FIITJEE

Solutions to JEE(Main) -2023

Test Date: 31st January 2023 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A :** Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (1 – 10)*** *contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. If 1000 droplets of water of surface tension 0.07N/m, having same radius 1mm each, combine to from a single drop. In the process the released surface energy is –

(Take $\pi = \frac{22}{7}$)

(A) 8.8×10^{-5}J (B) 7.92×10^{-6}J

(C) 7.92×10^{-4}J (D) 9.68×10^{-4}J

Q2. The initial speed of a projectile fired from ground is u. At the highest point during its motion, the speed of projectile is $\frac{\sqrt{3}}{2}$u. The time of flight of the projectile is:

(A) $\frac{2u}{g}$ (B) $\frac{u}{g}$

(C) $\frac{\sqrt{3}u}{g}$ (D) $\frac{u}{2g}$

Q3. The amplitude of 15 $\sin(1000\pi t)$ is modulated by 10 $\sin(4\pi t)$ signal. The amplitude modulated signal contains frequency (ies) of

A. 500 Hz
B. 2 Hz
C. 250 Hz
D. 498 Hz
E. 502 Hz

Choose the correct answer from the options given below:

(A) A and B Only (B) B Only

(C) A Only (D) A, D and E Only

Q4. A bar magnet with a magnetic moment 5.0 Am^2 is placed in parallel position relative to a magnetic field of 0.4 T. The amount of required work done in turning the magnet from parallel to antiparallel position relative to the direction is _____.

(A) 4 J (B) 2 J

(C) 1 J (D) zero

Q5. Given bellow are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : The beam of electrons show wave nature and exhibit interference and diffraction.

Reason R : Davisson Germer Experimentally verified the wave nature of electrons.

In the light of the above statements, choose the most approprlate answer from the options given below :

(A) Both A and R are correct but R is Not the correct explanation of A
(B) Both A and R are correct and R is the correct explanation of A
(C) A is correct but R is not correct
(D) A is not correct but R is correct

Q6. At a certain depth "d" below surface of earth, value of acceleration due to gravity becomes four times that of its value at a height 3R above earth surface. Where R is Radius of earth (Take R = 6400km). The depth d is equal to

(A) 640 km
(B) 4800 km
(C) 2560 km
(D) 5260 km

Q7. Spherical insulating ball and a spherical metallic ball of same size and mass are dropped from the same height. Choose the correct statement out of the following {Assume negligible air friction }

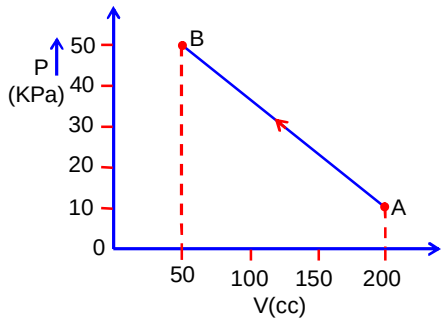
(A) Time taken by them to reach the earth's surface will be independent of the properties of their materials
(B) Metal ball will reach the earth's surface earlier than the insulating ball
(C) Both will reach the earth's surface simultaneously.
(D) Insulating ball will reach the earth's surface earlier than the metal ball

Q8. If R, X_L, and X_C represent resistance, inductive reactance and capacitive reactance. Then which of the following is dimensionless :

(A) $\frac{R}{\sqrt{X_L X_C}}$
(B) $R\frac{X_L}{X_C}$
(C) $R\ X_L\ X_C$
(D) $\frac{R}{X_L X_C}$

Q9. The pressure of a gas changes linearly with volume from A to B as shown in figure. If no heat is supplied to or extracted from the gas then change in the internal energy of the gas will be

(A) 6 J
(B) -4.5 J
(C) zero
(D) 4.5 J



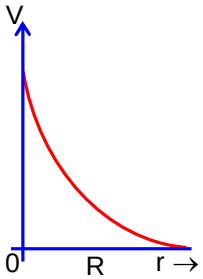
Q10. The correct relation between $\gamma = \frac{c_p}{c_v}$ and temperature T is :

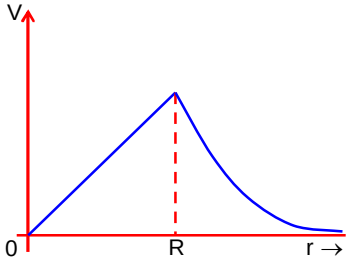
(A) $\gamma \alpha \frac{1}{\sqrt{T}}$
(B) $\gamma \alpha T$
(C) $\gamma \alpha \frac{1}{T}$
(D) $\gamma \alpha T^o$

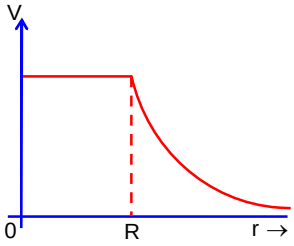
Q11. If a source of electromagnetic radiation having power 15kW produces 10^{16} photons per second, the radiation belongs to a part of spectrum is. (Take Plank constant h = 6×10^{-34} Js)

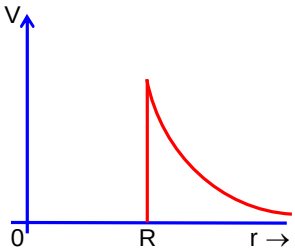
(A) Radio waves
(B) Micro waves
(C) Gamma rays
(D) Ultraviolet rays

Q12. Which of the following correctly represents the variation of electric potential (V) of a charged spherical conductor of radius (R) with radial distance (r) from the centre ?

(A) 

(B) 

(C) 

(D) 

Q13. The effect of increase in temperature on the number of electrons in conduction band (n_e) and resistance of a semiconductor will be as :

(A) n_e decreases, resistance increases
(B) Both n_e and resistance increase
(C) Both n_e and resistance decrease
(D) n_e increases, resistance decreases

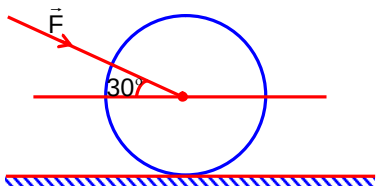
Q14. A free neutron decays into a proton but a free proton does not decay into neutron. This is because

(A) neutron is an uncharged particle
(B) proton is a charged particle
(C) neutron is a composite particle made of a proton and an electron
(D) neutron has larger rest mass than proton

Q15. Two polaroide A and B are placed in such a way that the pass-axis of polaroids are perpendicular to each other. Now, another polaroid C is placed between A and B bisecting angle between them. If intensity of upolarized light is I_0 then intensity of transmitted light after passing through polaroid B will be :

(A) $\frac{I_0}{4}$
(B) $\frac{I_0}{2}$
(C) zero
(D) $\frac{I_0}{8}$

Q16. As shown in figure, a 70kg garden roller is pushed with a force of $\vec{F} = 200\,N$ at and angle of 30° with horizontal. The normal reaction on the roller is (Given g = 10 ms^{-2})



(A) $200\sqrt{3}\,N$
(B) $800\sqrt{2}\,N$
(C) 800 N
(D) 8000 N

Q17. A rod with circular cross-section area $2cm^2$ and length 40cm is wound uniformly with 400 turns of an insulated wire. If a current of 0.4 A flows in the wire windings, the total magnetic flux produced inside windings is $4\pi \times 10^{-6}$ Wb. The relative permeability of the rod is
(Given : Permeability of vacuum $\mu_0 = 4\pi \times 10^{-7} NA^{-2}$)

(A) 12.5 (B) $\frac{32}{5}$

(C) $\frac{5}{16}$ (D) 125

Q18. The drift velocity of electrons for a conductor connected in an electrical circuit is V_d . The conductor is now replaced by another conductor with same material and same length but double the area of cross section. The applied voltage remains same. The new drift velocity of the electrons will be

(A) $2V_d$ (B) $\frac{V_d}{2}$

(C) V_d (D) $\frac{V_d}{4}$

Q19. 100 balls each of mass m moving with speed v simultaneously strike a wall normally and reflected back with same speed. In time t s. The total force exerted by the balls on the wall is

(A) $\frac{mv}{100t}$ (B) $\frac{100m\upsilon}{t}$

(C) 200 mvt (D) $\frac{200m\upsilon}{t}$

Q20. The maximum potential energy of a block executing simple harmonic motion is 25J. A is amplitude of oscillation. At $A/2$, the kinetic energy of the block is

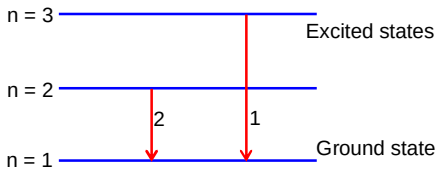
(A) 18.75 J (B) 9.75 J
(C) 12.5 J (D) 37.5 J

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

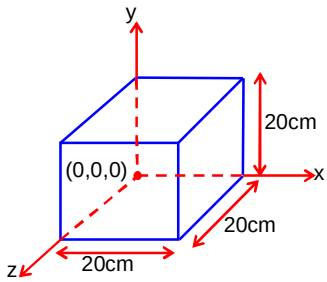
Q1. For hydrogen atom λ_1 and λ_2 are the wavelengths corresponding to the transitions 1 and 2 respectively as shown in figure. The ratio of λ_1 and λ_2 is $\frac{x}{32}$. The value of x is _____.



Q2. A lift of mass M = 500 kg is descending with speed of 2 ms^{-1}. Its supporting cable begins to slip thus allowing it to fall with a constant acceleration of $2ms^{-2}$. The kinetic energy of the lift at the end of fall through to a distance of 6m will be _____kJ.

