FIITJEE

Solutions to JEE(Main) -2023

Test Date: 30th January 2023 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (1 – 10)*** *contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. A person has been using spectacles of power -1.0 dioptre for distant vision and a separate reading glass of power 2.0 dioptres. What is the least distance of distinct vision for this person?

(A) 30cm (B) 40cm

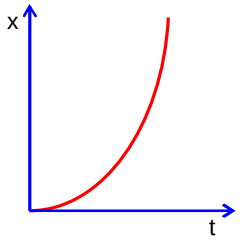
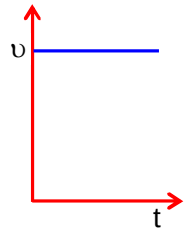
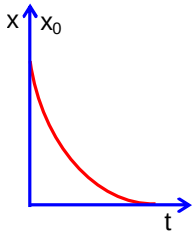
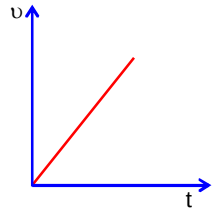
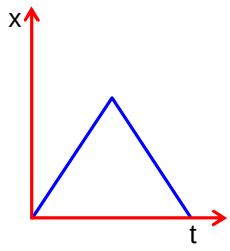
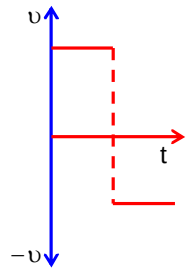
(C) 50cm (D) 10cm

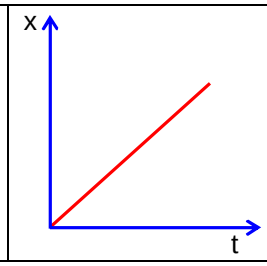
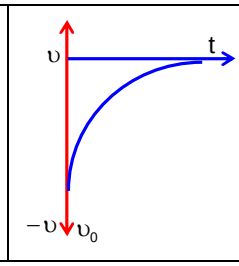
Q2. Two isolated metallic solid spheres of radii R and 2R are charged such that both have same charge density σ. The sphere are then connected by a conducting wire. If the new charge density of the bigger sphere is σ'. The ratio $\frac{\sigma'}{\sigma}$ is :

(A) $\frac{5}{3}$ (B) $\frac{5}{6}$

(C) $\frac{4}{3}$ (D) $\frac{9}{4}$

Q3. Match Column – I with Column – II :

Column-I (x-t graphs)		**Column-II (v-t graphs)**	
A.		I.	
B.		II.	
C.		III.	

D.		IV.	

Choose the correct answer from the options given below :

(A) A – I, B – III, C – IV, D – II
(B) A – II, B – IV, C – III, D – I
(C) A – I, B – II, C – III, D – IV
(D) A – II, B – III, C – IV, D – I

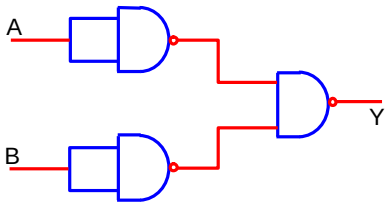
Q4. A small object at rest, absorbs a light pulse of power 20mW and duration 300ns. Assuming speed of light as $3\times10^{8}\,m/s$, the momentum of the object becomes equal to :

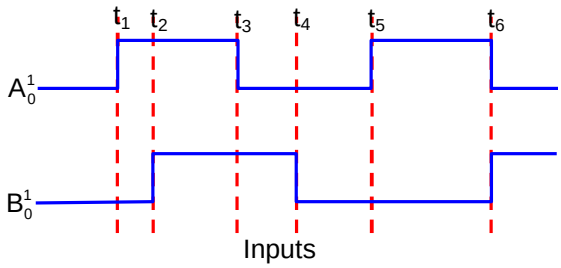
(A) $0.5\times10^{-17}\,kg\,m/s$
(B) $3\times10^{-17}\,kg\,m/s$
(C) $1\times10^{-17}\,kg\,m/s$
(D) $2\times10^{-17}\,kg\,m/s$.

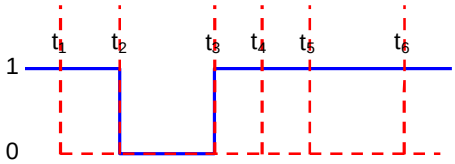
Q5. In a series LR circuit with $X_L = R$, power factor is P_1. If a capacitor of capacitance C with $X_C = X_L$ is added to the circuit the power factor becomes P_2. The ratio of P_1 to P_2 will be :

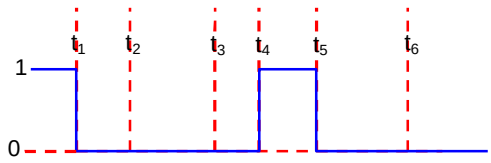
(A) $1:\sqrt{2}$
(B) 1 : 1
(C) 1 : 2
(D) 1 : 3

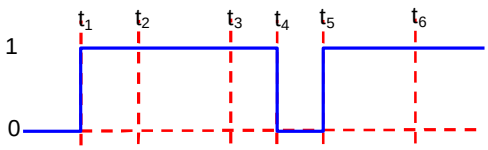
Q6. The output waveform of the given logical circuit for the following inputs A and B is shown below, is :

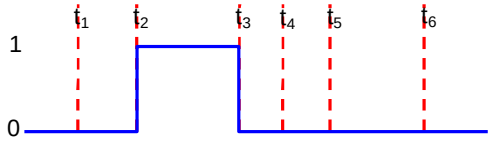




(A) 

(B) 

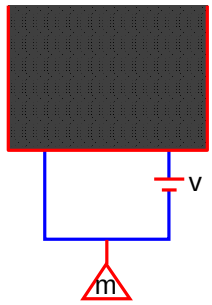
(C) 

(D) 

Q7. The magnetic moments associated with two closely wound circular coils A and B of radius r_A = 10cm and r_B = 20cm respectively are equal if : (Where N_A, I_A and N_B, I_B are number of turn and current of A and B respectively)

(A) $2N_A I_A = N_B I_B$
(B) $N_A = 2N_B$
(C) $N_A I_A = 4N_B I_B$
(D) $4N_A I_A = N_B I_B$

Q8. A massless square loop, of wire of resistance $10\,\Omega$, supporting a mass of 1g, hangs vertically with one of its sides in a uniform magnetic field of 10^3 G, directed outward in the shaded region. A dc voltage V is applied to the loop. For what value of V, the magnetic force will exactly balance the weight of the supporting mass of 1g ?
(If sides of the loop = 10cm, g = 10 ms^{-2})

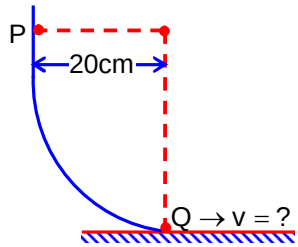


(A) 10V
(B) 100 V
(C) 1 V
(D) $\frac{1}{10}$V

Q9. The charge flowing in a conductor changes with time as $Q(t) = \alpha t - \beta t^2 + \gamma t^3$. Where α, β and γ are constants. Minimum value of current is :

(A) $\alpha - \frac{\beta^2}{3\gamma}$
(B) $\alpha - \frac{3\beta^2}{\gamma}$
(C) $\alpha - \frac{\gamma^2}{3\beta}$
(D) $\beta - \frac{\alpha^2}{3\gamma}$

Q10. As per the given figure, a small ball P slides down the quadrant of a circle and hits the other ball Q of equal mass which is initially at rest. Neglecting the effect of friction and assume the collision to be elastic, the velocity of ball Q after collision will be :
($g = m/s^2$)



(A) 0
(B) 0.25 m/s
(C) 4 m/s
(D) 2 m/s

Q11. A sinusoidal carrier voltage is amplitude modulated. The resultant amplitude modulated wave has maximum and minimum amplitude of 120V and 80V respectively. The amplitude of each sideband is :
(A) 15V
(B) 5V
(C) 20V
(D) 10V

Q12. Speed of an electron in Bohr's 7th orbit for Hydrogen atom is 3.6×10^6 m/s. The corresponding speed of the electron in 3rd orbit, in m/s is :
(A) $\left(7.5\times10^6\right)$
(B) $\left(8.4\times10^6\right)$
(C) $\left(1.8\times10^6\right)$
(D) $\left(3.6\times10^6\right)$

Q13. The pressure (P) and temperature (T) relationship of an ideal gas obeys the equation $PT^2 =$ constant. The volume expansion coefficient of the gas will be :
(A) $\frac{3}{T^2}$
(B) $3T^2$
(C) $\frac{3}{T}$
(D) $\frac{3}{T^3}$

Q14. Choose the correct relationship between poisson ratio (σ), bulk modulus (K) and modulus of rigidity (η) of a given solid object :
(A) $\sigma = \frac{3K-2\eta}{6K+2\eta}$
(B) $\sigma = \frac{6K-2\eta}{3K-2\eta}$
(C) $\sigma = \frac{3K+2\eta}{6K+2\eta}$
(D) $\sigma = \frac{6K+2\eta}{3K-2\eta}$

Q15. If the gravitational field in the space is given as $\left(-\frac{K}{r^2}\right)$. Taking the reference point to be at $r = 2\text{cm}$ with gravitational potential V = 10J/kg. Find the gravitational potential at r = 3cm is SI unit (Given, that K = 6 Jcm/kg)

(A) 9 (B) 12
(C) 10 (D) 11

Q16. Electric field in a certain region is given by $\vec{E} = \left(\frac{A}{x^2}\hat{i} + \frac{3}{y^3}\hat{j}\right)$. The SI unit of A and B are :

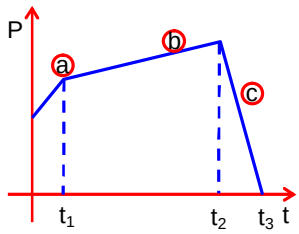
(A) Nm^3C; Nm^2C (B) Nm^3C^{-1}; Nm^2C^{-1}
(C) Nm^2C^{-1}; Nm^3C^{-1} (D) Nm^2C; Nm^3C

Q17. The height of liquid column raised in a capillary tube of certain radius when dipped in liquid A vertically is, 5cm. If the tube is dipped in a similar manner in another liquid B of surface tension and density double the values of liquid A, the height of liquid column raised in liquid B would be _____m.

(A) 0.20 (B) 0.5
(C) 0.05 (D) 0.10

Q18. The figure represents the momentum time (p-t) curve for a particle moving along an axis under the influence of the force. Identify the regions on the graph where the magnitude of the force is maximum and minimum respectively?

If $(t_3 - t_2) < t_1$



(A) a and b (B) b and c
(C) c and a (D) c and b

Q19. A ball of mass 200g rests on a vertical post of height 20m. A bullet of mass 10g, traveling in horizontal direction, hits the centre of the ball. After collision both travels independently. The ball hits the ground at a distance 30m and the bullet at a distance of 120m from the foot of the post. The value of initial velocity of the bullet will be (if g = 10 m/s^2) :

(A) 120 m/s (B) 60 m/s
(C) 400 m/s (D) 360 m/s

Q20. Heat is given to an ideal gas in an isothermal process.

A. Internal energy of the gas will decrease.
B. Internal energy of the gas will increase.
C. Internal energy of the gas will not change.
D. The gas will do positive work.
E. The gas will do negative work.

