PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

1. A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws, is

(1) $\frac{6}{11}$ (2) $\frac{1}{6}$

(3) $\frac{5}{11}$ (4) $\frac{5}{6}$

Ans. **(3)**

Sol. Required Probability = $\frac{5}{6}\times\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+.........$

$$= \frac{5}{36}\left[1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+.........\right] = \frac{5}{36}\times\frac{1}{1-\frac{25}{36}} = \frac{5}{11}$$

*2. If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P. then the common ratio of the G.P. is equal to

(1) 4 (2) 7

(3) 6 (4) 5

Ans. **(3)**

Sol. Let the G.P. is a, ar, ar^2, ...

$\Rightarrow \quad \frac{a(r^{64}-1)}{r-1} = 7\cdot\frac{a(r^{64}-1)}{r^2-1}$

$\Rightarrow \quad r + 1 = 7 \Rightarrow r = 6$

3. If $f(x)=\begin{cases}2+2x, & -1\le x<0\\ 1-\frac{x}{3}, & 0\le x\le 3\end{cases}$; $g(x)=\begin{cases}-x, & -3\le x\le 0\\ x, & 0<x\le 1\end{cases}$, then range of (fog)(x) is

(1) [0, 3) (2) [0, 1]

(3) [0, 1) (4) (0, 1]

Ans. **(2)**

Sol. $f(x)=\begin{cases}2+2x, & -1\le x<0\\ 1-\frac{x}{3}, & 0\le x\le 3\end{cases}$ and $g(x)=\begin{cases}-x, & -3\le x\le 0\\ x, & 0<x\le 1\end{cases}$

$\Rightarrow \quad f(g(x))=\begin{cases}1+\frac{x}{3}, & -3\le x\le 0\\ 1-\frac{x}{3}, & 0<x\le 1\end{cases}$

$\Rightarrow$ Range of f(g(x)) is [0, 1].

*4. In an A.P., the sixth term $a_6 = 2$. If the product $a_1a_4a_5$ is the greatest, then the common difference of the A.P. is equal to

(1) $\frac{2}{3}$ (2) $\frac{8}{5}$

(3) $\frac{5}{8}$ (4) $\frac{3}{2}$

Ans. (2)*

Sol. Let the A.P. is a, a + d, a + 2d, ……..

$\Rightarrow$ $a + 5d = 2$

$a_1.a_4.a_5 = a(a + 3d)(a + 4d)$

$\Rightarrow$ $a_1.a_4.a_5 = (2 - 5d)(2 - 2d)(2 - d) = f(d)$

$f(d) = -10d^3 + 34d^2 - 32d + 8$

$\Rightarrow$ $f'(d) = -30d^2 + 68d - 32$

$f'(d) = 0 \Rightarrow d = \frac{2}{3}$ and $\frac{8}{5}$

$f''(d) < 0$ for $d = \frac{8}{5}$ and $f''(d) > 0$ for $d = \frac{2}{3}$

*** Note :** f(d) attains local maximum at $d = \frac{8}{5}$ and not the absolute maximum value.

5. Consider the function $f:\left[\frac{1}{2},1\right]\to R$ defined by $f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$. Consider the statements :

(I) The curve y = f(x) intersects the x-axis is exactly at one point.

(II) The curve y = f(x) intersects the x-axis at $x = \cos\frac{\pi}{12}$

Then

(1) Both (I) and (II) are correct (2) Both (I) and (II) are incorrect

(3) Only (I) is correct (4) Only (II) is correct

Ans. (1)

Sol. $f:\left[\frac{1}{2},1\right]\to R,\ f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$

$f'(x) = 3\sqrt{2}(2x+1)(2x-1) > 0\ \forall\ x \in \left(\frac{1}{2},1\right]$

$\Rightarrow$ f(x) is strictly increasing in $\left[\frac{1}{2},1\right]$

$f\left(\frac{1}{2}\right) < 0,\ f(1) > 0$

$\Rightarrow$ f(x) = 0 has exactly one solution in $\left[\frac{1}{2},1\right]$

Also $f(x) = 0 \Rightarrow 4x^3 - 3x = \frac{1}{\sqrt{2}}$

Now put $x = \cos\theta \Rightarrow \cos3\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow$ $x = \cos\frac{\pi}{12}$.