Q3. A solid sphere of mass 1 kg rolls without slipping on a plane surface. Its kinetic energy is 7×10^{-3}J. The speed of the centre of mass of the sphere is _____cm s^{-1}.

Q4. Expression from an electric field is given by $\vec{E} = 4000x^2\hat{i}\frac{V}{m}$. The electric flux through the cube of side 20cm when placed in electric field (as shown in the figure) is V cm.



Q5. An inductor of 0.5mH, a capacitor of $20\mu F$ and resistance of 20Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the amplitude of current of the circuit is $\sqrt{x}$ A. The value of x is –

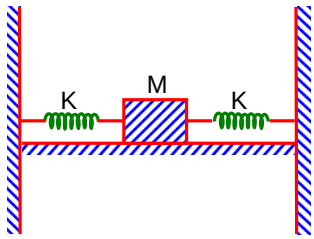
Q6. In a medium the speed of light wave decreases to 0.2 times to its speed in free space. The ratio of relative permittivity to the refractive index of the medium is x : 1. The value of x is_____.
(Given speed of light in free space = 3×10^8 ms^{-1} and for the given medium $\mu_r = 1$)

Q7. A thin rod having a length of 1m and area of cross-section $3\times10^{-6}m^2$ is suspended vertically from one end. The rod is cooled from $210^\circ C$ to $160^\circ C$. After cooling, a mass M is attached at the lower end of the rod such that the length of rod again becomes 1m. Young's modulus and coefficient of linear expansion of the rod are 2×10^{11} N m^{-2} and 2×10^{-5} K^{-1}, respectively. The value of M is _____kg.
(Take g = 10 ms^{-2})

Q8. Two identical cells, when connected either in parallel or in series gives same current in an external resistance 5Ω. The internal resistance of each cell will be _____Ω.

Q9. The speed of a swimmer is 4km h^{-1} in still water. If the swimmer makes his strokes normal to the flow of river of width 1km, he reaches a point 750m down the stream on the opposite bank. The speed of the river water is _____km h^{-1}

Q10. In the figure giver below, a block of mass M = 490g placed on a frictionless table is connected with two springs having same spring constant (K = 2 N m^{-1}). If the block is horizontally displaced through 'X' m then the number of complete oscillations it will make in 14π seconds will be_____.



PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. The correct order of basicity of oxides of vanadium is

(A) $V_2O_3 > V_2O_5 > V_2O_4$ (B) $V_2O_3 > V_2O_4 > V_2O_5$

(C) $V_2O_5 > V_2O_4 > V_2O_3$ (D) $V_2O_4 > V_2O_3 > V_2O_5$

Q2. Match List I with List II

List-I		List-II	
A.	XeF_4	I.	See-saw
B.	SF_4	II.	Square planar
C.	NH_4^+	III.	Bent T- shaped
D.	BrF_3	IV.	Tetrahedral

Choose the correct answer from the options given below:

(A) A- IV, B-I, C-II, D- III (B) A-II, B-I, C-IV, D-III

(C) A-II, B-I, C-III, D-IV (D) A-IV, B-III, C-II, D-I

Q3. Choose the correct set of reagents for the following conversion

Trans (Ph-CH=CH-CH_3) $\rightarrow$ cis(Ph-CH=CH-CH_3)

(A) Br_2.alc.KOH,$NaNH_2$,Na(LiqNH_3)

(B) Br_2, aq. KOH, $NaNH_2$,H_2 Lindlar Catalyst

(C) Br_2,aq.KOH,$NaNH_2$,Na(LiqNH_3)

(D) Br_2,alc.KOH,$NaNH_2$,H_2Lindlar Catalyst

Q4. Which of the following artificial sweeteners has the highest sweetness value in comparison to cane suagr?

(A) Aspartame (B) Saccharin

(C) Alitame (D) Sucralose

Q5. Consider the following reaction

$$\text{Propanal} + \text{Methanal} \xrightarrow[\substack{\text{(ii) } \Delta \\ \text{(iii) NaCN} \\ \text{(iv)} H_3O^+}]{\text{(i) dil NaOH}} \underset{(C_5H_8O_3)}{\text{Product}}$$

The correct statement for product B is. It is

(A) Racemic mixture and is neutral

(B) Racemic mixture and give a gas with saturated $NaHCO_3$ solution

(C) Optically active and adds one mole of bromine

(D) Optically active alcohol and is neutral

Q6. The methods NOT involved in concentration of ore are

A. Liquation

B. Leaching

C. Electrolysis

D. Hydraulic washing

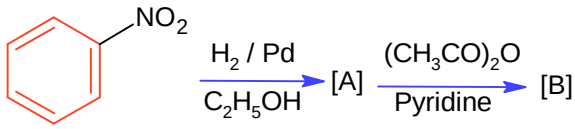
E. Froth floation

Choose the correct answer from the options given below :

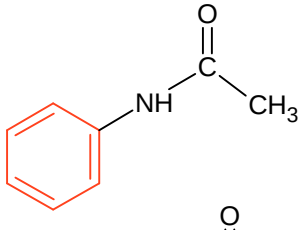
(A) A and C only (B) B, D and E only

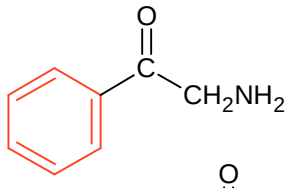
(C) B, D and C only (D) C, D and E only

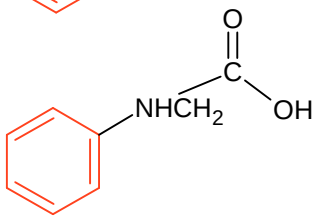
Q7.

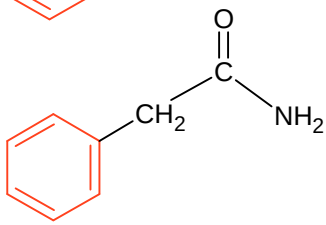


Consider the above reaction and identify the product B.

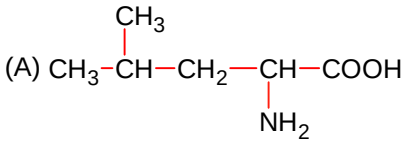
(A) 

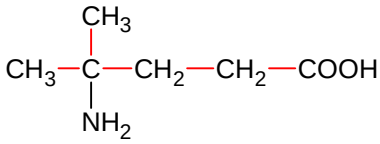
(B) 

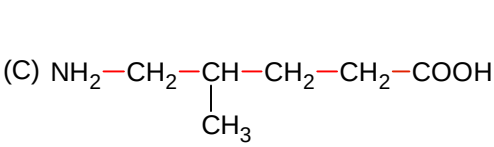
(C) 

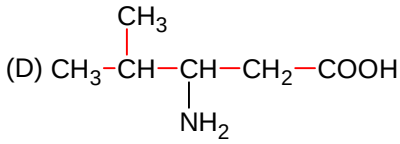
(D) 

Q8. A protein'X' with molecular weight of 70,000 u, on hydrolysis gives amino acids. One of these amino acids is

(A) 

(B) 

(C) 

(D) 

Q9. The correct increasing order of the ionic radii is

(A) $K^+ < S^{2-} < Ca^{2+} < Cl^-$

(B) $Ca^{2+} < K^+ < Cl^- < S^{2-}$

(C) $Cl^- < Ca^{2+} < K^+ < S^{2-}$

(D) $S^{2-} < Cl^- < Ca^{2+} < K^+$

Q10. Which one of the following statements is correct for electrolysis of brine solution?

(A) H_2 is formed at anode

(B) O^{2-} is formed at cathode

(C) Cl_2 is formed at cathode

(D) OH^- is formed at cathode

Q11. Identify X,Y and Z in the following reaction (Equation not balanced)

$$ClO^{\bullet} + NO_2 \rightarrow \underline{X} \xrightarrow{H_2O} \underline{Y} + \underline{Z}$$

(A) X = $ClNO_2$, Y= HCl, Z = HNO_3

(B) X = $ClNO_3$, Y= Cl_2, Z = NO_2

(C) X = $ClONO_2$, Y= HOCl, Z = NO_2

(D) X = $ClONO_2$, Y= HOCl , Z = HNO_3

Q12. Cobalt choride when dissolved in water forms pink colored complex X which has octahedral geometry. This solution on treating with conc. HCl forms deep blue complex. Y which has a Z geometry, X, Y and Z, respectively, are

(A) $X = \left[Co(H_2O)_6\right]^{2+}, Y = \left[CoCl_4\right]^{2-}, Z = \text{Tetrahedral}$

(B) $X = \left[Co(H_2O)_4 Cl_2\right]^{+}, Y = \left[CoCl_4\right]^{2-}, Z = \text{Tetrahedral}$

(C) $X = \left[Co(H_2O)_6\right]^{3-}, Y = \left[CoCl_6\right]^{3-}, Z = \text{Octahedral}$

(D) $X = \left[Co(H_2O)_6\right]^{2+}, Y = \left[CoCl_6\right]^{3-}, Z = \text{Octahedral}$

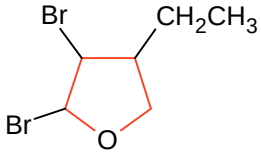
Q13. Which transition in the hydrogen spectrum would have the same wavelength as the Balmer type transition from n = 4 to n= 2 of He+ spectrum

