FIITJEE

Solutions to JEE(Main) -2023

Test Date: 30th January 2023 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (1 – 10)*** *contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. A vehicle travels 4km with speed of 3km/h and another 4km with speed of 5km/h, then its average speed is

(A) 3.50 km/h (B) 3.75 km/h
(C) 4.25 km/h (D) 4.00 km/h

Q2. A thin prism P_1 with an angle 6° and made of glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without average deviation. The angle of prism P_2 is

(A) 1.3° (B) 4.5°
(C) 6° (D) 7.8°

Q3. An electron accelerated through a potential difference V_1 has a de-Broglie wavelength of λ. When the potential is changed to V_2, its de-Broglie wavelength increases by 50%. The value of $\left(\frac{V_1}{V_2}\right)$ is equal to

(A) 4 (B) $\frac{3}{2}$
(C) $\frac{9}{4}$ (D) 3

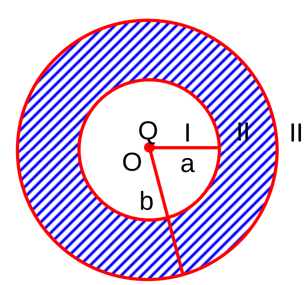
Q4. Match List I with List II:

List-I		List-II	
A.	Attenuation	I.	Combination of a receiver and transmitter.
B.	Transducer	II.	Process of retrieval of information from the carrier wave at receiver
C.	Demodulation	III.	Converts one form of energy into another
D.	Repeater	IV.	Loss of strength of a signal while propogating through a medium.

Choose the correct answer from the options given below :

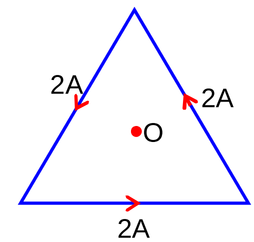
(A) A – IV, B – III, C – II, D – I (B) A – IV, B – III, C – I, D – II
(C) A – I, B – II, C – III, D – IV (D) A – II, B – III, C – IV, D – I

Q5. As shown in the figure, a point charge Q is placed at the centre of conducting spherical shell of inner radius a and outer radius b. The electric field due to charge Q in three different regions I, II and III is given by :
(I : r < a, II : a < b, III : r > b)



(A) $E_I \neq 0, E_{II} = 0, E_{III} \neq 0$
(B) $E_I = 0, E_{II} = 0, E_{III} = 0$
(C) $E_I = 0, E_{II} = 0, E_{III} \neq 0$
(D) $E_I \neq 0, E_{II} = 0, E_{III} = 0$

Q6. As shown in the figure, a current of 2A flowing in an equilateral triangle of side $4\sqrt{3}$ cm. The magnetic field at the centroid O of the triangle is (Neglect the effect of earth's magnetic field)

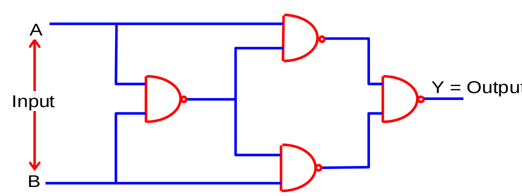


(A) $3\sqrt{3}\times10^{-5}$ T
(B) $4\sqrt{3}\times10^{-4}$ T
(C) $4\sqrt{3}\times10^{-5}$ T
(D) $\sqrt{3}\times10^{-4}$ T

Q7. A point source of 100W emits light with 5% efficiency. At a distance of 5m from the source, the intensity produced by the electric field component is :
(A) $\frac{1}{10\pi}\frac{W}{m^2}$
(B) $\frac{1}{20\pi}\frac{W}{m^2}$
(C) $\frac{1}{2\pi}\frac{W}{m^2}$
(D) $\frac{1}{40\pi}\frac{W}{m^2}$

Q8. A force is applied to a steel wire 'A', rigidly clamped at one end. As a result elongation in the wire is 0.2mm. If same force is applied to another steel wire 'B' of double the length and a diameter 2.4 times that of the wire 'A', the elongation in the wire 'B' will be (wires having uniform circular cross sections)
(A) 6.06×10^{-2}mm
(B) 3.0×10^{-2}mm
(C) 6.9×10^{-2}mm
(D) 2.77×10^{-2}mm

Q9. The output Y for the input A and B of circuit is given by truth table of the shown circuit is:



(A)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(B)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(C)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(D)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Q10. A flask contains hydrogen and oxygen in the ratio of 2 : 1 by mass at temperature 27°C. The ratio of average kinetic energy per molecule of hydrogen and oxygen respectively is :
(A) 4 : 1
(B) 1 : 4
(C) 2 : 1
(D) 1 : 1

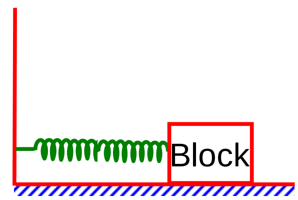
Q11. Match List I with List II :

List I	List II
A. Torque	I. $kg\ m^{-1}\ s^{-2}$
B. Energy density	II. $Kg\ ms^{-1}$
C. Pressure gradient	III. $kg\ m^{-2}\ s^{-2}$
D. Impulse	IV. $Kg\ m^{2}\ s^{-2}$

Choose the correct answer from the options given below :

(A) A – IV, B – I, C – III, D – II
(B) A – IV, B – I, C – II, D – III
(C) A – I, B – IV, C – III, D – II
(D) A – IV, B – III, C – I, D – II

Q12. For a simple harmonic motion in a mass spring system shown, the surface is frictionless. When the mass of the block is 1 kg, the angular frequency is ω_1. When the mass block is 2kg the angular frequency is ω_2. The ratio ω_2 / ω_1 is



(A) $1/\sqrt{2}$
(B) $1/2$
(C) 2
(D) $\sqrt{2}$

Q13. Other is labeled as **Reason R**

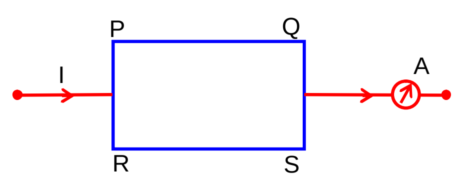
Assertion A : Efficiency of a reversible heat engine will be highest at -273° C temperature of cold reservoir.

Reason R : The efficiency of carnot's engine depends not only on temperature of cold reservoir but it depends on the temperature of hot reservoir too and is given as $n = \left(1 - \frac{T_2}{T_2}\right)$.

In the light of the above statements, choose the correct answer from the options given below

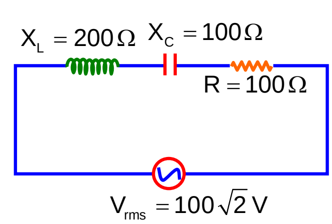
(A) **A** is true but **R** is false
(B) **A** is false but **R** is true
(C) Both **A** and **R** are true and **R** is the correct explanation of **A**
(D) Both **A** and **B** are true but **R** is **NOT** the correct explanation of **A**

Q14. A current carrying rectangular loop PQRS is made of uniform wire. The length PR = QS = 5cm and PQ = RS = 100 cm. If ammeter current reading change from I to 2I, the ratio of magnetic forces per unit length on the wire PQ due to wire RS in the two cases respectively $\left(f_{PQ}^{I} : f_{PQ}^{2I}\right)$ is :



(A) 1 : 3
(B) 1 : 2
(C) 1 : 4
(D) 1 : 5

Q15. In the given circuit, rms value of current (I_{rms}) through the resistor R is :



(A) $\frac{1}{2}$A
(B) 2A
(C) $2\sqrt{2}$ A
(D) 20A

Q16. A machine gun of mass 10kg fires 20g bullets at the rate of 180 bullets per minute with a speed of 100 ms^{-1} each. The recoil velocity of gun is

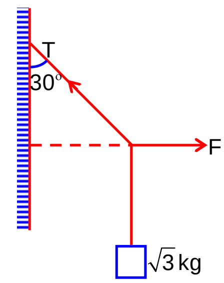
(A) 1.5 m/s
(B) 0.6 m/s
(C) 2.5 m/s
(D) 0.02 m/s

Q17. An object is allowed to fall from a height R above the earth, where R is the radius of earth. Its velocity when it strikes the earth's surface, ignoring air resistance, will be

(A) $\sqrt{2gR}$ (B) $\sqrt{gR}$

(C) $\sqrt{\frac{gR}{2}}$ (D) $2\sqrt{gR}$

Q18. A block of $\sqrt{3}$ kg is attached to a string whose other end is attached to the wall. An unknown force F is applied so that the string makes an angle of 30° with the wall. The tension T is :
(Given g = 10 ms^{-2})



(A) 15 N (B) 10 N
(C) 20 N (D) 25 N

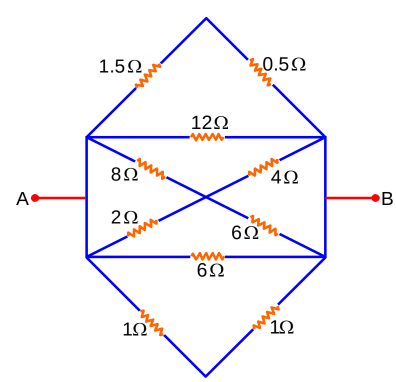
Q19. Given below two statements : one is labeled as **Assertion A** and the other is labeled as **Reason R**

Assertion A : The nuclear density of nuclides $^{10}_{5}B$, $^{6}_{3}Li$, $^{56}_{26}Fe$, $^{20}_{10}Ne$ and $^{209}_{83}Bi$ can be arranged as $\rho^{N}_{Bi} > \rho^{N}_{Fe} > \rho^{N}_{Ne} > \rho^{N}_{B} > \rho^{N}_{Id}$

Reason R : The radius R of nucleus is related to its mass number A as $R = R_0 A^{1/3}$, where R_0 is a constant.

In the light of the above statements, choose the **correct** answer from the options given below
(A) A is true but **R** is false
(B) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**
(C) Both **A** and **R** are true and **R** is the correct explanation of **A**
(D) **A** is false but **R** is true

Q20. The equivalent resistance between A and B is_______.