6. Let A be a square matrix such that $AA^T = I$. Then $\frac{1}{2}A\left[\left(A+A^T\right)^2+\left(A-A^T\right)^2\right]$ is equal to

(1) $A^2 + I$ (2) $A^2 + A^T$

(3) $A^3 + I$ (4) $A^3 + A^T$

Ans. **(4)**

Sol. $A.A^T = I \Rightarrow (A + A^T)^2 = A^2 + (A^T)^2 + 2I$

$(A - A^T)^2 = A^2 + (A^T)^2 - 2I$

$\Rightarrow \frac{1}{2}A\left[\left(A+A^T\right)^2+\left(A-A^T\right)^2\right] = A^3 + A^T$

7. Let R be a relation on $Z \times Z$ defined by (a, b) R (c, d) if and only if ad – bc is divisible by 5. Then R is

(1) Reflexive and transitive but not symmetric
(2) Reflexive, symmetric and transitive
(3) Reflexive and symmetric but not transitive
(4) Reflexive but neither symmetric nor transitive

Ans. **(3)**

Sol. $\forall$ (a, b) $\in I \times I$, ab – ba = 0 $\Rightarrow$ (a, b) R(a, b) $\Rightarrow$ R is reflexive

(a, b) R(c, d) $\Rightarrow$ ad – bc is divisible by 5 $\Rightarrow$ cb – da is divisible by 5

$\Rightarrow$ (c, d) R(a, b) $\Rightarrow$ R is symmetric

If (a, b) R(c, d) and (c, d) R (e, f) $\Rightarrow$ ad – bc = 5λ and cf – de = 5μ

$\Rightarrow$ af – be is not necessarily a multiple of 5 $\Rightarrow$ R is not transitive.

8. Suppose $f(x)=\frac{\left(2^x+2^{-x}\right)\tan x\sqrt{\tan^{-1}\left(x^2-x+1\right)}}{\left(7x^2+3x+1\right)^3}$. Then the value of f'(0) is equal to

(1) π (2) $\frac{\pi}{2}$

(3) 0 (4) $\sqrt{\pi}$

Ans. **(4)**

Sol. $f(x)=\frac{\left(2^x+2^{-x}\right)\tan x\sqrt{\tan^{-1}\left(x^2-x+1\right)}}{\left(7x^2+3x+1\right)^3}$

$\ln f(x)=\ln\left(2^x+2^{-x}\right)+\ln(\tan x)+\frac{1}{2}\ln\left(\tan^{-1}\left(x^2-x+1\right)\right)-3\ln\left(7x^2+3x+1\right)$

$\Rightarrow \frac{f'(x)}{f(x)}=\frac{\left(2^x-2^{-x}\right)\ln 2}{2^x+2^{-x}}+\frac{\sec^2 x}{\tan x}+\frac{(2x-1)}{2\left(\tan^{-1}\left(x^2-x+1\right)\right)\left(1+\left(x^2-x+1\right)^2\right)}-\frac{3(14x+3)}{7x^2+3x+1}$

$\Rightarrow$ f'(0) = $\sqrt{\pi}$

9. For $x \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, if y(x) = $\int \frac{\text{cosec}x+\sin x}{\text{cosec}x\sec x+\tan x\sin^2 x}dx$ and $\lim\limits_{x\to\left(\frac{\pi}{2}\right)^-} y(x)=0$ then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (2) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(3) $\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$ (4) $\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Ans. **(3)**

Sol. $y(x) = \int \frac{\text{cosec}x + \sin x}{\text{cosec}x \sec x + \tan x \cdot \sin^2 x} dx$

$\Rightarrow y(x) = \int \frac{(1+\sin^2 x)\cos x}{1+\sin^4 x} dx$

Put sinx = t $\Rightarrow$ cosx dx = dt

$\Rightarrow \int \frac{(1+t^2)dt}{1+t^4} = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t - \frac{1}{t}}{\sqrt{2}}\right) + c$

