A	18	C	19.	B	20.	B

SECTION - B

1.	4	2.	2	3.	15	4.	3
5.	4	6.	270	7.	36	8.	200
9.	4	10.	17				

PART – C (MATHEMATICS)

SECTION - A

1.	B	2.	C	3.	D	4.	A
5.	C	6.	B	7.	B	8.	D
9.	B	10.	B	11.	D	12.	D
13.	B	14.	ABD	15.	A	16.	D
17.	D	18	A	19.	C	20.	D

SECTION - B

1.	14	2.	10	3.	5	4.	9
5.	3000	6.	11	7.	603	8.	4
9.	9	10.	461				

FIITJEE

Solutions to JEE (Main)-2023

PART – A (PHYSICS)

SECTION – A

Sol1. $\left(x - At\right)^2 + \left(y - \frac{t}{B}\right)^2 = a^2$

$x = At$

$A = \frac{x}{t} = \frac{L}{T} = LT^{-1}$

$y = \frac{t}{B} = B = \frac{t}{y} = \frac{T}{L} = TL^{-1}$

$x = At$

$= A\left(\frac{1}{T}\right) = LT\left(\frac{1}{T}\right) \Rightarrow L$

$A = LT \quad y = \frac{t}{B}$

$\frac{T^{-1}}{L^{-1}T^{-1}} = \frac{LT}{LT} = L^{-1}$

$B = L^{-1}T^{-1}$

Sol2.

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}\cdot B$	$A\cdot\bar{B}$	$\bar{A}B \cdot^{+} A\cdot\bar{B}$
1	0	0	1	0	1	1
0	1	1	0	1	0	1
1	1	0	0	0	0	0 option 2
0	0	1	1	0	0	0

Sol3. Using Bernoulli's Principle

$\Rightarrow P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \Rightarrow \left(P_1 - P_2\right) = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$

$\Rightarrow \left(P_1 - P_2\right)A = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)A \Rightarrow Mg = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)A$

$\Rightarrow 5.4 \times 10^6 = \frac{1}{2} \times 1.2 \times [v_2^2 - v_1^2] \times 500$

$\Rightarrow 10.8 \times 10^3 = \frac{1}{2} \times 1.2 \times \left(v_2 - v_1\right)\left(v_2 + v_1\right)$

Since $v_1 + v_2 = 2 \times 300 \Rightarrow v_2 - v_1 = 30 m/s$

$\Rightarrow \left(\frac{v_2 - v_1}{v}\right) \times 100 = \frac{30}{300} \times 100 = 10\%$

Sol4. For rough surface :-

$mg\sin\theta - \mu mg\cos\theta = ma$

$g\sin 45\ \mu g\cos 45 = a$

$\frac{g}{\sqrt{2}} - \frac{\mu g}{\sqrt{2}} = a$

$\frac{g}{\sqrt{2}}(1-\mu) = a$ (1)

for smooth surface :-

$mg\sin\theta = ma$

$\frac{g}{\sqrt{2}} = a'$ (2) T = t

$S_1 = S_2$

$\frac{1}{2}a^1t^2 = \frac{1}{2}a(nt)^2$

$\left(\frac{g}{\sqrt{2}}\right)t^2 = \frac{g}{\sqrt{2}}(1-\mu)(n^2)t^2$

$\frac{g}{\sqrt{2}} = \frac{gn^2}{\sqrt{2}} - \frac{g}{\sqrt{2}}\mu n^2$

$1 = n^2 - \mu n^2$

$\mu n^2 = n^2 - 1$

$\mu = 1 - \frac{1}{n^2}$

Sol5. $T^2 \propto R^3 \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R}{R/4}\right)^2$

$\Rightarrow T_2^2 = \frac{T_1^2}{64} = \frac{(24)^2}{64} = 9 \Rightarrow T_2 = 3\,\text{hours}$

Sol6. (A) Directly proportional to the length of the potentiometer wire.
(B) Inversely proportional to the potential gradient of the wire.

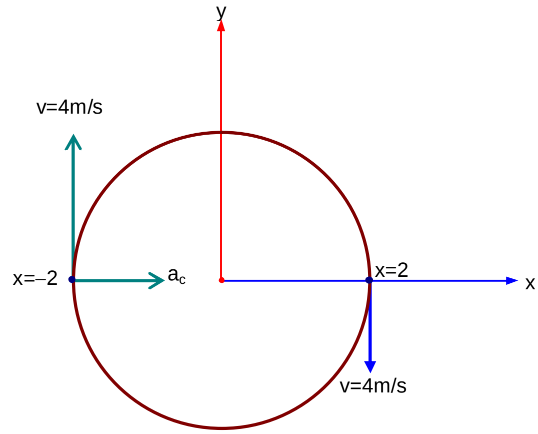
Sol7. $50 = v \times 10 \Rightarrow v = 5m/s$

$v = u + at \Rightarrow 5 = 0 + a \times 20$

$\Rightarrow a = \frac{1}{4}m/s^2 \Rightarrow F = ma = 20 \times \frac{1}{4} = 5N$

Sol8. $a_c = \frac{v^2}{r} = \frac{(4)^2}{2} = 8m/s^2\hat{i}$

$v = 4m/s\hat{j}$



Sol9. Resolving power = $\frac{2\mu\sin\theta}{1.22\lambda}$

Sol10. $V_{max} = 14mv$

$V_{min} = 6mv$

$\mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$

$\mu = \frac{14-6}{14+6} = \frac{8}{20} = 0.4$

Sol11. V = 220v

R = 40Ω

$I = \frac{V}{R} = \frac{220}{40} = \frac{11}{2} = 5.5\Omega$

$$I = \frac{220}{\sqrt{40^2 + \left[\left(50\times10^{-3}\right)^2 - \left(0.5\times10^{-6}\right)^2\right]}}$$

$$= \frac{220}{\sqrt{1600 + \left[\left(\frac{50}{1000}\right)^2 - \left(\frac{5}{10^7}\right)^2\right]}}$$

Option 3.

The rms current in current (b) can never be larger than that in (a).

Sol12. $W = \vec{F}.\vec{S} = |\vec{F}||\vec{S}|\cos\theta$

Sol13. $Q_1 = ms\Delta T = 0.6\times2222.3\times12 = 16000.56J$

Remaining heat $= 184000 - 16000.56 = 167999.44J$

Let mass of ice melted be m gm.

