FIITJEE

Solutions to JEE(Main) -2023

Test Date: 1st February 2023 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (1 – 10)*** *contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

PART – A (PHYSICS)

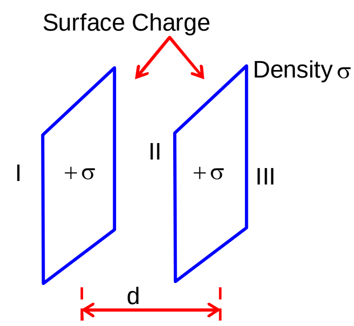
SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. The average kinetic energy of a molecule of the gas is
(A) Proportional to absolute temperature
(B) dependent on the nature of the gas
(C) proportional to pressure
(D) proportional to volume

Q2. Let σ be the uniform surface charge density of two infinite thin plane sheets shown in figure. Then the electric fields in the three different region E_1, E_{II} and E_{III} are.



(A) $\vec{E}_I = -\frac{\sigma}{\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{\sigma}{\epsilon_0}\hat{n}$

(B) $\vec{E}_I = \frac{\sigma}{2\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{\sigma}{2\epsilon_0}\hat{n}$

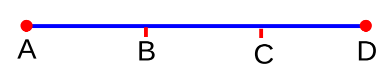
(C) $\vec{E}_I = \frac{2\sigma}{\epsilon_0}\hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{2\sigma}{\epsilon_0}\hat{n}$

(D) $\vec{E}_I = 0, \vec{E}_{II} = \frac{\sigma}{\epsilon_0}\hat{n}, E_{III} = 0$

Q3. $\left(P + \frac{a}{V^2}\right)(V - b) = RT$ represents the equation of state of some gases. Where P is the pressure, V is the volume, T is the temperature and a,b,R are the constants. The physical quantity, which has dimensional formula as that of $\frac{b^2}{a}$, will be :
(A) Compressibility
(B) Bulk modulus
(C) Energy density
(D) Modulus of rigidity

Q4. If earth has a mass nine times and radius twice to that of a planet P. Then $\frac{\upsilon_e}{3}\sqrt{x}\ ms^{-1}$ will be the minimum velocity required by a rocket to pull out of gravitational force of P, where υ_e is escape velocity on earth. The value of x is
(A) 2
(B) 18
(C) 3
(D) 1

Q5. An object moves with speed υ_1, υ_2 and υ_3 along a line segment AB, BC and CD respectively as shown in figure. Where AB = BC and AD = 3AB, then average speed of the object will be :



(A) $\frac{(\upsilon_1 + \upsilon_2 + \upsilon_3)}{3\upsilon_1\upsilon_2\upsilon_3}$

(B) $\frac{(\upsilon_1 + \upsilon_2 + \upsilon_3)}{3}$

(C) $\frac{\upsilon_1\upsilon_2\upsilon_3}{3(\upsilon_1\upsilon_2 + \upsilon_2\upsilon_3 + \upsilon_3\upsilon_1)}$

(D) $\frac{3\upsilon_1\upsilon_2\upsilon_3}{(\upsilon_1\upsilon_2 + \upsilon_2\upsilon_3 + \upsilon_3\upsilon_1)}$

Q6. Match List I with List II :

List-I		List-II	
A.	Intrinsic semiconductor	I.	Fermi-level near the valence band
B.	n-type semiconductor	II.	Fermi-level in the middle of valence and conduction band
C.	p-type semiconductor	III.	Fermi-level near the conduction band
D.	Metals	IV.	Fermi-level inside the conduction band

Choose the correct answer from the options given below :

(A) A – II, B – II, C – III, D – IV
(B) A – I, B – II, C – III, D – IV
(C) A – III, B – I, C – II, D – IV
(D) A – II, B – III, C – I, D – IV

Q7. 'n' polarizing sheets are arranged such that each makes an angle 45° with the preceeding sheet. An unpolarized light of intensity I is incident into this arrangement. The output intensity is found to be $I/64$. The value of n will be :

(A) 6
(B) 5
(C) 4
(D) 3

Q8. A child stands on the edge of the cliff 10 m above the ground and throws a stone horizontally with an initial speed of 5 ms^{-1}. Neglecting the air resistance, the speed with which the stone hits the ground will be ______ms^{-1}
(given, g = 10 ms^{-2}).

(A) 15
(B) 25
(C) 30
(D) 20

Q9. A sample of gas at temperature T is adiabatically expanded to double its volume. The work done by the gas in the process is (given $= \gamma = \frac{3}{2}$):

(A) $W = RT\left[2-\sqrt{2}\right]$
(B) $W = \frac{R}{T}\left[2-\sqrt{2}\right]$
(C) $W = TR\left[\sqrt{2}-2\right]$
(D) $W = \frac{T}{R}\left[\sqrt{2}-2\right]$

Q10. A steel wire with mass per unit length $7.0\times10^{-3}\,kgm^{-1}$ is under tension of 70 N. The speed of transverse waves in the wire will be :

(A) 10 m/s
(B) 100 m/s
(C) 50 m/s
(D) $200\,\pi$ m / s

Q11. Match List I with List II :

List-I		List-II	
A.	Microwaves	I.	Radio active decay of the nucleus
B.	Gamma rays	II.	Rapid acceleration and deceleration of
C.	Radio waves	III.	Inner shell electrons
D.	X-rays	IV.	Klystron valve

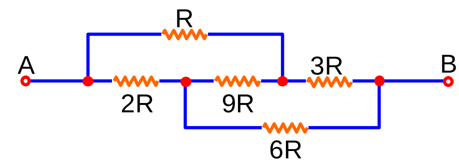
Choose the correct answer from the options given below :

(A) A – I, B – II, C – III, D – IV
(B) A – IV, B – III, C – II, D – I
(C) A – IV, B – I, C – II, D – III
(D) A – I, B – III, C – IV, D – II

Q12. A proton moving with one tenth of velocity of light has a certain de Broglie wavelength of λ. An alpha particle having certain kinetic energy has the same de-Brogle wavelength λ. The ratio of kinetic energy of proton and that of alpha particle is :

(A) 1 : 2
(B) 1 : 4
(C) 2 : 1
(D) 4 : 1

Q13. The equivalent resistance between A and B of the network shown in figure :



(A) 21R
(B) 14R
(C) $11\frac{2}{3}R$
(D) $\frac{8}{3}R$

Q14. Which of the following frequencies does not belong to FM broadcast.
(A) 99 MHz
(B) 89 MHz
(C) 106 MHz
(D) 64 MHz

Q15. The mass of proton, neutron and helium nucleus are respectively 1.0073u, 1.0087u and 4.0015u. The binding of helium nucleus is :
(A) 28.4 MeV
(B) 56.8 MeV
(C) 7.1 MeV
(D) 14.2 MeV

Q16. A block of mass 5kg is placed at rest on a table of rough surface. Now, if a force of 30N is applied in the direction parallel to surface of the table, the block slides through a distance of 50m in an interval of time 10s. Coefficient of kinetic friction is (given, g = $10ms^{-2}$):
(A) 0.60
(B) 0.50
(C) 0.25
(D) 0.75

Q17. Match List I with List II :

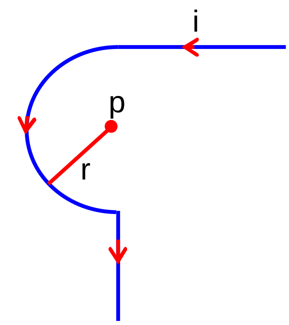
List I	List II
A. AC generator	I. Presence of both L and C
B. Transformer	II. Electromagnetic Induction
C. Resonance phenomenon to occur	III. Quality factor
D. Sharpness of resonance	IV. Mutual Induction

Choose the correct answer from the options given below :
(A) A – IV, B – II, C – I, D – III
(B) A – IV, B – III, C – I, D – II
(C) A – II, B – IV, C – I, D – III
(D) A – II, B – I, C – III, D – IV

Q18. A mercury drop of radius 10^{-3} m is broken into 125 equal size droplets. Surface tension of mercury is 0.45 Nm^{-1}. The gain in surface energy is :
(A) 5×10^{-5} J
(B) 28×10^{-5} J
(C) 17.5×10^{-5} J
(D) 2.26×10^{-5} J

Q19. Given below are two statements :
Statement I : Acceleration due to gravity is different at different places on the surface of earth.
Statement II : Acceleration due to gravity increases as we go down below the earth's surface.
In the light of the above statements, choose the correct answer from the options given below
(A) Statement I is true but Statement II is false
(B) Statement I is false but Statement II is true
(C) Both Statement I and Statement II are true
(D) Both Statement I and Statement II are false

Q20. Find the magnetic field at the point P in figure. The curved portion is a semicircle connected to two long straight wires.



(A) $\frac{\mu_0 i}{2r}\left(1+\frac{2}{\pi}\right)$

(B) $\frac{\mu_0 i}{2r}\left(1+\frac{1}{\pi}\right)$

(C) $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{\pi}\right)$

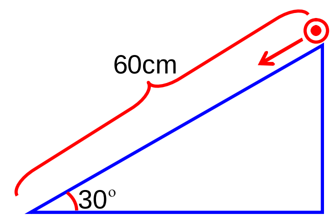
(D) $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{2\pi}\right)$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. A solid cylinder is released from rest from the top of an inclined plane of inclination 30° and length 60cm. If the cylinder rolls without slipping, its speed upon reaching the bottom of the inclined plane is _______ ms^{-1}.
(Given g = 10 ms^{-2})



Q2. Two equal positive point charges are separated by a distance 2a. The distance of a point from the centre of the line joining two charges on the equatorial line (perpendicular bisector) at which force experienced by a test charge q_0 becomes maximum is $\frac{a}{\sqrt{x}}$. The value of x is ________.

Q3. A light of energy 12.75 eV is incident on a hydrogen atom in its ground state. The atom absorbs the radiation and reaches to one of its excited states. The angular momentum of the atom in the excited state is $\frac{x}{\pi}\times10^{-17}$ eVs. The value of x is _______(use h = 4.14×10^{-15} evs, c = $3\times10^{8}ms^{-1}$).

Q4. A small particle moves to position $5\hat{i}-2\hat{j}+\hat{k}$ from its initial position $2\hat{i}+3\hat{j}-4\hat{k}$ under the action of force $5\hat{i}+2\hat{j}+7\hat{k}$ N. The value of work done will be _________J.

Q5. A charge particle of $2\,\mu C$ accelerated by a potential difference of 100V enters a region of uniform magnetic of magnitude 4mT at right angle to the direction of field. The charge particle completes semicircle of radius 3cm inside magnetic field. The mass of the charge particle is_________ $\times10^{-18}$ kg.

Q6. A series LCR circuit is connected to an ac source of 220V, 50Hz. The circuit contain a resistance R = $100\,\Omega$ and an inductor of inductive reactance $X_L = 79.6\Omega$. The capacitance of the capacitor needed to maximize the average rate at which energy is supplied will be _______ μF.

Q7. A thin cylindrical rod of length 10cm is placed horizontally on the principle axis of a concave mirror of focal length 20cm. The rod is placed in a such a way that mid point of the rod is at 40cm from the pole of mirror. The length of the image formed by the mirror will be $\frac{x}{3}$cm. The value of x is ______.