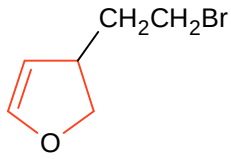
(A) n = 2 to n = 1
(B) n = 1 to n= 2
(C) n = 1 to n= 3
(D) n = 3 to n = 4

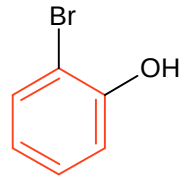
Q14. Nd^{2+} = _____

(A) $4f^4$
(B) $4f^2 6s^2$
(C) $4f^4 6s^2$
(D) $4f^3$

Q15. An organic compound 'A' with empirical formula C_6H_6O gives sooty flame on burning. Its reaction with bromine solution in low polarity solvent results in high yield of B. B is

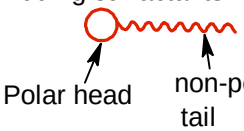
(A) 

(B) 

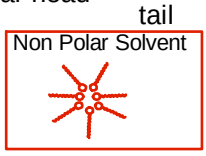
(C) 

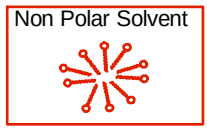
(D) 

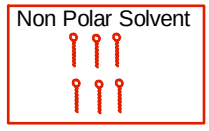
Q16. Adding surfactants in non polar solvent, the micelles structure will look like

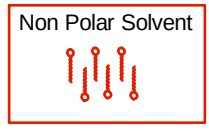


(Surfactant structure)

(a) 

(b) 

(c) 

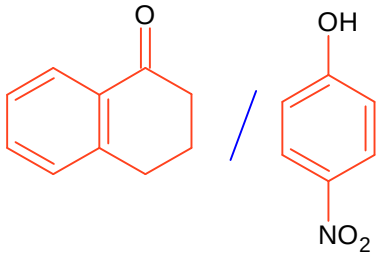
(d) 

(A) d
(B) b
(C) c
(D) a

Q17. H_2O_2 acts as a reducing agent in

(A) $2NaOCl + H_2O_2 \rightarrow 2NaCl + H_2O + O_2$
(B) $Mn^{2+} + 2H_2O_2 \rightarrow MnO_2 + 2H_2O$
(C) $2Fe^{2+} + 2H^+ + H_2O_2 \rightarrow 2Fe^{3+} + 2H_2O$
(D) $Na_2S + 4H_2O_2 \rightarrow Na_2SO_4 + 4H_2O$

Q18. Match items of column I and II

Column- I (Mixture of compounds)	**Column-II (Separation Techique)**
A. H_2O / CH_2Cl_2	i. Crystallization
B. 	ii. Differential solvent extraction
C. Kerosene / Naphthalene	iii. Column chromatography

D. $C_6H_{12}O_6$ /NaCl — iv. Fractional Distillation

Correct match is:

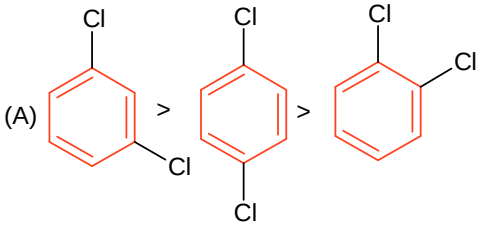
(A) A- i, B-iii, C-ii, D- iv

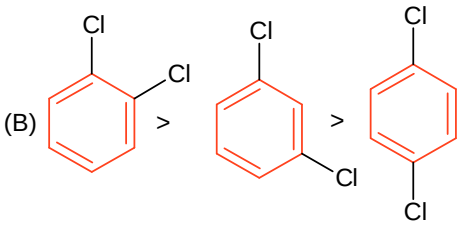
(B) A-iii, B-iv, C-ii, D-i

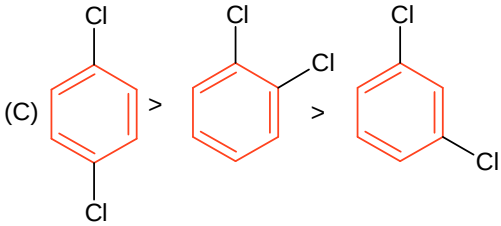
(C) A-ii, B-iii, C-iv, D-i

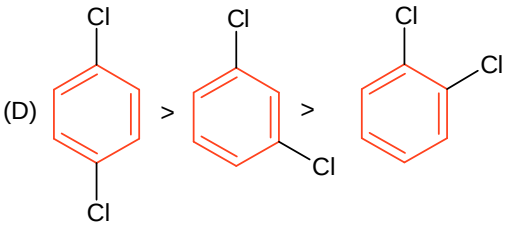
(D) A-ii, B-iv, C-i, D-iii

Q19. The correct order of melting points of dichloroenzenes is

(A) 

(B) 

(C) 

(D) 

Q20. When Cu^{2+} ion is treated with KI, a white precipitate, X appears in solution. The solution is titrated with sodium thiosulphate, the compound Y is formed. X and Y respectively are.

(A) X = Cu_2I_2 Y = $Na_2S_4O_6$

(B) X = CuI_2 Y = $Na_2S_4O_6$

(C) X = CuI_2 Y = $Na_2S_2O_3$

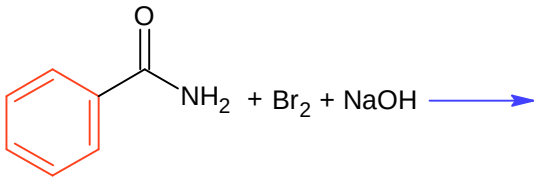
(D) X = Cu_2I_2 Y = $Na_2S_4O_5$

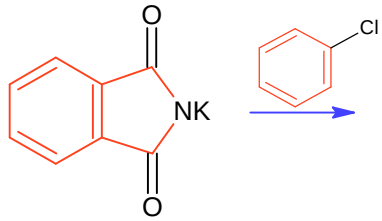
SECTION - B

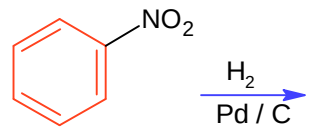
(Numerical Answer Type)

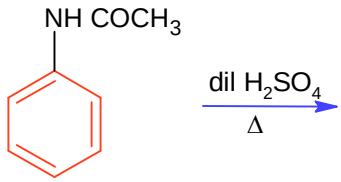
This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. How many of the transformations given below would result in aromatic amines?

(A) 

(B) 

(C) 

(D) 

Q2. On complete combustion, 0.492 g of an organic compound gave 0.792 g of CO_2. The % of carbon in the organic compound is _____(Nearest integer)

Q3. The total pressure of a mixture of non- reacting gases X (0.6g) and Y (0.45g) in a vessel is 740 mm of Hg. The partial pressure of the gas X is_____ mm of Hg. (Nearest Integer)
(Given: molar mass X = 20 and Y = 45 g mol^{-1})

Q4. The oxidation state of phosphorous in hypophosphoric acid is +_____.

Q5. For reaction: $SO_2(g)+\frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$ $K_p = 2\times10^{12}$ at 27^oC and 1 atm pressure. The K_C for the same reaction is _____$\times10^{13}$ (Nearest integer)
(Given $R = 0.082 L\ atm K^{-1} mol^{-1}$)

Q6. At 27^oC, a solution containing 2.5g of solute in 250.0 mL of solution exerts an osmotic pressure of 400 Pa. The molar mass of the solute is_____ g mol^{-1} (Nearest integer)
(Given: R = 0.083 L bar K^{-1} mol^{-1})

Q7. A $\rightarrow$B
The rate constants of the above reaction at 200 K and 300K are 0.03 min^{-1} and 0.05 min^{-1} respectively.The activation energy for the reaction is _____J (Nearest integer)

(Given : ln 10 = 2.3)
R = 8.3 JK^{-1} mol^{-1}
log 5 = 0.70
log 3 = 0.48
log 2 = 0.30)

Q8. Zinc reacts with hydrochloric acid to give hydrogen and zinc chloride. The volume of hydrogen gas produced at STP from the reaction of 11.5 g of zinc with excess HCl is _____ L (Nearest integer)
(Given: Molar mass of Zn is 65.4g mol^{-1} and Molar volume of H_2 at STP = 22.7L)

Q9. The logarithm of equilibrium constant for the reaction $Pd^{2+} + 4Cl^- \rightleftharpoons PdCl_4^{2-}$ is_____(Nearest integer)
Given: $\frac{2.303RT}{F} = 0.06V$
$Pd^{2+}_{(aq)} + 2e^- \rightleftharpoons Pd(s) \quad E^0 = 0.83V$
$PdCl_4^{2-}(aq) + 2e^- \rightleftharpoons Pd(s) + 4Cl^-(aq) \quad E^0 = 0.65V$

Q10. The enthalpy change for the conversion of $\frac{1}{2}Cl_2(g)$ to $Cl^-(aq)$ is (–)_____kJ mol^{-1} (Nearest integer)
Given: $\Delta_{dis}H^0_{Cl^-_{2(g)}} = 240 kJmol^{-1}$, $\Delta_{eg}H^0_{Cl_{(g)}} = -350 kJmol^{-1}$
$\Delta_{hyd}H^0_{Cl^-_{(g)}} = -380 kJmol^{-1}$

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. For all $z \in C$ on the curve $C_1 : |z| = 4$, let the locus of the point $z + \frac{1}{z}$ be the curve C_2. Then :

(A) the curves C_1 and C_2 intersect at 4 points

(B) the curve C_1 lies inside C_2

(C) the curves C_1 and C_2 intersect at 2 points

(D) the curve C_2 lies inside C_1

Q2. The value of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{(2+3\sin x)}{\sin x(1+\cos x)} dx$ is equal to