$\Rightarrow 167999.44 - m\times336000$

$\Rightarrow m = 0.49kg \approx 0.5kg$

$\Rightarrow$ mass of ice = 0.1 kg

Mass of water = 0.5 kg

$$\Rightarrow \frac{\text{mass of ice}}{\text{mass of water}} = \frac{0.1}{0.5} = \frac{1}{5}$$

Sol14. Electromagnetic waves are neutral.

$$\frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Sol16. for four turns

$$\ell = 4 \times 2\pi r_1$$

$$\mu_1 = \frac{\ell}{8\pi}$$

$$B_1 = \frac{\mu_0 NI}{2\mu_1} = \frac{\mu_0 NI}{2\left(\frac{\ell}{8\pi}\right)} \begin{matrix} = \dfrac{4\pi\mu_0 I \times 4}{\ell} \\ = \dfrac{16\pi\mu_0 I}{\ell} \end{matrix} \qquad \text{- (1)}$$

for single turn

$$\ell = 1 \times 2\pi r_2$$

$$r_2 = \frac{2}{2\pi}$$

$$B_2 = \frac{\mu_0 NI}{2\mu_2} \Rightarrow \frac{\mu_0 I \times I}{2\left(\frac{\ell}{2\pi}\right)} \Rightarrow \frac{\mu_0 \pi I}{\ell} \qquad \text{- (2)}$$

$1 \div 2$

$$\frac{B_1}{B_2} = \frac{16\pi\mu_0 I \times \ell}{\ell \times \mu_0 \pi I}$$

$$\frac{B_2}{B_2} = 16 \Rightarrow B_2 = \frac{32}{16} = 2T \qquad \text{- (3) option}$$

Sol17. $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2mk}}$

$$\lambda = \frac{h}{\sqrt{2mev}} \qquad \begin{matrix} m_\alpha = 4mp \\ q_\alpha = 2qp \end{matrix}$$

$$\frac{\lambda_{(\alpha)} \dfrac{h}{\sqrt{2(4mp)(2qp)V}}}{\lambda_P = \dfrac{h}{\sqrt{2mp\,qpV}}}$$

$$\frac{\lambda_\alpha}{\lambda_P} = \sqrt{\frac{2mp\,qp\,V}{2(4mp)(2qp)V}}$$

$$= \sqrt{\frac{1}{8}} \quad = \sqrt{\frac{1}{m}}$$

m = 8 Option – 2

Sol18. $A = 25\,cm^2 = 25 \times \left(10^{-2}\right)^2$

Resistance $= 10\,\Omega$

$a = 5 \times 10^{-2} m$

Velocity = $\dfrac{5 \times 10^{-2} m}{1} = 5 \times 10^{-2} m/s$

$\varepsilon = \dfrac{-d\phi}{dt} = \dfrac{40 \times 25 \times 10^{-4}}{1}$

$= 1000 \times 10^{-4} = 0.1V$

$R = 100\,\Omega$

$Wd = \dfrac{\varepsilon^2}{R} \Rightarrow \dfrac{0.12}{10} \Rightarrow \dfrac{1}{10 \times 10 \times 10} \Rightarrow 1 \times 10^{-3}\,\tau$ Option – 2

Sol19. $V_{rms} = \sqrt{\dfrac{3RT}{m}}$

$V_{avg} = \sqrt{\dfrac{8RT}{\pi m}}$

$V_{rms} = \left(\sqrt{\dfrac{\alpha+5}{\alpha}}\right) V_{avg}$

$\sqrt{\dfrac{3RT}{m}} = \left(\sqrt{\dfrac{\alpha+5}{\alpha}}\right)\left(\sqrt{\dfrac{8RT}{\pi m}}\right)$

Squaring :-

$\dfrac{3RT}{m} = \dfrac{(\alpha+5)8RT}{\alpha\pi m}$

$3\pi\alpha = 8\alpha + 40$

$3 \times \dfrac{22}{7}\alpha = 8\alpha + 40$

$66\alpha = 56\alpha + 280$

$10\alpha = 280$ Option – 2

$\alpha = 28$

Sol20. $q = 2 \times 10^{-2} C$

$\varepsilon = 30 Nc^{-1}$

$P = \hat{L} + 2\hat{j} + O\hat{R}$

$S = \left(O\hat{i} + O\hat{j} + O\hat{k}\right)$

Wd = Fd

$= \left[\left(2 \times 10^{-2}\right)(30)_{\hat{L}}\right]\left[-\hat{L} - 2\hat{J} - O\hat{k}\right]$

$\Rightarrow \left[60 \times 10^{-2}\hat{L}\right]\left[-\hat{L} - 2\hat{J}\right]$

$\Rightarrow -\left(0.6\hat{L}\right)\left(-\hat{L} - 2\hat{j}\right)$

$\Rightarrow -0.6J$

$\Rightarrow -0.6 \times 1000 mJ$

$= -600 mJ$ Option – 1

SECTION – B

Sol1. $m = 250\,g = \frac{250}{1000} = 0.25\,kg$

$F = -25x$
$V_{max} = 4$ m/s
F = ma.
0.25a = -25x
$\frac{a}{4} = -25x$
a = -100x
$\omega^2 = 100 \Rightarrow \omega = 10$
$\omega A = 4$
$A = \frac{4}{10} \Rightarrow \frac{2}{5} = 0.4\,m$
$A = 40\,cm$

Sol2. Inductor, $L = 2\mu H = 2\times10^{-6} H$

$X_L = \omega L = 2\pi fL$

$= 2\pi\times7\times10^{-3}\times2\times10^{-6}$

$= 2\times\frac{22}{7}\times7\times10^{-3}\times2\times10^{-6}.$

$= 88\times10^{-8}$

$\omega L = \frac{1}{\omega_C}$

$C = \frac{1}{\omega^2 L}$

$\Rightarrow \frac{1}{\omega L.\,\omega}$

$\Rightarrow \frac{1}{88\times10^{-2}2\pi\times f}$

$= \frac{1}{88\times10^{-2}\times\frac{22}{7}\times7\times10^{-3}}$

$\Rightarrow \frac{1}{88\times44\times10^{-5}}$

$\Rightarrow \frac{10^5}{3872}$

$\Rightarrow x = 3872$

Sol3. $\left(\frac{E}{r+5}\right)\times5 = 200x$ ……………………(i)

$\left(\frac{E}{r+15}\right)\times15 = 300x$ ……………………(ii)

$\Rightarrow r = 5\Omega$

Sol4. $\mu_1 = \sqrt{2.8\times1} = \sqrt{2.8}$

$\mu_2 = \sqrt{6.8\times1} = \sqrt{6.8}$

$\Rightarrow \mu_1 \sin\hat{i} = \mu_2.\sin(90-i)$

$\Rightarrow \tan i = \frac{\mu_2}{\mu_1} = \sqrt{\frac{6.8}{2.8}} = \sqrt{\frac{2.8+4}{2.8}}$

$\Rightarrow \tan i = \left(1+\frac{10}{7}\right)^{1/2}$

$$\Rightarrow i = \tan^{-1}\left(1+\frac{10}{7}\right)^{1/2} \Rightarrow \theta = 7$$