Q8. The amplitude of a particle executing SHM is 3cm. The displacement at which its kinetic energy will be 25% more than the potential energy is :
________cm.

Q9. A certain pressure 'P' is applied to 1 litre of water and 2 litre of a liquid separately. Water gets compressed to 0.1% whereas the liquid gets compressed to 0.03%. The ratio of Bulk modulus of water to that liquid is $\frac{3}{x}$. The value of x is ________.

Q10. In an experiment to find emf of a cell using potentiometer, the length of null point for a cell of emf 1.5 V is found to be 60cm. If this cell is replaced by another cell of emf E, the length-of null point increases by 40cm. The value of E is $\frac{x}{10}$ V. The value of x is _______.

PART – B (CHEMISTRY)

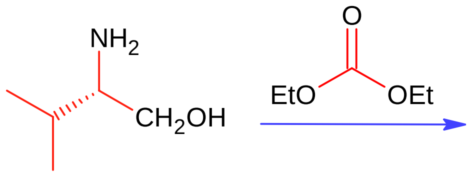
SECTION - A

(One Options Correct Type)

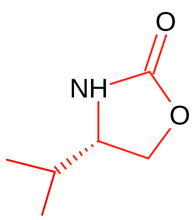
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

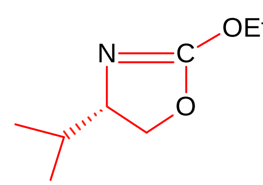
Q1. How can photochemical smog be controlled?
(A) By using catalyst.
(B) By complete combustion of fuel.
(C) By using catalytic convertors in the automobiles / industry.
(D) By using tall chimneys.

Q2. In the following reaction, 'A' is



'A'
Major product

(A)


(B)


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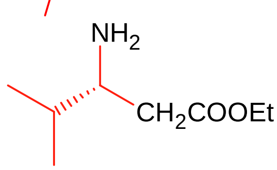

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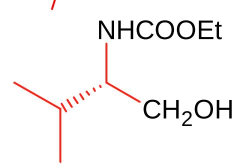


Q3. But-2-yne is reacted separately with one mole of Hydrogen as shown below:

$$B \xleftarrow[\text{liq. } NH_3]{Na} CH_3-C\equiv C-CH_3 \xrightarrow[\Delta]{Pd / C} \underline{A}$$

A. A is more soluble than B.
B. The boiling point & melting point of A are higher and lower than B respectively.
C. A is more polar than B because dipole moment of A is zero.
D. Br_2 adds easily to B than A.
Identify the incorrect statements from the options given below:
(A) A, C & D only
(B) A and B only
(C) B, C and D only
(D) B and C only

Q4. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**
Assertion A: Hydrogen is an environment friendly fuel.
Reason R: Atomic number of hydrogen is 1 and it is a very light element.
In the light of the above statements, choose the correct answer from the options given below
(A) **A** is true but **R** is false
(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**
(C) Both **A** and **R** are true and **R** is the correct explanation of **A**
(D) **A** is false but **R** is true

Q5. Which of the following are the example of double salt?
A. $FeSO_4.(NH_4)_2SO_4.6H_2O$
B. $CuSO_4.4NH_3.H_2O$
C. $K_2SO_4.Al_2(SO_4)_3.24H_2O$
D. $Fe(CN)_2.4KCN$
Choose the correct answer
(A) A and C only
(B) A and B only
(C) B and D only
(D) A, B and D only

Q6. Choose the correct statement(s):
A. Beryllium oxide is purely acidic in nature.
B. Beryllium carbonate is kept in the atmosphere of CO_2
C. Beryllium sulphate is readily soluble in water.
D. Beryllium shows anomalous behaviour.
Choose the correct answer from the options given below:
(A) B, C and D only
(B) A, B and C only
(C) A only
(D) A and B only

Q7. A solution of $FeCl_3$ when treated with $K_4[Fe(CN)_6]$ gives a prussiun blue precipitate due to the formation of
(A) $Fe_4[Fe(CN)_6]_3$
(B) $Fe_3[Fe(CN)_6]_2$
(C) $K[Fe_2(CN)_6]$
(D) $Fe[Fe(CN)_6$

Q8. Match List I with List II

List-I		**List-II**	
A.	Slaked lime	I.	NaOH
B.	Dead burnt plaster	II.	$Ca(OH)_2$
C.	Caustic soda	III.	$Na_2CO_3,10H_2O$
D.	Washing soda	IV.	$CaSO_4$

Choose the correct answer from the options given below:
(A) A- III, B-II, C-IV, D- I
(B) A-III, B-IV, C-II, D-I
(C) A-II, B-IV, C-I, D-III
(D) A-I, B-IV, C-II, D-III

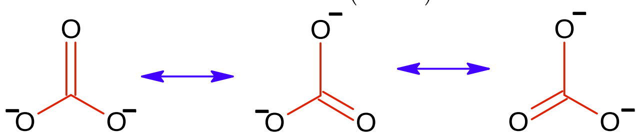
Q9. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**
Assertion A: Amongst He, Ne, Ar and Kr; 1g of activated charcoal adsorbs more of Kr.
Reason R: The critical volume $V_c(cm^3\ mol^{-1})$ and critical pressure P_C (atm) is highest for Krypton but the compressibility factor at critical point Z_C is lowest for Krypton.
In the light of the above statements, choose the correct answer from the options given below
(A) Both **A** and **R** are true and **R** is the correct explanation of **A**
(B) **A** is true but **R** is false
(C) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**
(D) **A** is false but **R** is true

Q10. Match List I with List II

List-I		**List-II**	
A.	Tranquilizers	I.	Anti blood clotting
B.	Aspirin	II.	Salvarsan
C.	Antibiotic	III.	Antidepressant drug
D.	Antiseptic	IV.	soframicine

Choose the correct answer from the options given below:
(A) A- IV, B-II, C-I, D-III
(B) A-III, B-I, C-II, D-IV
(C) A-II, B-IV, C-I, D-III
(D) A-II, B-I, C-III, D-IV

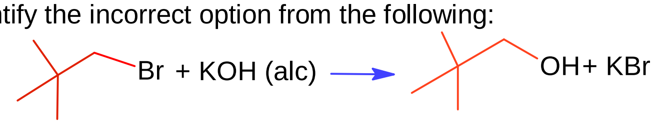
Q11. Resonance in carbonate ion $\left(CO_3^{2-}\right)$ is

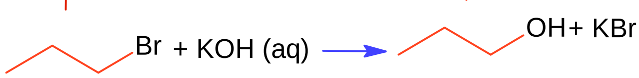


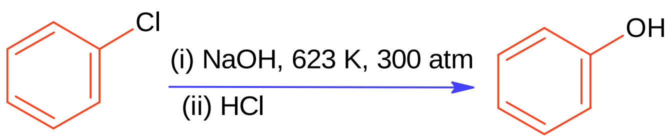
Which of the following is true?
(A) Each structure exists for equal amount of time.
(B) CO_3^{2-} has a single structure i.e. resonance hybrid of the above three structures.
(C) It is possible to identify each structure individually by some physical or chemical method.
(D) All these structure are in dynamic equilibrium with each other.

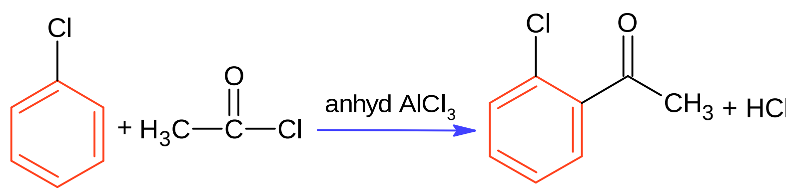
Q12. Which of the following complex will show largest splitting of d- orbtials?
(A) $\left[Fe(CN)_6\right]^{3-}$ (B) $\left[Fe(NH_3)_6\right]^{3+}$
(C) $\left[Fe(C_2O_4)_3\right]^{3-}$ (D) $[FeF_6]^{3-}$

Q13. Identify the incorrect option from the following:

(A) 

(B) 

(C) 

(D) 

Q14. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**
Assertion A: In an Ellingham diagram, the oxidation of carbon to carbon monoxide shows a negative slope with respect to temperature.
Reason R: CO tends to get decomposed at higher temperature.
In the light of the above statements, choose the correct answer from the options given below
(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**
(B) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
(C) **A** is correct but **R** is not correct
(D) **A** is not correct but **R** is correct

Q15. Highest oxidation state of Mn in exhibited in Mn_2O_7. The correct statements about Mn_2O_7 are
(A) Mn is tetrahedrally surrounded by oxygen atoms.
(B) Mn is octahedrally surrounded by oxygen atoms.
(C) Contains Mn-O-Mn bridge
(D) Contains Mn-Mn bond.
Choose the correct answer from the options given below:
(A) B and D only (B) A and C only
(C) A and D only (D) B and C only

Q16. Match List I with List II

List-I (Test)		List-II(Functional group / Class of compound	
A.	Molisch's Test	I.	Peptide
B.	Biuret Test	II.	Carbohydrate
C.	Carbylamine Test	III.	Primary amine
D.	Schiffs Test	IV.	Aldehyde

Choose the correct answer from the options given below:

(A) A- III, B-IV, C-I, D-II
(B) A-III, B-IV, C-II, D-I
(C) A-II, B-I, C-III, D-IV
(D) A-I, B-II, C-III, D-IV

Q17. Given below are two statements:

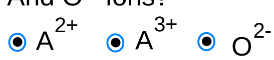
Statement I: Chlorine can easily combine with oxygen to form oxides: and the product has a tendency to explode.

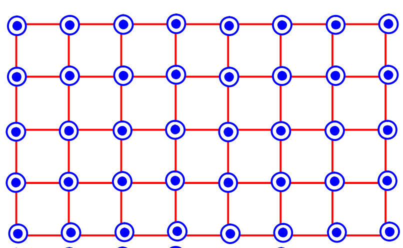
Statement II: Chemical reactivity of an element can be determined by its reaction with oxygen and halogens.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(A) Statement **I** is false but Statement **II** is true
(B) Both the Statement **I** and **II** are true
(C) Both the statement **I** and **II** are false
(D) Statement **I** is true but Statement **II** is false

Q18. Which of the following represents the lattice structure of $A_{0.95}O$ containing A^{2+}, A^{3+} And O^{2-} ions?



A.