(A) $\frac{1}{2}\Omega$
(B) $\frac{1}{3}\Omega$
(C) $\frac{2}{3}\Omega$
(D) $\frac{3}{2}\Omega$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

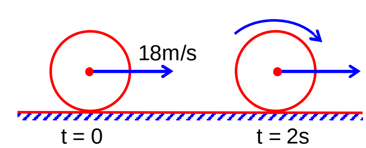
Q1. In a Young's double slit experiment, the intensities at two points, for the path differences $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$ (λ being the wavelength of light used) are I_1 and I_2 respectively. If I_0 denotes the intensity produced by each one of the individual slits, then $\frac{I_1 + I_2}{I_0} =$ ________.

Q2. A body of mass 2kg initially at rest. It starts moving unidirectionally under the influence of a source of constant power P. Its displacement in 4s is $\frac{1}{3}\alpha^2\sqrt{P}$ m. The value of α will be_____.

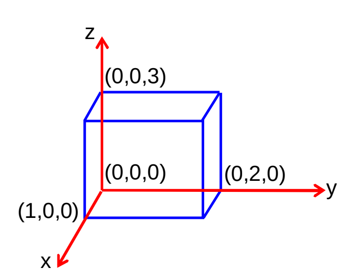
Q3. A stone tied to 180cm long string at its end is making 28 revolutions in horizontal circle in every minute. The magnitude of acceleration of stone is $\frac{1936}{x}\text{ms}^{-2}$. The value of x_______.
(Take $\pi = \frac{22}{7}$)

Q4. In an ac generator, a rectangular coil of 100 turns each having area $14 \times 10^{-2}\text{m}^2$ is rotated at 360 rev/min about an axis perpendicular to a uniform magnetic field of magnitude 3.0 T. The maximum value of the emf produced will be _________V.
(Take $\pi = \frac{22}{7}$)

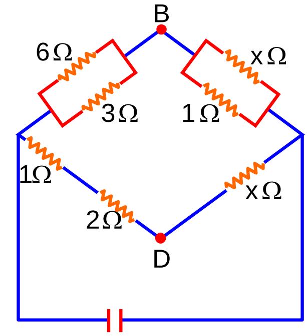
Q5. A uniform disc of mass 0.5kg and radius r is projected with velocity 18m/s at t = 0s on a rough horizontal surface. It starts off with a purely sliding motion at t = 0s. After 2s it acquires a purely rolling motion (see figure). The total kinetic energy of the disc after 2s will be______J (given, coefficient of friction is 0.3 and g = 10m/s^2).



Q6. As shown in figure, a cuboid lies in a region with electric field $E = 2x^2\hat{i} - 4y\hat{j} + 6\hat{k}\,{}^{N}\!/\!_{C}$. The magnitude of charge within the euboid is $n \in_0$ C. The value of n is ________
(if dimension of euboid is $1 \times 2 \times 3\text{m}^3$).



Q7. If the potential difference between B and D is zero, the value of x is $\frac{1}{n}\Omega$. The value of n is ________.



Q8. A faulty thermometer reads $5°C$ in melting ice and $95°C$ in steam. The correct temperature on absolute scale will be ______K when the faulty thermometer reads $41°C$.

Q9. The velocity of a particle executing SHM varies with displacement (x) as $4\upsilon^2 = 50 - x^2$. The time period of oscillations is $\frac{x}{7}$s. The value of x is _________.

(Take $\pi = \frac{22}{7}$)

Q10. A radioactive nucleus decays by two different process. The half life of the first process is 5 minutes and that of the second is 30s. The effective half-life of the nucleus is calculated to be $\frac{\alpha}{11}$s. The value of α is __________.

PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Which of the following reaction is correct?

(A) $2LiNO_3 \longrightarrow 2Li + 2NO_2 + O_2$
(B) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$
(C) $2LiNO_3 \xrightarrow{\Delta} 2LiNO_2 + O_2$
(D) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 2N_2O_4 + O_2$

Q2. The water quality of a pond was analysed and its BOD was found to be 4. The pond has
(A) Highly polluted water
(B) Slightly polluted water
(C) Very clean water
(D) Water has high amount of fluoride compounds

Q3. Match List I with List II

List-I(Mixture)		List-II (Separation Technique)	
A.	$CHCl_3 + C_6H_5NH_2$	I.	Steam distillation
B.	$C_6H_{14} + C_5H_{12}$	II.	Differential extraction
C.	$C_6H_5NH_2+H_2O$	III.	Distillation
D.	Organic compound in water	IV.	Fractional distillation

Choose the correct answer from the options given below:
(A) A- III, B-I, C-IV, D- II
(B) A-IV, B-I, C-III, D-II
(C) A-II, B-I, C-III, D-IV
(D) A-III, B-IV, C-I, D-II

Q4. Boric acid is solid, whereas BF_3 is gas at room temperature because of
(A) Strong ionic bond in Boric acid
(B) Strong covalent bond in BF_3
(C) Strong van der Waal's interaction in Boric acid
(D) Strong hydrogen bond in Boric acid

Q5. The wave function (Ψ) of 2s is given by

$$\psi_{2s} = \frac{1}{2\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{1/2}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$$

At $r = r_0$, radial node is formed. Thus, r_0 in terms of a_0

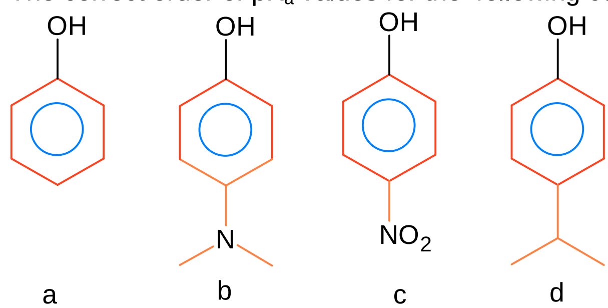
(A) $r_0 = 2a_0$
(B) $r_0 = \frac{a_0}{2}$
(C) $r_0 = 4a_0$
(D) $r_0 = a_0$

Q6. The Cl-Co-Cl bond angle values in a fac-$[Co(NH_3)_3Cl_3]$ complex is / are:
(A) 90^o
(B) 90^o & 180^o
(C) 90^o&120^o
(D) 180^o

Q7. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R.**
Assertion A: Antihistamines do not affect the secretion of acid in stomach.
Reason R: Antiallergic and antacid drugs work on different receptors.
In the light of the above statements, choose the **correct** answer from the options given below:

(A) A is false but R is true
(B) A is true but R is false
(C) Both A and R are true but R is not the correct explanation of A
(D) Both A and R are true and R is the correct explanation of A

Q8. The correct order of pK_a values for the following compounds is:

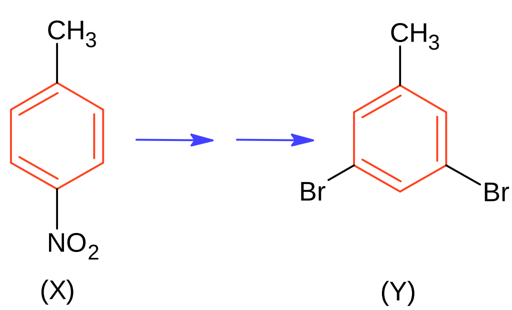


(A) b > d >a >c
(B) c > a > d > b
(C) b > a >d > c
(D) a > b > c > d

Q9. Given below are two statements:
Statement I: During Electroyltic refining, the pure metal is made to act as anode and its impure metallic form is used as cathode.
Statement II: During the Hall-Heroult electrolysis process, purified Al_2O_3 is mixed with Na_3AlF_6 to lower the melting point of the mixture.
In the light of the above statements, choose the **most appropriate** answer from the options given below:
(A) Both Statement I and **Statement II** are correct
(B) Statement **I** is correct but Statement **II** is incorrect
(C) Statement **I** is incorrect but Statement **II** is correct
(D) Both statement **I** and Statement **II** are incorrect

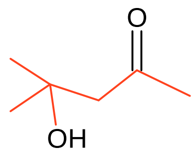
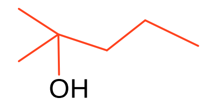
Q10. Formulae for Nessler's reagent is;
(A) $KHgI_3$
(B) K_2HgI_4
(C) HgI_2
(D) KHg_2I_2

Q11.



In the above conversion of compound (X) to product (Y), the sequence of reagents to be used will be:
(A) $Br_2(aq)$ (ii) $LiAlH_4$ (iii) H_3O^+
(B) Br_2,Fe (ii) Fe, H^+ (iii) $LiAlH_4$
(C) Fe, H^+ (ii) $Br_2(aq)$ (iii) HNO_2 (iv) H_3PO_2
(D) Fe, H^+ (ii) $Br_2(aq)$ (iii) HNO_2 (iv) CuBr

Q12. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A:  can be easily reduced using Zn-Hg / HCl to 

Reason R: Zn-Hg/HCl is used to reduce carbonyl group to $–CH_2–$ group.

In the light of the above statements, choose the **correct** answer from the options given below:

(A) Both A and R are true but R is not the correct explanation of A
(B) A is true but R is false
(C) A is false but R is true
(D) Both A and R are true and R is the correct explanation of A

Q13. Bond dissociation energy of "E-H" bond of the "H_2E" hydrides of group 16 elements (given below), follows order.
A. O
B. S
C. Se
D. Te
Choose the correct from the options given below :
(A) A > B >D > C
(B) A > B > C > D
(C) D > C > B > A
(D) B > A > C > D

Q14. Match List I with List II

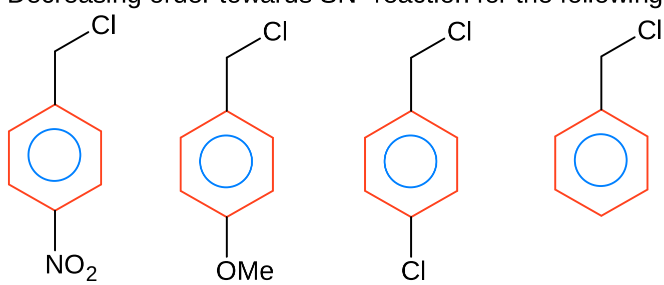
List-I(complexes)		List-II (Hybridisation)	
A.	$[Ni(CO)_4]$	I.	sp^3
B.	$[Cu(NH_3)_4]^{2+}$	II.	dsp^2
C.	$[Fe(NH_3)_6]^{2+}$	III.	sp^3d^2
D.	$[Fe(H_2O)_6]^{2+}$	IV.	d^2sp^3

(A) A- II, B-I, C-III, D- IV
(B) A-II, B-I, C-IV, D-III
(C) A-I, B-II, C-III, D-IV
(D) A-I, B-II, C-IV, D-III

Q15. Maximum number of electrons that can be accommodated in shell with n= 4 are:
(A) 16
(B) 72
(C) 32
(D) 50