Sol5. $\frac{v^2}{R} = \frac{dv}{dt} \Rightarrow \int_{15}^{v} \frac{dv}{v^2} = \int_0^t \frac{dt}{R}$

$$\Rightarrow \frac{1}{15} - \frac{1}{v} = \frac{t}{R} \Rightarrow \frac{1}{v} = \frac{R-15t}{15R} \Rightarrow v = \frac{15R}{R-15t} = \frac{dx}{dt}$$

$$\Rightarrow 15R\int_0^t \frac{dt}{(R-15t)} = \int_0^{300\pi} dx$$

$$\Rightarrow \frac{15R}{-15}\left[\ell n(R-15t)\right]\Big|_0^t = 300\pi \Rightarrow R\ell n\left(\frac{R}{R-15t}\right) = 300\pi$$

$$\Rightarrow \ell n\left(\frac{R}{R-15t}\right) = \pi/2 \Rightarrow \frac{R}{R-15t} = e^{\pi/2} \Rightarrow R-15t = Re^{-\pi/2}$$

$$\Rightarrow t = \frac{R}{15}\left(1-e^{-\pi/2}\right) = 40\left(1-e^{-\pi/2}\right)$$

Sol6. $F = \eta A.\frac{\Delta v}{\Delta y} \Rightarrow 0.1 = 5\times10^{-3}\times0.2\times\frac{v}{0.25\times10^{-3}}$

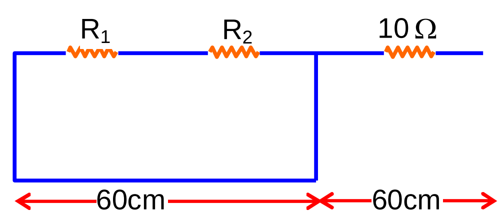
$$\Rightarrow v = 25\times10^{-3} m/s$$

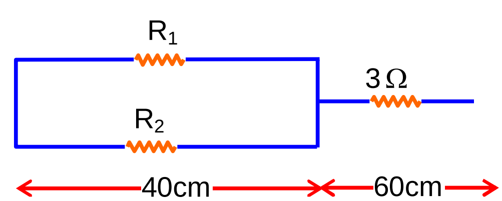
Sol7. Using the concept of balanced wheatstone bridge condition.

$$\Rightarrow \frac{R_1+R_2}{10} = \frac{60}{40} = \frac{3}{2} \Rightarrow R_1+R_2 = 15$$

$$\frac{R_1R_2}{(R_1+R_2)\times3} = \frac{40}{60} = \frac{2}{3}$$

$$\Rightarrow R_1R_2 = 30$$





Sol8. $V = 2ar^2 + b$

$$E = -\frac{dv}{dx} = -4ar$$

$$E.4\pi r^2 = \frac{q_{in}}{\varepsilon_0}$$

$$\varepsilon.4\pi r^2\varepsilon_0 = q_{in}$$

$$q_{in} = (-4ar)\varepsilon_0.4\pi r^2$$

$$\Rightarrow -16a\varepsilon_0 r^3\pi$$

$$\frac{dq_{in}}{dr} = -48a\varepsilon_0 r^3\pi$$

$v = \frac{4}{3}\pi r^3$

$\frac{dv}{dr} = 4\pi r^2$

$\delta = \frac{dq}{dv} = \frac{-4\delta a \varepsilon_0 r^2}{4\pi r^2} = -12\varepsilon_0 a$

$-\lambda a \varepsilon_0 \Rightarrow -12\varepsilon_0 a$

$\lambda = 12$

Method -II

$E = \frac{-dV}{dr} = -4ar = \frac{\rho r}{3\varepsilon_0} \Rightarrow \rho = -12a\varepsilon_0 \Rightarrow \lambda = 12$

Sol9. $\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{5.25}{5.00} = \frac{h}{h_1}$

Least count = $1MSD - 1VSD = \frac{1}{20} - \frac{49}{50\times 20} = 0.01mm$

$\ell n(\mu) = \ell n(h) - \ell n(h') \Rightarrow \frac{d\mu}{\mu} = \frac{dh}{h} - \frac{dh'}{h'}$

$\Rightarrow d\mu = \left[\frac{0.01}{5.25} + \frac{0.01}{5.00}\right] \times \frac{5.25}{5.00} = \frac{41}{10} \times 10^{-3}$

$\Rightarrow x = 41$

Sol10. $\Delta L = \int \tau dt = \int_0^2 mg \times v\cos\theta dt$

$\Rightarrow \Delta L = 10\sqrt{2}\left(\frac{t^2}{2}\right)\Big|_0^2 = 20\sqrt{2} = \sqrt{800} \Rightarrow k = 800$

PART – B (CHEMISTRY)

SECTION – A

Sol1. Ni is used in hydrogenation of unsaturated fat to make edible oil. Hydride of silicon is electron-precise and neither electron deficint, nor electron rich.

Sol2.

Calamine	– $ZnCO_3$
Siderite	– $FeCO_3$
Sphalerite	– ZnS
Malachite	– $CuCO_3.Cu(OH)_2$

Sol3.

$CH_3-CH_2-CH(CH_3)-CH_2Br \longrightarrow CH_3-CH_2-C(CH_3)=CH_2$ (1 prodct)

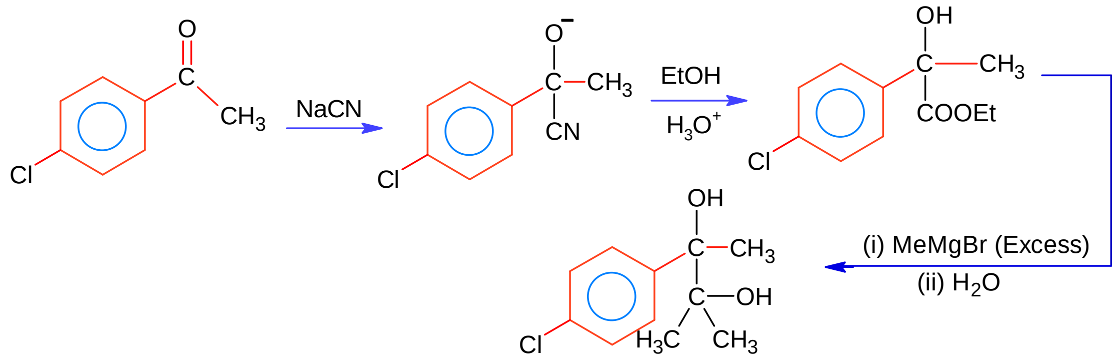
$CH_3-CH(Br)-CH_3 \longrightarrow CH_3-CH=CH_2$ (1 prodct)

$CH_3-CH_2-CH_2-CH(Br)-CH_3 \longrightarrow \underset{\text{(Cis + Trans)}}{CH_3-CH_2-CH=CH-CH_3} + CH_3-CH_2-CH_2-CH=CH_2$ (3 product)

$CH_3-CH_2-C(CH_3)_2-CH(Br)-CH_3 \longrightarrow CH_3-CH_2-C(CH_3)_2-CH=CH_2$ (1 prodct)

Sol4. The growth of fish gets inhibited if the concentration of dissolved oxygen in water is less than 6ppm and biological oxygen demend in clean water should be less than 5ppm.