(A) $-2+3\sqrt{3}+\log_e \sqrt{3}$

(B) $\frac{10}{3}-\sqrt{3}+\log_e \sqrt{3}$

(C) $\frac{10}{3}-\sqrt{3}-\log_e \sqrt{3}$

(D) $\frac{7}{2}-\sqrt{3}-\log_e \sqrt{3}$

Q3. Let a differentiable function f satisfy $f(x)+\int_3^x \frac{f(t)}{t} dt = \sqrt{x+1}, x \geq 3$. Then $12f(8)$ is equal to :

(A) 34

(B) 1

(C) 19

(D) 17

Q4. If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\leq x$, is $[2,6)$, then its range is

(A) $\left(\frac{5}{37},\frac{2}{5}\right] - \left\{\frac{9}{29},\frac{27}{109},\frac{18}{89},\frac{9}{53}\right\}$

(B) $\left(\frac{5}{37},\frac{2}{5}\right]$

(C) $\left(\frac{5}{26},\frac{2}{5}\right]$

(D) $\left(\frac{5}{26},\frac{2}{5}\right] - \left\{\frac{9}{29},\frac{27}{109},\frac{18}{89},\frac{9}{53}\right\}$

Q5. Let R be a relation on $N \times N$ defined by $(a,b)R(c,d)$ if any only if $ad(b-c) = bc(a-d)$. Then R is

(A) reflexive and symmetric but not transitive

(B) transitive but neither reflexive nor symmetric

(C) symmetric and transitive but not reflexive

(D) symmetric but neither reflexive nor transitive

Q6. Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2},0\right)$ and directrix $y = -\frac{1}{2}$. Then $S = \left\{x \in R : \tan^{-1}\left(\sqrt{f(x)}\right) + \sin^{-1}\left(\sqrt{f(x)+1}\right) = \frac{\pi}{2}\right\}$:

(A) contains exactly one element
(B) is an infinite set
(C) contains exactly two elements
(D) is an empty set

Q7. Let $y = f(x) = \sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right)\right)\right)$. Then, at x = 1

(A) $2y'+3\pi^2y=0$
(B) $\sqrt{2}y'-3\pi^2y=0$
(C) $y'+3\pi^2y=0$
(D) $2y'+\sqrt{3}\pi^2y=0$

Q8. If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296, respectively, then the sum of common ratios of all such GPs is

(A) $\frac{9}{2}$
(B) 3
(C) 7
(D) 14

Q9. For the system of linear equations

$x+y+z=6$

$\alpha x+\beta y+7z=3$

$x+2y+3z=14$

which of the following is NOT true?

(A) For every point $(\alpha,\beta)\neq(7,7)$ on the line $x-2y+7=0$, the system has infinitely many solutions

(B) If $\alpha=\beta=7$, then the system has no solution

(C) There is a unique point (α,β) on the line $x+2y+18=0$ for which the system has infinitely many solutions

(D) If $\alpha=\beta$ and $\alpha\neq 7$, then the system has a unique solution

Q10. Let $\alpha\in(0,1)$ and $\beta=\log_e(1-\alpha)$. Let $P_n(x)=x+\frac{x^2}{2}+\frac{x^3}{3}+....+\frac{x^n}{n}, x\in(0,1)$. Then the integral $\int_0^{\alpha}\frac{t^{50}}{1-t}dt$ is equal to

(A) $P_{50}(\alpha)-\beta$
(B) $\beta-P_{50}(\alpha)$
(C) $\beta+P_{50}(\alpha)$
(D) $-(\beta+P_{50}(\alpha))$

Q11. If the maximum distance of normal to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1, b<2$, from the origin is 1, then the eccentricity of the ellipse is :

(A) $\frac{1}{\sqrt{2}}$
(B) $\frac{\sqrt{3}}{2}$
(C) $\frac{1}{2}$
(D) $\frac{\sqrt{3}}{4}$

Q12. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is

(A) $\frac{2}{7}$
(B) $\frac{5}{7}$
(C) $\frac{5}{6}$
(D) $\frac{3}{7}$

Q13. If $\sin^{-1}\frac{\alpha}{17}+\cos^{-1}\frac{4}{5}-\tan^{-1}\frac{77}{36}=0, 0<\alpha<13$, then $\sin^{-1}(\sin\alpha)+\cos^{-1}(\cos\alpha)$ is equal to

(A) π (B) $16-5\pi$
(C) 16 (D) 0

Q14. A wire of length 20 m is to be cut into two pieces. A piece of length ℓ_1 is bent to make a square of area A_1 and the other piece of length ℓ_2 is made into a circle of area A_2. If $2A_1+3A_2$ is minimum then $(\pi\ell_1):\ell_2$ is equal to :

(A) 3 : 1 (B) 4 : 1
(C) 1 : 6 (D) 6 : 1

Q15. Let $A=\begin{pmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A+I)^{11}$ is equal to :

(A) 4097 (B) 4094
(C) 2050 (D) 6144

Q16. Let a circle C_1 be obtained on rolling the circle $x^2+y^2-4x-6y+11=0$ upwards 4 units on the tangent T to it at the point $(3,2)$. Let C_2 be the image of C_1 in T. Let A and B be the centres of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x-axis. Then the area of the trapezium AMNB is:

(A) $4(1+\sqrt{2})$ (B) $3+2\sqrt{2}$
(C) $2(1+\sqrt{2})$ (D) $2(2+\sqrt{2})$

Q17. Let $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that $|\vec{a}+\vec{b}+\vec{c}|=|\vec{a}+\vec{b}-\vec{c}|$ and $\vec{b}\cdot\vec{c}=0$.

Consider the following two statements :

(i) $|\vec{a}+\lambda\vec{c}|\geq|\vec{a}|$ for all $\lambda\in R$

(ii) $\vec{a}$ and $\vec{c}$ are always parallel.

Then,

(A) both (i) and (ii) are correct (B) only (ii) is correct
(C) neither (i) nor (ii) is correct (D) only (i) is correct

Q18. $(S1)(p\Rightarrow q)\vee(p\wedge(\sim q))$ is a tautology

$(S2)((\sim p)\Rightarrow(\sim q))\wedge((\sim p)\vee q)$ is a contradiction. Then

(A) only (S1) is correct (B) both (S1) and (S2) are correct
(C) both (S1) and (S2) are wrong (D) only (S2) is correct

Q19. The number of real roots of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$, is :

(A) 0 (B) 3
(C) 1 (D) 2

Q20. Let the shortest distance between the lines $L:\frac{x-5}{-2}=\frac{y-\lambda}{0}=\frac{z+\lambda}{1}, \lambda\geq 0$ and $L_1: x+1=y-1=4-z$ be $2\sqrt{6}$. If (α,β,γ) lies on L, then which of the following is NOT possible?

(A) $\alpha+2\gamma=24$ (B) $2\alpha-\gamma=9$
(C) $\alpha-2\gamma=19$ (D) $2\alpha+\gamma=7$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest inte ger value.

Q1. If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

is 3, then α is equal to…….

Q2. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11, is equal to…….

Q3. Let for $x \in R$, $f(x) = \frac{x+|x|}{2}$ and $g(x) = \begin{cases} x, & x < 0 \\ x^2, & x \geq 0 \end{cases}$. Then area bounded by the curve $y = (fog)(x)$ and the lines $y = 0, 2y - x = 15$ is equal to…….

Q4. Let 5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is……..

Q5. Let the line $L: \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point $R(1, -1, -3)$ on the line L. If α is the area of triangle PQR, then α^2 is equal to……..

Q6. Let θ be the angle between the planes $P_1 : \vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 9$ and $P_2 : \vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 15$. Let L be the line that meets P_2 at the point $(4, -2, 5)$ and makes an angle θ with the normal of P_2. If α is the angle between L and P_2, then $(\tan^2\theta)(\cot^2\alpha)$ is equal to…….

Q7. Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a term $\beta x^{-\alpha}, \beta \in N$. Then α is equal to…….

Q8. The remainder on dividing 5^{99} by 11 is…….

Q9. Let $a_1, a_2, ..., a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then $12\left(\frac{1}{\sqrt{a_{10}} + \sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}} + \sqrt{a_{12}}} + ... + \frac{1}{\sqrt{a_{17}} + \sqrt{a_{18}}}\right)$ is equal to…….

Q10. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$. Then $(\vec{a} \cdot \vec{b})^2$ is equal to……..