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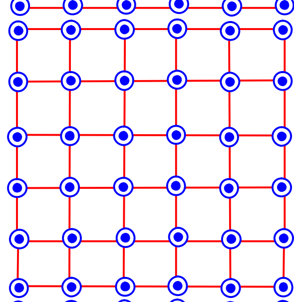

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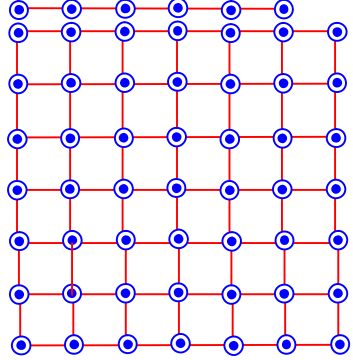
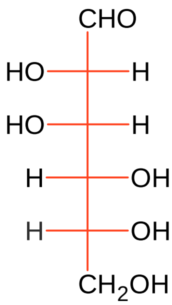
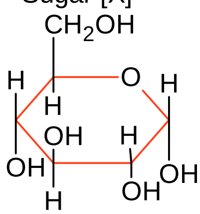


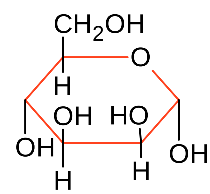
(A) B only
(B) A only
(C) A and B only
(D) B and C only

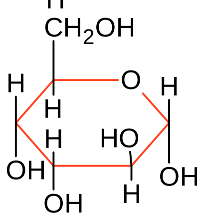
Q19. The correct representation in six membered pyranose from the following sugar [X] is

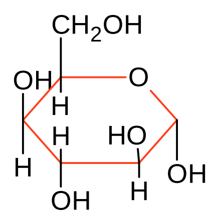


Sugar [X]

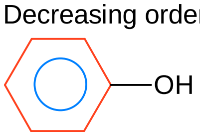
(A) 

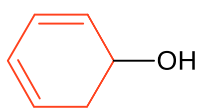
(B) 

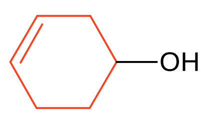
(C) 

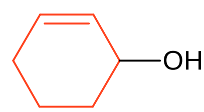
(D) 

Q20. Decreasing order of dehydration of the following alcohols is


a


b


c


d

(A) a > d > b > c
(B) b > d > c > a
(C) d > b> c > a
(D) b > a > d > c

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. Sum of oxidation states of bromine in bromic acid and perbromic acid is_______.

Q2. At 25°C, the enthalpy of the following process are given:

$H_2(g)+O_2(g) \longrightarrow 2OH(g) \quad \Delta H^o = 78\,kJmol^{-1}$

$H_2(g)+1/2O_2(g) \longrightarrow H_2O(g) \quad \Delta H^o = -242\,kJmol^{-1}$

$H_2(g) \longrightarrow 2H(g) \quad \Delta H^o = 436\,kJmol^{-1}$

$1/2O_2(g) \longrightarrow O(g) \quad \Delta H^o = 249\,kJmol^{-1}$

What would be the value of X for the following reaction?___________ (Nearest integer)

$H_2O(g) \rightarrow H(g) + OH(g) \Delta H^o = X\ kJmol^{-1}$

Q3. 25 mL of an aqueous solution of KCl was found a require 20 mL of 1M $AgNO_3$ solution when titrated using K_2CrO_4 as an indicator. What is the depression in freezing point of KCl solution of the given concentration? ___________(Nearest integer)
(Given: Kr = 2.0 K kg mol^{-1})
Assume 1) 100% ionization and 2) density of the aqueous solution as 1 g mL^{-1}

Q4. At what pH, given half cell $MnO_4^-(0.1M)\,|\,Mn^{2+}(0.001M)$ will have electrode potential of 1.282V? _________. (Nearest Integer)

Given $E^o_{MnO_4^-|Mn^{2+}} = 1.54V, \frac{2.303RT}{F} = 0.059V$

Q5. (i) $X(g) \rightleftharpoons Y(g)+Z(g) \qquad K_{P_1} = 3$

(ii) $A(g) \rightleftharpoons 2B(g) \qquad K_{P_2} = 1$

If the degree of dissociation and initial concentration of both the reactants X(g) and A(g) are equal, then the ratio of the total pressure at equilibrium $\left(\frac{p_1}{p_2}\right)$ is equal to x:1. The value of x is ___________(Nearest integer)

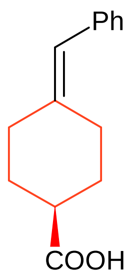
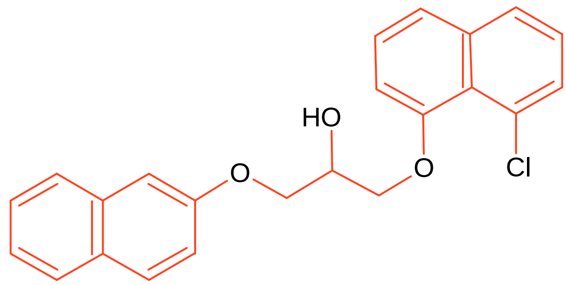
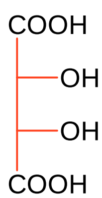
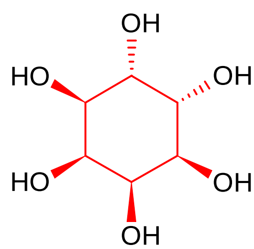
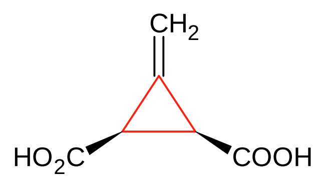
Q6. Number of isomeric compounds with molecular formula $C_9H_{10}O$ which (i) do not dissolve in NaOH (ii) do not dissolve in HCl. (iii) do not give orange precipitate with 2,4-DNP (iv) on hydrogenation give identical compound with molecular formula $C_9H_{12}O$ is_______.

Q7. Electrons in a cathode ray tube have been emitted with a velocity of 1000 ms^{-1}. The number of following statement which is / are true about the emitted radiation is ___________.

Given: $h = 6\times10^{-34}Js$, $m_e = 9\times10^{-31}kg$

(A) The de-Broglie wavelength of the electron emitted is 666.67 nm.
(B) The characteristic of electrons emitted depend upon the material of the electrodes of the cathode ray tube.
(C) The cathode rays starts from cathode and move towards anode.
(D) The nature of the emitted electron depends on the nature of the gas present in cathode ray tube.

Q8. The total number of chiral compound/s from the following is__________.

Q9. The density of 3 M solution of NaCl is 1.0 g mL-1. Molality of the solution is_______ $\times 10^{-2}$m.(nearest integer)
Given: Molar mass of Na and Cl is 23 and 35.5 g mol^{-1} respectively.

Q10. A and B are two substance undergoing radioactive decay in a container. The half life of A is 15 min and that of B is 5 min. if the initial concentration of B is 4 times that of A and they both start decaying at the same time, how much time will it take for the concentration of both of them to be same?____________ min.

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx}+\frac{x+a}{y-2}=0,\ y(1)=0$ is 4π.

Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and and Q on the curve C intersect x-axis at points R and S respectively, then the length of the line segment RS is

(A) $2\sqrt{3}$
(B) 2
(C) $\frac{4\sqrt{3}}{3}$
(D) $\frac{2\sqrt{3}}{3}$

Q2. The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5, then the sum of cubes of the remaining two observations is

(A) 1456
(B) 1792
(C) 1072
(D) 1216

Q3. Let S be the set of all solutions of the equation $\cos^{-1}(2x)-2\cos^{-1}\left(\sqrt{1-x^2}\right)=\pi$. $x\in\left[-\frac{1}{2},\frac{1}{2}\right]$.

Then $\sum_{x\in S}2\sin^{-1}\left(x^2-1\right)$ is equal to

(A) $\pi-\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$
(B) $\frac{-2\pi}{3}$
(C) 0
(D) $\pi-2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Q4. Let R be a relation on $\mathbb{R}$, given by $R=\left\{(a,b): 3a-3b+\sqrt{7} \text{ is an irrational number}\right\}$. Then R is

(A) reflexive but neither symmetric nor transitive
(B) reflexive and transitive but not symmetric
(C) reflexive and symmetric but not transitive
(D) an equivalence relation

Q5. The negation of the expression $q\vee\left((\sim q)\wedge p\right)$ is equivalent to

(A) $(\sim p)\wedge(\sim q)$
(B) $(\sim p)\vee q$
(C) $(\sim p)\vee(\sim q)$
(D) $p\wedge(\sim q)$

Q6. In a binomial distribution $B(n, p)$, the sum and the product of the mean and the variance are 5 and 6 respectively, then $6(n+p-q)$ is equal to

(A) 52
(B) 53
(C) 50
(D) 51

Q7. For a triangle ABC, the value of $\cos 2A + \cos 2B + \cos 2C$ is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct?

(A) $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$ (B) perimeter of ΔABC is $18\sqrt{3}$

(C) $\overrightarrow{MA} \cdot \overrightarrow{MB} = -18$ (D) area of ΔABC is $\frac{27\sqrt{3}}{2}$

Q8. If $y = y(x)$ is the solution curve of the differential equation

$\frac{dy}{dx} + y\tan x = x\sec x, 0 \le x \le \frac{\pi}{3}, y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(A) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$ (B) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

(C) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$ (D) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

Q9. If the center and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively (α, β) and γ, then $3(\alpha + \beta + \gamma)$ is equal to

(A) 11 (B) 9
(C) 10 (D) 12

Q10. If the orthocentre of the triangle, whose vertices are $(1, 2), (2, 3)$ and $(3, 1)$ is (α, β), then the quadratic equation whose roots are $\alpha + 4\beta$ and $4\alpha + \beta$, is

(A) $x^2 - 19x + 90 = 0$ (B) $x^2 - 18x + 80 = 0$
(C) $x^2 - 20x + 99 = 0$ (D) $x^2 - 22x + 120 = 0$

Q11. Let $f(x) = 2x + \tan^{-1} x$ and $g(x) = \log_e\left(\sqrt{1+x^2} + x\right), x \in [0, 3]$. Then

(A) there exists $\hat{x} \in [0, 3]$ such that $f'(\hat{x}) < g'(\hat{x})$

(B) min $f'(x) = 1 +$ max $g'(x)$

(C) there exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x), \forall x \in (x_1, x_2)$

(D) max $f(x) >$ max $g(x)$

Q12. Let $S = \left\{x : x \in R \text{ and } \left(\sqrt{3} + \sqrt{2}\right)^{x^2-4} + \left(\sqrt{3} - \sqrt{2}\right)^{x^2-4} = 10\right\}$. Then $n(S)$ is equal to

(A) 4 (B) 2
(C) 6 (D) 0

Q13. Let S denote the set of all real values of λ such that the system of equations

$\lambda x + y + z = 1$

$x + \lambda y + z = 1$

$x + y + \lambda z = 1$

Is in consistent, then $\sum_{\lambda \in S}\left(|\lambda|^2 + |\lambda|\right)$ is equal to

(A) 6 (B) 4
(C) 2 (D) 12

Q14. The sum to 10 terms of the series

$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+.....$ is

(A) $\frac{56}{111}$
(B) $\frac{55}{111}$
(C) $\frac{59}{111}$
(D) $\frac{58}{111}$

Q15. The combined equation of the two lines $ax+by+c=0$ and $a'x+b'y+c'=0$ can be written as $(ax+by+c)(a'x+b'y+c')=0$.