Sol5.



Sol6. $\left[FeF_6\right]^{3-}$; $Fe^{3+} \rightarrow d^5$ configuration (WL) $\Rightarrow t_{2g}{}^3\, eg^2$

Number of unpaired electrons = 5

Magnetic moment $= \sqrt{5(5+2)} = \sqrt{35}$ B.M

$\left[CoF_6\right]^{3-}$; $Co^{3+} \rightarrow d^6$ configuration (WL) $\Rightarrow t_{2g}{}^4\, eg^2$

Number of unpaired electrons = 4

Magnetic moment $= \sqrt{4(4+2)} = \sqrt{24}$ B.M

$\left[Co\left(C_2O_4\right)_3\right]^{3-}$; $Co^{3+} \rightarrow d^6$ (SL) $\Rightarrow t_{2g}{}^6\, eg^0$

Number of unpaired electrons = 0

Mapnetic moment = 0

Sol7. $I^- + H_2O_2 \longrightarrow I_2(A) + H_2O$

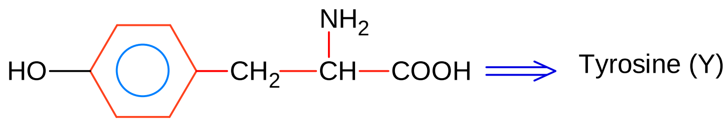
I_2 + starch (Indicator) $\longrightarrow$ Blue adsorption complex

Sol8. Hydrolysis of above tetrapeptide will give the following amino acids:

$Ph-CH_2-CH(NH_2)-COOH \Longrightarrow$ Phenylalanine (F)

$CH_3-CH(CH_3)-CH_2-CH(NH_2)-COOH \Longrightarrow$ Leucine (L)

$COOH-CH_2-CH(NH_2)-COOH \Longrightarrow$ Aspartic Acid (D)



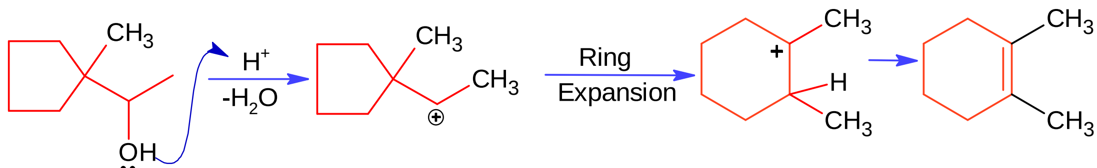
Sol9. Equanil is used for treating depression and hypertension.

Sol10. The first ionization energy of $B \rightarrow 801$ KJ / mole
$Al \rightarrow 577$ KJ / mole
$Ga \rightarrow 579$ KJ / mole
Electronic configuration of Ga $\rightarrow$ [Ar] $3d^{10}4s^{2}4p^{1}$

Sol11. $C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + y/2H_2O$

$$\left.\begin{aligned} x + \frac{y}{4} &= 9.5 \\ \frac{y}{2} &= 3 \end{aligned}\right\} \rightarrow x = 8,\ y = 6$$

Sol12.



Sol13. Neoprene – Elastomer
Polyester – Fibre
Polystryrene – Thermoplastic
Urea formaldehyde Resin – Thermosetting

Sol14. $\Delta U = q + w$

$\Delta G = \Delta H - T\Delta S$

$\Delta S = \frac{q_{reversible}}{T}$

$\Delta H = \Delta U + \Delta(nRT)$

Sol15. Sc does not show +4 oxidation state.
Rest statement are true.

Sol16. According to definition of the process mentioned.

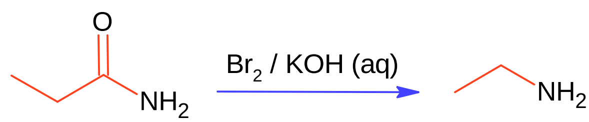
Sol17. $CO \Rightarrow$ Bond order $= \frac{10-4}{2} = 3$

$NO^{+} \Rightarrow$ Bond order $= \frac{10-4}{2} = 3$

$O_2^{2-} \Rightarrow$ Bond order $= \frac{10-8}{2} = 1$

Sol18. A solution of CrO_5 in amyl alcohol has blue colour.

Sol19.



Sol20. $i = \dfrac{\text{Normal Molar Mass}}{\text{Abnormal Molar Mass}}$

K_f Cryosocopic constant

Solution with same osmotic pressure are known as isotonic solution.

The composition of azeotropic mixture and the vapour above it are same.

SECTION – B

Sol1. Denticity of tartarate ion is copper tartarate complex is 4

Sol2. Unit of rate constant $\Rightarrow (\text{conc})^{1-n}$ time^{-1}

$K = 4.6 \times 10^{-5}\, \text{Lmol}^{-1}\text{s}^{-1} = 4.6 \times 10^{-5}\left(\text{moll}^{-1}\right)^{-1} \sec^{-1}$

$(l - n) = -1 \Rightarrow n = 2$

Sol3. $Na + H_2O \longrightarrow NaOH + 1/2H_2$

Number of moles of NaOH = number of moles of $Na = \dfrac{0.69}{23} = 0.03$ mole

Number of moles of HCl required = number of moles of NaOH

$\dfrac{73}{36.5} \times V(\ell) = 0.03$

$\Rightarrow v = 1.5 \times 10^{-2}\ell = 15\text{ml}$

Sol4. $2LiNO_3 \xrightarrow{\Delta} Li_2O + 2NO_2 + 1/2O_2$

Sol5. The desired reaction can be obtained by

(ii) + 3 $\times$(iii) –(i)

So $K_{eq} = \dfrac{K_2 \times K^3}{K_1} = \dfrac{1.6 \times 10^{12} \times \left(10^{-13}\right)^3}{4 \times 10^5} = 4 \times 10^{-33}$

Sol6. $r_{He^+} = r_{1,H} \times \dfrac{n^2}{z} = 0.6 \times \dfrac{(3)^2}{2} = 2.7\,\overset{o}{A} = 270\,\text{pm}$

Sol7. Number of spheres in 1 unit cell of hcp = 6

Number of teterahedral voids in 1 unit cell of hcp = 12

Number of octahedral voids in 1 unit cell of hcp = 6

One unit cell of hcp contains 18 voids

Number of voids = $\dfrac{18}{6} \times 6.02 \times 10^{23} \times 0.02 \sim 36 \times 10^{21}$

Sol8. Let molar mass of compound = x gm/ mole

Mass of compound = 0.01x gm

Number of carbon = number of moles of CO_2

$$\frac{0.01x \times 0.6}{12} = \frac{4.4}{44}$$

$\Rightarrow$ x = 200 gm / mole

Sol9. Acidic oxides are N_2O_3 , NO_2, Cl_2O_7 and SO_2.
N_2O, CO and NO are neutral oxide
CaO and Na_2O are basic oxides.