Choose the correct answer from the options given below :

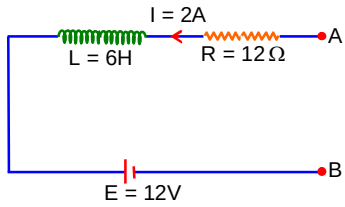
(A) A and E only (B) C and D only
(C) C and E only (D) B and D only

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. As per the figure, if $\frac{dI}{dt} = -1A/s$ then the value of V_{AB} at this instant will be _____V.

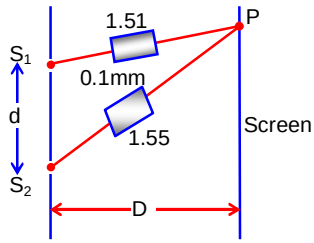


Q2. In a screw gauge, there are 100 divisions on the circular scale and the main scale moves by 0.5mm on a complete rotation of the circular scale. The zero of circular scale lies 6 divisions below the line of graduation when two studs are brought in contact with each other. When a wire is placed between the studs, 4 linear scale divisions are clearly visible while 46th division the circular scale coincide with the reference line, The diameter of the wire is _____$\times 10^{-2}$mm.

Q3. A horse rider covers half the distance with 5m/s speed. The remaining part of the distance was traveled with speed 10m/s for half the time and with speed 15m/s for other half of the time. The mean speed of the rider averaged over the whole time of motion is $\frac{x}{7}$m/s. The value of x is _____.

Q4. The general displacement of a simple harmonic oscillator is $x = A\sin\omega t$. Let T be its time period. The slope of its potential energy (U) – time (t) curve will be maximum when $t = \frac{T}{\beta}$. The value of β is _____.

Q5. In Young's double slit experiment, two slits S_1 and S_2 are 'd' distance apart and the separation from slits to screen is D (as shown in figure). Now if two transparent slabs of equal thickenss 0.1mm but refractive index 1.51 and 1.55 are introduced in the path of beam ($\lambda = 4000\,\mathring{A}$) from S_1 and S_2 respectively. The central bright fringe spot will shift by _____number of fringes.



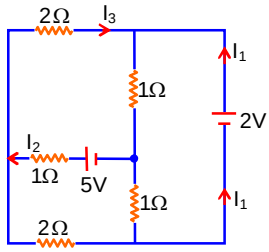
Q6. A point source of light is placed at the centre of curvature of a hemispherical surface. The source emits a power of 24 W. The radius of curvature of hemisphere is 10cm and the inner surface is completely reflecting. The force on the hemisphere due to the light falling on it is _____$\times 10^{-8}$N.

Q7. In an experiment for estimating the value of focal length of converging mirror, Image of an object placed at 40cm from the pole of the mirror is formed at distance 120cm from the pole of the mirror. These distances are measured with a modified scale in which there are 20 small divisions in 1cm. The value of error in measurement of focal length of the mirror is $\frac{1}{k}$cm. The value of K is _____.

Q8. A capacitor of capacitance $900\,\mu F$ is charged by a 100V battery. The capacitor is disconnected from the battery and connected to another uncharged identical capacitor such that one plate of uncharged capacitor connected to positive plate and another plate of uncharged capacitor connected to negative plate of the charged capacitor. The loss of energy in this process is measured as $x \times 10^{-2}$ J. The value of x is _____.

Q9. A thin uniform rod of length 2m, cross sectional area 'A' and density 'd' is rotated about an axis passing through the centre and perpendicular to its length with angular velocity ω. If value of ω in terms of its rotational kinetic energy E is $\sqrt{\frac{\alpha E}{Ad}}$ then value of α is_____.

Q10. In the following circuit, the magnitude of current I_1, is _____ A.



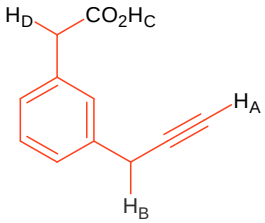
PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. What is the correct order of acidity of the protons marked A-D in the given compounds?



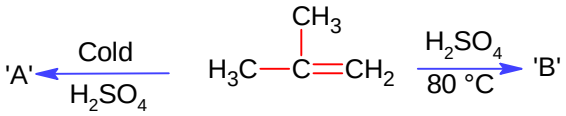
(A) $H_C > H_D > H_A > H_B$
(B) $H_C > H_A > H_D > H_B$
(C) $H_D > H_C > H_B > H_A$
(D) $H_C > H_D > H_B > H_A$

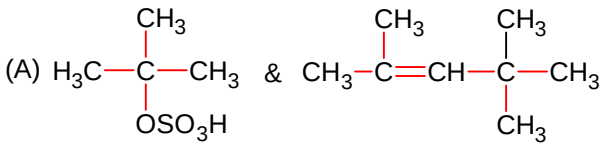
Q2. Which of the following is correct order of ligand field strength?

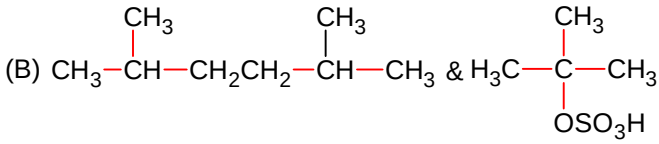
(A) $CO < en < NH_3 < C_2O_4^{2-} < S^{2-}$
(B) $S^{2-} < NH_3 < en < CO < C_2O_4^{2-}$
(C) $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$
(D) $NH_3 < en < CO < S^{2-} < C_2O_4^{2-}$

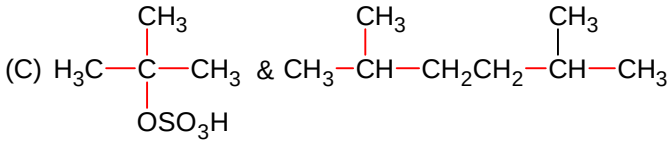
Q3. In the extraction of copper, its sulphide ore is heated in a reverberatory furnace after mixing with silica to:
(A) remove calcium as $CaSiO_3$
(B) Decrease the temperature needed for roasting of Cu_2S
(C) Separate CuO as $CuSiO_3$
(D) Remove FeO as $FeSiO_3$

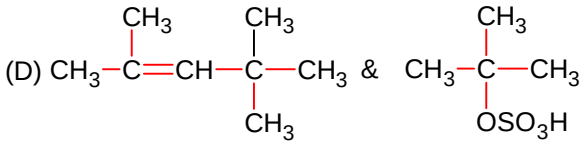
Q4. The major products 'A' and 'B', respectively, are



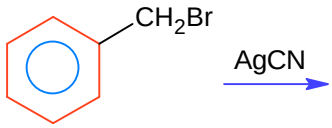
(A) 

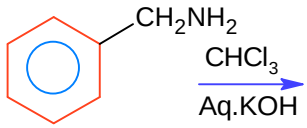
(B) 

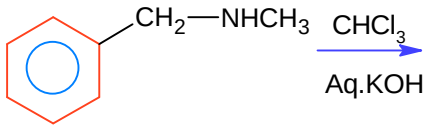
(C) 

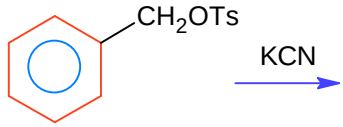
(D) 

Q5. Benzyl isocyanide can be obtained by:

(A) 

(B) 

(C) 

(D) 

Choose the **correct** answer from the option given below:

(A) only B
(B) B and C
(C) A and B
(D) A and D

Q6. During the qualitative analysis of SO_3^{2-} using dilute H_2SO_4, SO_2 gas is evolved which turns $K_2Cr_2O_7$ solution (acidified with dilute H_2SO_4)

(A) Red
(B) Green
(C) Blue
(D) Black

Q7. Lithium aluminium hydride can be prepared from the reaction of

(A) LiCl, Al and H_2
(B) LiH and $Al(OH)_3$
(C) LiH and Al_2Cl_6
(D) LiCl and Al_2H_6

Q8. Match **List I** with **List II**

List-I (molecules / ions)		**List-II (No of lone pairs of e^- on central atom)**	
A.	IF_7	I.	Three
B.	ICl_4^-	II.	One
C.	XeF_6	III.	Two
D.	XeF_2	IV.	Zero

Choose the **correct** answer from the options given below:

(A) A- IV, B-I, C-II, D- III
(B) A-II, B-III, C-IV, D-I
(C) A-IV, B-III, C-II, D-I
(D) A-II, B-I, C-IV, D-III

Q9. Given below are two statements: one is labelled as **Assertion(A)** and the other is labelled as **Reason (R).**

Assertion (A): In expensive scientific instruments, silica gel is kept in watch-glasses or in semipermeable membrane bags.

Reason (R): Silica gel adsorbs moisture from air via adsorption thus protects the instrument from water corrosion (rusting) and / or prevents malfunctioning.

In the light of the above statements, choose the **correct** answer from the options given below:

(A) (A) is true but (R) is false
(B) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of (A)
(C) Both **(A)** and (R) are true and **(R)** is the correct explanation of (A)
(D) **(A)** is false but (R) is true

Q10. Match **List I** with **List II**

List-I (atomic number)		**List-II(block of periodic table)**	
A.	37	I.	p-block
B.	78	II.	d-block
C.	52	III.	f-block
D.	65	IV.	s-block

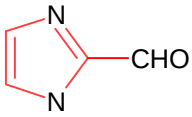
Choose the **correct** answer from the options given below:

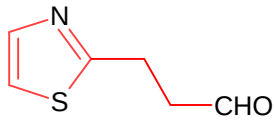
(A) A- I, B-III, C-IV, D- II
(B) A-IV, B-III, C-II, D-I
(C) A-II, B-IV, C-I, D-III
(D) A-IV, B-II, C-I, D-III

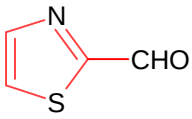
Q11. Caprolactam when heated at high temperature in presence of water, gives
(A) Dacron
(B) Nylon 6
(C) Teflon
(D) Nylon 6,6

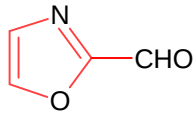
Q12. The alkaline earth metal sulphate(s) which are readily soluble in water is/ are
(A) $BeSO_4$
(B) $MgSO_4$
(C) $CaSO_4$
(D) $SrSO_4$
(E) $BaSO_4$
Choose the **correct** answer from the options given below:
(A) B only
(B) B and C
(C) A and B
(D) A only

Q13. Which of the following compounds would give the following set of qualitatiave analysis?
(i) Fehling's Test: Positive
(ii) Na fusion extract upon treatment with sodium nitroprusside gives a blood red colour but not Prussian blue.