FIITJEE

KEYS to JEE (Main)-2023

PART – A (PHYSICS)

SECTION - A

1.	C	2.	B	3.	D	4.	A
5.	B	6.	B	7.	D	8.	A
9.	D	10.	D	11.	C	12.	C
13.	D	14.	D	15.	D	16.	C
17.	C	18.	C	19.	D	20.	A

SECTION – B

1.	27	2.	7	3.	10	4.	640
5.	242	6.	5	7.	60	8.	5
9.	3	10.	20				

PART – B (CHEMISTRY)

SECTION - A

1.	B	2.	B	3.	D	4.	C
5.	B	6.	A	7.	A	8.	A
9.	B	10.	D	11.	D	12.	A
13.	A	14.	A	15.	D	16.	D
17.	A	18	C	19.	C	20.	A

SECTION - B

1.	3	2.	44	3.	555	4.	4
5.	1	6.	62250	7.	2520	8.	4
9.	6	10.	610				

PART – C (MATHEMATICS)

SECTION - A

1.	A	2.	B	3.	D	4.	B
5.	D	6.	C	7.	A	8.	C
9.	A	10.	D	11.	B	12.	B
13.	A	14.	D	15.	A	16.	A
17.	D	18	A	19.	C	20.	A

SECTION - B

1.	5	2.	710	3.	72	4.	2997
5.	180	6.	9	7.	2	8.	9
9.	8	10.	36				

FIITJEE

Solutions to JEE (Main)-2023

PART – A (PHYSICS)

SECTION – A

Sol1. Using conservation of Volume of the system

$$\frac{4}{3}\pi r^3 \times 1000 = \frac{4}{3}\pi R^3 \Rightarrow R = 10r$$

Energy Released = $\Delta U = U_i - U_f = N4\pi r^2\sigma - 4\pi R^2\sigma$

$$\Rightarrow \Delta U = \sigma \times 4\pi \times 90r^2 = 0.07 \times 4 \times \frac{22}{7} \times 90 \times \left(10^{-3}\right)^2 = 4 \times 22 \times 90 \times 10^{-8}$$

$$\Rightarrow \Delta U = 7.92 \times 10^{-5}\ J$$

Sol2. $4\cos\theta = \frac{\sqrt{3}}{2}u \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^o$

$$T = \frac{2u\sin\theta}{g} = \frac{2u\frac{1}{2}}{g} = \frac{u}{g}$$

Sol3. C = 15sin (1000 π t)
M = 10sin (4 π t)

$f_m = \frac{4\pi}{2\pi} = 2$

Sol4. $W = 2\ MB = 2 \times 5 \times \frac{4}{10} = 4J$

Sol5. Basic Concept

Sol6. According to question, we can write

$$g' = g\left(1 - \frac{d}{R}\right) = \frac{g}{4} \text{ and } g' = g\left(\frac{1}{1+\frac{h}{R}}\right)^2 = \frac{g}{16}$$

$$\Rightarrow \frac{3}{4} = \frac{d}{R} \Rightarrow d = \frac{3}{4}R$$

Sol7. $T = \sqrt{\frac{2h}{g_{eff}}}$

$g_{eff} \downarrow \ T \uparrow$

Sol8. Basic Concept of dimension

Sol9. $Q = 0 = W + U$

$W = -\Delta U \Rightarrow$ Area under the curve

$\frac{1}{2}(10+50)\times 150 = \Delta U$

$\Delta U = 4.5\,J$

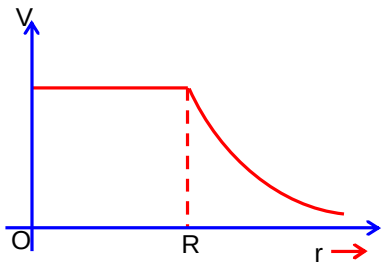
Sol10. $\gamma = \frac{C_P}{C_V} \Rightarrow \gamma\ \alpha\ T^0$

Sol11. $P = 15\,kW = nh\nu$

$15\times 10^3 = 10^{16}\times 6\times 10^{-34}\times V$

$\nu = 2.5\times 10^{21}\,Hz \Rightarrow$ Gamma rays

Sol12.



Sol13. $T\uparrow \Rightarrow$ number of electron(n_e) in CB $\uparrow$ and resistance decreases

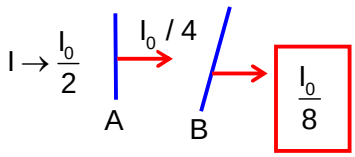
Sol14. Neutron has larges rest mass than proton

Sol15. $\theta = 45^o$

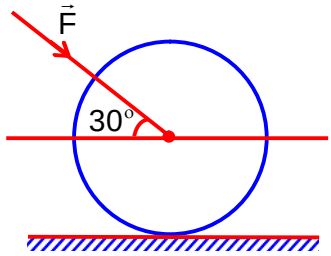
$I = I_0\cos^2\theta = \frac{I_0}{2}$

On passing through B,

$I' = \frac{I_0}{8}$



Sol16. $N = F\sin\theta + mg = 200\times\frac{1}{2} + 70\times 10 = 800\ N$



Sol17. $A = 2\,cm^2 \quad \ell = 40\,cm \quad n = 400 \quad I = 0.4A$

$|\phi| = BA\cos\theta = (\mu_0 nI)A$

$4\pi\times 10^{-6}\mu_r\, 4\pi\times 10^{-7}\times\frac{400}{40\times 10^{-2}}\times\frac{4}{10}\times 2\times 10^{-4} \Rightarrow \mu_r = 125$

Sol18. $v_d = \left(\frac{eE}{m}\right)\tau$

V_d is independent of Area

Sol19. $F = \frac{2mv}{t} \times 100 = \frac{200mv}{t}$

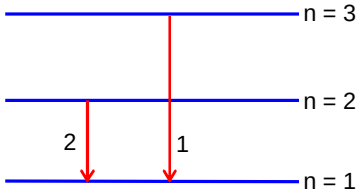
Sol20. $\frac{1}{2}kA^2 = 25J$

at $x = \frac{A}{2}$

$K = \frac{1}{2}k\left(A^2 - x^2\right) = \frac{1}{2}k\left(\frac{3}{4}A^2\right) = \frac{3}{4} \times 25 = 18.75$

SECTION – B

Sol1. $\frac{1}{\lambda_1} = R\left(1 - \frac{1}{9}\right) \Rightarrow \lambda_1 = \frac{9}{8R}$



$\lambda_2 = \frac{4}{3R}$

$\frac{\lambda_1}{\lambda_2} = \frac{9 \times 3}{8 \times 4} = \frac{27}{32} \Rightarrow n = 27$

Sol2. $M = 500kg$ and $u = 2m/s$

$v^2 = u^2 + 2as = 2^2 + 2(2)6 = 28$

$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 500 \times 28 = 7000J$

Sol3. $K = \frac{1}{2}mv_{com}^2 + \frac{1}{2}I\omega^2 \Rightarrow 7 \times 10^{-3} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mv^2\right) \Rightarrow 7 \times 10^{-3} = \frac{7}{10} \times 1 \times v^2$

$\Rightarrow 10^{-2} = v^2 \Rightarrow v = 10cm/s$

Sol4. $\vec{E} = 4000x^2$

$\phi = 4000\left(20 \times 10^{-2}\right)^2 \times 400 \times 10^{-4} = 4000 \times 400 \times 10^{-4} \times 400 \times 10^{-4} = 640$ Vcm

Sol5. $L = 0.5 \times 10^{-3}H$, $V = 220V$, $C = 20 \times 10^{-6}F$ and $R = 20\Omega$

$I = \frac{V}{R} = \frac{220}{20}$

Amplitude of current = $I\sqrt{2} = \sqrt{242}\,A \Rightarrow x = 242$

Sol6. $v = \frac{c}{n} \Rightarrow 0.2c = \frac{c}{n} \Rightarrow n = 5$

$n = \sqrt{\mu_r \varepsilon_r} \Rightarrow \varepsilon_r = 25 \Rightarrow \frac{\varepsilon_r}{n} = 5:1$

Sol7. $\ell = 1\text{m}$, $A = 3\times10^{-6}\,\text{m}^2$ and $\Delta T = 50\text{K}$

$$Y = \frac{F\ell}{A\Delta\ell}$$

$$L = L_0(1+\alpha\Delta T) \Rightarrow \frac{L-L_0}{L_0} = \alpha\Delta T \Rightarrow \frac{L-1}{1} = 2\times10^{-5}\times(-50)$$

$$\Rightarrow L = 1-10^{-3} = 1-10^{-3}\,\text{m}$$

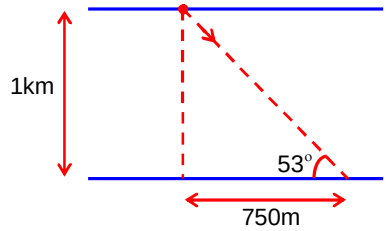
$$2\times10^{11} = \frac{Mg\times1m}{3\times10^{-6}\times10^{-3}} \Rightarrow 2\times10^{11} = \frac{M\times10}{3\times10^{-9}}$$

$$\Rightarrow 6\times10 = M \Rightarrow M = 60\text{kg}$$

Sol8. $\dfrac{E}{\frac{r}{2}+R} = \dfrac{2E}{2r+R}$

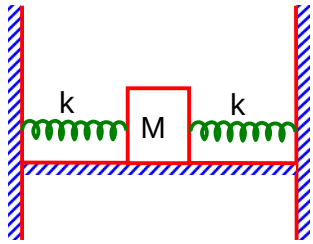
$$\Rightarrow 2Er + ER = \frac{2rE}{2} + 2ER \Rightarrow Er = ER \Rightarrow r = R = 5\Omega$$

Sol9. $\dfrac{v_{man}}{v_{riv}} = \dfrac{4}{3} \Rightarrow v_{riv} = 3\text{km/h}$



Sol10. $T = 2\pi\sqrt{\dfrac{m}{2k}} = 2\pi\sqrt{\dfrac{490\times10^{-3}}{2\times2}} = \dfrac{2\pi}{2}\times7\times10^{-1}$