$\Rightarrow y(x) = \frac{1}{\sqrt{2}} \tan^{-1}\left(-\frac{\cos^2 x}{\sqrt{2}\sin x}\right) + c$

$\lim_{x \to \frac{\pi}{2}^-} y(x) = 0 \Rightarrow c = 0$

$\Rightarrow y(x) = \frac{1}{\sqrt{2}} \tan^{-1}\left(-\frac{\cos^2 x}{\sqrt{2}\sin x}\right) \Rightarrow y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \tan^{-1}\left(-\frac{1}{2}\right)$

10. A function y = f(x) satisfies f(x) sin2x + sinx – (1 + $\cos^2$x) f'(x) = 0 with condition f(0) = 0. Then, $f\left(\frac{\pi}{2}\right)$ is equal to

(1) 1
(2) 2
(3) 0
(4) –1

Ans. **(1)**

Sol. f(x) . sin2x + sinx – (1 + $\cos^2$x) f'(x) = 0

$\Rightarrow$ (1 + $\cos^2$x) f'(x) – (sin2x) f(x) = sinx

$\Rightarrow \frac{d}{dx}(f(x)(1 + \cos^2 x)) = \sin x$

$\Rightarrow$ f(x) (1 + $\cos^2$x) = –cosx + c, As f(0) = 0 $\Rightarrow$ c = 1

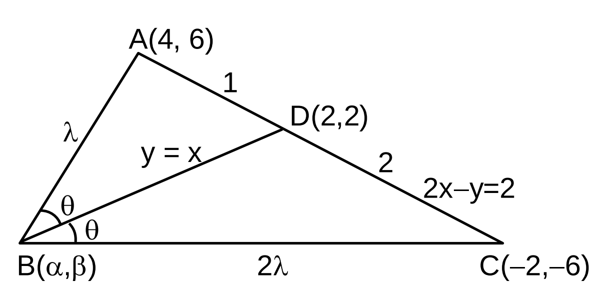
$\Rightarrow f(x) = \frac{1-\cos x}{1+\cos^2 x} \Rightarrow f\left(\frac{\pi}{2}\right) = 1$

*11. In a ΔABC, suppose y = x is the equation of the bisector of the angle B and the equation of the side AC is 2x – y = 2. If 2AB = BC and the points A and B are respectively (4, 6) and (α, β), then α + 2β is equal to

(1) 42
(2) 45
(3) 39
(4) 48

Ans. **(1)**

Sol. Let BD is angle bisector of ∠B



$\Rightarrow$ D is (2, 2)

Also $\frac{AD}{CD} = \frac{1}{2}$

$\Rightarrow$ C is (–2, –6)

Also (slope of AB) (slope of BC) = 1

$\Rightarrow \left(\frac{\beta - 6}{\alpha - 4}\right)\left(\frac{\beta + 6}{\alpha + 2}\right) = 1$

Also α = β $\Rightarrow$ α = β = 14

$\Rightarrow$ α + 2β = 42

*12. If α, $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ is the solution of $4\cos\theta + 5\sin\theta = 1$, then the value of $\tan\alpha$ is

(1) $\frac{\sqrt{10}-10}{6}$ (2) $\frac{\sqrt{10}-10}{12}$

(3) $\frac{10-\sqrt{10}}{12}$ (4) $\frac{10-\sqrt{10}}{6}$

Ans. **(2)**

Sol. $4\cos\theta + 5\sin\theta = 1$ has a solution $\theta = \alpha$, $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow$ $4 + 5\tan\alpha = \sec\alpha$
$\Rightarrow$ $16 + 25\tan^2\alpha + 40\tan\alpha = \sec^2\alpha$
$\Rightarrow$ $24\tan^2\alpha + 40\tan\alpha + 15 = 0$
$\Rightarrow$ $\tan\alpha = \frac{-10-\sqrt{10}}{12}, \frac{-10+\sqrt{10}}{12}$

$\tan\alpha = \frac{-\sqrt{10}-10}{12}$ is rejected.