Q16. $KMnO_4$ oxidises I^- in acidic and neutral / faintly alkaline solution respectively, to
(A) IO_3^- & I_2
(B) IO_3^- & IO_3^-
(C) I_2 & IO_3^-
(D) I_2 & I_2

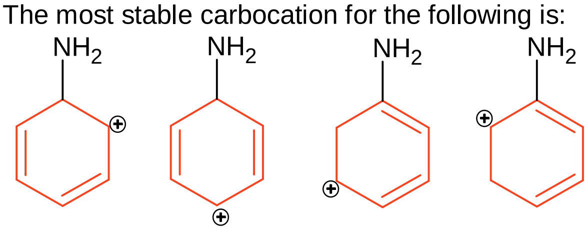
Q17. Decreasing order towards SN^1 reaction for the following compounds is:



a b c d

(A) a > c > d >b
(B) b > d > c > a
(C) a > b > c > d
(D) d > b > c > a

Q18. The most stable carbocation for the following is:



a b c d

(A) b
(B) a
(C) c
(D) d

Q19. Chlorides of which metal are soluble in organic solvents:

(A) Be
(B) Ca
(C) K
(D) Mg

Q20. 1 L, 0.02 M solution of $[Co(NH_3)_5SO_4]$ Br is mixed with 1L, 0.02 M solution of $[Co(NH_3)_5Br]SO_4$. The resulting solution is divided into two equal parts (X) and treated with excess of $AgNO_3$ solution and $BaCl_2$ solution respectively as shown below:

1L Solution (X) + $AgNO_3$ solution (excess) $\rightarrow$ Y

1L Solution (X) + $BaCl_2$ solution (excess) $\rightarrow$ Z

The number of moles of Y and Z respectively are

(A) 0.02, 0.01
(B) 0.01, 0.02
(C) 0.01, 0.01
(D) 0.02, 0.02

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. The graph of $\log\frac{x}{m}$ vs log p for an adsorption process is a straight line inclined at an angle of 45^o with intercept equal to 0.6020. The mass of gas adsorbed per unit mass of adsorbent at the pressure of 0.4 atm is _______ $\times 10^{-1}$(Nearest integer)
Given: log 2 = 0.3010

Q2. Lead storage battery contains 38% by weight solution of H_2SO_4. The van't Hoff factor is 2.67 at this concentration. The temperature in Kelvin at which the solution in the battery will freezes is __________(Nearest integer)
Given K_f = 1.8 K kg mol^{-1}.

Q3. An organic compound undergoes first order decomposition. If the time taken for the 60% decomposition is 540s, then the time required for 90% decomposition will be is ________s. (Nearest integer)
Given: ln 10 = 2.3; log 2 =0.3

Q4. Iron oxide FeO, crystallises in a cubic lattice with a unit cell edge length of 5.0 $\mathring{A}$. If density of the FeO in the crystal is 4.0 g cm^{-3}, then the number of FeO units present per unit cell is______________. (Nearest integer)
Given: Molar mass of Fe and O is 56 and 16g mol^{-1} respectively.
$N_A = 6.0\times10^{23}$ mol^{-1}.

Q5. The strength of 50 volume solution of hydrogen peroxide is ___________ g / L (Nearest integer).
Given:
Molar mass of H_2O_2 is 34 g mol^{-1}
Molar volume of gas at STP = 22.7L

Q6. A short peptide on complete hydrolysis produces 3 moles of glycine (G), two moles of leucine (L) and two moles of valine (V) per mole of peptide. The number of peptide linkages in it are ____________.

Q7. Consider the following equation:
$2SO_2(g)+O_2(g) \rightleftharpoons 2SO_3(g), \Delta H = -190\,KJ$
The number of factors which will increase the yield of SO_3 at equilibrium from the following is_______________.
A. Increasing temperature
B. Increasing pressure
C. Adding more SO_2
D. Adding more O_2
E. Addition of catalyst

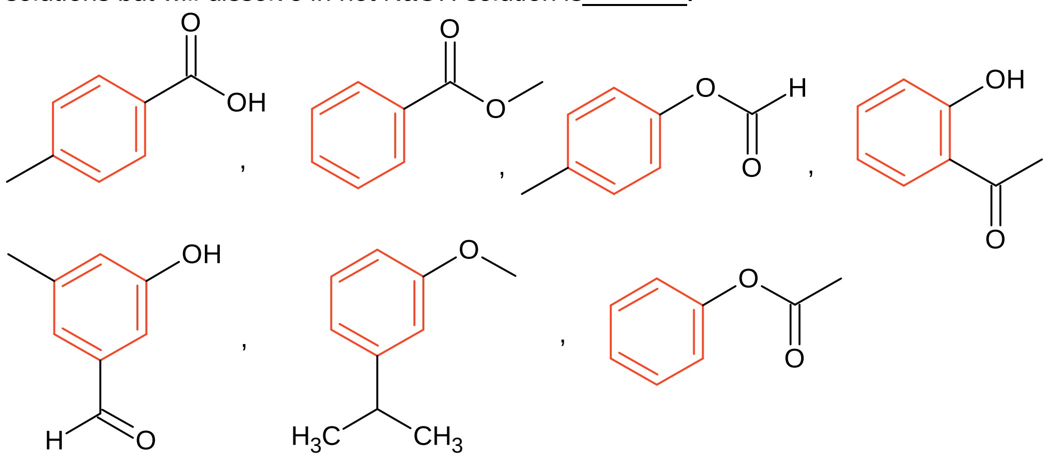
Q8. The electrode potential of the following half cell at 298 K
$X\,|\,X^{2+}(0.001M)\,||\,Y^{2+}(0.01M)\,|\,Y$ is ___________ $\times 10^{-2}$ V (Nearest integer)
Given: $E^0_{X^{2+}|X} = -2.36V$

$E^0_{Y^{2+}|Y} = +0.36V$

$\frac{2.303RT}{F} = 0.06V$

Q9. 1 mole of ideal gas is allowed to expand reversibly and adiabatically from a temperature of 27°C. The work done is 3 kJ mol^{-1}. The final temperature of the gas is _______ K (Nearest integer). Given C_V = 20 J mol^{-1} K^{-1}.

Q10. Number of compounds from the following which will not dissolve in cold $NaHCO_3$ and NaOH solutions but will dissolve in hot NaOH solution is_______.



PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Let f, g and h be the real valued functions defined on R as

$$f(x)=\begin{cases}\dfrac{x}{|x|}, x\neq 0\\ 1, x=0\end{cases}, g(x)=\begin{cases}\dfrac{\sin(x+1)}{(x+1)} & , x\neq -1\\ 1 & , x=-1\end{cases}$$

and $h(x)=2[x]-f(x)$, where $[x]$ is the greatest integer $\leq x$. Then the value of $\lim\limits_{x\to 1} g(h(x-1))$ is

(A) 0
(B) $\sin(1)$
(C) –1
(D) 1

Q2. Let $\vec{a}$ and $\vec{b}$ be two vectors. Let $|\vec{a}|=1, |\vec{b}|=4$ and $\vec{a}\cdot\vec{b}=2$. If $\vec{c}=(2\vec{a}\times\vec{b})-3\vec{b}$, then the value of $\vec{b}\cdot\vec{c}$ is

(A) –48
(B) –24
(C) –60
(D) –84

Q3. The solution of the differential equation $\dfrac{dy}{dx}=-\left(\dfrac{x^2+3y^2}{3x^2+y^2}\right), y(1)=0$ is

(A) $\log_e|x+y|-\dfrac{2xy}{(x+y)^2}=0$
(B) $\log_e|x+y|+\dfrac{2xy}{(x+y)^2}=0$
(C) $\log_e|x+y|+\dfrac{xy}{(x+y)^2}=0$
(D) $\log_e|x+y|-\dfrac{xy}{(x+y)^2}=0$

Q4. Let $\lambda\in R, \vec{a}=\lambda\hat{i}+2\hat{j}-3\hat{k}, \vec{b}=\hat{i}-\lambda\hat{j}+2\hat{k}$.

If $\left(\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)\right)\times\left(\vec{a}-\vec{b}\right)=8\hat{i}-40\hat{j}-24\hat{k}$, then $\left|\lambda\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right|^2$ is equal to

(A) 136
(B) 132
(C) 140
(D) 144

Q5. If a plane passes through the points $(-1,k,0),(2,k,-1),(1,1,2)$ and is parallel to the line $\dfrac{x-1}{1}=\dfrac{2y+1}{2}=\dfrac{z+1}{-1}$, then the value of $\dfrac{k^2+1}{(k-1)(k-2)}$ is

(A) $\dfrac{6}{13}$
(B) $\dfrac{13}{6}$
(C) $\dfrac{5}{17}$
(D) $\dfrac{17}{5}$

Q6. $\lim\limits_{n\to\infty}\frac{3}{n}\left\{4+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+...+\left(3-\frac{1}{n}\right)^2\right\}$ is equal to

(A) $\frac{19}{3}$ (B) 12

(C) 0 (D) 19

Q7. Let $a_1=1, a_2, a_3, a_4, ...$ be consecutive natural numbers.

Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{1}{1+a_2a_3}\right)+....+\tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to

(A) $\cot^{-1}(2022)-\frac{\pi}{4}$ (B) $\tan^{-1}(2022)-\frac{\pi}{4}$

(C) $\frac{\pi}{4}-\tan^{-1}(2022)$ (D) $\frac{\pi}{4}-\cot^{-1}(2022)$

Q8. A vector $\vec{v}$ in the first octant is inclined to the x-axis at 60^0, to the y-axis at 45^0 and to the z-axis at an acute angle. If a plane passing through the points $(\sqrt{2}, -1, 1)$ and (a, b, c) is normal to $\vec{v}$, then

(A) $a+b+\sqrt{2}c=1$ (B) $\sqrt{2}a+b+c=1$

(C) $a+\sqrt{2}b+c=1$ (D) $\sqrt{2}a-b+c=1$

Q9. If the functions $f(x)=\frac{x^3}{3}+2bx+\frac{ax^2}{2}$ and $g(x)=\frac{x^3}{3}+ax+bx^2, a\neq 2b$ have a common extreme point, then $a+2b+7$ is equal to :

(A) 3 (B) 6

(C) $\frac{3}{2}$ (D) 4

Q10. Let q be the maximum integral value of p in $[0, 10]$ for which the roots of the equation $x^2-px+\frac{5}{4}p=0$ are rational. Then the area of the region $\left\{(x, y): 0\le y\le (x-q)^2, 0\le x\le q\right\}$ is

(A) 164 (B) $\frac{125}{3}$

(C) 243 (D) 25

Q11. Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2+y^2=8$ and $y^2=16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to

(A) 76 (B) 81

(C) 72 (D) 64

Q12. The number of ways of selecting two numbers a and b, $a\in\{2, 4, 6, ..., 100\}$ and $b\in\{1, 3, 5, ..., 99\}$ such that 2 is the remainder when $a+b$ is divided by 23 is

(A) 268 (B) 108

(C) 54 (D) 186

Q13. For $\alpha, \beta \in R$, suppose the system of linear equations

$x - y + z = 5$

$2x + 2y + \alpha z = 8$

$3x - y + 4z = \beta$

has infinitely many solutions. Then α and β are the roots of

(A) $x^2 + 18x + 56 = 0$
(B) $x^2 - 10x + 16 = 0$
(C) $x^2 - 18x + 56 = 0$
(D) $x^2 + 14x + 24 = 0$

Q14. Let $x = \left(8\sqrt{3} + 13\right)^{13}$ and $y = \left(7\sqrt{2} + 9\right)^{9}$. If $\left[t\right]$ denotes the greatest integer $\le t$, then

(A) $\left[x\right]$ is even but $\left[y\right]$ is odd
(B) $\left[x\right]$ is odd but $\left[y\right]$ is even
(C) $\left[x\right]$ and $\left[y\right]$ are both odd
(D) $\left[x\right] + \left[y\right]$ is even

Q15. Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots, a_{100}$ is 25. Then S is

(A) $\{99\}$
(B) $\{9\}$
(C) N
(D) ϕ

Q16. The parabolas : $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line y = 1. If a, b, c, d, e, f are positive real numbers and a, b, c are in G.P., then

(A) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.
(B) d, e, f are in G.P.
(C) d, e, f are in A.P.
(D) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Q17. If P is a 3 x 3 real matrix such that $P^T = aP + (a-1)I$, where a > 1, then

(A) $|\text{Adj } P| > 1$
(B) P is a singular matrix
(C) $|\text{Adj } P| = \frac{1}{2}$
(D) $|\text{Adj } P| = 1$

Q18. The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is :

(A) $\left[\sqrt{5}, \sqrt{10}\right]$
(B) $\left[\sqrt{5}, \sqrt{13}\right]$
(C) $\left[2\sqrt{2}, \sqrt{11}\right]$
(D) $\left[\sqrt{2}, \sqrt{7}\right]$

Q19. Consider the following statements :

P : I have fever

Q : I will not take medicine

R : I will take rest

The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to :

(A) $(P \vee Q) \wedge ((\sim P) \vee R)$
(B) $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee R)$
(C) $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee \sim R)$
(D) $(P \vee \sim Q) \wedge (P \vee \sim R)$

Q20. Let $a, b, c > 1$, a^3, b^3 and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444, then abc is equal to:

(A) 343
(B) 216
(C) $\frac{125}{8}$
(D) $\frac{343}{8}$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. If the value of real number a > 0 for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then β is equal to……

Q2. The number of seven digits odd numbers, that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is…………

Q3. Let A be the area of the region $\left\{(x,y): y \ge x^2, y \ge (1-x)^2, y \le 2x(1-x)\right\}$. Then 540 A is equal to…….

Q4. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colour is q. If $p : q = m : n$, where m and n are coprime, then $m + n$ is equal to……..

Q5. If $\int \sqrt{\sec 2x - 1}\, dx = \alpha \log_e \left|\cos 2x + \beta + \sqrt{\cos 2x\left(1 + \cos \frac{1}{\beta} x\right)}\right| + \text{constant}$, then $\beta - \alpha$ is equal to……..

Q6. Let a line L pass through the point $P(2,3,1)$ and be parallel to the line $x + 3y - 2z - 2 = 0 = x - y + 2z$. If the distance of L from the point $(5,3,8)$ is α, then $3\alpha^2$ is equal to……..

Q7. Let $A = \{1, 2, 3, 5, 8, 9\}$. Then the number of possible functions $f : A \to A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to…………

Q8. 50^{th} root of a number x is 12 and 50^{th} root of another number y is 18. Then the remainder obtained on dividing $(x + y)$ by 25 is……..

Q9. Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C\left(\sqrt{2}, \sqrt{3}\right)$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to……….

Q10. The 8^{th} common term of the series
$S_1 = 3 + 7 + 11 + 15 + 19 + ...,$
$S_2 = 1 + 6 + 11 + 16 + 21 + ...$
is…………

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KEYS to JEE (Main)-2023

PART – A (PHYSICS)

SECTION - A

1.	B	2.	B	3.	C	4.	A
5.	A	6.	A	7.	D	8.	C
9.	B	10.	D	11.	A	12.	A
13.	C	14.	C	15.	B	16.	B
17.	B	18.	C	19.	D	20.	C

SECTION – B

1.	3	2.	4	3.	125	4.	1584
5.	54	6.	12	7.	2	8.	313
9.	88	10.	300				

PART – B (CHEMISTRY)

SECTION - A

1.	B	2.	C	3.	D	4.	D
5.	A	6.	A	7.	D	8.	A
9.	C	10.	B	11.	C	12.	C
13.	B	14.	D	15.	C	16.	C
17.	B	18	C	19.	A	20.	C

SECTION - B

1.	16	2.	243	3.	1350	4.	4
5.	150	6.	6	7.	3	8.	275
9.	150	10.	3				

PART – C (MATHEMATICS)

SECTION - A

1.	D	2.	A	3.	B	4.	C
5.	B	6.	D	7.	BD	8.	C
9.	B	10.	C	11.	C	12.	B
13.	C	14.	D	15.	C	16.	D
17.	D	18	A	19.	B	20.	B

SECTION - B

1.	13	2.	240	3.	25	4.	14
5.	1	6.	158	7.	432	8.	23
9.	24	10.	151				

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Solutions to JEE (Main)-2023

PART – A (PHYSICS)

SECTION – A

Sol1. $\langle v \rangle = \frac{2v_1v_2}{v_1+v_2} = \frac{2\times3\times5}{3+5} = \frac{15}{4}\text{km/hr} = 3.75\,\text{km/hr}$

Sol2. For dispersion without deviation,

$(\mu_1 - 1)\times A_1 = (\mu_2 - 1)\times A_2$

$\Rightarrow \mu_1 = 1.54, \quad A_1 = 6^o, \mu_2 = 1.72$

So, $(1.54-1)\times A_1 = (1.72-1)\times A_2$

$\Rightarrow 0.54\times 6^o = (1.72-1)\times A_2$

$\Rightarrow A_2 = 4.5^o$

Sol3. At V_1, wavelength $= \lambda$

At V_2, wavelength $= \lambda + 50\%$ of $\lambda = \frac{3\lambda}{2}$

As we know, debroglie wavelength

$\lambda = \frac{h}{\sqrt{2meV}} \propto \frac{1}{\sqrt{V}}$

$\therefore \frac{\lambda}{\left(\frac{3\lambda}{2}\right)} = \sqrt{\frac{V_2}{V_1}} \Rightarrow \frac{2}{3} = \sqrt{\frac{V_2}{V_1}} \Rightarrow \frac{V_1}{V_2} = \frac{9}{4}$

Sol4. A – IV, B – III, C – II, D – I

Sol5. Using Gauss's Law

$\phi = \frac{Q_{enclosed}}{\varepsilon_0}$

For conductor, $Q_{enclosed} = 0$

$\therefore E = 0, \quad a \le r \le b$

$E \ne 0, \quad r < a \;\&\; r > b$

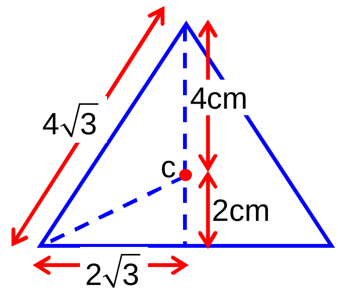
Sol6. B at centroid $=\left\{\frac{\mu_0 I}{4\pi d}\left[\sin\theta_1+\sin\theta_2\right]\right\}\times 3$

$\theta_1=\theta_2=60^o$

$d=2\,cm$

$\therefore B=\frac{\left(4\pi\times 10^{-7}\right)\times 2}{4\pi\times 2\times 10^{-2}}\left[\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right]\times 3$

$=3\sqrt{3}\times 10^{-5}\,T$



Sol7. Intensity (I) $=\frac{\text{Power}(P)}{\text{Area}(A)}$

$$I=\frac{(100W)0.05}{4\pi\times(5)^2\,m^2}=\frac{1}{20\pi}\ W/m^2$$

Sol8. For wire – A

$\frac{F}{A}=\left(\frac{\Delta\ell}{\ell}\right)Y$

$\Rightarrow \frac{F}{\pi r^2}=\frac{0.2}{\ell}\times Y \qquad$ - (i)

For wire – B

$\frac{F}{\pi(5.76)r^2}=\left(\frac{\Delta\ell}{2\ell}\right)Y \qquad$ - (ii)

$\Rightarrow$ From (i) & (ii), we get

$\frac{5.76}{1}=\frac{0.2\times 2}{\Delta\ell}$

$\Delta\ell=\frac{0.4}{5.76}=6.9\times 10^{-2}\,mm$

Sol9. From Truth table, we can say that it is NAND Gate.

Sol10. Average kinetic energy is equal to $\frac{3}{2}kT$

or, $\langle KE\rangle\propto T$

So $\frac{\langle KE\rangle_{H_2}}{\langle KE\rangle_{O_2}}=\frac{1}{1}$ or $1:1$

Sol11. Torque $=F\times r=kg\,m^2/s^2$

Energy density $=\frac{\text{Energy}}{\text{vol}}=\frac{kgm^2/s^2}{m^3}=kgm^{-1}s^{-2}$

Pressure gradient $=\frac{\Delta P}{\Delta x}=\frac{ML^{-1}T^{-2}}{L}=kgm^{-2}s^{-2}$

Impulse $=F\Delta t=kg\,m/s$ or $kg\,m\,s^{-1}$.