The equation of the angle bisectors of the lines represented by the equation $2x^2+xy-3y^2=0$ is

(A) $x^2-y^2-10xy=0$
(B) $3x^2+5xy+2y^2=0$
(C) $3x^2+xy-2y^2=0$
(D) $x^2-y^2+10xy=0$

Q16. Let $f(x)=\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}, x\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$. If α and β respectively are the maximum and the minimum values of f, then

(A) $\alpha^2-\beta^2=4\sqrt{3}$
(B) $\alpha^2+\beta^2=\frac{9}{2}$
(C) $\beta^2+2\sqrt{\alpha}=\frac{19}{4}$
(D) $\beta^2-2\sqrt{\alpha}=\frac{19}{4}$

Q17. The value of $\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+....+\frac{1}{49!2!}+\frac{1}{51!1!}$ is :

(A) $\frac{2^{50}}{51!}$
(B) $\frac{2^{51}}{50!}$
(C) $\frac{2^{51}}{51!}$
(D) $\frac{2^{50}}{50!}$

Q18. Let the image of the point $P(2,-1,3)$ in the plane $x+2y-z=0$ be Q. Then the distance of the plane $3x+2y+z+29=0$ from the point Q is

(A) $\frac{22\sqrt{2}}{7}$
(B) $3\sqrt{14}$
(C) $\frac{24\sqrt{2}}{7}$
(D) $2\sqrt{14}$

Q19. The shortest distance between the lines

$\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$ and $\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$ is

(A) $5\sqrt{3}$
(B) $6\sqrt{3}$
(C) $7\sqrt{3}$
(D) $4\sqrt{3}$

Q20. $\lim\limits_{n\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+...+\frac{1}{2n}\right]$ is equal to

(A) 0

(B) $\log_e\left(\frac{2}{3}\right)$

(C) $\log_e 2$

(D) $\log_e\left(\frac{3}{2}\right)$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7, is…………

Q2. Let $\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}, \vec{\omega} = 2\alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha > 0$. If the minimum value of the scalar triple product $[\vec{u}\,\vec{v}\,\vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u}\cdot\hat{i}|^2 = \frac{m}{n}$ where m and n are coprime natural numbers, then m + n is equal to………..

Q3. The remainder, when $19^{200} + 23^{200}$ is divided by 49, is………

Q4. $A(2,6,2), B(-4,0,\lambda), C(2,3,-1)$ and $D(4,5,0), |\lambda| \le 5$ are the vertices of a quadrilateral ABCD. If its area is 18 square units, then $5 - 6\lambda$ is equal to………..

Q5. Let $f : R \to R$ be a differentiable function such that $f'(x) + f(x) = \int_0^2 f(t)\,dt$. If $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to………

Q6. If $\int_0^1 (x^{21} + x^{14} + x^7)(2x^{14} + 3x^7 + 6)^{1/7}\,dx = \frac{1}{\ell}(11)^{m/n}$ where $\ell, m, n \in N$, m and n are coprime then $\ell + m + n$ is equal to……..

Q7. Let A be the area bounded by the curve $y = x|x-3|$, the x-axis and the ordinates $x = -1$ and x = 2. Then 12 A is equal to………

Q8. Let $a_1 = 8, a_2, a_3, \ldots, a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is…….

Q9. The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is…….

Q10. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to…….

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KEYS to JEE (Main)-2023

PART – A (PHYSICS)

SECTION - A

1.	A	2.	A	3.	A	4.	A
5.	D	6.	D	7.	A	8.	A
9.	A	10.	B	11.	C	12.	D
13.	D	14.	D	15.	A	16.	B
17.	C	18.	D	19.	A	20.	D

SECTION – B

1.	2	2.	2	3.	828	4.	40
5.	144	6.	40	7.	32	8.	2
9.	1	10.	25				

PART – B (CHEMISTRY)

SECTION - A

1.	C	2.	A	3.	DROP	4.	B
5.	A	6.	A	7.	A	8.	C
9.	B	10.	B	11.	B	12.	A
13.	A	14.	C	15.	B	16.	C
17.	B	18	B	19.	B	20.	B

SECTION - B

1.	12	2.	499	3.	3	4.	3
5.	12	6.	2	7.	2	8.	2
9.	364	10.	15				

PART – C (MATHEMATICS)

SECTION - A

1.	C	2.	C	3.	B	4.	A
5.	A	6.	A	7.	D	8.	D
9.	D	10.	C	11.	D	12.	A
13.	A	14.	B	15.	A	16.	D
17.	A	18	B	19.	B	20.	C

SECTION - B

1.	514	2.	3501	3.	29	4.	11
5.	1	6.	63	7.	62	8.	754
9.	50400	10.	14				

FIITJEE
Solutions to JEE (Main)-2023

PART – A (PHYSICS)

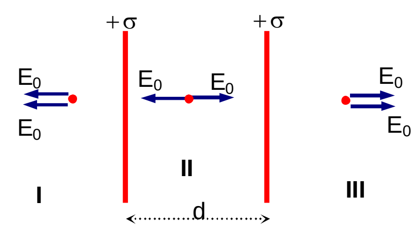
SECTION – A

Sol1. The average kinetic energy of a molecule of the gas $= \frac{3}{2}kT$

$\Rightarrow$The average kinetic energy of a molecule of the gas $\alpha\ T$ (absolute temperature)

Sol2. $E_0 = \frac{\sigma}{2\in_0}$

$\vec{E}_I = -2E_0\hat{n} = -\frac{\sigma}{\in_0}\hat{n}$, $\vec{E}_{II} = \vec{0}$ and $\vec{E}_{III} = 2E_0\hat{n} = \frac{\sigma}{\in_0}\hat{n}$



Sol3. Dimension of b = = $V \equiv \left[M^0L^3T^0\right]$

$PV^2 = a \equiv \left[\frac{MLT^{-2}}{L^2}\right] \times L^6 = \left[ML^5T^{-2}\right] \Rightarrow \frac{b^2}{a} = \frac{\left[M^0L^6T^0\right]}{\left[ML^5T^{-2}\right]} \equiv \left[M^{-1}LT^{-2}\right]$

Compressibility $= \frac{1}{V}\frac{dV}{dP} = \frac{1}{P} \equiv \left[M^{-1}LT^{-2}\right]$

Sol4. Given, $M_P \Rightarrow$ Actual mass of Planet

$M_e = 9M_P \Rightarrow \frac{M_e}{M_P} = 9$

$R_P \Rightarrow$ Actual radius of Planet

$R_e = 2R_P \Rightarrow \frac{R_e}{R_P} = 2$

We know escape velocity $= \sqrt{\frac{2GM}{r}}$

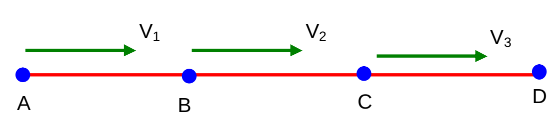
$\therefore \frac{v_e}{v_P} = \sqrt{\frac{M_e}{M_P} \times \frac{R_P}{R_e}} = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}$

Hence $v_P = \frac{v_e\sqrt{2}}{3} = \frac{v_e}{3}\sqrt{x} \Rightarrow x = 2$

Sol5. $AB = BC = CD = x$ **and** $S = ut$

$t_{AB} = \frac{x}{v_1}, t_{BC} = \frac{x}{v_2}$ **and** $t_{CD} = \frac{x}{v_3}$

$\therefore$ total time $= \frac{3x}{v_{avg}} = t_{AB} + t_{BC} + t_{CD}$



$$= \frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}$$

$$\Rightarrow \frac{3}{v_{avg}} = \frac{v_2v_3 + v_1 + v_2 + v_1v_2}{v_1v_2v_3}$$

$$\therefore v_{avg} = \frac{3v_1v_2v_3}{v_2v_3 + v_1v_2 + v_1v_3}$$

Sol6. Metals have Fermi level inside conduction bond
$\therefore$ D = IV
Intrinsic semiconductor $\rightarrow$ Fermi level b/w valence & conductive bond A = IF
P-typre semiconductor Fermi level near valence hard
C=I

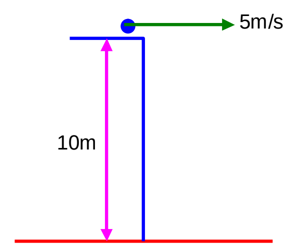
Sol7. 1^{st} Plate $\Rightarrow \frac{I_0}{2}$

2^{nd} plate $\Rightarrow \frac{I_0}{2}\cos^2\phi$

$\therefore$ n^{th} plate $\Rightarrow \frac{I_0}{2}\left(\cos^2\phi\right)^n = \frac{I_0}{2}(2)^n = \frac{I_0}{2^{n+1}} = \frac{I_0}{64}$

$\therefore$ Total no of plates = 6

Sol8. Using conservation of energy



$$\frac{1}{2}mv^2 + mgh = \frac{h}{2} = v_0^2$$

$$\Rightarrow \frac{2s}{2} + 20 \times 10 = \frac{1}{2}v_0^2$$

$$\Rightarrow 25 + 200 = v_0^2$$

$$\Rightarrow v_0^2 = 225 \Rightarrow v_0 = 15$$

Sol9. $PV^{\gamma} = \text{constant}$

$TV^{\gamma-1} = \text{constant}$

$TV^{\frac{1}{2}} = \text{constant}$

$T_1V_1^{1/2} = T_2(2V_1)^{1/2}$

$T_2 = \frac{T_1}{\sqrt{2}} \qquad \frac{nR}{r-1}(T_1 - T_2)$

Work done = $\frac{R\left(T - \frac{T}{\sqrt{2}}\right)}{\frac{3}{2} - 1} = \frac{R\left(T - \frac{T}{\sqrt{2}}\right)}{\frac{1}{2}} = RT\left(2 - \sqrt{2}\right)$

Sol10. $\mu = 7.0 \times 10^{-3}\, kg/m$

$T = 70$

$$\therefore v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{70}{7 \times 10^{-3}}} = \sqrt{10^4} = 100$$

Sol11. Micro wave $\rightarrow$ k Lystron value
Gamma Rays $\rightarrow$ Radio active decay of nucleus
Radio wave $\rightarrow$ Rapid acceleration & deceleration of e^-
X- ray $\rightarrow$inner shell electron

Sol12. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{m \times \varepsilon}}$

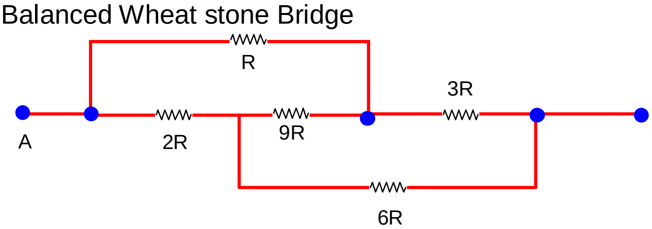
$\frac{h \times 10}{m_p} = \frac{v}{m_\alpha . v_\alpha} 4v\alpha = \frac{1 \times c}{10}$

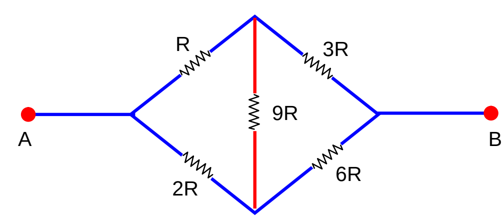
$v_\alpha = \frac{C}{40}$

$$\therefore \frac{(K6)_{proton} = \frac{1}{2} \times \frac{C^2}{100}}{(KE)_{\alpha particle} = \frac{1}{2} \times 4m \times \frac{C^2}{100 \times 16}} = \frac{4}{1}$$

$\therefore (KE)_p : (KE)_\alpha = 4:1$

Sol13. Balanced Wheat stone Bridge





$\frac{1}{R_{eq}} = \frac{1 \times 2}{4 \times 2} + \frac{1}{8} \Rightarrow R_{eq} = \frac{8}{3}$

Sol14. Range of FM broad cast = [88MHz, 108 MHz]
$\therefore$ 64 MHz

Sol15. $\therefore$ Helium $\Rightarrow$ 2 proton and 2 neutron

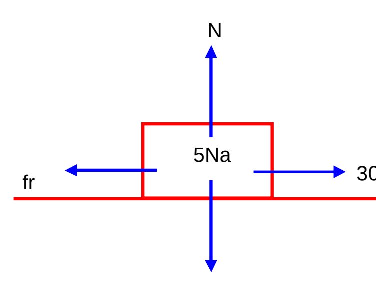
$\therefore \Delta m = |4.0015 - (2(1.0087 + 1.0073))| = |4.0016 - (2(2.016))| = |4.0016 - 4.032|$

$\Delta m = 0.0305$ amu $\Rightarrow$ mass defect

$\therefore BE = \Delta mc^2 = 0.0308 \times 9 \times 10^{16}$

$BE = 0.2745 \times 10^{16}$

Sol16. We know,



$S = \frac{1}{2}at^2$

$a = \frac{2s}{t^2} = \frac{2 \times 50}{10 \times 10} = 1 m/s$

$\therefore 30 - fr = 5 \times 1$

We know fr = μmg

$f_r = 25 \Rightarrow \mu \times 5 \times 10 = 25 \Rightarrow \mu = \frac{1}{2}$

Sol17. D. Sharpness of resonance $\rightarrow$ III Quality factor
C. Resonance phenomenon $\rightarrow$ I Presence of L & C.