13. $\lim_{x\to\frac{\pi}{2}}\left(\frac{1}{\left(x-\frac{\pi}{2}\right)^2}\int_{x^3}^{\left(\frac{\pi}{2}\right)^3}\cos\left(t^{1/3}\right)dt\right)$ is equal to

(1) $\frac{3\pi}{8}$ (2) $\frac{3\pi^2}{4}$

(3) $\frac{3\pi}{4}$ (4) $\frac{3\pi^2}{8}$

Ans. **(4)**

Sol. Let $\lim_{x\to\frac{\pi}{2}}\frac{\int_{x^3}^{(\pi/2)^3}\cos\left(t^{1/3}\right)dt}{\left(x-\frac{\pi}{2}\right)^2} = L$

Applying L-Hospital's Rule

$\Rightarrow$ $L = \lim_{x\to\frac{\pi}{2}}\frac{-\cos(x)3x^2}{2\left(x-\frac{\pi}{2}\right)} = \lim_{x\to\frac{\pi}{2}}\frac{\sin\left(x-\frac{\pi}{2}\right)3x^2}{2\left(x-\frac{\pi}{2}\right)}$

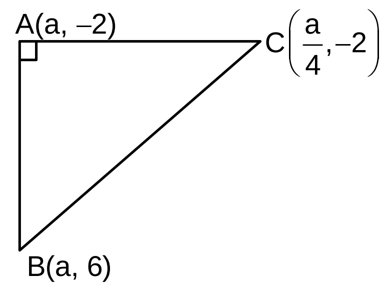
$\Rightarrow$ $L = \frac{3\pi^2}{8}$

*14. Let $\left(5, \frac{a}{4}\right)$ be the circumcenter of a triangle with vertices A(a, –2), B(a, 6) and $C\left(\frac{a}{4}, -2\right)$. Let α denote the circumradius, β denote the area and γ denote the perimeter of the triangle. Then $\alpha + \beta + \gamma$ is

(1) 30 (2) 53
(3) 62 (4) 60

Ans. **(2)**

Sol. As AB $\perp$ AC



$\Rightarrow$ circum-centre is mid-point of BC

$\Rightarrow \frac{a+\frac{a}{4}}{2} = 5 \Rightarrow a = 8$

$\Rightarrow$ vertices are A(8, –2), B(8, 6), C(2, –2)

$\Rightarrow \alpha = 5, \beta = 24, \gamma = 24$

$\Rightarrow \alpha + \beta + \gamma = 53$

15. Let A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ and $|2A|^3 = 2^{21}$ where $\alpha, \beta \in Z$. Then a value of α is

(1) 5 (2) 9
(3) 3 (4) 17

Ans. **(1)**

Sol. A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix} \Rightarrow |2A| = 2^3|A|$

$\Rightarrow |2A|^3 = 2^9.|A|^3 = 2^{21}$

$\Rightarrow |A| = 16 \Rightarrow \alpha^2 - \beta^2 = 16$

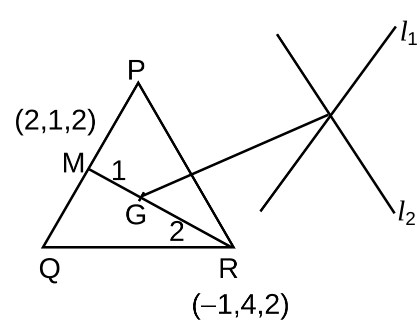
$\Rightarrow \alpha$ can be 5 for $\beta = \pm 3$.

16. Let PQR be a triangle with R (–1, 4, 2). Suppose M (2, 1, 2) is the mid point of PQ. The distance of the centroid of ΔPQR from the point of intersection of the lines $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is

(1) $\sqrt{99}$ (2) 9
(3) $\sqrt{69}$ (4) 69

Ans. **(3)**

Sol. G is centroid of ΔPQR



$\therefore G \equiv \left(\frac{2\times 2-1}{3}, \frac{2\times 1+4}{3}, \frac{2\times 2+2}{3}\right)$

G $\equiv$ (1, 2, 2)

Let x is the point of intersection of lines

$l_1 : \frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$

$l_2 : \frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$, then x $\equiv$ (2, –6, 0)

$\therefore$ distance between G and x = $\sqrt{69}$

17. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear. If $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}, \vec{b} + 6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a} + \alpha\vec{b} + \beta\vec{c} = \vec{0}$, then $\alpha + \beta$ is equal to