Sol12. $T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{k}} \qquad \rightarrow$ Time period of spring block system

$$\omega = \sqrt{\frac{k}{m}}$$

or $\omega \propto \frac{1}{\sqrt{m}}$

$$\therefore \frac{\omega_2}{\omega_1} = \frac{\sqrt{2\,kg}}{\sqrt{1kg}} = \frac{\sqrt{2}}{1}$$

Sol13. Both Reason & assertion are two & reason is the correct explanation of assertion.

Sol14. $\left(\frac{F_1}{\ell}\right) = F_{PQ}^{\ell} = \frac{\mu_0 I^2}{2\pi d} \& \left(\frac{F_2}{\ell}\right) = F_{PQ}^{2\ell} = \frac{\mu_0 (2I)^2}{2\pi d}$

$\therefore\ F_{PQ}^{\ell} : F_{PQ}^{2\ell} = 1:4$

Sol15. $I_{rms} = \frac{V_{rms}}{z} = \frac{V_{rms}}{\sqrt{R^2 + (x_C - x_L)^2}}$

$= \frac{200\sqrt{2}}{\sqrt{(100)^2 + (100)^2}}$

$= \frac{200\sqrt{2}}{100\sqrt{2}}$

$= 2$

Sol16. As $F_{ext} = 0$

So $\vec{P}_{net} = 0$

or momentum of gun = momentum of bullets

$\Rightarrow 10 \times V_{gun} = \frac{180}{60} \times \frac{20}{1000} \times 100$

$\therefore V_{gun} = 0.6\,m/s$

Sol17. Using conservation of mechanical energy :

$PE_i + KE_i = PE_f + KE_f$

$\Rightarrow \frac{-GMm}{2R} + 0 = \frac{-GMm}{R} + \frac{1}{2}mv^2$

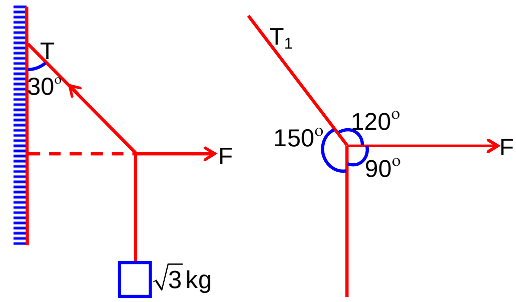
$\Rightarrow \frac{-GMm}{2R} + \frac{GMm}{R} = \frac{1}{2}mv^2$

$\Rightarrow \frac{GMm}{2R} = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

Sol18. For Translation equilibrium

$$\sum F_x = 0 \quad \& \quad \sum F_y = 0$$



For vertical

$$T_1 \cos 30^{\circ} = \sqrt{3}\,g$$

$$\Rightarrow T_1 \frac{\sqrt{3}}{2} = \sqrt{3}g$$

$$\Rightarrow T_1 = 20N$$

For Horizontal

$$T_1 \sin 30^{\circ} = F$$

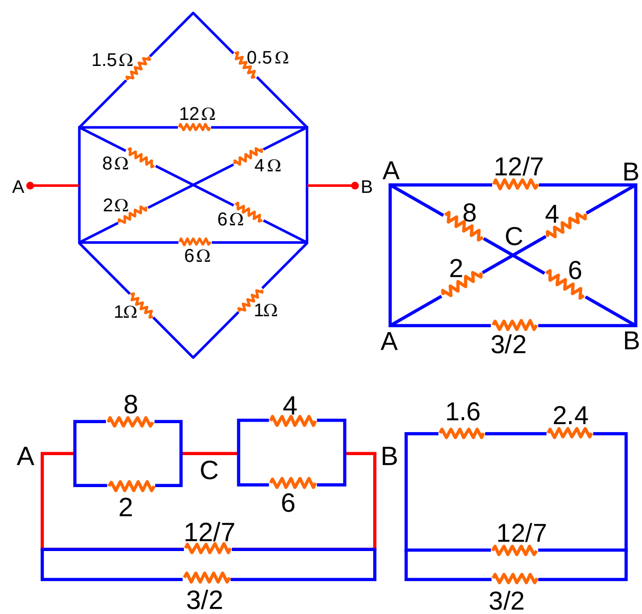
$$\Rightarrow 20 \times \frac{1}{2} = F$$

$$\Rightarrow F = 10N$$

Sol19. Assertion is false but Reason is True.

Sol20. $\frac{1}{R_{eq}} = \frac{1}{4} + \frac{7}{12} + \frac{2}{3} = \frac{3+7+8}{12} = \frac{18}{12}$

$$\therefore\ R_{eq} = \frac{12}{18}\Omega = \frac{2}{3}\Omega$$



SECTION – B

Sol1. As we know that the phase difference $(\Delta\phi)$ is related to path difference (Δx) as :

$$\Delta\phi = k\Delta x$$

So, $\quad \Delta\phi_1 = \frac{2\pi}{\lambda}.\frac{\lambda}{4} = \frac{\pi}{2}$

& $\quad \Delta\phi_2 = \frac{2\pi}{\lambda}.\frac{\lambda}{3} = \frac{2\pi}{3}$

Now resultant intensity is given as

$$I_1 = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\ \cos(\Delta\phi_1) = 2I_0$$

& $\quad I_2 = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\ \cos(\Delta\phi_2) = I_0$

$\therefore \quad \frac{I_1 + I_2}{I_0} = \frac{2I_0 + I_0}{I_0} = \frac{3}{1}$

Sol2. $P = Fv = ma\ v$

$$\Rightarrow \frac{P}{m} = v.\frac{dv}{dt}$$

$$\Rightarrow \frac{P}{m}dt = vdv$$

$$\frac{P}{m}t = \frac{v^2}{2} \Rightarrow v = \sqrt{\left(\frac{2P}{m}\right)}(t)^{1/2}$$

Also, $v = \frac{dx}{dt}$

So, $\frac{dx}{dt} = \sqrt{\frac{2P}{m}}\left(t^{1/2}\right)$

$$dx = \sqrt{\frac{2P}{m}}\int t^{1/2}dt$$

$$x = \frac{2}{3}\sqrt{\frac{2P}{m}}.t^{3/2}$$

$$x = \frac{1}{3} \times \sqrt{P}\sqrt{\frac{8}{2}} \times (4)^{3/2} = \frac{1}{3}.4^2\sqrt{P}$$

$$\therefore\ \alpha = 4$$

Sol3. $\ell = 180\text{cm} = 1.8\,\text{m}$

$\because$ In 60sec $\longrightarrow$ 28 revolution $= 28 \times 2\pi$ rad

$\therefore$ In 1sec $\longrightarrow \frac{28 \times 2\pi}{60}\frac{\text{rad}}{\text{sec}}$

or $\omega = \frac{14\pi}{15}\text{rad / sec}$

$$= \frac{14}{15} \times \frac{22}{7}$$

$$\omega = \frac{44}{15}\text{rad / sec}$$

$\therefore$ $a = \omega^2\ell$

$$= \frac{44 \times 44 \times 1.8}{225}$$

$$= \frac{1936 \times 1.8}{225}$$

$$= \frac{1936}{x}$$

$$\therefore\ x = \frac{225}{1.8} = 125$$

Sol4. As we know that

$\varepsilon_{max} = NBA\omega$

$$= (100) \times 3 \times \left(14 \times 10^{-2}\right) \times \left(\frac{360 \times 2\pi}{60}\right)$$

$$= 1584\,\text{Volt}$$

Sol5. As we know that

$f_r = m(-a)$

$\Rightarrow \mu mg = -ma$

$\Rightarrow a = -\mu g = -3 m/s^2$

& $u = 18 m/s$

So, at $t = 2 sec$, $v = 18 - 3 \times 2$

$v = 18 - 6 = 12 m/s$

Thus, $KE_{Total} = KE_{Translational} + KE_{rotational}$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{v}{r}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{v}{r}\right)^2$$

$$= \frac{3}{4}mv^2$$

$$= \frac{3}{4} \times 0.5 \times 144$$

$KE_{Total} = 54 J$

Sol6. From the diagram, we can conclude that sides of cuboid is 1m, 2m & 3m along x, y and z-axis.
As we know that

$$\text{flux}(\phi) = \vec{E} \cdot \vec{A}$$

$\phi_{x=0} = 0$ & $\phi_{x=1m} = 2(1)^2 . (2 \times 3) = 12$

$\Rightarrow \phi_{y=0} = 0$ & $\phi_{y=2m} = -4 \times 2 \times (1 \times 3) = -24$

& $\phi_{z=0} = 6 \times (1 \times 2) = 12$ & $\phi_{z=3} = 6 \times (1 \times 2) = 12$

$\therefore \phi_{Total} = (0 + 12) + (0 - 24) + (12 + 12) = 12$

Also $\phi = \dfrac{Q_{enclosed}}{\varepsilon_0}$

$$12 = \frac{Q}{\varepsilon_0} \Rightarrow Q = 12\varepsilon_0$$

$\therefore n = 12$

Sol7. If $V_B - V_D = 0$

or $V_B = V_D$

$\therefore$ Resistance is in Proportion

$$\frac{P}{Q} = \frac{R}{S}$$

$$\Rightarrow \frac{\left(\frac{6 \times 3}{6 + 3}\right)}{1 + 2} = \frac{\left(\frac{x \times 1}{x + 1}\right)}{x}$$

$$\Rightarrow \frac{2}{3} = \frac{x}{x(x + 1)}$$

$\Rightarrow 2x^2 + 2x = 3x$

$\Rightarrow 2x^2 - x = 0$

$\Rightarrow x(2x-1)=0$

$x=0, \quad x=\frac{1}{2}$

Thus $\quad x=\frac{1}{n}=\frac{1}{2}\Rightarrow n=2$

Sol8. Using Temperature relation :-

$$\frac{41^{o}C-5^{o}C}{95^{o}C-5^{o}C}=\frac{c-0^{o}C}{100^{o}C-0^{o}C}$$

$$\Rightarrow \frac{36}{90}=\frac{c}{100}$$

$$\Rightarrow x=40^{o}c$$

Thus temperature in Kelvin is $(40+273)k=313k$

Sol9. $4v^2=50-x^2$

$$v^2=\frac{50}{4}-\frac{x^2}{4}$$

$$v=\sqrt{\frac{50-x^2}{4}}$$

$$v=\frac{1}{2}\sqrt{\left(5\sqrt{2}\right)^2-x^2}$$

$$v=\omega\sqrt{A^2-x^2}$$

$\therefore \ \omega=\frac{1}{2},\qquad$ Now Time period, $T=\frac{2\pi}{\omega}$

$$T=\frac{2\pi}{1/2}=4\pi$$

$$T=4\times\frac{22}{7}=\frac{x}{7}$$

$$\therefore \ x=88$$

Sol10. In radioactive successive decay,

$$\lambda_{eff}=\lambda_1+\lambda_2$$

$$\frac{\ell n2}{T_{eff}}=\frac{\ell n2}{T_1}+\frac{\ell n2}{T_2}$$

$$\frac{1}{T_{eff}}=\frac{1}{5\times60}+\frac{1}{30}$$

$$\frac{1}{T_{eff}}=\frac{1+10}{300}$$

$$T_{eff}=\frac{300}{11}=\frac{\alpha}{11}$$

$$\therefore \ \alpha=300$$

PART – B (CHEMISTRY)

SECTION – A

Sol1. $4LiNO_3 \longrightarrow 2Li_2O + 4NO_2 + O_2$

Sol2. BOD value < 5 $\Rightarrow$ clean water

Sol3. (A) Due to large difference in B.pt of $CHCl_3$ & $C_6H_5NH_2$ these can be separated by simple distillation.
(B) Due to very small difference in B.pt of C_6H_{14} and C_5H_{12} these can be separated by fractional distillation.
(C) Mixture of aniline and water can be separated by steam distillation because aniline is steam volatile and it is insoluble in water.
(D) Generally organic compounds are water insoluble so these can be separated by using differential extraction technique.