(A) 

(B) 

(C) 

(D) 

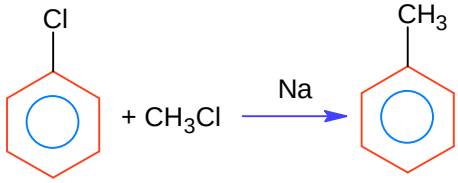
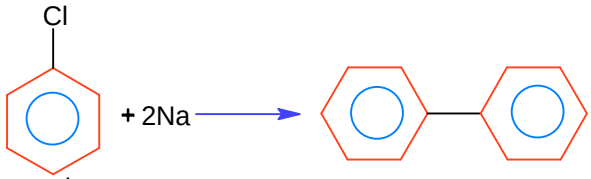
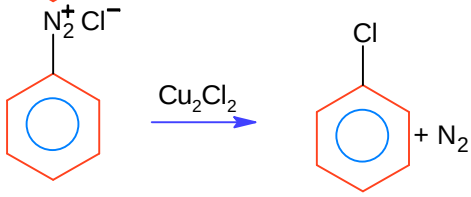
Q14. In the wet tests for identification of various cations by precipitation, which transition element cation doesn't belong to group IV in qualitative inorganic analysis?
(A) Fe^{3+}
(B) Co^{2+}
(C) Zn^{2+}
(D) Ni^{2+}

Q15. To inhibit the growth of tumours, identify the compounds used from the following:
A. EDTA
B. Coordination Compounds of Pt
C. D- Penicillamine
D. Cis- Platin
Choose the correct answer from the option given below :
(A) A and C only
(B) B and D only
(C) C and D only
(D) A and B only

Q16. Formation of photochemical smog involves the following reaction in which A, B and C are respectively.
(i) $NO_2 \xrightarrow{hv} A + B$
(ii) $B + O_2 \rightarrow C$
(iii) $A + C \rightarrow NO_2 + O_2$
Choose the correct answer from the options given below:
(A) N, O_2 & O_3
(B) NO, O & O_3
(C) O, NO & NO_3^-
(D) O, N_2O & NO

Q17. Amongst the following compounds, which one is an antacid?
(A) Terfenadine
(B) Brompheniramine
(C) Meprobamate
(D) Ranitidine

Q18. Match **List I** with **List II**

List-I		**List-II**
A.		I. Fittig reaction
B.		II. Wurtz Fittig reaction
C.		III. Finkelstein reaction
D.	$C_2H_5Cl + NaI \rightarrow C_2H_5I + NaCl$	IV. Sandmeyer reaction

Choose the **correct** answer from the options given below:

(A) A- II, B-I, C-IV, D- III
(B) A-IV, B-II, C-III, D-I
(C) A-II, B-I, C-III, D-IV
(D) A-III, B-II, C-IV, D-I

Q19. Given below are two statements: one is labelled as **Assertion(A)** and the other is labelled as **Reason (R).**

Assertion (A): Ketoses give Seliwanoff's test faster than Aldoses.

Reason (R): Ketoses undergo β-elimination followed by formation furfural.

In the light of the above statements, choose the **correct** answer from the options given below:

(A) **(A)** is true but **(R)** is false
(B) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**
(C) **(A)** is false but **(R)** is true
(D) Both **(A)** and (R) are true and **(R)** is the correct explanation of **(A)**

Q20. For OF_2 molecule consider the following:

A. Number of lone pairs on oxygen is 2,
B. FOF angle is less than 104.5^o.
C. Oxidation state of O is -2.
D. Molecule is bent 'V' shaped.
D. Molecular geometry is linear.

Correct options are:

(A) A, C, D only
(B) A, B, D only
(C) B, E, A only
(D) C, D, E only

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. When 2 litre of ideal gas expands isothermally into vacuum to a total volume of 6 litre, the change in internal energy is _____J. (Nearest integer)

Q2. The number of electrons involved in the reduction of permagnate to manganese dioxide in acidic medium is_____.

Q3. A solution containing 2 g of a non- volatile solute in 20 g of water boils at 373.52K. The molecular mass of the solute is _____ g mol^{-1}.(Nearest integer)
Given, water boils at 373K, Kb for water = 0.52 K kg mol^{-1}

Q4. Consider the cell
$Pt_{(s)} \mid H_2(g,1atm) \mid H^+(aq.1M) \mid\mid Fe^{3+}_{(aq)}, Fe^{2+}_{(aq)} \mid\mid Pt_{(s)}$
When the potential of the cell is 0.712 V at 298 K, the ratio $\left[Fe^{2+}\right]/\left[Fe^{3+}\right]$ is_____.
(Nearest integer)
Given: $Fe^{3+} + e^- = Fe^{2+}, E^0 Fe^{3+}, Fe^{2+} \mid Pt = 0.771$
$\frac{2.303RT}{F} = 0.06V$

Q5. If compound A reacts with B following first order kinetics with rate constant $2.011\times10^{-3}s^{-1}$. The time taken by A (in seconds) to reduce from 7 g to 2 g will be _____.
(Nearest integer)
$[\log 5 = 0.698, \log 7 = 845, \log 2 = 0.301]$

Q6. Some amount of dichloromethane (CH_2Cl_2) is added to 671.141 mL of chloroform ($CHCl_3$) to prepare 2.6×10^{-3}M solution of CH_2Cl_2(DCM). The concentration of DCM is _____ppm (by mass)
Given:
Atomic mass : C = 12
H=1
Cl = 35.5
Density of $CHCl_3$ = 1.49 g cm^{-3}

Q7. A trisubstituted compound 'A' $C_{10}H_{12}O_2$ gives neutral $FeCl_3$ test positive. Treatment of compound 'A' with NaOH and CH_3Br gives $C_{11}H_{14}O_2$, with hydroidic acid gives methyl iodide and with hot conc. NaOH gives a compound B $C_{10}H_{12}O_2$. Compound 'A' also decolorizes alkaline $KMnO_4$. The number of π bond/s present in the compound 'A' is_____.

Q8. The energy of one mole of photons of radiation of frequency 2×10^{12} Hz in J mol^{-1} is _____
(Nearest integer)
[Given ; h= 6.626×10^{-34}Js $N_A = 6.022\times10^{23}mol^{-1}$]

Q9. A 300 mL bottle of soft drink has 0.2 M CO_2 dissolved in it. Assuming CO_2 behaves as an ideal gas, the volume of the dissolved CO_2 at STP is_____ mL (Nearest integer)
Given: At STP molar volume of an ideal gas is 22.7 L mol^{-1}

Q10. 600mL of 0.01 M HCl is mixed with 400mL of 0.01 M H_2SO_4. The pH of the mixture is _____ $\times 10^{-2}$. (Nearest integer)
[Given log 2 = 0.30
log 3 = 0.48
log 5= 0.69
log7 = 0.84
log 11 = 1.04]

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. If an unbiased die, marked with $-2,-1,0,1,2,3$ on its faces, is thrown five times, then the probability that the product of the outcomes is positive, is :

(A) $\frac{881}{2592}$ (B) $\frac{27}{288}$

(C) $\frac{521}{2592}$ (D) $\frac{440}{2592}$

Q2. The minimum number of elements that must be added to the relation $R=\{(a,b),(b,c)\}$ on the set $\{a,b,c\}$ so that it becomes symmetric and transitive is :

(A) 4 (B) 5

(C) 7 (D) 3

Q3. Let a unit vector $\widehat{OP}$ make angles α,β,γ with the positive directions of the co-ordinate axes OX, OY, OZ respectively, where $\beta\in\left(0,\frac{\pi}{2}\right)$. If $\widehat{OP}$ is perpendicular to the plane through points $(1,2,3),(2,3,4)$ and $(1,5,7)$, then which one of the following is true?

(A) $\alpha\in\left(\frac{\pi}{2},\pi\right)$ and $\gamma\in\left(\frac{\pi}{2},\pi\right)$ (B) $\alpha\in\left(0,\frac{\pi}{2}\right)$ and $\gamma\in\left(\frac{\pi}{2},\pi\right)$

(C) $\alpha\in\left(\frac{\pi}{2},\pi\right)$ and $\gamma\in\left(0,\frac{\pi}{2}\right)$ (D) $\alpha\in\left(0,\frac{\pi}{2}\right)$ and $\gamma\in\left(0,\frac{\pi}{2}\right)$

Q4. If $\tan15^0+\frac{1}{\tan75^0}+\frac{1}{\tan105^0}+\tan195^0=2a$, then the value of $\left(a+\frac{1}{a}\right)$ is :

(A) 2 (B) $4-2\sqrt{3}$

(C) $5-\frac{3}{2}\sqrt{3}$ (D) 4

Q5. Let the solution curve $y=y(x)$ of the differential equation $\frac{dy}{dx}-\frac{3x^5\tan^{-1}(x^3)}{(1+x^6)^{3/2}}y=2x\ \exp.\left\{\frac{x^3-\tan^{-1}x^3}{\sqrt{(1+x^6)}}\right\}$ pass through the origin. Then $y(1)$ is equal to :

(A) $\exp\left(\frac{1-\pi}{4\sqrt{2}}\right)$ (B) $\exp\left(\frac{\pi-4}{4\sqrt{2}}\right)$

(C) $\exp\left(\frac{4-\pi}{4\sqrt{2}}\right)$ (D) $\exp\left(\frac{4+\pi}{4\sqrt{2}}\right)$

Q6. Let the system of linear equations

$x + y + kz = 2$

$2x + 3y - z = 1$

$3x + 4y + 2z = k$

have infinitely many solutions. Then the system

$(k+1)x + (2k-1)y = 7$

$(2k+1)x + (k+5)y = 10$

has :

(A) unique solution satisfying $x - y = 1$

(B) unique solution satisfying $x + y = 1$

(C) no solution

(D) infinitely many solutions

Q7. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a} = \alpha\vec{b} - \hat{n}, (\alpha \neq 0)$ and $\vec{b} \cdot \vec{c} = 12$, then $\left|\vec{c} \times \left(\vec{a} \times \vec{b}\right)\right|$ is equal to :

(A) 9

(B) 12

(C) 6

(D) 15

Q8. Let $y = x + 2, 4y = 3x + 6$ and $3y = 4x + 1$ be three tangent lines to the circle $(x-h)^2 + (y-k)^2 = r^2$. Then $h + k$ is equal to :

(A) 6

(B) $5\sqrt{2}$

(C) $5\left(1+\sqrt{2}\right)$

(D) 5

Q9. If the coefficient of x^{15} in the expansion of $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$, where a and b are positive real numbers, then for each such ordered pair (a,b) :

(A) a = b

(B) a = 3b

(C) ab = 3

(D) ab = 1

Q10. If $a_n = \frac{-2}{4n^2 - 16n + 15}$, then $a_1 + a_2 + ... + a_{25}$ is equal to :

(A) $\frac{49}{138}$

(B) $\frac{50}{141}$

(C) $\frac{52}{147}$

(D) $\frac{51}{144}$

Q11. Among the statements :

$(S1)\ ((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

$(S2)\ ((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

(A) neither $(S1)$ nor $(S2)$ is a tautology

(B) only $(S2)$ is a tautology

(C) both $(S1)$ and $(S2)$ are tautologies

(D) only $(S1)$ is a tautology

Q12. If $P(h,k)$ be a point on the parabola $x = 4y^2$, which is nearest to the point $Q(0, 33)$, then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to :

(A) 8 (B) 4
(C) 2 (D) 6

Q13. If $[t]$ denotes the greatest integer $\le t$, then the value of $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx$ is :

(A) $e^8 - e$ (B) $e^8 - 1$
(C) $e^7 - 1$ (D) $e^9 - e$

Q14. The number of points on the curve $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$ at which the normal lines are parallel to $x + 90y + 2 = 0$ is :

(A) 3 (B) 2
(C) 0 (D) 4

Q15. The line ℓ_1 passes through the point $(2,6,2)$ and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line ℓ_1 and the line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is :

(A) $\frac{19}{3}$ (B) 7
(C) 9 (D) $\frac{13}{3}$

Q16. The coefficient of x^{301} in $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + ... + x^{500}$ is :

(A) ${}^{500}C_{301}$ (B) ${}^{501}C_{302}$
(C) ${}^{500}C_{300}$ (D) ${}^{501}C_{200}$

Q17. Suppose $f : R \to (0, \infty)$ be a differentiable function such that $5f(x + y) = f(x) \cdot f(y), \forall\, x, y \in R$. If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to :

(A) 6825 (B) 6525
(C) 6875 (D) 6575

Q18. If the solution of the equation $\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha + \sqrt{\beta}}{2}\right)$, where α, β are integers, then $\alpha + \beta$ is equal to :

(A) 4 (B) 3
(C) 6 (D) 5

Q19. Let $A = \begin{pmatrix} m & n \\ p & q \end{pmatrix}$, $d = |A| \neq 0$ and $|A - d(\text{Adj } A)| = 0$. Then

(A) $1 + d^2 = m^2 + q^2$ (B) $1 + d^2 = (m + q)^2$
(C) $(1 + d)^2 = m^2 + q^2$ (D) $(1 + d)^2 = (m + q)^2$

Q20. A straight line cuts off the intercepts $OA = a$ and $OB = b$ on the positive directions of x-axis and y-axis respectively. If the perpendicular from origin O to this line makes an angle of $\frac{\pi}{6}$ with positive direction of y-axis and the area of ΔOAB is $\frac{98}{3}\sqrt{3}$, then $a^2 - b^2$ is equal to :

(A) 196

(B) $\frac{392}{3}$

(C) $\frac{196}{3}$

(D) 98

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. Number of 4-digit numbers (the repeation of digits is allowed) which are made using the digits 1, 2, 3 and 5 and are divisible by 15, is equal to……..