(1) –25 (2) 35
(3) –30 (4) 30

Ans. **(2)**

Sol. $\lambda_2\left(\vec{a}+5\vec{b}=\lambda_1\vec{c}\right)$ …(1)

$\vec{b}+6\vec{c}=\lambda_2\vec{a}$ …(2)

Add (1) and (2)

$\Rightarrow \left(5\lambda_2+1\right)\vec{b}=\left(\lambda_1\lambda_2-6\right)\vec{c}$

$\because$ $\vec{b}$ and $\vec{c}$ are non-collinear

$\Rightarrow \lambda_2=-\frac{1}{5}$ and $\lambda_1=-30$

$\Rightarrow \vec{a}+5\vec{b}+30\vec{c}=0 \Rightarrow \alpha = 5, \beta = 30 \therefore \alpha + \beta = 35$

*18. If $z=\frac{1}{2}-2i$ is such that $|z + 1| = \alpha z + \beta(1 + i)$, $i=\sqrt{-1}$ and $\alpha, \beta \in R$, then $\alpha + \beta$ is equal to

(1) –1 (2) –4

(3) 2 (4) 3

Ans. **(4)**

Sol. $z=\frac{1}{2}-2i$, $|z + 1| = \alpha z + \beta(1 + i)$; $\alpha, \beta \in R$

$\Rightarrow \alpha(-2) + \beta = 0$ …(i)

and $|z+1|^2=\left(\frac{\alpha}{2}+\beta\right)^2$

$\Rightarrow \left(\frac{3}{2}\right)^2+(2)^2=\left(\frac{5\alpha}{2}\right)^2 \Rightarrow \alpha=\pm1$

$\Rightarrow \alpha = 1$ and $\beta = 2 \Rightarrow \alpha + \beta = 3.$

19. Let O be the origin and the position vectors of A and B be $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is

(1) $\frac{3}{2}\sqrt{34}$ (2) $\frac{3}{2}\sqrt{31}$

(3) $\frac{2}{3}\sqrt{34}$ (4) $\frac{2}{3}\sqrt{31}$

Ans. **(3)**

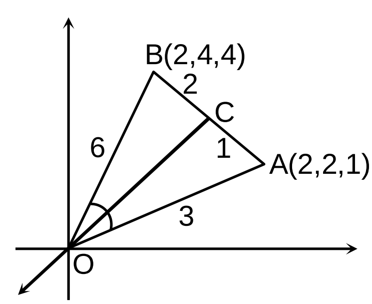
Sol. $\because$ OC bisects $\angle AOB$

$\therefore$ C divides AB in the ratio of 1 : 2

$\Rightarrow C \equiv \left(\frac{2+2\times2}{3},\frac{4+2\times2}{3},\frac{4+2\times1}{3}\right)$

$C \equiv \left(2,\frac{8}{3},2\right)$

OC = $\frac{2}{3}\sqrt{34}$



20. If the value of the integral $\int_{-\pi/2}^{\pi/2}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2023}}}\right)dx=\frac{\pi}{4}(\pi+a)-2$, then the value of a is

(1) 2 (2) $-\frac{3}{2}$

(3) $\frac{3}{2}$ (4) 3

Ans. **(4)**

Sol. Let I = $\int_{-\pi/2}^{\pi/2}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2023}}}\right)dx=\frac{\pi}{4}(\pi+a)-2$

$\Rightarrow$ I = $\int_0^{\pi/2} x^2\cos x\,dx+\int_0^{\pi/2}(1+\sin^2 x)dx$

I = $\frac{\pi^2}{4}-2+\frac{3\pi}{4}=\frac{\pi}{4}(\pi+3)-2 \Rightarrow a=3$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

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21. A line with direction ratio 2, 1, 2 meets the lines x = y + 2 = z and x + 2 = 2y = 2z respectively at the points P and Q. If the length of the perpendicular from the point (1, 2, 12) to the line PQ is *l*, then l^2 is _____ .