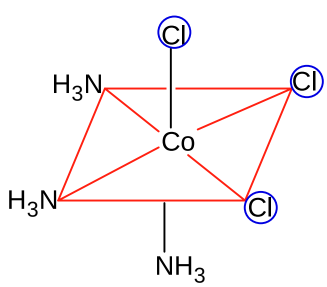
Sol4. In Boric acid strong H-bonding are present therefore it exist as solid.

Sol5. At node $\psi_{2s} = 0$

$$2 - \frac{r_0}{a_0} = 0$$

$$\Rightarrow r_0 = 2a_0$$

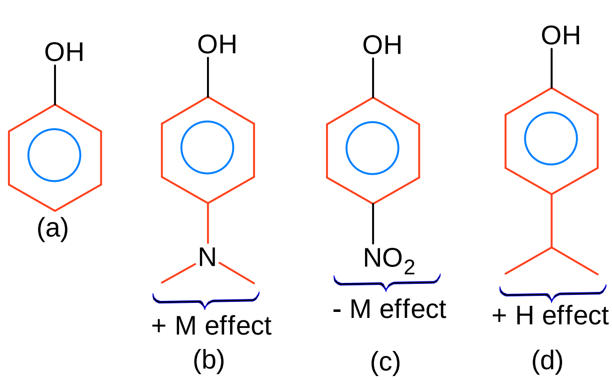
Sol6.



Cl—Co—Cl Bond angle = 90°

Sol7. Antihistamines do not affect the secretion of acid in stomach because antiallergic and aniacid drugs work on different receptors.

Sol8.



So the correct order of acidic strength is c > a > d > b

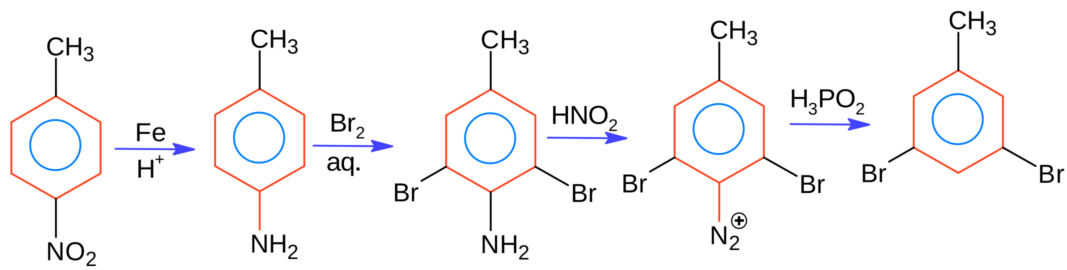
$\because$ pka $\propto \dfrac{1}{\text{acidic strength}}$

$\therefore$ correct order of pKa is b> d > a > c

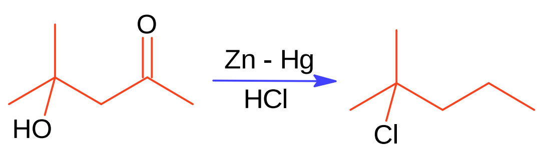
Sol9. In electrolytic refining, the pure metal is used as cathode and impure metal is used as anode. Na_3AlF_6 is added during electrolysis of Al_2O_3 to lower the melting point and increase conductivity.

Sol10. Nessler's reagent is K_2HgI_4

Sol11.



Sol12.



If acid sensitive group is present then side reaction can also occurs.

Sol13.

$$\xrightarrow{\text{H—O} \quad \text{H—S} \quad \text{H—Se} \quad \text{H—Te}}$$

$\because$ extent of overlapping decreases

$\therefore$ Bond dissociation energy also decreases.

Sol14.

	$[Ni(CO)_4]$	$[Cu(NH_3)_4]^{2+}$	$[Fe(NH_3)_6]^{2+}$	$[Fe(H_2O)_6]^{2+}$
Hybridisation	Sp^3	dsp^2	Sp^3d^2	d^2sp^3

Sol15. Maximum no of electrons in a shell = $2n^2 = 2(4)^2 = 32$

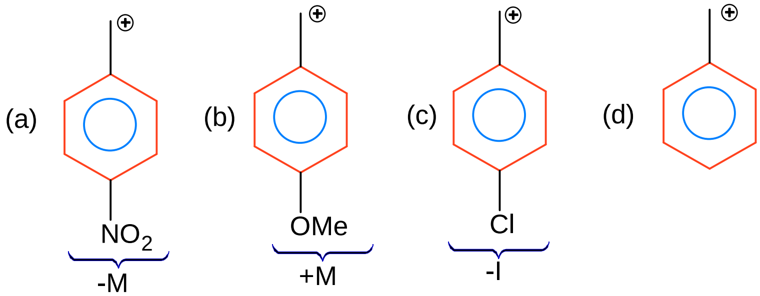
Sol16. Oxidation of $I^{\ominus}$ in acidic medium

$$2MnO_4^{\ominus} + 10\,I^{\ominus} + 16H^{\oplus} \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$$

Oxidation of $I^{\ominus}$ in neutral / faintly alkaline solution

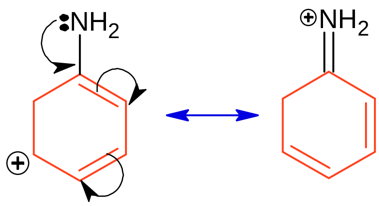
$$2MnO_4^{\ominus} + I^{\ominus} + H_2O \rightarrow 2MnO_2 + 2\overset{\ominus}{O}H + IO_3^{\ominus}$$

Sol17. Rate of SN^1 reaction $\propto$ stability of $C^{\oplus}$



Stability of $C^{\oplus} \longrightarrow b > d > c > a$

Sol18.



Most stable due to extended conjugation.

Sol19. $BeCl_2$ are covalent in nature and soluble in organic solvent.

Sol20. $\underset{0.01\,mol}{[Co(NH_3)_5\,SO_4]Br} + AgNO_3 \longrightarrow \underset{0.01\,mol}{AgBr}$

$\underset{0.01\,mol}{[Co(NH_3)_5\,Br]\,SO_4} + BaCl_2 \longrightarrow \underset{0.01\,mol}{BaSO_4}$

SECTION – B

Sol1.

$$\log\frac{x}{m} = \frac{1}{n}\log p + \log k$$

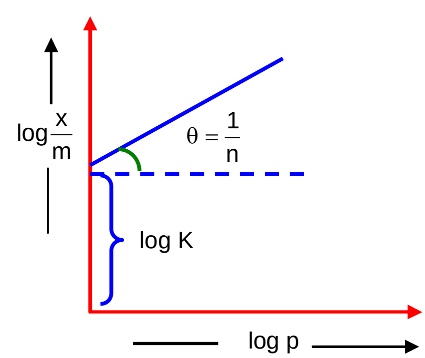
$$\frac{1}{n} = \tan 45^{o} = 1$$

$$\log k = 0.6020 \Rightarrow k = 4$$

$$\log\frac{x}{m} = \log\left(k.p^{\frac{1}{n}}\right)$$

$$\frac{x}{m} = kp^{\frac{1}{n}} \Rightarrow \frac{x}{m} = 4(0.4)^{1}$$

$$\frac{x}{m} = 1.6 = 16\times10^{-1}$$



Sol2.

$$\Delta T_f = iK_f m$$

$$i = 2.67$$

$$k_f = 1.8K\ kgmol^{-1}$$

$$m = \frac{\text{moles of solute}}{\text{mass of solvent}} = \frac{38\times1000}{98\times62}$$

$$\Delta T_f = \frac{2.67\times1.8\times38\times1000}{98\times62}$$

$\Delta T_f = 30.05^{o}C$

So, freezing point of solution = 273–30=243K

Sol3. For 1st order reaction rate constant is

$$k = \frac{1}{t}\ln\frac{[A]_0}{[A]_t} \Rightarrow t = \frac{1}{k}\ln\frac{[A]_0}{[A]_t}$$

$$\frac{t_1}{t_2} = \frac{\frac{1}{k}\ell n\frac{[A]_0}{0.4[A]_0}}{\frac{1}{k}\ell n\frac{[A]_0}{0.1[A]_0}} \Rightarrow \frac{540}{t_2} = \frac{\ell n\frac{10}{4}}{\ell n10}$$

$$\frac{540}{t_2} = \frac{\ell n10 - \ell n4}{\ell n4} \Rightarrow t_2 = 1350\,\text{sec}$$

Sol4. $d = \frac{ZM}{N_A \times a^3} \Rightarrow 4 = \frac{Z \times 72}{6.0 \times 10^{23} \times 125 \times 10^{-24}}$

$Z = 4.166 \approx 4$

Sol5. Strength = molarity $\times$ molarmass

Strength of $H_2O_2 = \frac{50}{11.35} \times 34 \approx 150$

Sol6. Number of peptide linkage = amino acid-1
= 7-1
= 6

Sol7. $2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}; \Delta H = -190\,kJ$ the yield of SO_3 can be increases by

(B) Increasing pressure
(C) Adding more SO_2
(D) Adding more O_2

Sol8. Anode $X \longrightarrow X^{2+} + 2e^-$

Cathode $Y^{2+} + 2e^- \longrightarrow Y$

$$X + Y^{2+} \longrightarrow X^{2+} + Y$$

$$E^0_{cell} = E^0_C - E^0_A$$

$$= 0.36 - (-2.36) \approx 2.72\ V$$

$$E_{cell} = E^0_{cell} - \frac{0.0591}{2}\log\frac{0.001}{0.01}$$

$$E_{cell} = 275 \times 10^{-2}\,V$$

Sol9. For reversible adiabatic process

$w = \Delta U = nC_v\Delta T$

$-3000 = 1 \times 20 \times (T_2 - 300)$

$T_2 = 150K$

Sol10. Ester will not dissolve in cold $NaHCO_3$ and NaOH solution but dissolve in hot NaOH solution.