As $\omega^2 = \dfrac{1}{LC}$

B. Transformer $\rightarrow$ IV Mutual induction.
A. AC generator $\rightarrow$ EMI

Sol18. $\frac{4}{3}\pi R^3 = 125 \times \frac{4}{3}\pi r^3 \Rightarrow R = 5r$

$\therefore$ New radius $= \dfrac{10^{-3}}{5} m \Rightarrow r = 2 \times 10^{-4} m$

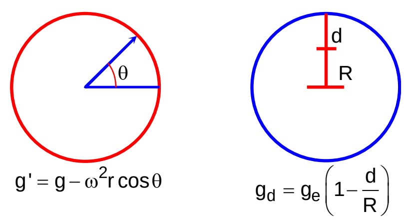
Surface energy = TA

$\therefore\ U_{Initial} = 0.45 \times 4\pi \times 10^{-8} = 180\pi \times 10^{-8}$

$U_{Final} = 0.45 \times 4\pi \times 10^{-8} = 1080\pi \times 10^{-8}$

$\therefore \Delta U = (1080 - 180)\pi \times 10^{-8} = 900\pi \times 10^{-8} = 9 \times 3.14 \times 10^{-6} = 28.26 \times 10^{-6}$

Sol19. Statement 1 is true.
Statement 2 is false.

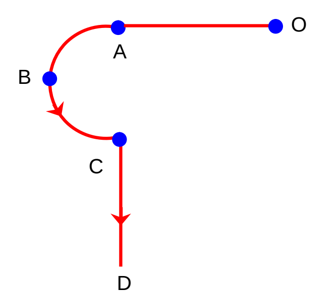


Sol20. $B_{(O \longrightarrow A)} = \dfrac{\mu_0 i}{4\pi r}$

$B_{(ABC)} = \dfrac{\mu_0 i}{4r}$

$B_{(CD)} = 0$

$\therefore \sum B = \dfrac{\mu_0 i}{2r}\left(\dfrac{1}{2} + \dfrac{1}{2\pi}\right)$



SECTION - B

Sol1. $I_{Cylinder} = \dfrac{mR^2}{2}$

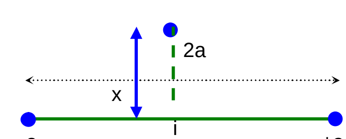
$\therefore$ Total KE at bottom $= \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2 = \dfrac{21}{22}mv^2 = \dfrac{1}{2} \times \dfrac{mR^2}{2} \times \dfrac{v^2}{R^2} = \dfrac{3}{4}mv^2$

$\therefore$ Conversation of energy $= \dfrac{3}{4}mv^2 = mg \times \dfrac{30}{100}$

$\Rightarrow v^2 = 4$
$\Rightarrow v = 2$

Sol2. x=2

As we know max force at $\dfrac{a}{\sqrt{2}}$



Sol3. $E = 12.75\text{eV} = 13.6\left[\frac{1}{1} - \frac{1}{n_2^2}\right]$

$12.75\text{eV} = 13.6\text{eV} - \frac{13.6}{n^2}$

$\frac{13.6}{n^2} = 0.95$

$n^2 = \frac{13.6 \times 100}{0.95 \times 100} \approx 16$

$n = 4$

Angular momentum $= \frac{nh}{2\pi} = \frac{4 \times h}{2\pi} = \frac{x}{\pi} \times 10^{-17}$

$x = 2h \times 10^{17} = 2 \times 4.14 \times 10^{-15} \times 10^{17}$

$x = 828$

Sol4. $W = \vec{F}.\vec{S} = \vec{F}.(\vec{r}_f - \vec{r}_i) = (5\hat{i} + 2\hat{j} + 7\hat{k})(5\hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} - 3\hat{j} + 4\hat{k}) = (5\hat{i} + 2\hat{j} + 7\hat{k})(3\hat{i} - 5\hat{j} + 5\hat{k})$

$\Rightarrow W = 15 - 10 + 35 = 40\text{J}$

Sol5. $r = \frac{mv}{qB} \Rightarrow \frac{3}{100} = \frac{m \times v}{2 \times 4 \times 10^{-9}} \Rightarrow 8 \times 3 \times 10^{-11} = mv \Rightarrow 24 \times 10^{-11} = mv$

$qV = \Delta KE$

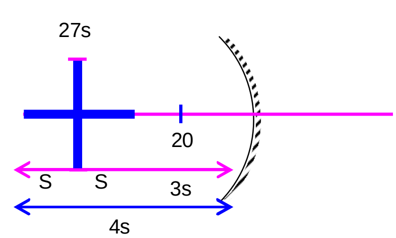
$2m \times qV = m^2v^2 = 24 \times 24 \times 10^{-22}$

$\Rightarrow m = \frac{24 \times 24 \times 10^{-22} \times 10^{-18}}{100 \times 2 \times 2 \times 10^{-4}} = 144 \times 10^{-18}$

Sol6. At resonance, any rate of energy supplied is max

$X_L = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega \times L} = \frac{1}{1000 \times \pi 70.6} \Rightarrow C = 40\mu F$

Sol7. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \leftarrow$ mirror formula



$\frac{1}{v_A} = \frac{-1}{20} + \frac{1}{35} = \frac{1}{5}\left(\frac{1}{7} - \frac{1}{4}\right) = \frac{1}{5}\left(\frac{-3}{28}\right)$

$v_A = \frac{-140}{3}$

$\frac{1}{v_B} = \frac{1}{5}\left(\frac{1}{9} - \frac{1}{4}\right) \Rightarrow v_B = -36$

$\therefore v_A - v_B = \frac{32}{3}$**,** Hence $x = 32$

Sol8. U = (constant) × $\sin^2 \omega t$
K = (constant) × $\cos^2 \omega t$
We know total energy = (constant) = K + U

$K = U\frac{(1.25)}{100} \times \frac{5}{4}$

$$A^2 - x^2 = \frac{5}{4}x^2 \Rightarrow 4A^2 - 4x^2 = 5x^2 \Rightarrow 4A^2 = 9x^2 \Rightarrow x = \frac{2}{3}A$$

Sol9. $B = \frac{-1}{V}\frac{dp}{dV}$

$$\therefore \frac{B_W}{B_L} = \left[-\frac{1}{(dP)_L}\left(\frac{dV}{V}\right)_L\right]\left[-(dP)_W\left(\frac{V}{dV}\right)_W\right] = \frac{0.03}{0.01} = \frac{3}{1}$$

Hence $\frac{3}{1} = \frac{3}{x} \Rightarrow x = 1$

Sol10. $\frac{E_1}{E_2} = \frac{x_1}{x_2} = \frac{40}{100} \Rightarrow \frac{E_1}{E_2} = \frac{3}{5}$

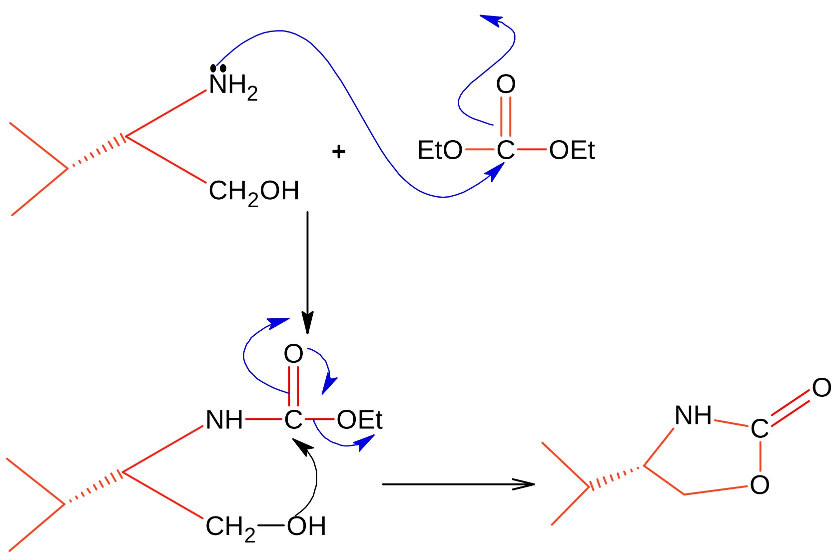
$$\Rightarrow E_2 = \frac{5}{3} \times \frac{15}{10} = \frac{25}{10} \Rightarrow x = 25$$

PART – B (CHEMISTRY)

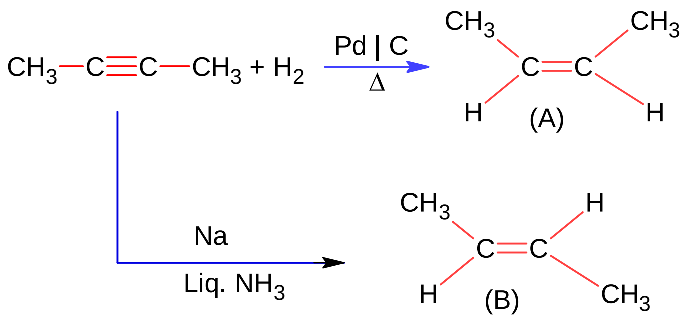
SECTION – A

Sol1. Photochemical smog is controlled by using catalytic converters in the automobile industry.

Sol2.



Sol3.



Correct options are not given because Trans is more stable than
Cis is polar and trans is non-polar
B.P of Cis form is greater due to polar nature.
M.P of trans form is greater due to packing efficiency.

Sol4. No pollution occurs by combustion of H_2 and very low density of H_2.

Sol5. Double salt contains two or more types of salts. Double salts are
$FeSO_4\ (NH_4)_2SO_4{,}6H_2O$ and $K_2SO_4.\ Al_2(SO_4)_3.2HH_2O$

Sol6. (A) Beryllium oxide is amphoteric in nature
(B) Beryllium carbonate is kept in atmosphere of CO_2 became it is thermally less stable.
(C) Beryllium sulphate is readily soluble in water due to high degree of hydration.
(D) Beryllium shows anamolous behaviour due to small size, high IE and high polarizing power.