Q2. The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observation, then $a+3b-5$ is equal to…….

Q3. If the equation of the plane passing through the point $(1,1,2)$ and perpendicular to the line $x-3y+2z-1=0=4x-y+z$ is $Ax+By+Cz=1$, then $140(C-B+A)$ is equal to…….

Q4. Let $f^1(x)=\frac{3x+2}{2x+3}, x\in R-\left\{\frac{-3}{2}\right\}$

For $n\geq 2$, define $f^n(x)=f^1 o\, f^{n-1}(x)$.

If $f^5(x)=\frac{ax+b}{bx+a}$, $\gcd(a,b)=1$, then a + b is equal to…..

Q5. Let $\sum_{n=0}^{\infty}\frac{n^3\left((2n)!\right)+(2n-1)(n!)}{(n!)\left((2n)!\right)}=ae+\frac{b}{e}+c$, where $a,b,c\in Z$ and $e=\sum_{n=0}^{\infty}\frac{1}{n!}$. Then a^2-b+c is equal to……

Q6. Let α be the area of the larger region bounded by the curve $y^2=8x$ and the lines $y=x$ and x = 2, which lies in the first quadrant. Then the value of 3α is equal to…….

Q7. Let $z=1+i$ and $z_1=\frac{1+i\bar{z}}{\bar{z}(1-z)+\frac{1}{z}}$. Then $\frac{12}{\pi}\arg(z_1)$ is equal to……..

Q8. If $\lambda_1<\lambda_2$ are two values of λ such that the angle between the planes $P_1:\vec{r}\cdot\left(3\hat{i}-5\hat{j}+\hat{k}\right)=7$ and $P_2:\vec{r}\cdot\left(\lambda\hat{i}+\hat{j}-3\hat{k}\right)=9$ is $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$, then the square of the length of perpendicular from the point $(38\lambda_1,10\lambda_2,2)$ to the plane P_1 is……..

Q9. Let $S=\{1,2,3,4,5,6\}$. Then the number of one-one functions $f:S\to P(S)$, where $P(S)$ denote the power set of S, such that $f(n)\subset f(m)$ where $n<m$ is…….

Q10. $\lim_{x\to 0}\frac{48}{x^4}\int_0^x\frac{t^3}{t^6+1}dt$ is equal to……

FIITJEE

KEYS to JEE (Main)-2023

PART – A (PHYSICS)

SECTION - A

1.	C	2.	B	3.	B	4.	D
5.	A	6.	C	7.	C	8.	A
9.	A	10.	D	11.	D	12.	B
13.	C	14.	A	15.	D	16.	C
17.	C	18.	D	19.	D	20.	B

SECTION – B

1.	30	2.	220	3.	50	4.	8
5.	10	6.	4	7.	32	8.	225
9.	3	10.	2				

PART – B (CHEMISTRY)

SECTION - A

1.	A	2.	C	3.	D	4.	A
5.	C	6.	B	7.	C	8.	C
9.	C	10.	D	11.	B	12.	C
13.	B	14.	A	15.	B	16.	B
17.	D	18	A	19.	A	20.	B

SECTION - B

1.	0	2.	3	3.	100	4.	10
5.	623	6.	221	7.	4	8.	798
9.	1362	10.	186				

PART – C (MATHEMATICS)

SECTION - A

1.	C	2.	C	3.	A	4.	D
5.	C	6.	B	7.	B	8.	D
9.	D	10.	B	11.	A	12.	D
13.	A	14.	D	15.	C	16.	D
17.	A	18	A	19.	D	20.	B

SECTION - B

1.	21	2.	37	3.	15	4.	3125
5.	26	6.	22	7.	9	8.	315
9.	3240	10.	12				

FIITJEE

Solutions to JEE (Main)-2023

PART – A (PHYSICS)

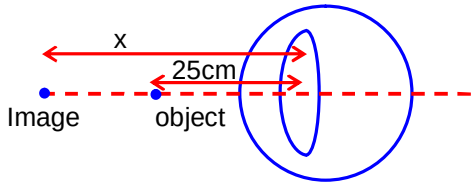
SECTION – A

Sol1. Power, $P_1 = -1\,D$ (concave lens)

$P_2 = +2D$ (Convex lens)

For least distance of distinct vision, convex lens is used :-

Here x = least distance of distinct vision.



Where x = least distance of distinct vision

Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{-x}-\frac{1}{-25}=\frac{1}{50}$$

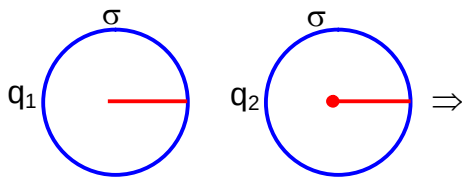
$$\Rightarrow \quad -\frac{1}{x}=\frac{1}{50}-\frac{1}{25}$$

$$\Rightarrow \quad -\frac{1}{x}=\frac{1-2}{50}=\frac{-1}{50} \quad \Rightarrow x = 50\,cm$$

Sol2. $q_1 = \sigma 4\pi R^2$

$q_2 = \sigma 4\pi(2R)^2$

After connecting the two spheres :-

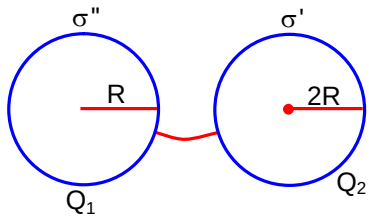


$$Q_2 = \frac{2R}{3R}\times(q_1+q_2) = \frac{2}{3}\left(4\sigma\pi R^2 + 16\sigma\pi R^2\right)$$

$$= \frac{40}{3}\sigma\pi R^2$$

$$\therefore \quad \sigma' = \frac{Q_2}{4\pi(2R)^2}$$

$$= \frac{40\sigma\pi R^2}{3\times 16\pi R^2} \qquad \Rightarrow \frac{\sigma'}{\sigma} = \frac{40}{48} = \frac{5}{6}$$



Sol3. $A \rightarrow II$ $\quad a > 0 \Rightarrow v\,\alpha\,t$

$B \rightarrow IV$ $\quad x = x_0 e^{-\alpha t}$

$$\Rightarrow V = \frac{dx}{dt} = -\alpha x_0 e^{-\alpha t}$$

$C \rightarrow III$ $\quad x\,\alpha\,t \Rightarrow V = constant$

$D \rightarrow I$ $\quad x\,\alpha\,t \Rightarrow V = constant$

$$\begin{pmatrix} V \rightarrow +ve \text{ for } 0 \text{ to } t/2 \\ V \rightarrow -ve \text{ for } t/2 \text{ to } t \end{pmatrix}$$

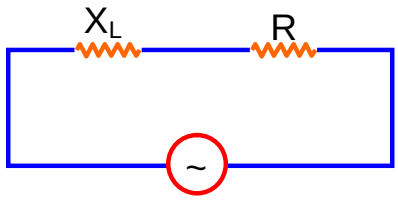
Sol4. Power = 20mW

t = 300nS

$\therefore$ Energy absorbed $= Pt = 300\times10^{-9}\times20\times10^{-3} = 6\times10^{-9}J$

$\therefore$ Momentum transferred = $\dfrac{\text{Energy}}{C} = \dfrac{6\times10^{-9}}{3\times10^{8}} = 2\times10^{-17}\text{kg.m/s}$

Sol5. $X_L = R$



Power factor $P_1 = \dfrac{R}{\sqrt{x_2^2+R^2}} = \dfrac{1}{\sqrt{2}}$

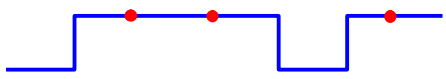
Here, $X_C = X_L = R$

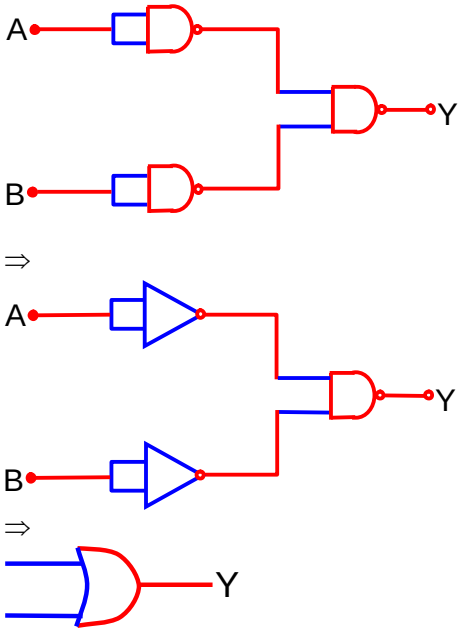
Power factor $P_2 = \dfrac{R}{\sqrt{R^2+(X_L-X_C)^2}} = 1$

$\therefore \dfrac{P_1}{P_2} = \dfrac{1}{\sqrt{2}}$

Sol6.