PART – C (MATHEMATICS)

SECTION – A

Sol1. LHL

$\lim\limits_{x\to1^-} g(h(x-1))$

$= \lim\limits_{x\to1^-} g\left(2[x-1]-\frac{(x-1)}{|x-1|}\right)$

$= \lim\limits_{x\to1^-} g(-2+1)$

$= \lim\limits_{x\to1^-} g(-1)$

$= 1$

RHL

$\lim\limits_{x\to1^+} g(h(x+1))$

$= \lim\limits_{x\to1^+} g\left(2[x-1]-\frac{(x-1)}{|x-1|}\right)$

$= \lim\limits_{x\to1^+} g(0-1)$

$= \lim\limits_{x\to1^+} g(-1)$

$= 1$

Sol2. $\vec{c} = \left(2\vec{a}\times\vec{b}\right)-3\vec{b}$

take dot product with $\vec{b}$

$\vec{c}\cdot\vec{b} = 0-3\left(\vec{b}\cdot\vec{b}\right)$

$= -3|b|^2 = -48$

Sol3. $y = vx$, homogeneous equation $\frac{dy}{dx} = v+\frac{xdv}{dx}$

$\Rightarrow \frac{dx}{x} = \frac{-\left(3+v^2\right)}{(v+1)^3}dv$

$\Rightarrow \int\frac{dx}{x} = -3\int\frac{dv}{(v+1)^3}-\int\frac{v^2dv}{(v+1)^3}+c$

$\Rightarrow \ell nx = -\int\frac{3dv}{(v+1)^3}-\int\frac{dv}{v+1}+2\int\frac{dv}{(v+1)^2}-\int\frac{dv}{(v+1)^3}+c$

$\Rightarrow \ell nx = -4\int\frac{dv}{(v+1)^3}-\int\frac{dv}{v+1}+2\int\frac{dv}{(v+1)^2}+c$

$\Rightarrow \ell nx = \frac{-4}{(-2)(v+1)^2}-\ell n(v+1)\ -\frac{2}{(v+1)}+c$

$\Rightarrow \ell nx+\ell n\left(\frac{y}{x}+1\right) = \frac{-2x}{x+y}+\frac{2x^2}{(x+y)^2}+c$

$\Rightarrow \ell n(x+y) = \frac{-2x(x+y)+2x^2}{(x+y)^2}+c$

$\Rightarrow \ell n(x+y) = \frac{-2xy}{(x+y)^2+c}$

$y(1) = 0 \Rightarrow c = 0$

$\Rightarrow \ell n(x+y) = \frac{-2xy}{(x+y)^2}$

Sol4. Use $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\cdot\vec{c}\right)\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{c}$

$$\left(\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)\right)\times\left(\vec{a}-\vec{b}\right)=\left(\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)\right)\left(\vec{a}\times\vec{b}\right)-\left(\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)\right)\left(\vec{a}+\vec{b}\right)$$

$$\Rightarrow 8\hat{i}-40\hat{j}-24\hat{k}=\left(\left|\vec{a}\right|^2-\left|\vec{b}\right|^2\right)\left(\vec{a}\times\vec{b}\right)-0$$

$$\Rightarrow 8\left(\hat{i}-5\hat{j}-3\hat{k}\right)=\left(\lambda^2+4+9-\left(1+\lambda^2+4\right)\right)\left(\vec{a}\times\vec{b}\right)$$

$$\Rightarrow \hat{i}-5\hat{j}-3\hat{k}=\vec{a}\times\vec{b}=\hat{i}\left(4-3\lambda\right)-\hat{j}\left(2\lambda+3\right)+\hat{k}\left(-\lambda^2-2\right)$$

$$\Rightarrow \lambda=1$$

$$\vec{a}+\vec{b}=2\hat{i}+\hat{j}-\hat{k}$$

$$\vec{a}-\vec{b}=3\hat{j}-5\hat{k}$$

$$\left|\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right|^2=\left|\vec{a}\times\vec{a}-\vec{a}\times\vec{b}+\vec{b}\times\vec{a}-\vec{b}\times\vec{b}\right|^2$$

$$=4\left|\vec{a}\times\vec{b}\right|^2$$

$$=4\left(1+25+9\right)=140$$

Sol5. line $\dfrac{x-1}{1}=\dfrac{y-\left(-1/2\right)}{1}=\dfrac{z-\left(-1\right)}{-1}$

Let $A\left(-1,k,0\right),B\left(2,k,-1\right),C\left(1,1,2\right)$

$$\overrightarrow{CA}=-2\hat{i}+\left(k-1\right)\hat{j}-2\hat{k}$$

$$\overrightarrow{CB}=\hat{i}+\left(k-1\right)\hat{j}-3\hat{k}$$

line $\perp$ normal $\Rightarrow\begin{vmatrix}1 & 1 & -1\\ -2 & k-1 & -2\\ 1 & k-1 & -3\end{vmatrix}=0$

$$\Rightarrow k=5$$

Sol6. $\lim\limits_{n\to\infty}\dfrac{3}{n}\left(\sum\limits_{r=0}^{n-1}\left(2+\dfrac{r}{n}\right)^2\right)$ convert to continuous integral

$$3\int_0^1\left(2+x\right)^2dx$$

$$\Rightarrow 3\left.\frac{\left(2+x\right)^3}{3}\right|_0^1$$

$$=3^3-2^3=19$$

Sol7. $t_r=\tan^{-1}\left(\dfrac{1}{1+a_r\,a_{r+1}}\right)$

$$=\tan^{-1}\left(\frac{a_{r+1}-a_r}{1+a_r\cdot a_{r+1}}\right)$$

$$=\tan^{-1}\left(a_{r+1}\right)-\tan^{-1}a_r$$

Put $r=1,2,.....2021$

Sum $= \tan^{-1} a_{2022} - \tan^{-1} a_1$

$= \tan^{-1}(2022) - \tan^{-1} 1$

$= \tan^{-1}(2022) - \frac{\pi}{4}$

or $= \frac{\pi}{2} - \cot^{-1} 2022 - \frac{\pi}{4}$

$= \frac{\pi}{4} - \cot^{-1}(2022)$

Sol8. Direction cosine of vectors $\pm(\cos 60^0, \pm\cos 45^0, \cos\theta)$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \frac{1}{2}$ as θ is acute.

Equation of plane $\frac{1}{2}(x - \sqrt{2}) + \frac{1}{\sqrt{2}}(y+1) + \frac{1}{2}(z-1) = 0$

$\Rightarrow x - \sqrt{2} + \sqrt{2}y + \sqrt{2} + z - 1 = 0$

$\Rightarrow x + \sqrt{2}y + z = 1$

$\Rightarrow (a,b,c)$ lies in it $\Rightarrow a + \sqrt{2}b + c = 1$

Sol9. At extreme point slope is '0'.

So $f'(x) = 0$ and $g'(x) = 0$, will have common solution.

$f'(x) = x^2 + ax + 2b$

$g'(x) = x^2 + 2bx + a$

$x = 1$, is the common root.

Sol10. $D = P^2 - 4 \cdot \frac{5}{4}P$

$= P^2 - 5P$

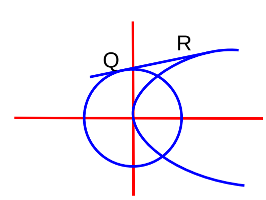
$= P(P-5)$

$P \in [0, 10]$, for D to perfect square P can be 0, 9

So, q = 9

now required area = $\int_0^9 (x-9)^2\, dx = 243$

Sol11. $y = mx + \frac{4}{m}$ is tangent to $x^2 + y^2 = 8$



$\Rightarrow 2\sqrt{2} = \left|\frac{\frac{4}{m}}{\sqrt{1+m^2}}\right| \Rightarrow m^2(1+m^2) = 2$

$m = \pm 1$

$y = x + 4$

For parabola at (x_1, y_1)

$yy_1 = 8(x + x_1)$

$\Rightarrow$ compare $(x_1, y_1) = R(4,8)$

For circle at (x_2, y_2)

$xx_2 + yy_2 = 8$

$(x_2, y_2) = Q(-2, 2)$

Sol12. a + b will be an odd number

$a + b \in \{25, 71, 117, 163\}$

$(a,b) = (2,23), (4,21),....., (2, 4, 1) \to 12$ cases

$(2,69), (4,67),....., (7, 0, 1) \to 35$ cases

$(18,99), (20,97),....., (100, 17) \to 92$ cases

$(64,99), ..., (100,63) \to 19$ cases

Sol13. use cramer's rule

$$\Delta = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0;\ \Delta_3 = \begin{vmatrix} 1 & -1 & 5 \\ 2 & 2 & 8 \\ 3 & -1 & \beta \end{vmatrix} = 0$$

$\alpha = 0, \beta = 14$

Sol14. Let $x = \left(8\sqrt{3} + 13\right)^{13} = I + f$

where $I = [x]$

$0 \le f < 1$

Let $f' = \left(8\sqrt{3} - 13\right)^{13}$

It will be a fraction

now $I + f - f' = \left(8\sqrt{3} + 13\right)^{13} - \left(8\sqrt{3} - 13\right)^{13}$

$I + f - f' = 2(\text{Natural number})$

$\Rightarrow f - f'$ must be an integer

So, $f = f' = 0$

So, $I = 2$ (Natural number)

Similarly $[y]$ is an even number

So $[x] + [y]$ is an even number

Sol15. Pattern is $a_1, a + 1, a_1 + 2, ..., a_1 + 99$

Mean deviation remains unchanged while shifting the pattern.