Sol10. $E_{cell} = E^0_{cell} - \frac{0.0591}{n}\log Q$

At equilibrium

$$O = E^0_{cell} - \frac{0.0591}{n}\log k_{eq}$$

$$\Rightarrow E^0_{cell} = \frac{0.0591}{2}\log 10^{20}$$

$$\Rightarrow E^0_{Sn^{2+}|sn} - (-0.76) = \frac{0.0591 \times 20}{2}$$

$$\Rightarrow E^0_{sn^{2+}|Sn} = -0.17\,V$$

$$\therefore E^0_{sn|sn^{2+}} = 0.17\,V = 17 \times 10^{-2}\,V$$

PART – C (MATHEMATICS)

SECTION – A

Sol1. $x = 2 \Rightarrow f(1) + 2f(2) = 6f(2)$

$$\Rightarrow f(2) = \frac{1}{4}$$

$$x = 3 \Rightarrow f(1) + 2f(2) + 3f(3) = 12f(3)$$

$$\Rightarrow f(3) = \frac{1}{6}$$

$$x = 4 \Rightarrow f(1) + 2f(2) + 3f(3) + 4f(4) = 20f(4)$$

$$\Rightarrow f(4) = \frac{1}{8}$$

$$\Rightarrow f(n) = \frac{1}{2n}, n \in \mathbb{N}, n \ge 2$$

$$\therefore \frac{1}{f(2022)} + \frac{1}{f(2028)} = 2(2022 + 2028) = 8100$$

Sol2. $K = {}^{99}C_1 + {}^{99}C_3 + ... + {}^{99}C_{99} = \frac{2^{99}}{2} = 2^{98}$

$$a = {}^{200}C_{100} \cdot 2^{100}\left(\frac{1}{\sqrt{2}}\right)^{100} = {}^{200}C_{100} \cdot 2^{50}$$

$$\therefore \frac{{}^{200}C_{99}K}{a} = \frac{{}^{200}C_{99} \cdot 2^{98}}{{}^{200}C_{100} \cdot 2^{50}} = \frac{25 \cdot 2^{50}}{101} \Rightarrow (\ell, n) = (50, 101)$$

Sol3. $I=\int_1^2 \frac{t^4+1}{t^6+1}dt=\int_1^2 \frac{t^4+1-t^2+t^2}{t^6+1}dt$

$=\int_1^2 \frac{\left(t^4+1-t^2\right)+t^2}{\left(t^2+1\right)\left(t^4+1-t^2\right)}dt=\int_1^2 \frac{\left(t^4+1-t^2\right)}{\left(t^2+1\right)\left(t^4+1-t^2\right)}dt+\int_1^2 \frac{t^2}{\left(t^3\right)^2+1}$

$=\int_t^2 \frac{dt}{t^2+1}+\frac{1}{3}\int_1^2 \frac{3t^2}{\left(t^3\right)^2+1}$

$=\left[\tan^{-1}t\right]_1^2+\frac{1}{3}\left[\tan^{-1}\left(t^3\right)\right]_1^2$

$=\tan^{-1}2-\frac{\pi}{4}+\frac{1}{3}\tan^{-1}8-\frac{\pi}{4}\cdot\frac{1}{3}$

$=\tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$

Sol4. $\Delta=\begin{vmatrix} e^t & e^{-t}\left(\sin t-2\cos t\right) & e^{-t}\left(-2\sin t-\cos t\right) \\ e^t & e^{-t}\left(2\sin t+\cos t\right) & e^{-t}\left(\sin t-2\cos t\right) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{vmatrix}$

$R_1\to R_1-R_2$ and $R_2\to R_2-R_3$

$\therefore \Delta=\begin{vmatrix} 0 & e^{-t}\left(-\sin t-3\cos t\right) & e^{-t}\left(-3\sin t+\cos t\right) \\ 0 & e^{-t}\cdot 2\sin t & e^{-t}\left(-2\cos t\right) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{vmatrix}$

$=6e^{-t}\neq 0\ \forall\ t\in\mathbb{R}$

$\therefore$ Invertible for $\mathbb{R}$

Sol5. For parabola :

Tangent at point t is $yt=\frac{3}{2}\left(x+\frac{t^2}{3}\right)$

$\therefore \frac{3}{2t}=-\frac{1}{2}\Rightarrow t=-3$

$\therefore P=\left(3,-3\right)$

For ellipse tangent at $\left(x_1, y_1\right)$ is $\frac{xx_1}{4}+\frac{yy_1}{1}=1$

Product of Slopes = $\frac{-x_1}{4y_1}=-1\Rightarrow x_1=4y_1$

Also, $\frac{x_1^2}{4}+y_1^2=1$

Solve to get $\left(x_1, y_1\right)=\left\{\underset{Q}{\left(\frac{4}{\sqrt{5}},\frac{1}{\sqrt{5}}\right)},\underset{R}{\left(\frac{-4}{\sqrt{5}},\frac{-1}{\sqrt{5}}\right)}\right\}$

$$\text{Area }(PQR) = \frac{1}{2}\left\|\begin{matrix} 3 & -3 & 1 \\ \frac{4}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 1 \\ \frac{-4}{\sqrt{5}} & \frac{-1}{\sqrt{5}} & 1 \end{matrix}\right\| = 3\sqrt{5}$$

Sol6. a R b means $2a + 3b = 5m$

$a, b \in \{5k, 5k+1, 5k+2, 5k+3, 5k+4\}$

For $2a + 3b$ to be a multiple of 5, both a and b must be of the same type, i.e.,

$a = 5k, b = 5\ell$

or $a = 5k+1, b = 5\ell + 1$

.... and so on.

$a\,R\,a \Rightarrow 2a + 3a = 5a$ (a multiple of 5)

Hence reflexive.

Again, given a R b

$\Rightarrow 2a + 3b = 5m$, i.e. a, b are of same type $\Rightarrow$ b, a are of same type

$\Rightarrow 2b + 3a$ is a multiple of 5

$\Rightarrow$ symmetric relation.

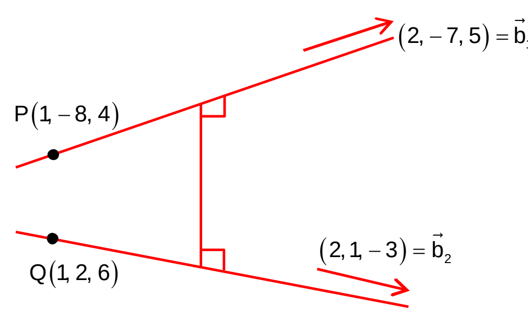
Again, $\left.\begin{matrix} a\,R\,b \Rightarrow \text{a and b are of same type} \\ b\,R\,c \Rightarrow \text{b and c are of same type} \end{matrix}\right\} \Rightarrow$ a and c are of same type

$\Rightarrow$ transitive relation $\Rightarrow$ a R b

Hence, the given relation is an equivalence relation.