Number of oscillations $= \dfrac{7}{10}\pi = \dfrac{14\pi}{7\pi}\times10 = 20$

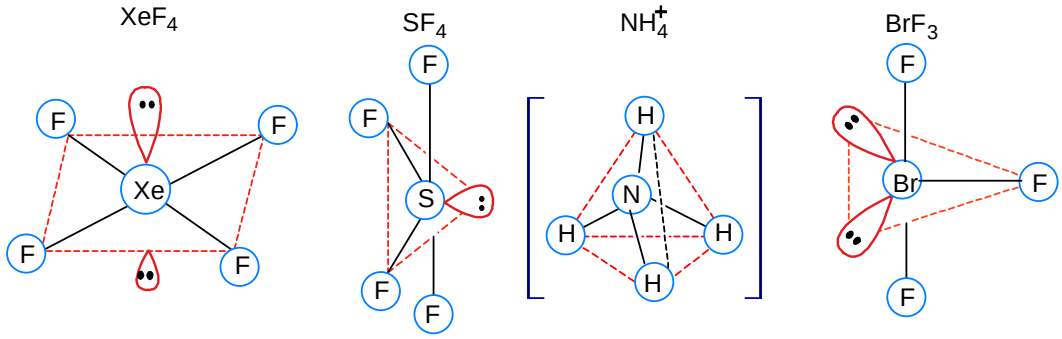


PART – B (CHEMISTRY)

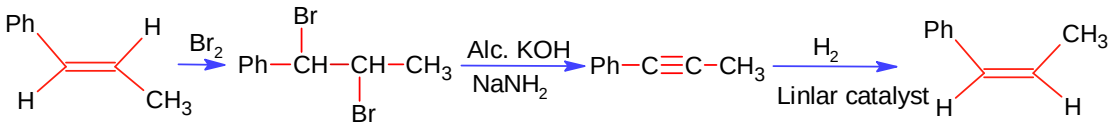
SECTION – A

Sol1. Basic nature of oxides $\propto \dfrac{1}{\text{oxidation state of central atom}}$

Sol2.

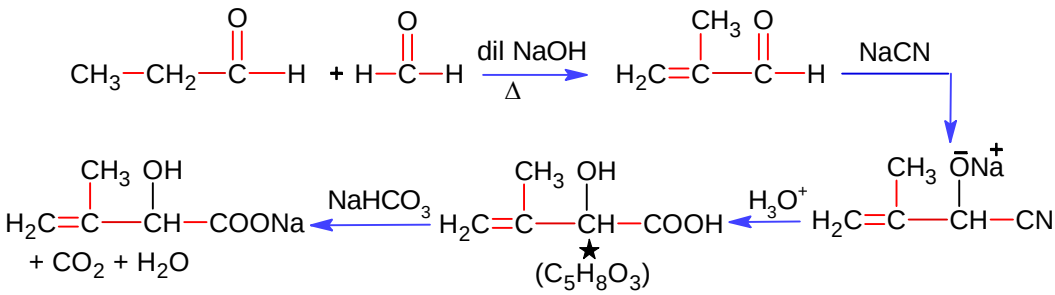


Sol3.



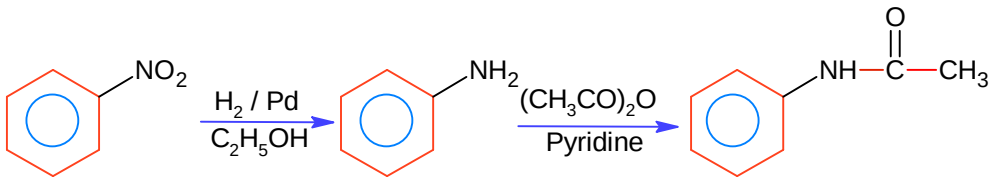
Sol4. Sweetness value order with respect to cane sugar is
Alitame > Sucralose > Saccharin > Aspartame

Sol5.

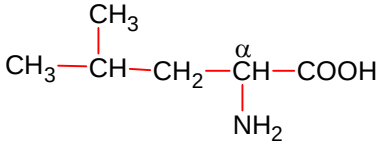


Sol6. Methods involving in concentration of ore are-
froth flotation, Hydraulic washing, Leaching, electromagnetic separation.

Sol7.



Sol8.



Remaining options are not α-amino acid.

Sol9. For isoelectronic species

Ionic radii $\propto$ -ve charge $\propto \dfrac{1}{+\text{ve charge}}$

Sol10. Brine solution (aq NaCl)

or

$NaCl + H_2O$

$\Rightarrow$ Cation – Na^+ & H^+

Anion – $OH^{\ominus}$ & $Cl^{\ominus}$

At anode - $2Cl^{\ominus}_{(aq)} \longrightarrow Cl_{2(g)} + 2e^-$

At cathode - $2H_2O_{(\ell)} + 2e^- \longrightarrow H_{2(g)} + 2OH^{\ominus}_{(aq)}$

Sol11. $ClO^{\bullet} + NO_2 \longrightarrow ClONO_2 \xrightarrow{+H_2O} HOCl + HNO_3$

Sol12. $CoCl_4 + 6H_2O \longrightarrow \left[Co(H_2O)_6\right]Cl_2 \xrightarrow{\text{Conc HCl}} \left[CoCl_4\right]^{2-}$

Sol13. $\lambda_H = \lambda_{He^+}$

$$R(1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right] = R(2)^2\left[\frac{1}{2^2}-\frac{1}{4^2}\right]$$

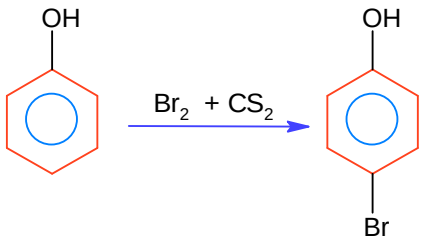
$$\frac{1}{n_1^2}-\frac{1}{n_2^2} = \frac{1}{2^2}-\frac{1}{4^2}$$

On comparing $n_1 = 1$ & $n_2 = 2$

Sol14. ${}_{60}Nd = [Xe]4f^4\ 5d^0 6s^2$

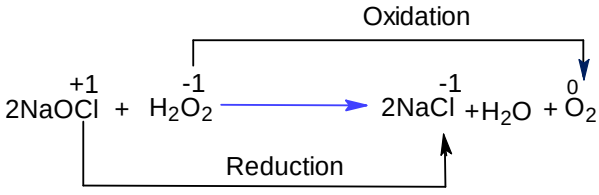
${}_{60}Nd^{2+} = [Xe]4f^4\ 5d^0 6s^0$

Sol15.



Sol16. Polar part of surfactant directed towards polar particle.

Sol17.

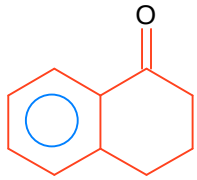


NaOCl $\rightarrow$ oxidising agent & $H_2O_2 \longrightarrow$ Redusing agent

Sol18. (A) Since $d_{CH_2Cl_2} > d_{H_2O}$ therefore they can be separated by differential solvent extraction.

(B) Intermolecular Hydrogenbonding is present in

So it can be separated from



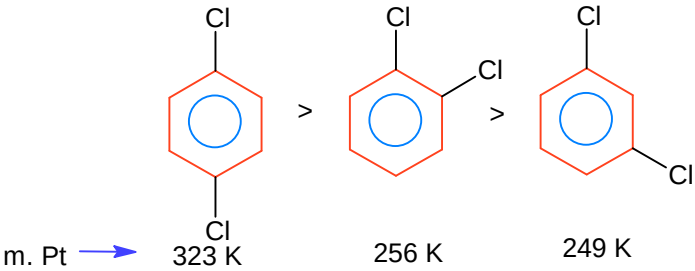
by

Column chromatography.

(C) Due to difference in boiling point of kerosene and Naphthalene, it can be separated by fractional distillation

(D) Since NaCl is an ionic compound so they can be separated from $C_6H_{12}O_6$ by crystallization.

Sol19.



Sol20. $Cu^{2+} + 2KI \longrightarrow CuI_2 \downarrow + 2K^+$

CuI_2 is unstable.

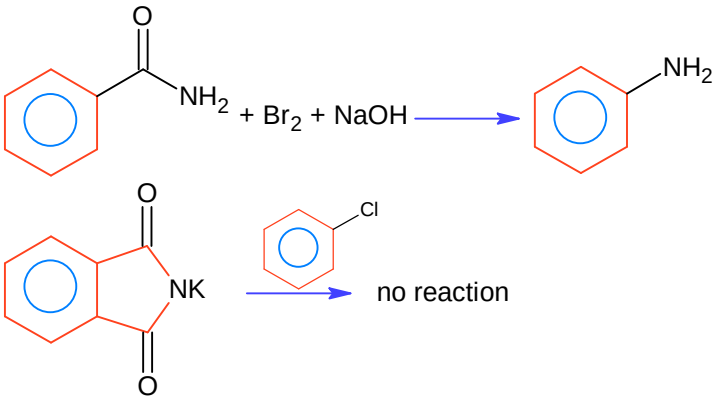
$2CuI_2 \longrightarrow \underset{(x)}{Cu_2I_2} + I_2$

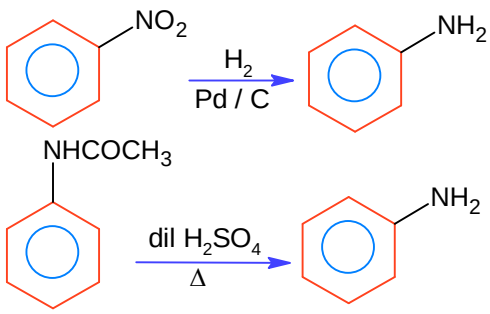
$KI + I_2 \longrightarrow KI_3$

$KI_3 + Na_2S_2O_3 \longrightarrow KI + \underset{(Y)}{Na_2S_4O_6}$

SECTION – B

Sol1.