A	B	Y = A + B
0	0	0
1	0	1
1	1	1
0	1	1
0	0	0
1	0	1
0	1	1

$\Rightarrow$ 



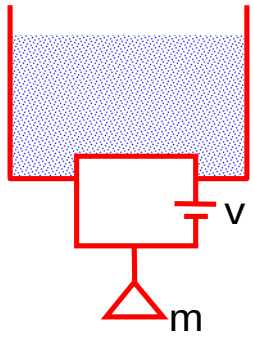
Sol7. $\mu = N\,iA = Ni\pi r^2$

As, $\mu_A = \mu_B$

$\Rightarrow N_A I_A 100 = N_B I_B 400$

$\Rightarrow N_A I_A = 4N_B I_B$

Sol8. At equilibrium –



$$mg = i\ell B$$

$$\Rightarrow mg = \frac{v}{R}\ell B$$

$$\therefore \quad v = \frac{mgR}{\ell B}$$

$$= \frac{10^{-3}\times 10\times 10}{10^{-1}\times 10^{3}\times 10^{-4}}$$

$$\left\{1G = 10^{-4}\,T\right\}$$

$$= \frac{10^{-1}}{10^{-2}} = 10v$$

Sol9. $Q_t = \alpha t - \beta t^2 + \gamma^{t^3}$

$$i_t = \frac{dQ}{dt} = \alpha - 2\beta t + 3\gamma^{t^2}$$

For minimum $i, = \frac{di}{dt} = 0$

$$\Rightarrow -2\beta + 6\gamma t = 0$$

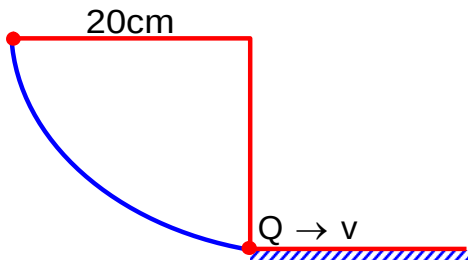
$$\Rightarrow t = \frac{2\beta}{6\gamma} = \frac{\beta}{3\gamma}$$

$$= \alpha - 2\beta\cdot\frac{\beta}{3\gamma} + 3\gamma\cdot\frac{\beta^2}{9\gamma^2}$$

$$= \alpha - \frac{2}{3}\frac{\beta^2}{\gamma} + \frac{\beta^2}{3\gamma}$$

$$= \alpha - \frac{\beta^2}{3\gamma}$$

Sol10. For equal manses colliding elastically, velocities are interchanged.



$$V_P = \sqrt{2gR} = \sqrt{2\times 10\times\frac{20}{100}} = 2m/s$$

$$\therefore \quad V_Q = 2m/s$$

Sol11. $V_{max} = V_c + V_m = 120$ - (i)

$V_{min} = V_c - V_m = 80$ - (ii)

From (i) and (ii) :-

$V_c = 100$

$V_m = 20$

$$\mu = \frac{V_m}{V_c} = \frac{20}{100} = 0.2$$

$$\therefore \text{ Amplitude of side band } = \frac{\mu Ac}{2}$$

$$= 0.2\times\frac{100}{2} = 10v$$

Sol12. As $V_n = \frac{z}{n}v_0$

$$\therefore \quad \frac{V_3}{V_7} = \frac{7}{3} \Rightarrow V_3 = \frac{7}{3}V_7$$

$$= \frac{7}{3} \times 3.6 \times 10^6 = 8.4 \times 10^6 \text{m/s}$$

Sol13. PT^2 = constant

$$\frac{dv}{v} = \gamma dT \Rightarrow \gamma = \frac{1}{v}\frac{dv}{dT}$$

Using $PV = nRT \Rightarrow P = \frac{nRT}{V}$

$$\Rightarrow \frac{nRT}{v}T^2 = \text{const}(k)$$

$$\Rightarrow V = \frac{nR}{k}T^3$$

$$\Rightarrow \frac{dv}{dT} = 3T^2\frac{nR}{k} = 3(nRT)\frac{T}{k}$$

$$\Rightarrow \frac{dv}{dT} = \frac{3Pv.T}{PT^2} = 3\frac{v}{T}$$

$$\Rightarrow Y = \frac{1}{v}\frac{dv}{dT} = \frac{3}{T}$$

Sol14. Poisson ratio (σ)

Bulk modulus $= K$

Modulus of rigidity = n

$Y = 2n(1+\sigma)$ - (i)

and $Y = 3k(1-2\sigma)$ - (ii)

$$\therefore \quad 2n(1+\sigma) = 3k(1-2\sigma)$$

$$2n + 2n\sigma = 3k - 6k\sigma$$

$$\sigma(2n + 6k) = 3k - 2n$$

$$\therefore \quad \sigma = \frac{3k - 2n}{2n + 6k}$$

Sol15. Gravitational, $g = \frac{-k}{r^2}$; where $K = 6\text{Jcm/kg}$.

At $r = 2\text{cm}, V = 10\text{J/kg}$

as $dv = -\vec{g}.d\vec{r}$

$$\Rightarrow \int dv = -\int g dr$$

$$\int_{10}^{v} dv = -\int_{2}^{3}\left(-\frac{k}{r^2}\right)dr$$

$$\Rightarrow v - 10 = \left[-\frac{k}{r}\right]_2^3$$

$$V = 10 - k\left\{\frac{1}{3} - \frac{1}{2}\right\}$$
$$= 10 - 6\left\{\frac{2-3}{6}\right\}$$
$$= 11v$$

Sol16. $\vec{E} = \frac{A}{x^2}\hat{i} + \frac{B}{y3}\hat{j}$

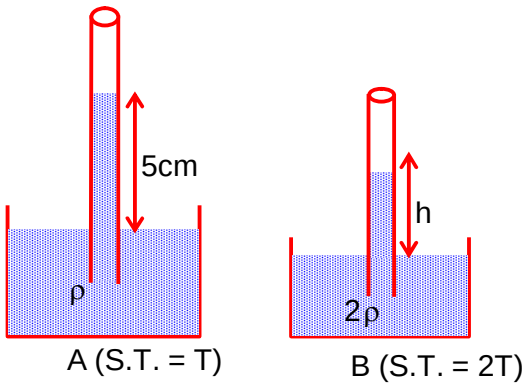
Unit of $A \rightarrow \frac{N}{C} \times m^2 = \frac{N.m^2}{C}$

Unit of $B \rightarrow \frac{N}{C} \times m^3 = \frac{N.m^3}{C}$

Sol17. $h = \frac{2T\cos 0}{r\rho g} \Rightarrow h\ \alpha \frac{T}{\rho}$

$\Rightarrow \frac{h}{5} = \frac{2T}{2\rho} \times \frac{\rho}{T} = 1$

$\Rightarrow h = 0.05m$

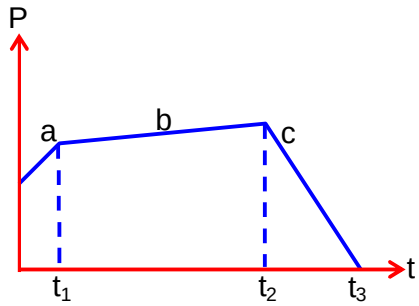


A (S.T. = T) B (S.T. = 2T)

Sol18. $F = \frac{dP}{dt}$

$F_{max} = \left(\frac{dP}{dt}\right)_{max} \Rightarrow$ region – C

$F_{min} = \left(\frac{dP}{dt}\right)_{min} \Rightarrow$ region – b



Sol19. $m_1 = 200g$

$m_2 = 10g$

Let velocities of ball and bullet after collision be

V_1 and V_2 respectively then –

$V_1 t = 30m$ - (i)

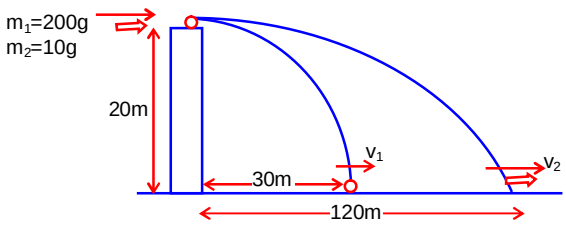
$V_2 t = 120m$ - (ii)

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 20}{10}} = 2\sec$

from (i) and (iii)

$\Rightarrow V_1 = 15m/s$

$V_2 = 60m/s$



From COLM :- $m_2 u = m_1 V_1 + m_2 V_2$

$\Rightarrow 10u = 200 v_1 + 10 v_2$

$\Rightarrow u = 20 v_1 + v_2 = 360 m/s$

Sol20. In an isothermal process –

$\Delta T = 0 \quad \therefore \Delta U = 0 \Rightarrow (C)$

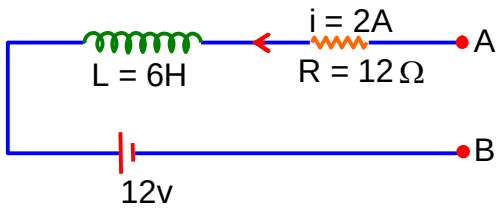
$\Delta Q = \Delta U + W$

$\Rightarrow \Delta Q = W \Rightarrow (D)$

SECTION – B

Sol1. $\frac{di}{dt} = -1\,A/s$

Using KVL –



$V_A - iR - L\frac{di}{dt} - 12 = V_B$

$\therefore \quad V_A - V_B = 12 + 2\times 12 + 6\times(-1)$

$= 30v$

Sol2. Pitch = 0.5 mm.

$\Rightarrow 100\ CSD = 0.5 mm$

$\therefore \quad LC = 1\,CSD = \frac{0.5}{100} mm = 5\mu m$

Zero. Error = $6 \times LC = 6 \times 5\mu m = 30\mu m$

Diameter of wire = Linear scale reading + n (L.C.) – Zero error

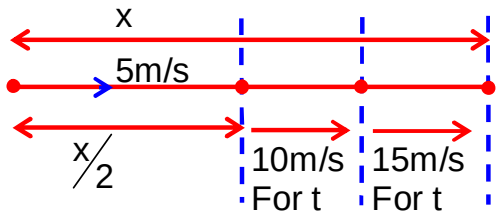
$= 4 \times 0.5 mm + 46(0.005) - 6(0.005)$

$= 2mm + \frac{200}{1000} mm$

$= 2.2mm$

$= 220 \times 10^{-2} mm$

Sol3. Average speed = $\frac{x}{\frac{x}{2\times 5} + 2t}$ - (i)



As, $10t + 15t = \frac{x}{2}$

$\therefore \quad t = \frac{x}{50}$ - (ii)

From (i) and (ii), av. Speed $= \frac{x}{\frac{x}{10} + \frac{x}{25}} = \frac{50}{7}$

Sol4. $x = A\sin\omega t$

P.E., $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2\omega t$; $k = m\omega^2$

$$\text{Stop} = \frac{dU}{dt} = kA^2 \sin\omega t.\cos\omega t.\ \omega$$

$$= kA^2\omega \sin\omega t \cos\omega t$$

$$\Rightarrow \frac{dU}{dt} = \frac{1}{2}kA^2\omega \sin 2\omega t$$

As, $\frac{dU}{dt}$ is maximum $\Rightarrow \sin 2\omega t = +1$

$$\Rightarrow 2\omega t = \pi/2$$

$$\Rightarrow t = \frac{\pi}{4\omega}$$

As, slope is maximum at $t = \frac{T}{\beta} \Rightarrow \frac{T}{\beta} = \frac{\pi}{4\omega}$

$$\Rightarrow \beta = \frac{4\omega T}{\pi} = \frac{4.2\pi}{T} \times \frac{T}{\pi} = 8$$

Sol5. $\Delta x_P = \Delta x_1 - \Delta x_2$

$$\Rightarrow \Delta x_P = (1.55 - 1)0.1 - (1.51 - 1)0.1$$

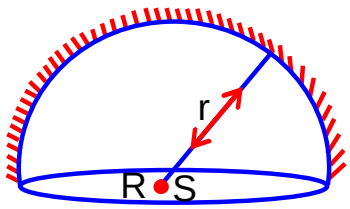
$$= (.55 - .51)0.1$$

$$= .04 \times 0.1 = 4 \times 10^{-3}\text{mm}$$

$$\therefore\ n = \frac{\Delta x}{\lambda} = \frac{4 \times 10^{-6}}{4 \times 10^{-7}} = 10$$

Sol6. Power of source, $P = 24\,W$

$$r = 10\,cm$$



Intensity on the surface $= \frac{P}{4\pi r^2}$

Pressure on the surface $= \frac{2I}{C}$

$\therefore$ Net force = Pressure × Projected Area

$$\Rightarrow F_{net} = \frac{2I}{C} \times \pi r^2$$

$$= \frac{2P.\pi r^2}{C.4\pi r^2} = \frac{P}{2C} = \frac{24}{2 \times 3 \times 10^8} = 4 \times 10^{-8}\text{N}$$