Sol7. $\overrightarrow{PQ} = (0, 10, 2)$

$$d = \left|\overrightarrow{PQ} \cdot \frac{\vec{b}_1 \times \vec{b}_2}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = 4\sqrt{3}$$

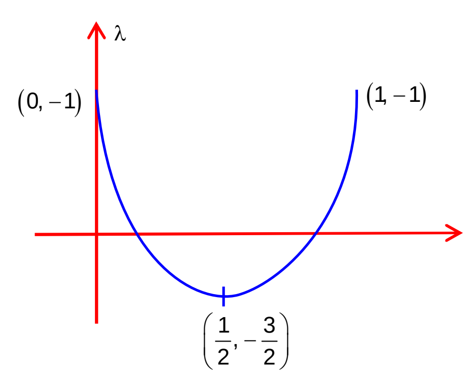


Sol8. Put $\cos^2 x = t$, then equation becomes

$(2t-1)^2 - 2(1-t)^2 - 2t = \lambda, 0 \le t \le 1$

$\Rightarrow 2t^2 - 2t - 1 = \lambda, 0 \le t \le 1$

$\Rightarrow -\frac{3}{2} \le \lambda \le 1$



Sol9. $I = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{x} dx$

Put $x = \frac{1}{z} \Rightarrow dx = -\frac{1}{z^2}dz$

$\therefore I = \int_2^{\frac{1}{2}} \frac{\tan^{-1}\frac{1}{z}}{\frac{1}{z}} \cdot \frac{-1}{z^2}dz = \int_{\frac{1}{2}}^{2} \frac{\frac{\pi}{2} - \tan^{-1}z}{z}dz$

$\Rightarrow 2I = \frac{\pi}{2} \cdot \int_{\frac{1}{2}}^{2} \frac{1}{z}dz = \frac{\pi}{2} \cdot 2\ell n2$

$\Rightarrow I = \frac{\pi}{2}\ell n2$

Sol10. We've $P(W_1) + P(W_2) + P(W_3) + ... = 1$

$\Rightarrow P + \frac{P}{2} + \frac{P}{4} + ... \infty = 1$

$\Rightarrow \frac{P}{1-\frac{1}{2}} = 1 \Rightarrow P = \frac{1}{2}$

$\therefore P(W_1) = \frac{1}{2}, P(W_2) = \frac{1}{2^2}, P(W_3) = \frac{1}{2^3} .... P(W_n) = \frac{1}{2^n}$

Now, $2k + 3\ell \in \{5, 7, 8, 9, 10, 11.....\}$

$= \mathbb{N} - \{1, 2, 3, 4, 6\}$

$\therefore P(B) = 1 - P(W_1) - P(W_2) - P(W_3) - P(W_4) - P(W_6)$

$= 1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16} - \frac{1}{64} = \frac{3}{64}$

Sol11. We've $\frac{dy}{dx} + \frac{y}{x\ell nx} = x$

IF = $e^{\int \frac{1}{x\ell nx}dx} = e^{\ell n(\ell nx)} = \ell nx$

$\therefore y \cdot \ell nx = \int x\ell nx\, dx$

$= \ell nx \cdot \frac{x^2}{2} - \frac{x^2}{4} + C$

$y(2) = 2 \Rightarrow 2\ell n2 = 2\ell n2 - 1 + c \Rightarrow c = 1$

At $x = e, y = \frac{e^2}{2} - \frac{e^2}{4} + 1 = \frac{e^2}{4} + 1$

Sol12. $f'(1) = 9, g'(1) = 3, f(2) = 12, g(2) = 4$

Given $f''(x) - g''(x) = 6x$

Integrate to get $f'(x) - g'(x) = 3x^2 + c$ ……….(i)

Integrate again : $f(x) - g(x) = x^3 + cx + d$ ……….(ii)

Put $x = 1$ in equation (i), $f'(1) - g'(1) = 3 + c$

$\Rightarrow c = 9 - 3 - 3 = 3$

Put $x = 2$ in equation (ii), $f(2) - g(2) = 8 + 2c + d$

$\Rightarrow d = 12 - 4 - 8 - 6 = -6$

$\therefore f(x)-g(x)=x^3+3x-6=h(x)$ (say).

$h(1)=-2, h\left(\frac{3}{2}\right)=\frac{15}{8}$

$\therefore$ By intermediate value theorem, $\exists\, x_0 \in \left(1,\frac{3}{2}\right)$

such that $h(x_0)=0$ or $f(x_0)=g(x_0)$

$\therefore$ option 1 is correct.

Again $h(-2)=20$ (option 2 is correct)

also, $|f'(x)-g'(x)|=|3x^2+3|$

$=3|x^2+1|<6\ \forall\ x\ t(-1,1)$ (option 3 is correct)

Further,

$|f(x)-g(x)|<8$

$\Rightarrow |x^3+3x-6|<8 \Rightarrow -8<x^3+3x-6<8$

$\Rightarrow 0<x^3+3x+2$ and $x^3+3x-14<0$

$\Rightarrow (x+2)(x^2-2x+7)<0$

$\Rightarrow x<-2$

But we've $-1<x<2$

$\therefore$ option 4 is not correct.

Sol13.

Form			Total Numbers
G			4! = 24
H			4! = 24
O			4! = 24
T	G		3! = 6
T	H		3! = 6
T	O	G	2! = 2
T	O	H	2! = 2
			88

Hence the word TOUGH has 89th position

Sol14.

A	B	$\sim A$	$\sim A \vee B$	$B \Rightarrow ((\sim A)\vee B)$	$\sim A \Rightarrow B$	$A \Rightarrow B$	$A \Leftrightarrow B$	1	2	3	4
F	F	T	T	T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	F	T	T	T	T
T	F	F	F	T	T	F	F	T	T	F	T
T	T	F	T	T	T	T	T	T	T	T	T

1, 2 and 4 are correct option.