Sol7. Formation of Prussian blue complex takes place which is $Fe_4[Fe(CN)_6]_3$

Sol8. Salaked lime $\longrightarrow Ca(OH)_2$
Dead burnt plaster $\longrightarrow CaSO_4$
Caustic soda $\longrightarrow$ NaOH.
Washing soda $\longrightarrow Na_2CO_3.10H_2O$

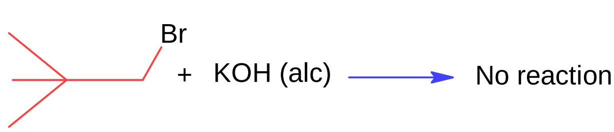
Sol9. Adsorption is directly proportion to Vander Waal's force of attraction.
$Z_C = \frac{3}{8}$ for all real gases.

Sol10. Tranquilizers $\longrightarrow$ Anti depressant drug
Aspirin $\longrightarrow$ Anti Blood clotting
Antibiotic $\longrightarrow$ Salvarsan
Antiseptic $\longrightarrow$ Soframincine.

Sol11. Resonating structure are hypothetical structure and resonance hybrid is real structure which is the actual representative.

Sol12. Splitting of d-orbitals takes place in pressure of strong field ligand and CN is the ligand in which maximum splitting occurs.

Sol13.



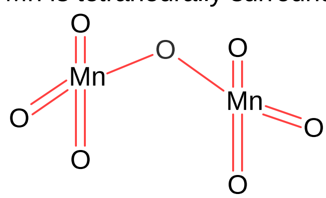
Elimination reaction takes place with alcohol KOH and β-hydrogen should be present.

Sol14. $2C(s) + O_2(g) \longrightarrow 2CO(g)$

ΔS_r^0 is +ve, $\Delta G_r^0 = \Delta H_r^0 - T.\Delta S_r^0$

As temperature increases ΔG_r^0 become more –ve thus it has lower tendency to get decomposed.

Sol15. Mn is tetrahedrally surrounded by oxygen atoms



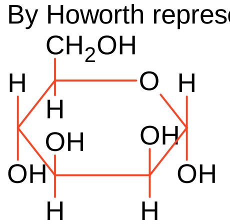
Sol16.

Test	**Functional group / class of compound.**
(A) Molisch's Test	(II) Carbolydrate
(B) Biuret test	(I) Peptide
(C) Carbylamine Test	(III) Primary amine
(D) Schiff's Test	(IV) Aldehyde.

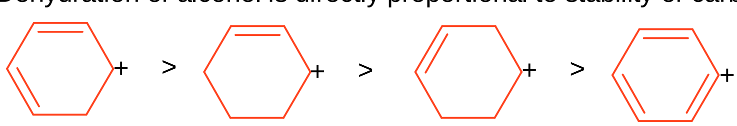
Sol17. Chlorine oxides : Cl_2O, ClO_2, Cl_2O_6, and Cl_2O_7 are highly reactive oxidizing agents and tend to explode.

Sol18. According to electrical neutrality principle in metal deficiency effect $3A^{2+}$ ions are replaced by $2A^{3+}$ ions, thus one vacant site per pair of A^{3+} is created.

Sol19. By Howorth representation, structure of manose is



Sol20. Dehydration of alcohol is directly proportional to stability of carbocation.



SECTION – B

Sol1. $HBrO_3$ (Bromic acid)

$O.S = +5$

$HBrO_4$ (Per bromic acid)

$O.S = +7$

Sum of the $O.S = 12$

Sol2. $2H_2O(g) \longrightarrow 2H_2(g) + O_2(g) + (242 \times 2)\,kJ/mol$

$H_2(g) + O_2(g) \longrightarrow 2OH + 78\,kJ/mol$

$H_2(g) \longrightarrow 2H + 430\,kJ/mol$

$$2H_2O \longrightarrow 2H + 2OH + 998\,kJ/mol$$

$$H_2O \longrightarrow H + OH + 998 \times \frac{1}{2} = 499\,kJ/mol$$

Sol3. $KCl \quad + AgNO_3 \longrightarrow AgCl + KNO_3$

$V = 25\,ml \qquad V = 20\,ml$

$M = 1M$

At equivalent point

M. mol of KCl = m mol of $AgNO_3$ = 20 m. mol

Volume of Solution = 25 ml

Mass of solution = 25 gm

Mass of solvent = 25– mass of solute

$= 25 - \left(20 \times 10^{-3} \times 74.5\right) = 23.57\,gm$

Molality of KCl $= \dfrac{20 \times 10^{-3}}{23.5 \times 10^{-3}} = 0.85$

i of KCl = 2 (100% ionization)

$\Delta T_f = i \times K_f \times m$

$= 2 \times 2 \times 0.085 = 3.4 \approx 3$

Sol4. $MnO_4^- + 8H^+ + 5e^- \rightleftharpoons Mn^{2+} + 4H_2O$

$$E = E^o - \frac{0.059}{5} \log \frac{\left[Mn^{2+}\right]}{\left[MnO_4^-\right]\left[H^+\right]^8}$$

$$1.282 = 1.54 - \frac{0.059}{5} \log \frac{10^{-3}}{10^{-1} \times \left[H^+\right]^8}$$

$$\frac{0.258 \times 5}{0.059} = \log \frac{10^{-2}}{\left[H^+\right]^8}$$

$21.86 = -2 + 8pH$

$pH = 2.98 \approx 3$

Sol5.

	$x(g)$	$\rightleftharpoons$	$Y(g)\ +$	$Z(g)$; $K_{p_1} = 3$
Initial moles	n		–	–
At equilibrium	$n - n\alpha$		$n\alpha$	$n\alpha$

$$K_{p_1} = \frac{\left(\dfrac{\alpha}{1+\alpha} \times p_1\right)^2}{\left(\dfrac{1-\alpha}{1+\alpha} \times p_1\right)} \Rightarrow 3 = \frac{\alpha^2 \times p}{1 - \alpha^2}$$

	$A(g) \rightleftharpoons$	$2B(g)$; $K_{p_2} = 1$	
Initial moles	n	–	
At equilibrium	$n - n\alpha$	$2n\alpha$	$P_{total} = P_2$

$$K_{p_2} = \frac{\left(\dfrac{2\alpha}{1+\alpha} \times p_2\right)^2}{\left(\dfrac{1-\alpha}{1+\alpha} \times P_2\right)} \Rightarrow 1 = \frac{4\alpha^2 \times p_2}{1 - \alpha^2}$$

$$\frac{K_{p_1}}{K_{p_2}} = \frac{p_1}{4p_2}$$

$$\frac{3}{1} = \frac{p_1}{4p_2} \left(\because p_1 : p_2 = 12:1\right)$$

$$x = 12$$

Sol6. As per information given in the question the compound should not be alcohol, aldehyde, ketone and it is aromatic as well

$\therefore$ Ph—CH=CH—OCH_3

Cis and trans form exist only.

Sol7. (A) $V_e = 1000 m/s,\ h = 6 \times 10^{-34} Js$

$M_e = 9 \times 10^{-31} Kg$

$$\lambda = \frac{h}{mv} = \frac{6 \times 10^{-34}}{9 \times 10^{-31} \times 1000} = 666.67 \times 10^{-9} M$$

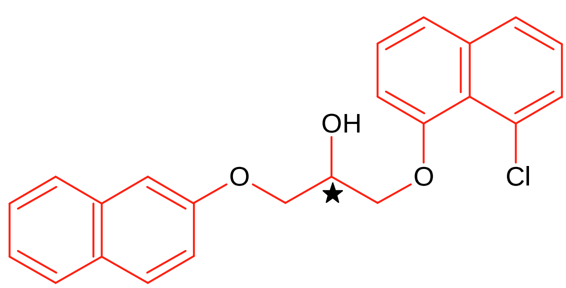
$= 666.67 nm$

(B) The characteristics of electrons emitted is independent of the material of the electrodes of the cathode ray tube.

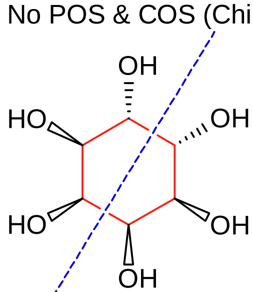
(C) The cathode rays start from cathode and move towards anode.

(D) The nature of the emitted electrons is independent of the nature of the gas present in the cathode ray tube.

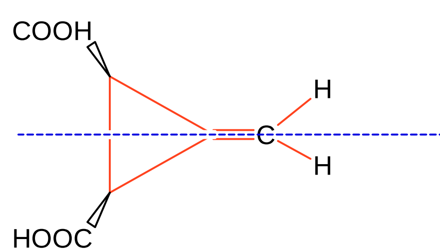
Sol8.



No POS & COS (Chiral)



(Achiral)



(Achiral)

Sol9. $$\text{Molality}(m) = \frac{1000 \times M}{1000 \times d - M \times \text{mol.wt. of solute}}$$

$$= \frac{1000 \times 3}{1000 \times 1 - (3 \times 58.5)} = 3.64$$

$$= 364 \times 10^{-2}$$

Sol10. For A

$C_t = Co.\, e^{-kt}$

$K = \frac{\ell n2}{t_{1/2}} = \frac{\ell n2}{15}$

$y = x.\, e^{-kt} = x.\; e^{-\left(\frac{\ell n2}{15}\right)t}$

For B

$K = \frac{\ell n2}{t_{1/2}} = \frac{\ell n2}{5}$

$y = 4.x.\quad e^{-\left(\frac{\ell n2}{5}\right)}t$

Now,

$x.\quad e^{-\left(\frac{\ell n2}{15}\right)t} = 4x.\; e^{-\left(\frac{\ell n2}{5}\right)t}$

$e^{t\left(\frac{\ell n2}{5} \times \frac{\ell n2}{15}\right)} = 4$

$t\left[\frac{\ell n2}{5} - \frac{\ell n2}{15}\right] = \ell n4$

$t = 15\,\text{min}$

PART – C (MATHEMATICS)

SECTION – A

Sol1. $\frac{dy}{dx} + \frac{x+a}{y-2} = 0$

$\Rightarrow (y-2)dy + (x+a)dx = 0$

$\Rightarrow (y-2)^2 + (x+a)^2 = c$

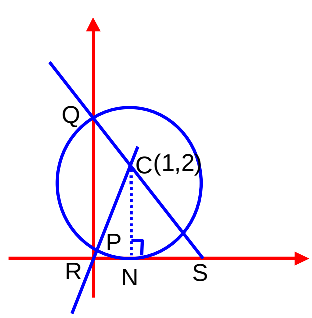
$y(1) = 0 \Rightarrow c = 4 + (1+a)^2$

$c : (y-2)^2 + (x+a)^2 = 4 + (1+a)^2$

Area enclosed by c is $4\pi \Rightarrow 1 + a = 0$

$\Rightarrow a = -1$

$c : (x-1)^2 + (y-2)^2 = 4$



$x = 0 \Rightarrow (y-2)^2 = 3$

$y = 2 \pm \sqrt{3}$

$P\left(0, 2-\sqrt{3}\right)$ & $Q\left(0, 2+\sqrt{3}\right)$

Equation QS

$y - 2 = -\sqrt{3}\left(x - 1\right)$

Cuts x axis at $S\left(1 + \frac{2}{\sqrt{3}}, 0\right)$

$NS = \frac{2}{\sqrt{3}}$

$\therefore RS = \frac{4}{\sqrt{3}}$

Sol2. $1 + 3 + 5 + x_4 + x_5 = 25$

$\Rightarrow x_4 + x_5 = 16$ ……….(i)

$\frac{1}{5}\left(1 + 9 + 25 + x_4^2 + x_5^2\right) - 25 = 8$

$\Rightarrow x_4^2 + x_5^2 = 33 \times 5 - 35 = 130$ ………..(ii)

$\therefore x_4 = 9$ & $x_5 = 7$ or $x_4 = 7$ and $x_5 = 9$

$\therefore x_4^3 + x_5^3 = 1072$

Sol3. $\cos^{-1}\left(2x\right) - 2\cos^{-1}\sqrt{1 - x^2} = \pi$

$\Rightarrow \cos^{-1} 2x = \pi + 2\cos^{-1}\sqrt{1 - x^2}$

as $0 \le \cos^{-1} y \le \pi$

$\therefore$ only possibility

$\cos^{-1} 2x = \pi$

$\Rightarrow 2x = \cos\pi = -1$

$\Rightarrow x = -\frac{1}{2}$

&

$2\cos^{-1}\sqrt{1 - x^2} = 0$

$\Rightarrow \cos^{-1}\sqrt{1 - x^2} = 0 \Rightarrow \sqrt{1 - x^2} = 1$

$\Rightarrow x = 0$

$\therefore$ no such value of x satisfy simultaneously.