Sol7. As $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow f = -30\text{cm}$

$$-\frac{df}{f^2} = -\frac{dv}{v^2} - \frac{d\mu}{u^2}$$

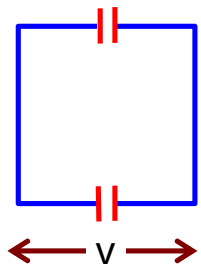
$$\Rightarrow \frac{df}{f^2} = \frac{dv}{v^2} + \frac{d\mu}{u^2}$$

$$dv = du = \frac{1\text{cm}}{20} = 0.05\,\text{cm}$$

$$\frac{df}{(30)^2} = \frac{0.05}{(120)^2} + \frac{0.05}{(40)^2}$$

$$\Rightarrow df = \frac{1}{32}cm \Rightarrow k = 32$$

Sol8. $C = 900\mu F, \quad V = 100v$



$$\therefore \quad Q = cv = 9\times10^{-2}C$$

$$U_i = \frac{1}{2}cv^2 = \frac{1}{2}\times900\times10^4 = 4.5\times10^6\times10^{-6}J = 4.5J$$

$$Q_f = Q_i$$

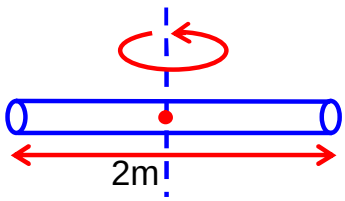
$$\Rightarrow 2cv = 9\times10^{-2}$$

$$\therefore \quad V = \frac{9\times10^{-2}}{2\times900\times10^{-6}} = 0.5\times10^2 = 50v$$

$$\therefore \quad U_f = 2\left(\frac{1}{2}cv^2\right) = 900\times10^{-6}\times2500 = 225\times10^{-2} = 2.25J$$

Loss in $U = 4.5 - 2.25 = 2.25J = 225\times10^{-2}J$

Sol9. Cross – sectional area = A
Density = d
mass of the rod, m = 2Ad.



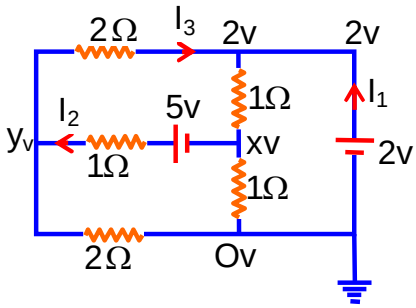
$$MI = \frac{m\ell^2}{12} = \frac{2Ad.4}{12} = \frac{2Ad}{3}$$

$\therefore$ Rotational kinetic energy

$$E = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2Ad}{3}\right)\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{3E}{Ad}} \Rightarrow \alpha = 3$$

Sol10. For x volt node –



$$\frac{x-2}{1} + \frac{x-0}{1} + \frac{x+5-y}{1} = 0$$

$$\Rightarrow 3x - y = -3$$

- (i)

For y volt node –

$$\frac{y-2}{2} + \frac{y}{2} + y - x - 5 = 0$$

$$4y - 2x = 12$$

$$2y - x = 6$$

- (ii)

From (i) & (ii) :- $x = 0, y = 3$

$$\therefore \quad I_1 = {}^{y}\!/_{2} = 1.5A$$

PART – B (CHEMISTRY)

SECTION – A

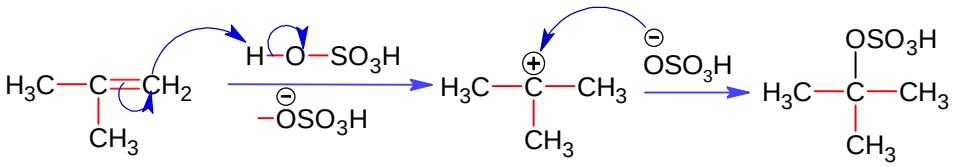
Sol1. Acidic strength $\propto$ stability of conjugate base

Sol2. According to spectrochemical series the correct order is $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$.

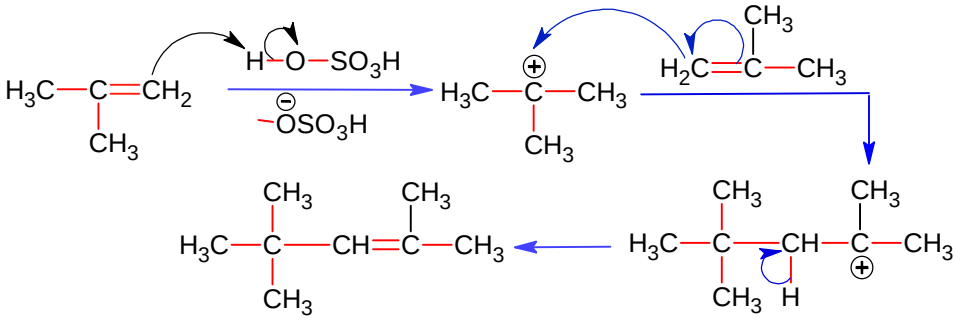
Sol3. The copper ore contains iron to remove this impurity , it is mixed with silica then FeO slags off as $FeSiO_3$.

$$FeO + SiO_2 \longrightarrow FeSiO_3$$

Sol4. For product 'A'



For product 'B'



Sol5.

$$Ph-CH_2-Br \xrightarrow{AgCN} Ph-CH_2-NC$$
(Benzyl isocyanide)

$$Ph-CH_2-NH_2 \xrightarrow[\text{aq. KOH}]{CHCl_3} Ph-CH_2-NC$$
(Benzyl isocyanide)

Sol6. $Cr_2O_7^{2-} + SO_3^{2-} \longrightarrow \underset{\text{(Green)}}{Cr^{3+}} + SO_4^{2-}$

Sol7. $8LiH + Al_2Cl_6 \longrightarrow 2LiAlH_4 + 6LiCl$

Sol8.

For central atom	IF_7	$ICl_4^{\ominus}$	XeF_6	XeF_2
Total no of valence e^-	7	7+1	8	8
No of bonded e^-	7	4	6	2
No of non- bonded e^-	0	4	2	6
No of lone pair e^-	0	2	1	3

Sol9. Silica gel prevents water corrosion and instrument malfunction by adsorbing moisture from the air.

Sol10.

f-block element → Lanthanoids → At no (57-71)

f-block element → Actinoids → At. no (89-103)

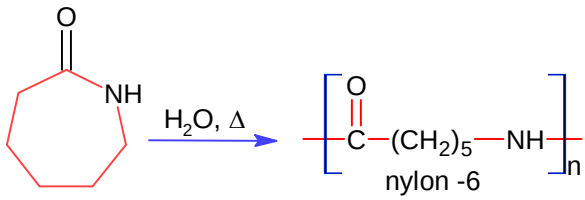
Therefore element with atomic no 65 belongs f-block

Atomic no 37 → [Kr] $5s^1$ → s-block element

Atomic no 52 → [kr] $5s^2$ $4d^{10}5p^4$ → p-block element

Atomic no 78 → [Xe] $6s^1$ $4f^{14}5d^9$ → d- block element

Sol11.



Sol12. Due to high hydration energy of Be^{2+} and Mg^{2+}, $BeSO_4$ and $MgSO_4$ are readily soluble in water.

Sol13. Aromatic aldehyde do not respond Fehling's test and both 'N' & 'S' must be present to obtain blood red colour because sodium nitroprusside give blood red color with S & N.

Sol14. Zn^{2+}, Co^{2+}, Ni^{2+} = IV^{th} group

Fe^{3+} = III^{rd} group.

Sol15. Cis- platin { Cis-$[Pt(NH_3)_2Cl_2]$} and coordination compounds of 'Pt' is used to inhibt the growth of tumours.

So16.

$$\underset{}{NO_{2(g)}} \xrightarrow{h\nu} \underset{(A)}{NO_{(g)}} + \underset{(B)}{O_{(g)}}$$

$$\underset{(B)}{O_{(g)}} + O_{2(g)} \rightleftharpoons \underset{(C)}{O_{3(g)}}$$

$$\underset{(A)}{NO_{(g)}} + \underset{(C)}{O_{3(g)}} \longrightarrow NO_{2(g)} + O_{2(g)}$$

Sol17. Ranitidine → Antacid

Meprobamate → Tranquilizer

Terfenadine → Antihistamine

Brompheniramine → Antihistamine

Sol18.

$$Ph - Cl + CH_3 - Cl \xrightarrow{2Na} Ph - CH_3 \text{ (Wurtz Fittig)}$$

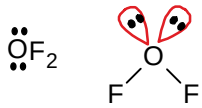
$$2Ph - Cl \xrightarrow{2Na} Ph - Ph \text{ (Fittig)}$$

$$Ph - \overset{\oplus}{N_2}\,\overset{\ominus}{Cl} \xrightarrow{Cu_2Cl_2} Ph - Cl + N_2 \text{ (Sandmeyer)}$$

$$C_2H_5 - Cl + NaI \longrightarrow C_2H_5I + NaCl \text{ (Finkelstein)}$$

Sol19. Seliwanoff's test is used to differentiate ketose and aldose. This test relies on the principle that the keto hexose are more rapidly dehydrated to form 5- Hydroxy methyl furfural when heated in acidic medium which on condensation with resorcinol, cherry red or brown red coloured complex is formed rapidly indicating a positive test.

Sol20.



No. of lone pair on 'O' = 2
FOF bond angle < 104^o
Oxidation state of 'O' = +2
Shape = v-shape
Geometry = Tetrahedral

SECTION – B

Sol1. For isothermal process $T_1 = T_2$
As we know that $\Delta U = nC_V(T_2 - T_1)$
So $\Delta U = 0$

Sol2.

$$\overset{+7}{MnO_4^{\ominus}} + 4H^{\oplus} + 3e^{\ominus} \longrightarrow \overset{+4}{MnO_2} + 2H_2O$$

Sol3. Given $K_b = 0.52\ K\ kg\ mol^{-1}$

$T_b^0 = 373K \quad \& \quad T_b = 373.52K$

So $\Delta T_b = 0.52K$

$\Delta T_b = k_b \times m$

$$0.52 = \frac{0.52 \times 2}{\text{Molar mass} \times 20 \times 10^{-3}}$$

Molar mass = $100\ g\ mol^{-1}$

Sol4. $Pt_{(s)} / H_2(g,1atm) / H^+(aq.1M) \,||\, Fe^{3+}_{(aq)}, Fe^{2+}_{(aq)} / Pt_{(s)}$

$$E^o_{cell} = E^o_{H_2/H^+} + E^o_{Fe^{3+}|Fe^{2+}} = (0 + 0.771) = 0.771V$$

$$E_{cell} = E^o_{cell} - \frac{0.0591}{1}\log\frac{Fe^{2+}}{Fe^{3+}}$$

$$0.712 = 0.771 - \frac{0.0591}{1}\log\frac{Fe^{2+}}{Fe^{3+}}$$

$$\log\frac{Fe^{2+}}{Fe^{3+}} \approx 1$$

On taking antilog $\frac{Fe^{2+}}{Fe^{3+}} = 10$

Sol5.