So, 0, 1, 2, 3,….,99

Mean = $\dfrac{99 \cdot 100}{2 \times 100} = 49 \cdot 5$

Mean deviation about mean

$$\Rightarrow \frac{2\left(\frac{1}{2} + \frac{3}{2} + ... + \frac{99}{2}\right)}{100} = \frac{50^2}{100} = 25\,.$$

It is true as given value. So, $a_i \in N$.

Sol16. $ax^2 + 2bx + cy = 0$

$y = 1, b^2 = ac$

$\Rightarrow ax^2 + 2\sqrt{ac}x + c = 0$

$\Rightarrow \left(\sqrt{a}x + \sqrt{c}\right)^2 = 0 \Rightarrow x = \dfrac{-\sqrt{c}}{\sqrt{a}}$

Now, $dx^2 + 2ex + fy = 0$

$x = -\sqrt{\dfrac{c}{a}}, y = 1$

$\Rightarrow \dfrac{d \cdot c}{a} - 2e\sqrt{\dfrac{c}{a}} + f = 0$

$\Rightarrow d \cdot c - 2 \cdot e\sqrt{ca} + fa = 0$

$\Rightarrow dc - 2eb + af = 0$

$\Rightarrow dc + af = 2eb$

$\Rightarrow \dfrac{d}{a} + \dfrac{f}{c} = \dfrac{2eb}{ac}$

$\Rightarrow \dfrac{d}{a} + \dfrac{f}{c} = \dfrac{2e}{b} \Rightarrow \dfrac{d}{a}, \dfrac{e}{b}, \dfrac{f}{c} \rightarrow AP$

Sol17. $P^T = \alpha P + (\alpha - 1)I$

$P = \alpha P^T + (\alpha - 1)I$

$\Rightarrow P = \alpha\left(\alpha P + (\alpha - 1)I\right) + (\alpha - 1)I$

$\Rightarrow \left(1 - \alpha^2\right)(P + I) = 0$

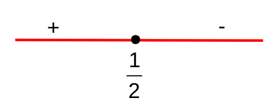
$\Rightarrow P = -I$

use $|\text{adj } A| = |A|^{n-1}$

Sol18. Domain $x \in [-2, 3]$

$f'(x) = \dfrac{1}{2\sqrt{2+x}} - \dfrac{1}{2\sqrt{3-x}} = \dfrac{\sqrt{3-x} - \sqrt{2+x}}{2\sqrt{2+x}\sqrt{3-x}}$

$f'(x)$ sign



Maxima at $x = \dfrac{1}{2}$

$f\left(\dfrac{1}{2}\right) = \sqrt{10}$

$f(-2) = \sqrt{5} = f(3)$

$f(x) \rightarrow \left[\sqrt{5}, \sqrt{10}\right]$

Sol19. $P \Rightarrow (\sim Q \wedge R)$

Use $p \Rightarrow q = \sim p \vee q$

So, $\sim P \vee (\sim Q \wedge R) = (\sim P \vee \sim Q) \wedge (\sim P \vee R)$

Sol20. $2b^3 = a^3 + c^3; \left(\frac{\log a}{\log c}\right)^2 = \frac{\log b}{\log a} \cdot \frac{\log c}{\log b}$

$\Rightarrow (\log c)^3 = (\log a)^3$

$\Rightarrow c = a$

$\Rightarrow 2b^3 = 2a^3$

$\Rightarrow a = b = c$

$T_1 = \frac{a + 4b + c}{3} = 2a;$

Common difference = $\frac{a - 8b + c}{10} = \frac{-3a}{5}$

Sum $\Rightarrow -444 = \frac{20}{2}\left(2 \cdot (2a) + (19)\left(-\frac{3a}{5}\right)\right)$

$\Rightarrow a = 6$

SECTION – B

Sol1. Let common root be 'α'

$\Rightarrow \frac{\alpha^2}{26a} = \frac{\alpha}{6} = \frac{1}{4a} \Rightarrow \alpha = \frac{13a}{13} = \frac{3}{2a}$

$\Rightarrow a^2 = \frac{9}{2 \cdot 13} \Rightarrow \frac{9}{2\beta} = \frac{9}{2 \cdot 13}$

$\Rightarrow \beta = 13$

Sol2. Pattern ....1 $\rightarrow \frac{6!}{2!3!}$ ways

.....................3 $\rightarrow \frac{6!}{3!}$ ways

.....................5 $\rightarrow \frac{6!}{3!2!}$ ways

total = $\frac{6!}{3!} \times 2 = 240$

Sol3. Plot $y = x^2$

$y = (1 - x)^2$

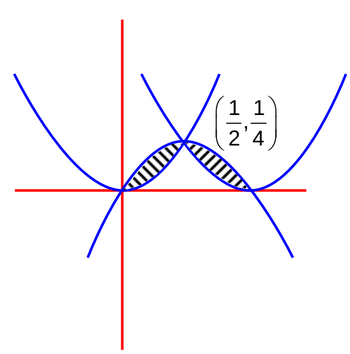
$y = -2x(x - 1)$

required region is

$2\int_0^{\frac{1}{2}} (2x - 2x^2 - x^2)\,dx$

$= 2\left[x^2 - x^3\right]_0^{\frac{1}{2}}$

$A = \frac{1}{4} \Rightarrow 540A = 135$



Sol4. $p = {}^6C_1 \cdot \left(\frac{1}{6}\cdot\frac{1}{6}\right) = \frac{1}{6}$

$q = {}^6C_2 \cdot 2!\left(\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}\right)\cdot 4$

here pattern is $aaab \to$ select ${}^6C_2 \cdot 2!$ arrange $\frac{4!}{3!} = 4$ each balls have probability $\frac{1}{6}$.

$\frac{p}{q} = \frac{1/6\cdot 6\cdot 6\cdot 6\cdot 6}{6\cdot 5\cdot 4}$

$\frac{p}{q} = \frac{9}{5} \Rightarrow m+n = 14$

Sol5. $\int\sqrt{\sec 2x - 1}\,dx = \int\sqrt{\frac{1-\cos 2x}{\cos 2x}}dx$

$= \int\frac{\sqrt{2}\sin x}{\sqrt{2\cos^2 x - 1}}\,dx$; put $\sqrt{2}\cos x = t$; $-\sqrt{2}\sin x\,dx = dt$

$\Rightarrow -\int\frac{dt}{\sqrt{t^2-1}} = -\ell n\left|t+\sqrt{t^2-1}\right| + c$

$= -\ell n\left|\sqrt{2}\cos x + \sqrt{2(\cos x)^2 - 1}\right| + c$

$= -\ell n\left|\sqrt{2}\cos x + \sqrt{\cos 2x}\right| + c$

$= -\frac{1}{2}\cdot\ell n\left|\sqrt{2}\cos x + \sqrt{\cos 2x}\right|^2 + c$

$= -\frac{1}{2}\cdot\ell n\left|2\cos^2 x + \cos 2x + 2\sqrt{2}\cos x\cdot\sqrt{\cos 2x}\right| + c$

$= -\frac{1}{2}\cdot\ell n\left|2\cos^2 x + \cos 2x + 2\sqrt{1+\cos 2x}\cdot\sqrt{\cos 2x}\right| + c$

$= -\frac{1}{2}\cdot\ell n\left|1 + 2\cos 2x + 2\sqrt{\cos 2x}\ \sqrt{1+(\cos 2x)}\right| + c$

$= -\frac{1}{2}\cdot\ell n\left|\cos 2x + \frac{1}{2} + \sqrt{\cos 2x}\sqrt{1+\cos 2x}\right| +$ constant, $\alpha = \frac{-1}{2}, \beta = \frac{1}{2}$

$\Rightarrow \beta - \alpha = 1$

Sol6. Let line be $\frac{x-2}{a} = \frac{y-3}{b} = \frac{z-1}{c}$

It will also satisfy $\Rightarrow a + 3b - 2c = 0$

$a - b + 2c = 0$

$\Rightarrow \frac{a}{-1} = \frac{b}{1} = \frac{c}{1}$

General pt on line $P(-\lambda+2, \lambda+3, \lambda+1)$

Direction ratio of $PQ(-\lambda-3, \lambda, \lambda-7)$

$PQ \perp$ line $\Rightarrow \lambda + 3 + \lambda + \lambda - 7 = 0 \Rightarrow \lambda = \frac{4}{3}$

$PQ = \sqrt{(\lambda+3)^2 + \lambda^2 + (\lambda-7)^2} = \sqrt{3\lambda^2 - 8\lambda + 58}$

$\Rightarrow \alpha = \sqrt{\frac{158}{3}}$

$\Rightarrow 3\alpha^2 = 158$

Sol7. $f(m \cdot n) = f(m) \cdot f(n)$

Let $m = 1, f(n) = f(1) \cdot f(n)$

$\Rightarrow f(n)(f(1) - 1) = 0$

$\Rightarrow f(1) = 1$

Let check for $m = n = 3$

$f(9) = (f(3))^2$

So $f(3)$ can be 1,3 $\Rightarrow f(9)$ will be decided automatically.

Now total ways = $1 \cdot 6 \cdot 2 \cdot 6 \cdot 6 \cdot 1 = 432$

Sol8. $x = 12^{50}, y = 18^{50}$

$\Rightarrow x + y = 12^{50} + 18^{50}$

$= 6^{50}\left(4^{25} + 9^{25}\right)$

$= (5+1)^{50}\left((5-1)^{25} + (10-1)^{25}\right)$

$= (25\lambda + 1)[25\mu - 1 + 25v - 1]$

or $(25\lambda + 1)[25x + 23]$

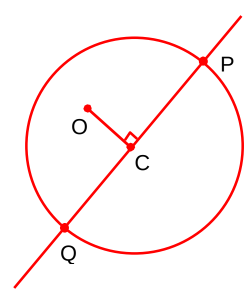
so, remainder is 23.

Sol9. PQ will be diameter $\frac{\sqrt{35}}{2} = \frac{1}{2} \cdot OC \cdot PC$

$\Rightarrow \sqrt{35} = \sqrt{5} \cdot PC \Rightarrow PC = \sqrt{7}$

$OP = OQ = \sqrt{12}$

So, $a_1^2 + b_1^2 + a_2^2 + b_2^2 = 12 + 12 = 24$



Sol10. 1st common term = 11

common difference = LCM of both 4 & 5 = 20

$t_8 = 11 + (8-1)20$

$= 11 + 140 = 151$