Sol4. $R : \left\{\left(a, b\right) : 3a - 3b + \sqrt{7} \text{ is an irrational number}\right\}$

$3\left(a - b\right) + \sqrt{7} \in$ Irrational number

$\Rightarrow 3\left(a - a\right) + \sqrt{7} \in$ Irrational

$\therefore$ R is reflexive.

Now, if $a - b = \frac{\sqrt{7}}{3}$ then

$3\left(a - b\right) + \sqrt{7} = 2\sqrt{7} \in$ irrational

but $3\left(b - a\right) + \sqrt{7} = -\sqrt{7} + \sqrt{7} = 0 \notin$ irrational

$\therefore$ R is not symmetric.

R is not transitive too.

Sol5.

P	Q	$\sim Q$	$(\sim Q \wedge P)$	$Q \vee (\sim Q \wedge P)$	$\sim P$	$\sim P \wedge \sim Q$
T	T	F	F	T	F	T
T	F	T	T	T	F	T
F	T	F	F	F	T	F
F	F	T	F	T	T	F

Sol6. In $B(n,p)$ $\bar{x} = np$

$\alpha^2 = npq$

given sum $np + npq = 5$

$\Rightarrow np(1+q) = 5$ ……………(i)

and $(np)(npq) = 6$

$n^2p^2q = 6$ …………(ii)

$$\frac{n^2p^2(1+q)^2}{n^2p^2q} = \frac{25}{6}$$

$$\Rightarrow q^2 + 2q + 1 = \frac{25}{6}q$$

$$\Rightarrow (3q-2)(2q-3) = 0$$

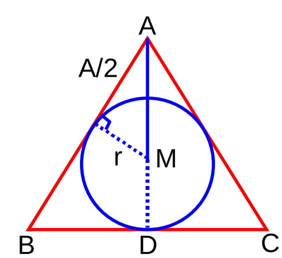
$$\Rightarrow q = \frac{2}{3} \text{ or } \frac{3}{2}$$

$$\Rightarrow p = \frac{1}{3}$$

Now, (i) $\rightarrow$ $n\frac{1}{3}\left(1+\frac{2}{3}\right) = 5 \Rightarrow n = 9$

$$\therefore 6(n+p-q) = 6\left(9+\frac{1}{3}-\frac{2}{3}\right) = 52$$

Sol7. $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$ which is minimum for $A = B = C = \frac{\pi}{3}$



r = 3

$AM = r\,\text{cosec}\,A/2 = 3\cos\frac{\pi}{6} = 6$ as ΔABC is equilateral

$\therefore$ M is centroid too.

If side length of Δ is a

then $AD = \frac{\sqrt{3}}{2}a$

$$AM = \frac{2AD}{3} = \frac{a}{\sqrt{3}}$$

$$6 = \frac{a}{\sqrt{3}} \Rightarrow a = 6\sqrt{3}$$

Area of $\Delta ABC = \frac{\sqrt{3}}{4}\left(6\sqrt{3}\right)^2 = 27\sqrt{3}$

Sol8. $\frac{dy}{dx} + y\tan x = x\sec x$

I.F.= $e^{\int \tan x\, dx} = e^{\ell n \sec x} = \sec x$

solution $y\sec x = \int x\sec^2 x\, dx$

$y\sec x = x\tan x - \ell n\sec x + c$

$1 = c$

$y\sec x = x\tan x - \ell n\sec x + 1$

$y\left(\frac{\pi}{6}\right) \Rightarrow y\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}\cdot\frac{1}{\sqrt{3}} - \ell n\left(\frac{2}{\sqrt{3}}\right) + 1$

$y = \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ell n\left(\frac{2}{e\sqrt{3}}\right)$

Sol9. $\left|\frac{z-2}{z-3}\right| = 2$

for $z = x + iy$

$(x-2)^2 + y^2 = 4\left[(x-3)^2 + y^2\right]$

$3x^2 + 3y^2 - 20x + 32 = 0$

$x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$

centre $(\alpha,\beta) = \left(\frac{10}{3}, 0\right)$ $\quad r = \sqrt{\frac{100}{9} - \frac{32}{3}} = \frac{2}{3}$

$\alpha = \frac{10}{3}, \beta = 0, \gamma = \frac{2}{3}$

$3(\alpha + \beta + \gamma) = 12$

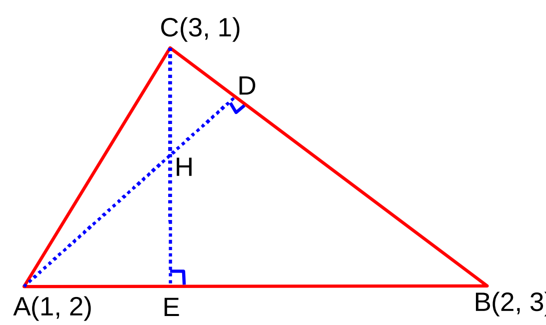
Sol10. Equation AD $y - 2 = \frac{1}{2}(x-1)$ ....(i)

Equation CE $y - 1 = -1(x-3)$ ....(ii)

Solving (i) & (ii) $H(\alpha,\beta) \equiv \left(\frac{5}{3}, \frac{7}{3}\right)$

$\alpha + 4\beta = 11$ & $4\alpha + \beta = 9$

$\therefore$ the required quadratic equation is $x^2 - 20x + 99 = 0$



Sol11. $f(x) = 2x + \tan^{-1} x$

$g(x) = \ell n\left(\sqrt{1+x^2} + x\right)$

Let $h(x) = 2x + \tan^{-1} x - \ell n\left(\sqrt{1+x^2} + x\right)$

$h'(x) = 2 + \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}} > 0 \ \forall x > 0$

$h''(x) = -\frac{2x}{\left(1+x^2\right)^2} + \frac{2x}{\left(1+x^2\right)^{3/2}}$

$$= \frac{2x}{\left(1+x^2\right)^{3/2}}\left[1-\frac{1}{\sqrt{1+x^2}}\right] > 0 \ \forall \ x \in [0,3]$$

$\therefore h'(x) > h'(0)$

$\Rightarrow f'(x) > g'(x) \ \forall x \in [0,3]$

and $f(x) > g(x) \forall x \in [0,3]$

$\therefore \max f(x) > \max g(x)$

Sol12. $S = \left\{x : x \in R \quad \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4} = 10\right\}$

Let $y = \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} \Rightarrow \frac{1}{y} = \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}$

$\therefore y + \frac{1}{y} = 10 \Rightarrow y = 5 \pm 2\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = 5+2\sqrt{6} = \left(\sqrt{3}+\sqrt{2}\right)^2$

$x^2 = 6 \Rightarrow x = \pm\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = 5-2\sqrt{6} = \left(\sqrt{3}-\sqrt{2}\right)^2 \Rightarrow x^2 - 4 = -2$

$x = \pm\sqrt{2}$

$n(5) = 4$

Sol13. $\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$\Rightarrow (\lambda+2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$\Rightarrow (\lambda+2)\begin{vmatrix} 1 & 1 & 1 \\ 0 & \lambda-1 & 0 \\ 0 & 0 & \lambda-1 \end{vmatrix} = 0$

$\Rightarrow (\lambda+2)(\lambda-1)^2 = 0$ for $\lambda = 1$ system will be consistent.

$\therefore \lambda = -2$

$\therefore \sum_{\lambda \in S}\left(|\lambda|^2 + |\lambda|\right) = 4 + 2 = 6$

Sol14. $t_n = \frac{n}{1+n^2+n^4} = \frac{n}{\left(n^2+n+1\right)\left(n^2-n+1\right)}$

$= \frac{1}{2}\left[\frac{1}{n^2-n+1} - \frac{1}{n^2+n+1}\right]$

$\therefore \Sigma t_n = \frac{1}{2}\left[1 - \frac{1}{n^2+n+1}\right]$

For n = 0

$$\Sigma t_n = \frac{1}{2}\left[1 - \frac{1}{111}\right] = \frac{55}{111}$$

Sol15. $2x^2 + xy - 3y^2 = 0$ ....(i)

Equation of angle bisectors between the lines of (i) is $\dfrac{x^2 - y^2}{2+3} = \dfrac{xy}{\frac{1}{2}}$

$\Rightarrow x^2 - y^2 = 10xy$

Sol16. $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$

$C_1 + C_2 + C_3$

$$f(x) = (2+\sin 2x)\begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 1 & 1+\cos^2 x & \sin 2x \\ 1 & \cos^2 x & 1+\sin 2x \end{vmatrix}$$

$R_3 - R_1, R_2 - R_1$

$$f(x) = (2+\sin 2x)\begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$f(x) = 2 + \sin 2x$

$\alpha = 2 + 1 = 3$ for $x = \dfrac{\pi}{4}$.