Sol15. $C = \left(\frac{4+a}{3}, \frac{-2+2b}{3}, \frac{-2}{3}\right)$

C lies on the given plane :

$2\left(\frac{4+a}{3}\right) - \left(\frac{2b-2}{3}\right) - \frac{2}{3} = 4$

$\Rightarrow a - b = 2$ ....(i)

$\therefore C = \left(\frac{6+b}{3}, \frac{2}{3}(b-1), \frac{-2}{3}\right)$

$OC^2 = 5 \Rightarrow b = -1, \frac{1}{5}$

$b = -1 \Rightarrow a = 1$ (from (i))

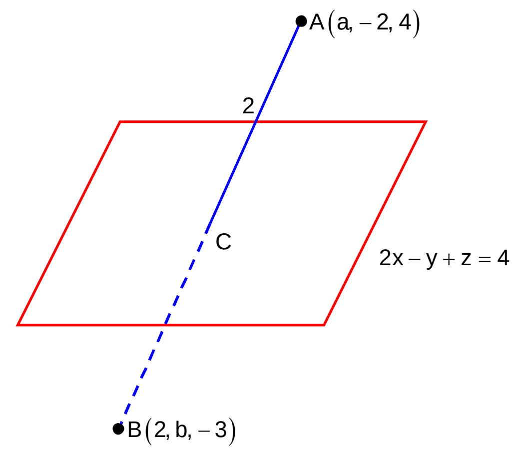
$b = \frac{1}{5} \Rightarrow a = \frac{11}{5}$ (from (ii))

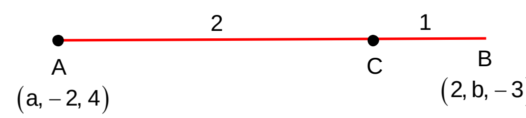
$\because ab < 0 \Rightarrow a = 1, b = -1$

$\therefore P = (2, -1, -3)$

and $C = \left(\frac{5}{3}, \frac{-4}{3}, -\frac{2}{3}\right)$

$\Rightarrow CP^2 = \frac{17}{3}$





Sol16. $\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$

$\Rightarrow (\vec{r} - \vec{c}) \times \vec{b} = 0 \Rightarrow \vec{r} = \vec{c} + \lambda\vec{b}$

Also, $\vec{r} \cdot \vec{a} = 0 \Rightarrow 0 = \vec{c} \cdot \vec{a} + \lambda\, \vec{b} \cdot \vec{a}$

$\Rightarrow \lambda = \frac{-\vec{c} \cdot \vec{a}}{\vec{b} \cdot \vec{a}} = -\frac{15}{3} = -5$

$\vec{r} = (7, -3, 4) - 5(1,1,1) = (2, -8, -1)$

$\therefore \vec{r} \cdot \vec{c} = (2, -8, -1) \cdot (7, -3, 4) = 34$

Sol17. Any point on first line : $(1+\lambda, 2+2\lambda, -3+\lambda)$

Any point on second line : $(a+2t, -2+3t, 3+t)$

Equate the respective coordinates and solve to get $\lambda = 22, a = -9$

$\therefore P = (23, 46, 19)$

$\therefore$ required distance = $19 - (-9) = 28$

Sol18. Let $\vec{c} = (\ell, m, n)$

Now, $\vec{c} \cdot (\vec{a} \times \vec{b}) = -25$

$\Rightarrow -3\ell + 4m + 5n = 5$ ....(i)

$\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4 \Rightarrow \ell + m + n = 4$ ....(ii)

$\vec{c} \cdot \hat{a} = 1 \Rightarrow 4\ell + 3m = 5$ ....(iii)

Solve equation (i), (ii) and (iii) to get $\ell = 2, m = -1, n = 3$

Now, $\vec{c}\cdot\hat{b} = \dfrac{5}{\sqrt{2}}$

Sol19. Let A = set of 3 digit numbers divisible by 3.

$n(A) = 300$.

B = set of 3-digit numbers divisible by 4

$n(B) = 225$

$n(A \cap B) = 75$

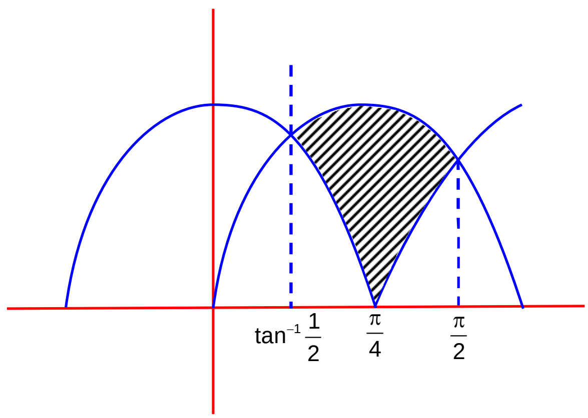
$n(A \cup B) = 300 + 225 - 75 = 450$

C = set of 3-digit numbers divisible by 48

$n(C) = 18$

$\therefore$ number of required numbers = 450 – 18 = 432.

Sol20.



$y = |\cos x - \sin x| = \sqrt{2}\left|\cos\left(x + \dfrac{\pi}{4}\right)\right|$

$$= \begin{cases} \sqrt{2}\cos\left(x + \dfrac{\pi}{4}\right), 0 \le x \le \dfrac{\pi}{4} \\ -\sqrt{2}\cos\left(x + \dfrac{\pi}{4}\right), \dfrac{\pi}{4} \le x \le \dfrac{\pi}{2} \end{cases}$$

Shaded area = $\displaystyle\int_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{4}} \sin x - (\cos x - \sin x)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x + (\cos x - \sin x)dx$

$= \left[-2\cos x - \sin x\right]_{\tan^{-1}\frac{1}{2}}^{\pi/4} + \left[\sin x\right]_{\pi/4}^{\pi/2} = \sqrt{5} - 2\sqrt{2} + 1$

SECTION – B

Sol1. Let $z = x + iy$

Now, $\dfrac{\alpha z - \bar{\alpha}\bar{z}}{z^2 - \bar{z}^2 - 112i} = 1 \Rightarrow 4y - 7x = xy - 28$ ....(i)

Also, $|z + 3i| = 4 \Rightarrow x^2 + (y+3)^2 = 16$ ....(ii)

Solve, (i) & (ii) to get $(x, y) = \{(4, -3), (0, -7)\}$

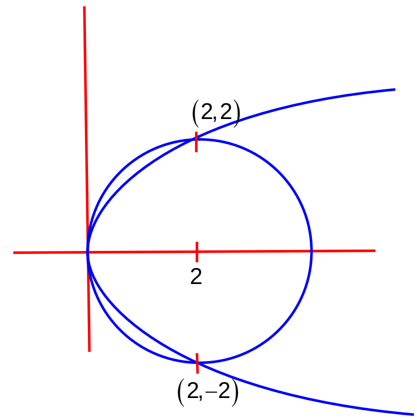
$$\therefore \sum_{z \in A \cap B} (\operatorname{Re} z - \operatorname{Im} z) = \{4-(-3)\} + \{0-(-7)\} = 14$$

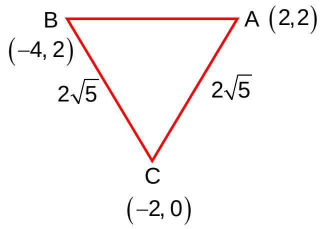
Sol2. Tangent on the parabola : $x - 2y + 2 = 0\,(L_1)$