Sol2. Mass of carbon in $0.792\,g$ of $CO_2 = \frac{12\times 0.792}{44}$

$= 0.216$

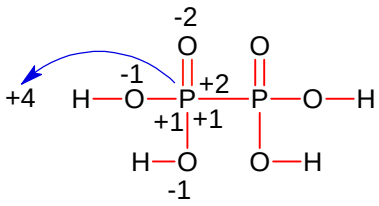
% of 'C' in 0.492 g of organic compound $= \frac{0.216\times 100}{0.492}$

$= 43.90\%$

Sol3. $P_x = X_x P_T$

$$P_x = \frac{\frac{0.6}{20}}{\frac{0.6}{20}+\frac{0.45}{45}}\times 740 = 555 \text{ mm Hg.}$$

Sol4.



Hypophosphoric acid

Sol5. $SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$

As we know that $K_p = K_C(RT)^{\Delta ng}$

$2\times 10^{12} = K_C(0.0821\times 300)^{-1/2}$

$K_C = 9.92\times 10^{12}$

or

$K_C = 0.992\times 10^{13}$

$K_C \approx 1$

Sol6. Osmotic pressure (π) = CRT

Let 'x' is the molar mass of the solute-

$$400\times 9.869\times 10^{-6}\text{atm} = \frac{2.5g\times 1000}{x\times 250(L)}\times 0.82 \text{ atm Lmol}^{-1}\text{ K}^{-1}\times 300K$$

$x = 62250$

Sol7. $\log\frac{K_2}{K_1}=\frac{E_a}{2.303R}\left\{\frac{1}{T_1}-\frac{1}{T_2}\right\}$

$\log\frac{0.05}{0.03}=\frac{E_a}{2.303\times 8.314}\left\{\frac{1}{200}-\frac{1}{300}\right\}$

$E_a=2519.88\,J\approx 2520\,J$

Sol8. $\underset{11.5g}{Zn}+2HCl\longrightarrow ZnCl_2+H_2\uparrow$

Moles of $Zn=\frac{11.5\,g}{65.4\,g\,mol^{-1}}=0.178\,mol=$moles of $H_{2(g)}$

Volume of $H_{2(g)}=0.178\times 22.7=3.99\,L$

Sol9. $Pd^{2+}_{(aq)}+2e^-\rightleftharpoons Pd_{(s)}$; $E^0=0.83V$

$PdCl^{2-}_{4(aq)}+2e^-\rightleftharpoons Pd_{(s)}+4Cl^{\Theta}_{(aq)}$; $E^0=0.65\,V$

Net cell reaction $Pd^{2+}_{(aq)}+4Cl^{\Theta}_{(aq)}\rightleftharpoons PdCl^{2-}_{4(aq)}$

$n=2$

$E^0_{cell}=E^0_c-E^0_A$

$=0.83-0.65$

$=0.18\,V$

$E^0_{cell}=\frac{0.0591}{n}\log K_{eq}$

$0.18=\frac{0.0591}{2}\log K_{eq}$

$\log k_{eq}=6.09$

Sol10. $\frac{1}{2}Cl_{2(g)}\longrightarrow Cl_{(g)}$; $\frac{240}{2}KJ$

$Cl_{(g)}+1e^-\longrightarrow Cl^{\Theta}_{(g)}$; $-350KJ$

$Cl^{\Theta}_{(g)}\longrightarrow Cl^{\Theta}_{(aq)}$; $-380kJ$

$\Delta H^0=\left(\frac{240}{2}-350-380\right)KJ/mol$

$\Delta H^0=-610KJ/mol$

PART – C (MATHEMATICS)

SECTION – A

Sol1. Let $z = 4e^{i\theta}$, $\therefore w = z + \frac{1}{z} = \frac{17}{4}\cos\theta + i\frac{15}{4}\sin\theta$

$\therefore$ locus of w is $\frac{x^2}{(17/4)^2} + \frac{y^2}{(15/4)^2} = 1$

which intersects the circle $x^2 + y^2 = 16$ at 4 points

Sol2. $I = \int_{\pi/3}^{\pi/2} \frac{2+3\sin x}{\sin x(1+\cos x)}dx = 2\int_{\pi/3}^{\pi/2} \frac{1}{\sin x(1+\cos x)}dx + 3\int_{\pi/3}^{\pi/2} \operatorname{cosec}^2 x - \operatorname{cosec} x\cot x\,dx$

$= 2\cdot\frac{1}{2}\int_{\pi/3}^{\pi/2} \frac{(1+\tan^2 x/2)\sec^2 x/2}{2\tan^2 x/2}dx + 3\int_{\pi/3}^{\pi/2} \operatorname{cosec}^2 x - \operatorname{cosec} x\cot x\,dx$

$= \int_{1/\sqrt{3}}^{1} \frac{1+t^2}{t}dt + 3\left[\operatorname{cosec} x - \cot x\right]_{\pi/3}^{\pi/2}$ (Let $\tan\frac{x}{2} = t$)

$= \frac{10}{3} + \ell n\sqrt{3} - \sqrt{3}$

Sol3. Differentiate w.r.t x to get,

$f'(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}$

IF $= e^{\int 1/x\,dx} = x$

$\therefore xf(x) = \int \frac{x}{2\sqrt{x+1}}dx$

$= \frac{(x+1)^{3/2}}{3} - \sqrt{x+1} + c$ ……..(i)

Also, $f(3) = 2$

From equation (i), $C = \frac{16}{3}$

$\therefore f(x) = \frac{\frac{(x+1)^{3/2}}{3} - \sqrt{x+1} + \frac{16}{3}}{x}$

$\therefore 12f(8) = 17$

Sol4. $f(x) = \begin{cases} \frac{2}{1+x^2}, & x \in [2,3) \\ \frac{3}{1+x^2}, & x \in [3,4) \\ \frac{4}{1+x^2}, & x \in [4,5) \\ \frac{5}{1+x^2}, & x \in [5,6) \end{cases}$

Sol5. $(a,b)R(c,d) \Rightarrow ad(b-c)=bc(a-d)$

$\Rightarrow cb(d-a)=da(c-b)$

$\Rightarrow (c,d)R(a,b)$. Hence symmetric

Not reflexive : $ab(b-a)\neq ba(a-b)$

Not transitive : $(2,3)R(3,2)\,\&\,(3,2)R(5,30)$

but $(2,3)\not R(5,30)$

Sol6. $\left(x+\frac{1}{2}\right)^2=\left(y+\frac{1}{4}\right)\Rightarrow y=x^2+x$

$\tan^{-1}\sqrt{x^2+x}+\sin^{-1}\sqrt{x^2+x+1}=\pi/2$

$0\le x^2+x+1\le 1$ (Domain of arcsin)

$\Rightarrow x^2+x\le 0$

Also, $x^2+x\ge 0$ (Domain of $\sqrt{g(x)}$)

$\therefore x^2+x=0\Rightarrow x=0,-1$

Sol7. Let $h(x)=\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}$

$\therefore h(1)=\frac{2\pi}{3}, h'(x)=\frac{\pi}{2\sqrt{2}}\left(-4x^3+5x^2+1\right)^{1/2}\cdot\left(-12x^2+10x\right)$

$y'=3\sin^2\left(\frac{\pi}{3}\cos h(x)\right)\cdot\cos\left(\frac{\pi}{3}\cosh(x)\right)\cdot\frac{\pi}{3}\left(-\sinh(x)\right)\cdot h'(x)$

$\therefore y'(1)=\frac{3\pi^2}{16}, y(1)=-\frac{1}{8}, h'(1)=-\pi$

$\therefore 2y'(1)+3\pi^2 y(1)=0$

Sol8. $a, ar, ar^2, ar^3,.....$

Now, $a^4r^6=1296\Rightarrow a=\frac{6}{r^{3/2}}$ ……(i)

Again, $a+ar+ar^2+ar^3=126$

From (i) $\left(r^{-3/2}+r^{3/2}\right)+\left(r^{-1/2}+r^{1/2}\right)=21$ ……(ii)

Let $r^{-1/2}+r^{1/2}=A$

$\therefore A^3=r^{-3/2}+r^{3/2}+3A$

Substitute in (ii) $A^3-3A+A=21\Rightarrow A=3$

$\therefore \sqrt{r}+\frac{1}{\sqrt{r}}=3\Rightarrow r+1=3\sqrt{r}$

$\Rightarrow r^2-7r+1=0$

$\therefore r_1+r_2=7$

Sol9. $\begin{bmatrix}1&1&1&:&6\\1&2&3&:&14\\\alpha&\beta&7&:&3\end{bmatrix}\sim\begin{bmatrix}1&1&1&:&6\\0&1&2&:&8\\0&\beta-\alpha&7-\alpha&:&3-6\alpha\end{bmatrix}\sim\begin{bmatrix}1&1&1&:&6\\0&1&2&:&8\\0&0&\alpha-2\beta+7&:&2\alpha-8\beta+3\end{bmatrix}$

(1) is not correct as for $\alpha-2\beta+7=0$; rank $\left[A\right]\neq$ rank$\left[A\,|\,b\right]$

Sol10. $\int_0^{\alpha}\frac{t^{50}-1+1}{1-t}=-\int_0^{\alpha}\left(1+t+....+t^{49}\right)+\int_0^{\alpha}\frac{1}{1-t}dt$