	A	+	B	$\longrightarrow$	P
Time = 0			7g		
Time = t			2g		

For the 1^{st} order kinetics

$$t = \frac{2.303}{k}\log\frac{[A_0]}{[A_t]}$$

$$t = \frac{2.303}{2.011\times10^{-3}}\log\frac{7}{2}$$

$$t = 622.989 \approx 623\,\text{Sec.}$$

Sol6. $M = \dfrac{n\times10^3}{V(\text{in ml})}$

$$2.6\times10^{-3} = \frac{x\times10^3}{85\times671.141} \qquad \left\{n = \frac{\text{Mass}}{\text{molar mass}}\right\}$$

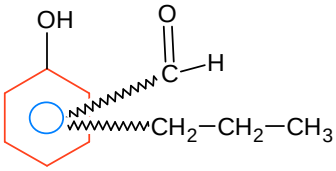
$x = 148322.161\times10^{-6}\,g \approx 0.148\,g$

Concentration in PPM for dichloromethane

$$PPM = \frac{0.148\times10^6}{1.49\times671.141 + 0.148g}$$

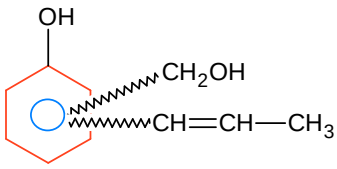
PPM = 148

Sol7. Neutral $FeCl_3$ test positive $\Rightarrow$ presence of phenol



$C_{10}H_{12}O_2$

or



$C_{10}H_{12}O_2$

Sol8. Energy of 1 photon $E = h\nu$

Energy of 1 mol photon $E = N_A h\nu$

$$E = 6.023\times10^{23}\times6.626\times10^{-34}\times2\times10^{12}\,J$$

$= 798.16J$

Sol9. $M = \dfrac{n}{v} \Rightarrow 0.2 = \dfrac{n\times1000}{300}$

$n_{CO_2} = 0.06$ mole

Volume of 0.06 mole CO_2 at STP = $0.06\times22.7L$

= 1.362 L

Or 1362 ml

Sol10.

	HCl	H_2SO_4
	600ml of 0.01 M	400 ml of 0.01M
No. of milimoles $\rightarrow$	600×0.01	400×0.01
	=6	= 4
Milimoles of H^+	=6	= 8

Total milimoles of H^+ = 14

$$\left[H^{\oplus}\right] = \frac{14}{1000} = 14\times10^{-3}$$

$$pH = -\log\left(14\times10^{-3}\right)$$

$pH = 1.86$ or 186×10^{-2}

PART – C (MATHEMATICS)

SECTION – A

Sol1. Let p be the probability of +ve outcome $P = \frac{{}^3C_1}{{}^6C_1} = \frac{1}{2}$

$q(\text{for} - \text{ve}) = \frac{{}^2C_1}{{}^6C_1} = \frac{1}{3}$

for +ve outcome = all +ve + 2(-ve) . 3(+ve) + 4(-ve) . 1(+ve)

$${}^5C_5 \cdot \left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{3}\right)^0 + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{3}\right)^2 + {}^5C_1 \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)^4 = \frac{521}{2592}$$

Sol2. Factors that to be added $\{(a,c),(c,a),(b,a),(c,b),(a,a),(b,b),(c,c)\}$

Sol3. Equation of plane $\begin{vmatrix} x-1 & y-2 & z-3 \\ 2-1 & 3-2 & 4-3 \\ 1-1 & 5-2 & 7-3 \end{vmatrix} = 0$

$\Rightarrow x - 4y + 3z = 2$

DC of normal $\pm\left(\frac{1}{\sqrt{26}}, \frac{-4}{\sqrt{26}}, \frac{3}{\sqrt{26}}\right)$

Since $\beta \in \left(0, \frac{\pi}{2}\right) \Rightarrow \cos\beta = +\text{ve}$

So DC $= \left(\frac{-1}{\sqrt{26}}, \frac{4}{\sqrt{26}}, \frac{-3}{\sqrt{26}}\right) = (\cos\alpha, \cos\beta, \cos\gamma)$

$\alpha, \gamma \in \left(\frac{\pi}{2}, \pi\right)$

Sol4. $\tan 75^0 = \cot 15^0$

$\tan(105^0) = \tan(90 + 15^0) = -\cot 15^0$

$\tan(195^0) = \tan(180 + 15^0) = \tan 15^0$

So, $\Rightarrow \tan 15^0 + \tan 15^0 - \tan 15^0 + \tan 15^0 = 2a$

$\Rightarrow a = \tan 15^0 = 2 - \sqrt{3}$

Now, $a + \frac{1}{a} = 2 - \sqrt{3} + 2 + \sqrt{3} = 4$

Sol5. $\frac{dy}{dx} - \frac{x^3 \cdot 3x^2 \cdot \tan^{-1} x^3}{\sqrt{1+x^6}\left(1+x^6\right)} y = 2x \cdot \exp\left(\frac{x^3 - \tan^{-1} x^3}{\sqrt{1+x^6}}\right)$

IF = $\exp\left(\int \frac{\left(-x^3\right)}{\sqrt{1+x^6}} \cdot \frac{3x^2 \tan^{-1} x^3}{\left(1+x^6\right)} dx\right)$

Let $\tan^{-1} x^3 = t \Rightarrow \frac{3x^2 \cdot dx}{1+x^6} = dt$

$$= \exp\left(\int \frac{(-\tan t)\cdot t\,dt}{\sec t}\right)$$

$$= e^{-\int t\cdot\sin t\,dt} = e^{(t\cos t - \sin t)}$$

So, $y\cdot\exp\left(\frac{\tan^{-1}x^3\cdot 1 - x^3}{\sqrt{1+x^6}}\right) = \int 2x\cdot dx + c$

It passes through origin, c = 0

$$\Rightarrow y\cdot\exp\left(\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}\right) = x^2$$

$$x = 1,\ y = \exp\left(\frac{1-\pi/4}{\sqrt{2}}\right)$$

Sol6. Using Cramer's rule

$$\Delta = \begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0 \Rightarrow k = 3$$

Now $4x + 5y = 7$

$7x + 8y = 10 \Rightarrow x = -2,\ y = 3$

Sol7. $\vec{n} \perp \vec{c}$

$\vec{a} = \alpha\vec{b} - \vec{n}$

$\vec{a}\cdot\vec{c} = \alpha\vec{b}\cdot\vec{c}$

$\vec{a}\cdot\vec{c} = 12\alpha$

$\vec{c}\times(\vec{a}\times\vec{b}) = (\vec{c}\cdot\vec{b})\vec{a} - (\vec{c}\cdot\vec{a})\vec{b}$

$= 12\vec{a} - 12\alpha\vec{b}$

$\vec{c}\times(\vec{a}\times\vec{b}) = -12\vec{n}$

$\left|\vec{c}\times(\vec{a}\times\vec{b})\right| = 12$

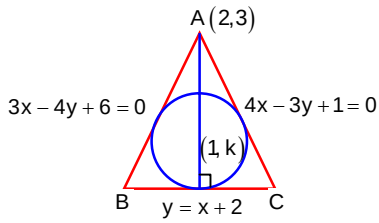
Sol8. Incentre lies on angle bisector of sides AB and AC.

$$\Rightarrow \frac{4x-3y+1}{5} = \pm\frac{(3x-4y+6)}{5}$$

$x - y = -1$ or $x + y = 5$

Angle bisector from A will be $\perp$ to BC.

So, (h,k) lies on $x + y = 5$ $\qquad h + k = 5$



Sol9. $\left(ax^3 + \frac{1}{b}x^{-1/3}\right)^5 \Rightarrow T_{r+1} = {}^{15}C_r\cdot(ax^3)^{15-r}\left(\frac{1}{b}\cdot x^{-\frac{1}{3}}\right)^r$

$$= {}^{15}C_r\cdot\frac{a^{15-r}}{b^r}\cdot x^{45-3r-\frac{r}{3}}$$

$$\Rightarrow 45 - 3r - \frac{r}{3} = 15 \Rightarrow 30 = \frac{10r}{3} \Rightarrow r = 9$$

$$\left(ax^{1/3}-\frac{1}{b}x^{-3}\right)^{15} \Rightarrow T_{r+1}={}^{15}C_r\cdot\frac{(-1)^r\cdot a^{15-r}}{b^r}\cdot x^{\frac{15-r}{3}-3r}$$

$$\Rightarrow -15=\frac{15-r-9r}{3}$$

$\Rightarrow -45=15-1$ or $\Rightarrow r=6$

According to question,

$$\Rightarrow {}^{15}C_9\cdot\frac{a^6}{b^9}={}^{15}C_6\cdot(-1)^6\cdot\frac{a^9}{b^6}$$

$\Rightarrow ab=1$

Sol10. $4n^2-16n+15=4n^2-10n-6n+15$

$=2n(2n-5)-3(2n-5)$

$=(2n-3)(2n-5)$

$=2n(2n-5)-3(2n-5)$

$=(2n-3)(2n-5)$

Now $\frac{-2}{4n^2-6n+15}=\frac{(2n-5)-(2n-3)}{(2n-3)(2n-5)}$

$=\frac{1}{2n-3}-\frac{1}{2n-5}$

Telescopic sum = $\frac{1}{(-1)}-\frac{(1)}{-3}+\left(\frac{1}{1}-\frac{1}{(-1)}\right)+\left(\frac{1}{3}-\frac{1}{1}\right)+\left(\frac{1}{6}-\frac{1}{3}\right)+....+....\left(\frac{1}{47}-\frac{1}{43}\right)$

Sum = $\frac{1}{3}+\frac{1}{47}=\frac{50}{147}$

Sol11.

p	**q**	**r**	$p\vee q$	$p\vee q\Rightarrow r$	$p\Rightarrow r$	S_1
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	T

So S_1 is not a tautology

p	**q**	**r**	$p\vee q$	$p\vee q\Rightarrow r$	$p\Rightarrow r$	$q\Rightarrow r$	$(p\Rightarrow r)\vee(q\Rightarrow r)$	S_2
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	T	F
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T	F
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

So S_2 is not a tautology

Sol12. Shortest distance will be along common normal

$y + tx = 2at + at^3$

$a = \frac{1}{16} \Rightarrow y + tx = \frac{t}{8} + \frac{t^3}{16}$

$(0,33) \Rightarrow 33 = \frac{t}{8} + \frac{t^3}{16}$

$\Rightarrow t^3 + 2t - 528 = 0$

$\Rightarrow (t-8)(t^2 - 8t + 66) = 0$

$\Rightarrow t = 8$; so $pt(at^2, 2at) = (4,1)$

Parabola $y^2 - 4y = 4x$

$\Rightarrow (y-2)^2 = 4(x+1)$

Parabola $y^2 - 4y = 4x$

$\Rightarrow (y-2)^2 = 4(x+1)$

Directrix = $\Rightarrow x + 1 = -1$

$x = -2$

So distance of $(4,1)$ from directrix = $4 + 2 = 6$ unit

Sol13. $\frac{(e-1)}{e}\int_1^2 3x^2 \cdot e^{[x]} \cdot e^{[x^3]} dx$

$1 < x < 2 \Rightarrow [x] = 1$

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$= \frac{e-1}{e}\int_1^8 e^1 \cdot e^{[t]} dt$

$= (e-1)\left[\int_1^2 e\,dt + \int_2^3 e^2 dt + \int_0^4 e^3 dt + ... + \int_7^8 e^7 dt\right]$

$= (e-1)(e + e^2 + ...e^7)$

Sol14. $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$

$\frac{dy}{dx} = 270x^4 - 540x^3 - 210x^2 + 360x + 210$

Slope of normal

$\Rightarrow \frac{-1}{90} = \frac{-1}{270x^4 - 540x^3 - 210x^2 + 360x + 210}$

$\Rightarrow 90 = 30(90x^2 - 18x^3 - 7x^2 + 12x + 7)$

$\Rightarrow 9x^2 - 18x^3 - 7x^2 + 12x + 4$

$\Rightarrow (x-1)(x-2)(3x+1)(3x+2) = 0$, four points

Sol15. $\ell_1 \Rightarrow \frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$