$\beta = 2 + \dfrac{\sqrt{3}}{2}$ for $x = \dfrac{\pi}{3}$

$\therefore \beta^2 - 2\sqrt{\alpha} = 4 + \dfrac{3}{4} + 2\sqrt{3} - 2\sqrt{3} = \dfrac{19}{4}$

Sol17. $\dfrac{1}{1!\,50!} + \dfrac{1}{3!\,48!} + \ldots\ldots + \dfrac{1}{51!\,1!}$

$= \dfrac{1}{51!}\left[\dfrac{51!}{1!\,50!} + \dfrac{51!}{3!\,48!} + \ldots\ldots. + \dfrac{51!}{51!\,1!}\right]$

$= \dfrac{1}{51!}\left[{}^{51}C_1 + {}^{51}C_3 + \ldots\ldots + {}^{51}C_{51}\right]$

$= \dfrac{2^{50}}{(51)!}$

Sol18. Image of $P(2, -1, 3)$ in $x + 2y - z = 0$ be $\dfrac{x-2}{1} = \dfrac{y+1}{2} = \dfrac{z-3}{-1} = \dfrac{-2(\cancel{2} - \cancel{2} - 3)}{1+4+1} = 1$

$x = 3, y = 1, z = 2$

$\therefore Q(3, 1, 2)$

$\therefore$ distance of $3x + 2y + z + 29 = 0$ from $Q = \left|\dfrac{9+2+2+29}{\sqrt{9+4+1}}\right| = \dfrac{42}{\sqrt{14}} = 3\sqrt{14}$

Sol19. $\dfrac{x-5}{1}=\dfrac{y-2}{2}=\dfrac{z-4}{-3}$ and $\dfrac{x+3}{1}=\dfrac{y+5}{4}=\dfrac{z-1}{5}$

$A\left(5\hat{i}+2\hat{j}+4\hat{k}\right)\ \ B\left(-3\hat{i}-5\hat{j}+\hat{k}\right)$

$\vec{n}_1=\hat{i}+2\hat{j}-3\hat{k}\quad \vec{n}_2=\hat{i}+4\hat{j}-5\hat{k}$

$$\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 1 & 4 & -5\end{vmatrix}=\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$\overrightarrow{AB}=-8\hat{i}-7\hat{j}-3\hat{k}$

$\therefore$ S.D. between the lines

$$=\left|\frac{\overrightarrow{AB}\cdot\left(\vec{n}_1\times\vec{n}_2\right)}{\left|\vec{n}_1\times\vec{n}_2\right|}\right|=\left|\frac{-8-7-3}{\sqrt{3}}\right|=6\sqrt{3}$$

Sol20. $\lim\limits_{n\to\infty}\left[\dfrac{1}{1+n}+\dfrac{1}{2+n}+\dfrac{1}{3+n}+....+\dfrac{1}{n+n}\right]$

$$\underset{n\to\infty}{\text{Lt}}\frac{1}{n}\sum_{r=1}^{n}\frac{n}{r+n}$$

$$=\underset{n\to\infty}{\text{Lt}}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{\left(\frac{r}{n}+1\right)}=\int_0^1\frac{dx}{x+1}=\ell n\left(x+1\right)=\ell n2$$

SECTION – B

Sol1. Let E(i) denote number of 3 digit numbers divisible by i.

$E\left(2\right)=450$

$E\left(3\right)=300$

$E\left(2\cap 3\right)=150$

$\therefore\left(2\cup 3\right)=450+300-150=600$

$E\left(2\cap 7\right)=64$

$E\left(3\cap 7\right)=43$

$E\left(2\cap 3\cap 7\right)=21$

$\therefore E\left\{\left(2\cap 7\right)\cup\left(3\cap 7\right)\right\}=64+43-21=86$

$\therefore$ Required numbers which are divisible by 2 or 3 but not by $7=600-86=514$

Sol2. $$\vec{v}\times\vec{w}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & 2 & -3\\ 2\alpha & 1 & -1\end{vmatrix}=\hat{i}-5\alpha\hat{j}-3\alpha\hat{k}$$

$\left[\vec{u}\ \vec{v}\ \vec{w}\right]=\vec{u}\cdot\left(\vec{v}\times\vec{w}\right)=\left|\vec{u}\right|\left|\vec{v}\times\vec{w}\right|\cos\theta=\alpha\sqrt{34\alpha^2+1}\cos\theta$

which is minimum for $\cos\theta=-1$

$\therefore -\alpha\sqrt{34\alpha^2+1}=-\alpha\sqrt{3401}$

$\therefore 34\alpha^2+1=3401$

$\Rightarrow \alpha^2 = 100 \Rightarrow \alpha = 10 > 0$

$\therefore \vec{u} = \lambda\left(\hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}\right)$

$\Rightarrow \left|\vec{u}\right| = \lambda\sqrt{3401}$

$\therefore \left|\vec{u}\cdot i\right|^2 = \lambda^2 = \dfrac{\alpha^2}{3401} = \dfrac{100}{3401} = \dfrac{m}{n}$

$\therefore m + n = 3501$

Sol3. $19^{200} + (23)^{200}$

$= (21-2)^{200} + (21+2)^{200}$

$= 2\left[(21)^{200} + {}^{200}C_2\,(21)^{198} + ... + {}^{200}C_{198}\,(21)^2 + 2^{200}\right]$

$\therefore$ All terms except 2^{201} is divisible by 49.

$2^{201} = \left(2^3\right)^{67} = (1+7)^{67}$

$= 1 + {}^{67}C_1(7) + \underbrace{{}^{67}C_2 7^2 + ... + {}^{67}C_{67} 7^{67}}_{\text{divisible by 49}}$

Now to check only

$1 + 67 \times 7 = 470$

$470 = 49 \times 9 + 29$

$\therefore$ Remainder = 29

Sol4. $A(2,6,2), B(-4,0,\lambda), C(2,3,-1)$ and $D(4,5,0)$

Area of quadrilateral ABCD

$= \dfrac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BD}\right| = 18$ given

$\Rightarrow \dfrac{1}{2}\left\|\begin{matrix} i & j & k \\ 0 & -3 & -3 \\ 8 & 5 & -\lambda \end{matrix}\right\| = 18$

$\Rightarrow \left|(3\lambda + 15)\hat{i} + 24\hat{j} - 24\hat{k}\right| = 36$

$\Rightarrow (3\lambda + 15)^2 + (24)^2 + (24)^2 = (36)^2$

$(3\lambda + 15)^2 = 144$

$\Rightarrow 3\lambda + 15 = \pm 12$

$\Rightarrow \lambda = -1$ or, -9 but $|\lambda| \le 5 \Rightarrow \lambda = -1$

$\therefore 5 - 6\lambda = 11$

Sol5. $f'(x) + f(x) = \int_0^2 f(t)\,dt$

I.F. = $e^{\int dx} = e^x$

$\therefore$ solution $e^x f(x) = \int a\, e^x dx$

where $a = \int_0^2 f(x)\,dx$

$\therefore e^x f(x) = a\, e^x + c$

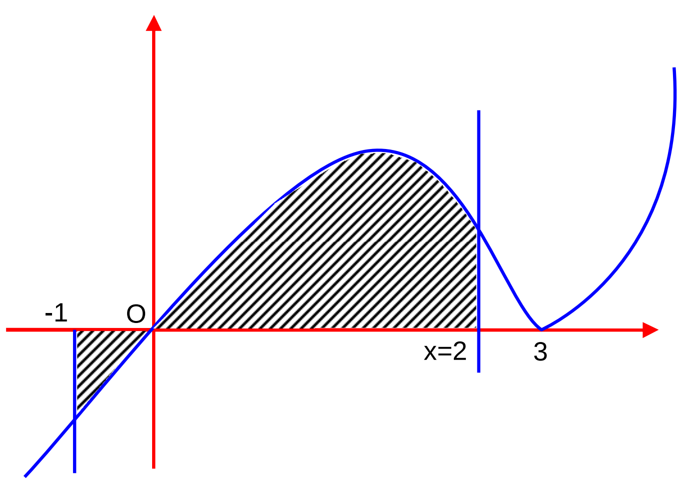
$f(0) = e^{-2} \Rightarrow e^{-2} = a + c$

$$\Rightarrow c = e^{-2} - a$$
$$e^x f(x) = ae^x + e^{-2} - a$$
$$\therefore f(x) = a + e^{-(x+2)} - ae^{-x} \text{ ....(i)}$$
$$a = \int_0^2 \left[a + e^{-(x+2)} - ae^{-x}\right]dx$$
$$a = e^{-2} - 1$$
$$\therefore f(x) = \left(e^{-2} - 1\right)\left(1 - e^{-x}\right) + e^{-(x+2)}$$
$$f(0) = e^{-2}$$
$$f(2) = -\left(1 - e^{-2}\right)^2 + e^{-4}$$
$$= -1 - e^{-4} + 2e^{-2} + e^{-4}$$
$$\therefore 2f(0) - f(2) = 1$$

Sol6. $\int_0^1 \left(x^{21} + x^{14} + x^7\right)\left(2x^{14} + 3x^7 + 6\right)^{1/7} dx$

$$= \int_0^1 \left(x^{20} + x^{13} + x^6\right)\left(2x^{21} + 3x^{14} + 6x^7\right)^{1/7} dx$$
Let $2x^{21} + 3x^{14} + 6x^7 = t^7$
$$\therefore 42\left(x^{20} + x^{13} + x^6\right)dx = 7t^6 dt$$
$$\Rightarrow \frac{1}{6}\int_0^{11^{\frac{1}{7}}} t^6 \cdot t\, dt = \left.\frac{t^8}{48}\right|_0^{11^{\frac{1}{7}}} = \frac{11^{8/7}}{48} = \frac{(11)^{m/n}}{\ell}$$
$$\ell = 48, m = 8, n = 7$$
$$\ell + m + n = 63$$

Sol7.



$$y = \begin{cases} x^2 - 3x\,;\, x \geq 3 \\ 3x - x^2\,;\, x < 3 \end{cases}$$

Required area = $\int_{-1}^{2} \left(3x - x^2\right)dx$

$$= \left|\int_{-1}^{0} \left(3x - x^2\right)dx\right| + \left|\int_{0}^{2} \left(3x - x^2\right)dx\right|$$

$= \left|\frac{3}{2}x^2 - \frac{x^3}{3}\right|_{-1}^{0} + \left|\frac{3}{2}x^2 - \frac{x^3}{3}\right|_{0}^{2}$

$= \left|\frac{3}{2} + \frac{1}{3}\right| + \left|6 - \frac{8}{3}\right| = \frac{11}{6} + \frac{10}{3} = \frac{31}{6}$

$\therefore 12A = 62$

Sol8. $a_1 = 8, a_2, a_3, ..., a_n$ in A.P.

$\frac{4}{2}\left[2 \times 8 + 3d\right] = 50 \Rightarrow d = 3$

$\frac{4}{2}\left[2 \times a_n - 3d\right] = 170 \Rightarrow a_n = 47$

$a_n = 47 = 8 + (n-1)3 \Rightarrow n = 14$

$\therefore$ middle terms are 7^{th} & 8^{th} term

$\therefore a_7 \cdot a_8 = (8 + 6 \times 3)(8 + 7 \times 3)$

$= 26 \times 29 = 754$

Sol9. ASSASSINATION

A→3 T→1

S→4 O→1

I→2

N→2 vowels AAAIIO

SS <u>AAAIIO</u> SSNTN

Required no. of arrangement = $\frac{8!}{4!\,2!} \times \frac{6!}{3! \times 2!} = 50400$

Sol10. $f(x) = x^2 + ax + b$

we have $a = g'(1)$ & $b = g''(2)$

$f'(x) = 2x + a$ & $f''(x) = 2$

$g(x) = (1 + a + b)x^2 + x(2x + a) + 2$

$= (a + b + 3)x^2 + ax + 2$

$g'(x) = 2(a + b + 3)x + a$

$g''(x) = 2(a + b + 3)$

Now, $a = g'(1) = 2(a + b + 3) + a$

$\Rightarrow a + b + 3 = 0$ …………..(i)

$\therefore g''(x) = 0 \ \forall \ x$

$b = g''(2) = 0$

$\therefore$ (i) $\Rightarrow a = -3$, $b = 0$

$\therefore f(x) = x^2 - 3x$

& $g(x) = -3x + 2$

$\therefore f(4) - g(4) = 4 - (-10) = 14$