$=-\left(\frac{\alpha^{50}}{50}+\frac{\alpha^{49}}{49}+.....+\frac{\alpha^{1}}{1}\right)+\left(\frac{\ell n(1-t)}{-1}\right)_0^{\alpha}$

$=-P_{50}(\alpha)-\ell n(1-\alpha)$

Sol11. $2x\sec\theta-by\,\text{cosec}\theta=4-b^2$

Distance from $(0,0)$ is $\frac{4-b^2}{\sqrt{4\sec^2\theta+b^2\,\text{cosec}^2\theta}}$

From max distance, $\tan^2\theta=b/2$

$\therefore 4-b^2=b+2\Rightarrow b=1\Rightarrow e=\frac{\sqrt{3}}{2}$

Sol12. $\frac{{}^5C_2+{}^6C_2}{{}^2C_2+{}^3C_2+{}^4C_2+{}^5C_2+{}^6C_2}=\frac{10+15}{1+3+6+10+15}=\frac{5}{7}$

Sol13. $\sin^{-1}\frac{\alpha}{17}=\tan^{-1}\frac{77}{36}-\tan^{-1}\frac{3}{4}=\tan^{-1}\frac{8}{15}=\sin^{-1}\frac{8}{17}\Rightarrow\alpha=8$

$\therefore\sin^{-1}(\sin 8)+\cos^{-1}(\cos 8)=3\pi-8+8-2\pi=\pi$

Sol14. $\ell_1+\ell_2=20\Rightarrow\frac{d\ell_1}{d\ell_2}=-1$

$S=2A_1+3A_2=\frac{\ell_1^2}{8}+\frac{3\ell_2^2}{4\pi}$

$\frac{dS}{d\ell}=0\Rightarrow\frac{2\ell_1}{8}+\frac{6\ell_2}{4\pi}\cdot\frac{d\ell_2}{d\ell_1}=0\Rightarrow\frac{\ell_1}{4}=\frac{6\ell_2}{4\pi}$

Sol15. Here we've $A^2=A$

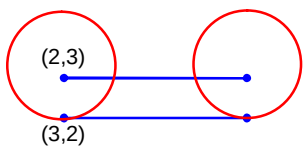
$\therefore A=A^2=A^3=A^4+.....$

Now, $(A+I)^{11}=(I+A)^{11}={}^{11}C_0I+{}^{11}C_1A+.....{}^{11}C_{10}A^{10}+{}^{11}C_{11}A^{11}$

$=\left(2^{11}-1\right)A+I=2047A+I$

Sum of diagonal elements (trace) = 2047 (1 + 4 – 3) + 3 = 4097

Sol16. $C=(2,3),\ r=\sqrt{2}$



Centre of G = A = $2+4\cdot\frac{1}{\sqrt{2}}$

$\therefore A=\left(2+2\sqrt{2},3+2\sqrt{2}\right)$

$B=\left(4+2\sqrt{2},1+2\sqrt{2}\right)$

$\therefore$ Area of trapezium = $\frac{1}{2}\left(4+4\sqrt{2}\right)\times 2=4\left(1+\sqrt{2}\right)$

Sol17. $\left|\vec{a}+\vec{b}+\vec{c}\right|^2=\left|\vec{a}+\vec{b}-\vec{c}\right|^2$

$\Rightarrow \vec{a}\cdot\vec{c}=0$

Again, $\left|\vec{a}+\lambda\vec{c}\right|^2 \geq |a|^2$

$\Rightarrow \lambda^2|c|^2>0$ which is true.

Sol18.

p	q	$p\Rightarrow q$	$\sim q$	$p\wedge\sim q$	S_1
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

P	q	$\sim p\Rightarrow\sim q$	$\sim p\vee q$	S_2
F	F	T	T	T
F	T	F	F	F
T	F	T	F	F
T	T	T	T	T

Sol19. $\sqrt{(x-1)(x-3)}+\sqrt{(x-3)(x+3)}=2\sqrt{\left(x-\frac{12}{4}\right)\left(x-\frac{1}{2}\right)}$

$\Rightarrow \sqrt{x-3}=0\Rightarrow x=3$ which is in domain.

Sol20. $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-2&0&1\\1&1&-1\end{vmatrix}=-\hat{i}-\hat{j}-2\hat{k}$

$\vec{a}_2-\vec{a}_1=6\hat{i}+(\lambda-1)\hat{j}+(-\lambda-4)\hat{k}$

$\left|\frac{-6-\lambda+1+2\lambda+8}{\sqrt{1+1+4}}\right|=2\sqrt{6}$

$\therefore |\lambda+3|=12\Rightarrow\lambda=9,-15$

$\alpha=-2k+5,\ \gamma=k-\lambda$

$\Rightarrow \alpha+2\gamma=5-2\lambda=-13,35$

SECTION – B

Sol1.

x_i	f_i	$d_i=x_i-5$	$f_id_i^2$	f_id_i
2	3	–3	27	–9
3	6	–2	24	–12
4	16	–1	16	–16
5	α	0	0	0
6	9	1	9	9
7	5	2	20	10
8	6	3	54	18

$$\sigma_x^2 = \sigma_d^2 = \frac{\Sigma f_i d_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2$$

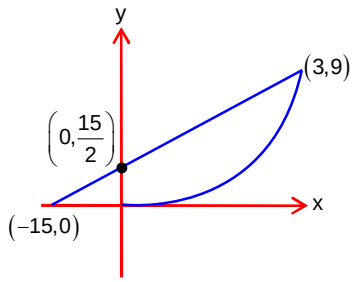
$$\Rightarrow \frac{150}{45+\alpha} - 0 = 3 \Rightarrow \alpha = 5$$

Sol2. Using PIE : divisible by 3 = 600
divisible by 11 = 164
divisible by 33 = 54

Sol3. $f(g(x)) = \begin{cases} g(x), & g(x) \geq 0 \\ 0, & g(x) < 0 \end{cases}$

$$= \begin{cases} x^2, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$A = \int_0^3 \left(\frac{x+15}{2} - x^2\right) dx + \frac{1}{2} \times \frac{15}{2} \times 15 = 72$$



Sol4. 2_ _ _ _ = 1296

3_ _ _ _ = 1296

40_ _ _ = 216

420_ _ = 36

422_ _ = 36

423_ _ = 36

424_ _ = 36

427_ _ = 36

4290_ = 6

$$\left.\begin{matrix} 4292\,0 \\ 4292\,2 \\ 4292\,3 \end{matrix}\right\} = 3$$

Sol5. $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1} = \lambda$

Intersection : $2(2\lambda+1) + (-\lambda-1) + 3(\lambda+3) = 16$

$\Rightarrow \lambda = 1$

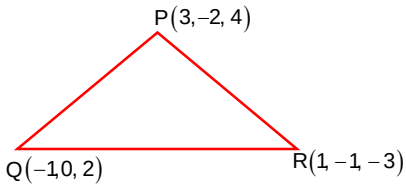
$\therefore P = (3, -2, 4)$

DR of QR $= (2\lambda, -\lambda, \lambda+6)$

DR of L $= (2, -1, 1)$

$\therefore 4\lambda + \lambda + \lambda + 6 = 0 \Rightarrow \lambda = -1$

$\therefore Q = (-1, 0, 2)$



$\overrightarrow{QR} \times \overrightarrow{QP} = -12\hat{i} - 24\hat{j}$

$\alpha = \frac{1}{2}\sqrt{144+576} \Rightarrow \alpha^2 = 180$

Sol6. $\cos\theta = \frac{\left(\hat{i}+\hat{j}+2\hat{k}\right)\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)}{6} = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \alpha = \frac{\pi}{6} \Rightarrow \left(\tan^2\theta\right)\left(\cot^2\alpha\right) = 3\cdot 3 = 9$

Sol7. $T_{r+1} = {}^{30}C_r \cdot \left(x^{2/3}\right)^{30-r} \cdot \left(\frac{2}{x^3}\right)^r$

$= {}^{30}C_r \cdot 2^r \cdot x^{\frac{60-11r}{3}}$

$\frac{60-11r}{3} < 0 \Rightarrow r > \frac{60}{11} = 6$

$T_7 = \underbrace{{}^{30}C_6 \cdot 2^6}_{\downarrow} x^{-2}$

a natural number

Sol8. $5^{99} = 625\times\left(5^5\right)^{19}$

$= 625\times\left(3124+1\right)^{19}$

$= 625\left(1+11k\right)^{19}$

$= 625\left(1+11k_1\right)$

$= 625+11k_2$

$= 9+11k_3$

Sol9. $a_5 = 2a_7 \Rightarrow a_1+4d = 2\left(a_1+6d\right) \Rightarrow a_1+8d = 0$ ……(i)

Also, $a_1+10d = 18$ ……..(ii)

Solve to get, $a_1 = -72, d = 9$

$S = \frac{12}{d}\left(\sqrt{a_{18}} - \sqrt{a_{10}}\right) = \frac{12\left(9-3\right)}{9} = 8$

$\left(\because a_{18} = a_1+17d = 81 \,\&\, a_{10} = a_1+9d = 9\right)$

Sol10. $\left|\vec{a}\right| = \sqrt{14}, \left|\vec{b}\right| = \sqrt{6}, \left|\vec{a}\times\vec{b}\right| = \sqrt{48}$

$\left|\vec{a}\times\vec{b}\right|^2 + \left|\vec{a}\cdot\vec{b}\right|^2 = \left|\vec{a}\right|^2\cdot\left|\vec{b}\right|^2$

$\Rightarrow \left|\vec{a}\cdot\vec{b}\right|^2 = 84-48 = 36$