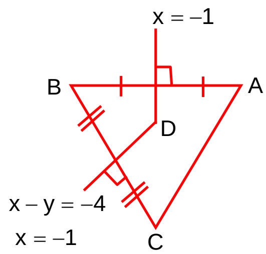
Tangent on the circle: $y = 2\,(L_2)$

given $x + y + 2 = 0\,(L_3)$

Solve L_1, L_2 and L_3 to get the triangle as







Perpendicular bisectors of the sides AB and BC intersects at $(-1, 3)$ (circumcentre)

$\therefore r^2 = CD^2 = 1 + 9 = 10$

Sol3. Let $A = \begin{bmatrix} a & h \\ h & g \end{bmatrix}$

given $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} a & h \\ h & g \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a + h & 2h + g \\ 3a + \frac{3}{2}h & 3h + \frac{3}{2}g \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$ ……….(i)

Also, $ag - h^2 = 2$ …………(ii)

Solve (i) & (ii) to get $a = \frac{3}{4}, g = 3, h = -\frac{1}{2}, \alpha = \frac{3}{2}, \beta = 3$

Now, $s = a + g = \frac{15}{4}$

$$\therefore \frac{\beta s}{\alpha^2} = \frac{3 \times \frac{15}{4}}{\frac{9}{4}} = 5$$

Sol4. $a_k = 4r_1^{k-1}$ $\quad r_1 < r_2$

$b_k = 4r_2^{k-1}$

$c_k = a_k + b_k$

$c_2 = 5 \Rightarrow r_1 + r_2 = \frac{5}{4}$ ....(i)

$c_3 = \frac{13}{4} \Rightarrow r_1^2 + r_2^2 = \frac{13}{16}$ ....(ii)

Solve to get $r_1 = \frac{3}{4}, r_2 = \frac{1}{2}$ or $r_1 = \frac{1}{2}, r_2 = \frac{3}{4}$

Since $r_1 < r_2 \Rightarrow r_1 = \frac{1}{2}, r_2 = \frac{3}{4}$

Now, $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4) = \sum_{k=1}^{\infty} a_k + \sum_{k=1}^{\infty} b_k - (12a_6 + 8b_4)$

$$= \frac{4}{1-\frac{1}{2}} + \frac{4}{1-\frac{3}{4}} - \left(12 \cdot 4\left(\frac{1}{2}\right)^5 + 8 \cdot 4 \cdot \left(\frac{3}{4}\right)^3\right) = 9$$

Sol5. $54 = 2^1 \times 3^3$ Total 4 digit numbers = 9000

Let A = set of 4 digit numbers divisible by 2

B = set of 4 digit numbers divisible by 3.

$n(A) = \frac{9000}{2} = 4500$

$n(B) = \frac{9000}{3} = 3000$

$n(A \cap B) = \frac{9000}{6} = 1500$

$n(A) - n(A \cap B) = 4500 - 1500 = 3000$

Sol6. Chord of contact of S is same as the given line

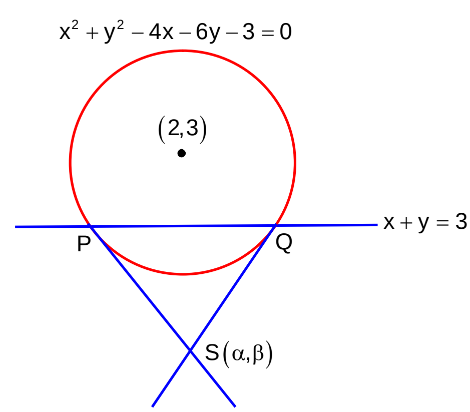
$x + y = 3$

$\therefore (\alpha - 2)x + (\beta - 3)y - 2\alpha - 3\beta - 3 = 0$ is same as

$x + y - 3 = 0$

$\therefore \alpha = -6, \beta = -5$

$\therefore 4\alpha - 7\beta = 11$



Sol7. $y_i = x_i + 50$

$\Rightarrow \bar{y} = \bar{x} + 50$

$\bar{x} = \left(\frac{41 \times 42}{2} - \frac{10 \times 11}{2}\right) \Big/ 31 = 26$

$\Rightarrow \bar{y} = \bar{x} + 50 = 76$

For combined data, mean $(\mu) = \frac{76 \times 31 + 26 \times 31}{62} = 51$

Variance $(\sigma^2) = \frac{\sum (z_i - z)^2}{62}$

$$= \frac{(61-51)^2 + (62-51)^2 + \ldots.. + (91-51)^2 + (11-51)^2 + (12-51)^2 + \ldots.. + (41-51)^2}{62}$$

$$= \frac{\left(10^2 + 11^2 + \ldots.40^2\right)\times 2}{62} = 705$$

$$\therefore \left|\bar{x} + \bar{y} - \sigma^2\right| = |26 + 76 - 705| = 603$$

Sol8. $y = \frac{x-a}{(x+b)(x-2)}$

$P(1,-3) \Rightarrow -3 = \frac{1-a}{(1+b)(-1)} \Rightarrow a + 3b = -2$ ……….(i)

$$\frac{dy}{dx} = \frac{(x+b)(x-2) - (x-a)(2x+b-2)}{(x+b)^2(x-2)^2}$$

$$\left.\frac{dy}{dx}\right|_p = \frac{-1-2b+ab}{(1+b)^2} = -4$$

$\Rightarrow -1 - 2b + ab + 4(1+b)^2 = 0$ ……….(ii)

Solve (i) and (ii) to get a = 1, b = -1 or a = 7, b = -3

a = 1, b = -1 is not possible as the curve becomes $y = \frac{1}{x-2}, x \neq 1$

$\Rightarrow P(1,-3)$ does not lie on it

$\therefore a + b = 7 - 3 = 4$

Sol9. $x^7 + 3x^5 - 13x^3 - 15x = 0$

$\Rightarrow x\left(x^6 + 3x^4 - 13x^2 - 15\right) = 0$

$\Rightarrow x\left(x^2+1\right)\left(x^2+5\right)\left(x^2-3\right) = 0$

$\therefore \alpha_1, \alpha_2 \in \left\{\pm\sqrt{5}i\right\}$

$\alpha_3, \alpha_4 \in \left\{\pm\sqrt{3}\right\}$

$\alpha_5, \alpha_6 \in \{\pm i\}$

$\alpha_7 = 0$

$\therefore \alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6 = 5 - (-3) + 1 = 9$

Sol10. Solve the given recurrence relation to get

$b_1 = 1, b_2 = 2, b_3 = 4, b_4 = 8, b_5 = 15, b_6 = 26, b_7 = 42, b_8 = 64, b_9 = 93, b_{10} = 130$

$$\therefore 2^8 \cdot S = 2^8 \cdot \sum_{n=1}^{10} \frac{b_n}{2^n} = 963$$

$$T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$$

$$\therefore 2^7 T = \sum_{n=1}^{8} n\, 2^{8-n} = 502$$

$$\therefore 2^7(2S - T) = 2^8 \cdot S - 2^7 \cdot T = 963 - 502 = 461$$