Shortest distance = $\left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}\right|$

$\left(\vec{a}_2 - \vec{a}_1\right) = 3\hat{i} + 10\hat{j} + 2\hat{k}$

$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix} = \hat{i}(-4) - \hat{j}(8) + \hat{k}(-8)$

Shortest distance = $\frac{|-12-80-16|}{\sqrt{16+64+64}} = 9$

Sol16. $(1+x)^{500} + x(1+x)^{499} + ... + x^{500}$

$= (1+x)^{500} \frac{\left[1-\left(\frac{x}{1+x}\right)^{501}\right]}{1-\frac{x}{1+x}}$; terms = 501

$= (1+x)^{501} - x^{501}$

Coefficient of $x^{301} \Rightarrow {}^{501}C_{301}$

Sol17. $f(x) = 5a^x$

$\Rightarrow x = 3, f(3) = 5a^3$

$\Rightarrow 320 = 5a^3$

$\Rightarrow a = 4$

$f(x) = 5 \cdot 4^x$

$= f(0) + f(1) + f(2) + f(3) + f(4) + f(5)$

$= \frac{5\left[4^6 - 1\right]}{4-1} = 6825$

Sol18. $\log_{\cos x} \cot x + 4 \cdot \log_{\sin x} \tan x = 1$

$\Rightarrow \log_{\cos x} \frac{\cos x}{\sin x} + 4\log_{\sin x} \frac{\sin x}{\cos x} = 1$

$= 1 - \log_{\cos x} \sin x + 4\left(1 - \log_{\sin x} \cos x\right) = 1$

Use $\log_b a = \frac{\log a}{\log b} = \frac{1}{\log_a b}$

Let $\log_{\cos x} \sin x = t$

$\Rightarrow 1 - t + 4\left(1 - \frac{1}{t}\right) = 1$

$\Rightarrow t^2 - 4t + 4 = 0$

$\Rightarrow t = 2$

$\Rightarrow \log_{\cos x} \sin x = 2$

$\Rightarrow \sin x = \cos^2 x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\sin x = \frac{-1+\sqrt{5}}{2}$

So, $\alpha + \beta = 4$

Sol19. $\text{adjA} = \begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$

$$A - d(\text{adjA}) = \begin{bmatrix} m & n \\ p & q \end{bmatrix} - d\begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$$

$$= \begin{bmatrix} m - dq & n(1+d) \\ p(1+d) & q - dm \end{bmatrix}$$

$$\Rightarrow \left|A - d(\text{adjA})\right| = (m - dq)(q - dm) - np(1+d)^2$$

$$= mq - d(m^2) - dq^2 + d^2mq - np(d^2 + 2d + 1) = 0$$

$$\Rightarrow d^2(-np + mq) + d(-2np - q^2 - m^2) + mq - np = 0$$

use $d = mq - np$

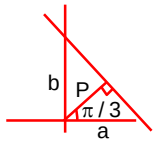
$$\Rightarrow d^3 - d\left((m+q)^2 + 2np - 2mq\right) + d = 0$$

$$\Rightarrow d^3 - d\left((m+q)^2 - 2d\right) + d = 0$$

$$\Rightarrow d^3 + 2d^2 + d = d(m+q)^2$$

$$\Rightarrow (d+1)^2 = (m+q)^2$$

Sol20. $\frac{x}{a} + \frac{y}{b} = 1$



$$\Rightarrow a = P\sec\frac{\pi}{3} = 2P$$

$$\Rightarrow b = P\sec\frac{\pi}{6} = \frac{2P}{\sqrt{3}}$$

$$\text{area} \Rightarrow \frac{1}{2}ab = \frac{98}{\sqrt{3}}$$

$$\Rightarrow \frac{1}{2}\frac{4P^2}{\sqrt{3}} = \frac{98}{\sqrt{3}} \Rightarrow P = 7$$

$$\text{now, } a^2 - b^2 = 4P^2 - \frac{4P^2}{3} = \frac{8}{3}P^2 = \frac{392}{3}$$

SECTION – B

Sol1. Unit place must be 5, let number be abc5. so cases are

$(1,3,3)5 \rightarrow 3$ ways

$(2,2,3)5 \rightarrow 3$ ways

$(1,1,2)5 \rightarrow 3$ ways

$(2,3,5)5 \rightarrow 6$ ways

$(2,2,5) \rightarrow 3$ ways

$(3,5,5) \rightarrow 3$ ways , so total 21 numbers.

Sol2. $\frac{\Sigma x}{7} = 8\,;\ \frac{\Sigma x^2}{7} - 8^2 = 16$

$\Sigma x = 56, \Sigma x^2 = 560$

now $a = \dfrac{56-14}{6} \Rightarrow a = 7$

$b = \dfrac{560-14^2}{6} - 7^2 \Rightarrow b = \dfrac{35}{3}$

Sol3. Required plane is $\pi \equiv a(x-1)+b(y-1)+c(z-2)=0$

$\Rightarrow a-3b+2c=0 \Rightarrow \dfrac{a}{-1}=\dfrac{b}{7}=\dfrac{c}{11}$

So, $\pi \Rightarrow -(x-1)-7(y-1)+11(z-2)=0$

$\Rightarrow -x+7y+11z=28$

Compare with $Ax+By+Cz=1$

$A=\dfrac{-1}{28}, B=\dfrac{7}{28}, C=\dfrac{11}{28}$

Sol4. $f^n(x) = f^n \text{of}^1(x)$

$f^2(x) = f^1\text{of}^1(x) = \dfrac{(2^2+3^2)x+2\cdot 2\cdot 3}{2^2+3^2+(2\cdot 2+3)x}$

Sum of coeff. of x numerator and denominator = $(2+3)^2$

$f^3(x) = f^1\text{of}^2(x) = \dfrac{(3^3+3\cdot 3\cdot 2^2)x+3\cdot 3^2\cdot 2+2\cdot 3}{3^3+3\cdot 3\cdot 2^2+(3\cdot 3^2\cdot 2+2^3)x}$

Coefficient of x in N^r and $D^r = (2+3)^3$

Similarly in $f^4(x) = (2+3)^4$

$f^5(x) = (2+3)^5$

Sol5. $t_n = \dfrac{n^3(2n)!+(2n-1)n!}{(n!)(2n)!} = \dfrac{n^3}{n!}+\dfrac{(2n)n!-n!}{2n!\cdot n!}$

$= \dfrac{n^2-1+1}{(n-1)!}+\dfrac{1}{(2n-1)!}-\dfrac{1}{2n!}$

$= \dfrac{n+1}{(n-2)!}+\dfrac{1}{(n-1)!}+\dfrac{1}{(2n-1)!}-\dfrac{1}{2n!}$

$= \dfrac{n-2+3}{(n-2)!}+\dfrac{1}{(n-1)!}+\dfrac{1}{(2n-1)!}-\dfrac{1}{2n!}$

$= \dfrac{1}{(n-3)!}+\dfrac{3}{(n-2)!}+\dfrac{1}{(n-1)!}+\dfrac{1}{(2n-1)!}-\dfrac{1}{(2n)!}$

Use $\Rightarrow e = 1+1+\dfrac{1}{2!}+\dfrac{1}{3!}+....$

$e^{-1} = 1-1+\dfrac{1}{2!}-\dfrac{1}{3!}+.....$

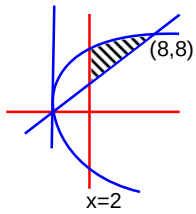
Sum = $e+3e+e-e^{-1}$

$= 5e - \frac{1}{e} \Rightarrow a = 5, b = 1$

$a^2 - b + c = 26$

Sol6. $\int_2^8 \left(\sqrt{8x} - x\right) dx$

$\alpha = \frac{22}{3}$



Sol7. $z = 1 + i,\ z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$

$z_1 = \frac{1 + i3}{1 - 2i}$

$\text{org} z_1 = \left[\tan^{-1} 3 - \tan^{-1}(-2)\right]$

$= \left[\tan^{-1} 8 + \tan^{-1} 2\right]$

$= \pi + \tan^{-1}\left(\frac{5}{1 - 6}\right)$

$= \pi + \tan^{-1}(-1) = \frac{3\pi}{4}$

$\Rightarrow \frac{12}{\pi} \arg z_1 = \frac{12}{\pi} \times \frac{3\pi}{4} = 9$

Sol8. Let $\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right) = \cos^{-1}\left(\frac{1}{5}\right) = \theta$

$\cos\theta = \left|\frac{3\lambda - 5 - 3}{\sqrt{35}\sqrt{\lambda^2 + 10}}\right| = \frac{1}{5} \Rightarrow \lambda = 5, \frac{25}{19}$

$\lambda_1 = \frac{25}{19}, \lambda_2 = 5$

$\text{Pt}(50, 50, 2) \Rightarrow \text{drop} \perp = \left|\frac{150 - 250 + 2 - 7}{\sqrt{9 + 25 + 1}}\right|$

$d = 3\sqrt{35}$

$d^2 = 315$

Sol9. $S = \{1, 2, 3, 4, 5, 6\}$

$P(S) = \{\phi, \{1\}, \{2\}, ... \{6\}, \{1, 2\} ...., \{1, 2, 3, 4, 5, 6\}\}$

$f(1)$ can be the set with cardinality 1 or null set.

Case I, $f(1) \in$ any of $\{1\}, \{2\}, ... \{6\}$; $f(2)$ will have two elements in which one is $f(1)$

$f(2) = \{-, f(1)\} \rightarrow$ 5 other possibilities

$f(3) = \{-, f(2), f(1)\} \rightarrow$ 4 ways

and so on, total ways = ${}^6C_1 \cdot {}^5C_1 \cdot {}^4C_1 \cdot {}^3C_1 \cdot {}^2C_1 \cdot 1 = 6! = 720$

Case II, $f(1) = \phi$

$f(2)$ cardinality can be 1 or 2.

Cardinality	f(2)	f(3)	f(4)	f(5)	f(6)
	1	2	3	4	5

$f(1) = \{-\}; f(4) = \{-, f(1), f(2), f(3)\}$

$f(2) = \{-, f(1)\}; f(5) = \{-, f(1), f(2), f(3), f(4)\}$

$f(3) = \{-, f(1), f(2)\}; f(6) = \{-, f(1), f(2), f(3), f(4)\}$

Total ways = $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

f(2)	f(3)	f(4)	f(5)	f(6)
1	2	3	4	6

total ways = $6 \cdot 5 \cdot 4 \cdot 3 \cdot 1$

1	2	3	4	5	6; ways = $6 \cdot 5 \cdot 4 \cdot {}^3C_2 \cdot 1$
1	2	4	5	6	6; ways = $6 \cdot 5 \cdot {}^4C_2 \cdot {}^2C_1 \cdot 1$
1	3	4	5	6	6; ways = $6 \cdot {}^5C_2 \cdot {}^3C_1 \cdot {}^2C_1 \cdot 1$
2	3	4	5	6	6; ways = ${}^6C_2 \cdot {}^4C_1 \cdot {}^3C_1 \cdot {}^2C_1 \cdot 1$

So total ways = 720 + 720 + 360.5 = 3240

Sol10. Apply Leibniz rule

$$\lim_{x \to 0} \frac{48 \cdot x^3}{4x^3\left(x^6 + 1\right)} = 12$$