Ans. **65**

Sol. L_1 : x = y + 2 = z

L_2 : x + 2 = 2y = 2z

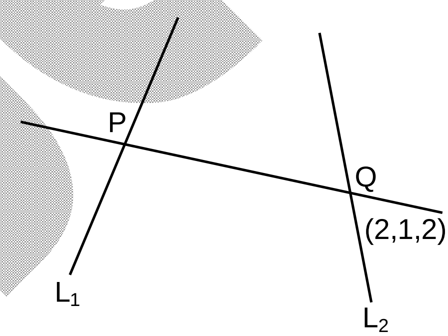
P ≡ (λ, λ – 2, λ)

Q ≡ $\left(\mu-2,\frac{\mu}{2},\frac{\mu}{2}\right)$

λ – μ + 2 = 2k

λ – $\frac{\mu}{2}$ – 2 = k $\Rightarrow$ λ = 6, μ = 4

λ – $\frac{\mu}{2}$ = 2k



Equation of PQ is $\frac{x-6}{2}=\frac{y-4}{1}=\frac{z-6}{2}$

Now length of perpendicular from the point (1, 2, 12) to the line PQ = $\sqrt{5^2+2^2+6^2}=\sqrt{65}$ = *l*

$\therefore$ l^2 = 65

22. The area (in sq. units) of the part of the circle $x^2 + y^2 = 169$ which is below the line 5x – y = 13 is $\frac{\pi\alpha}{2\beta}-\frac{65}{2}+\frac{\alpha}{\beta}\sin^{-1}\left(\frac{12}{13}\right)$, where α, β are coprime number. Then α + β is equal to ____ .

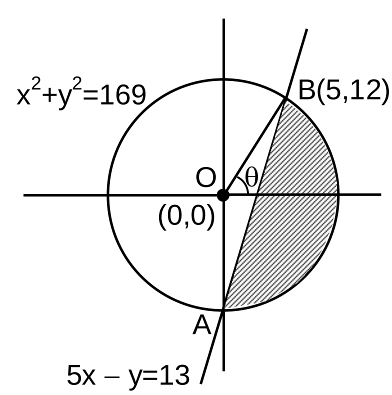
Ans. **171**

Sol. Area of shaded region = Area of sector $\widehat{AOB}$ – area of ΔAOB

= $\frac{(13)^2}{2}\left(\frac{\pi}{2}+\theta\right)-\frac{(13)^2}{2}\sin\left(\frac{\pi}{2}+\theta\right)$

= $\frac{169\pi}{2\times 2}-\frac{65}{2}+\frac{169}{2}\sin^{-1}\left(\frac{12}{13}\right)$ $\left(\because \sin\theta=\frac{12}{13}\right)$

$\Rightarrow$ α = 169, β = 2 $\Rightarrow$ α + β = 171



23. If the solution curve y = y(x) of the differential equation $(1 + y^2)(1 + \log_e x)\,dx + x\,dy = 0$, x > 0 passes through the point (1, 1) and $y(e) = \dfrac{\alpha - \tan\left(\frac{3}{2}\right)}{\beta + \tan\left(\frac{3}{2}\right)}$, then $\alpha + 2\beta$ is _____ .

Ans. **3**

Sol. $(1 + y^2)(1 + \ln x)\,dx + x\,dy = 0$

$\Rightarrow \int \frac{(1+\ln x)dx}{x} = -\int \frac{dy}{1+y^2}$

$\frac{(1+\ln x)^2}{2} = -\tan^{-1}(y) + c$

$\because$ curve passes through (1, 1)

$\Rightarrow c = \frac{1}{2} + \frac{\pi}{4}$

$\Rightarrow \tan^{-1} y = \frac{\pi}{4} + \frac{1}{2} - \frac{(1+\ln x)^2}{2}$

$y(e) = \frac{1-\tan(3/2)}{1+\tan(3/2)} \Rightarrow \alpha = \beta = 1$

$\alpha + 2\beta = 3$.

*24. If the mean and variance of the data 65, 68, 58, 44, 48, 45, 60, α, β, 60 where $\alpha > \beta$ are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to ______ .

Ans. **6344**

Sol. Mean of 10 observations (65, 68, 58, 44, 48, 45, 60, α, β, 60) is 56

i.e. $\bar{x} = 56$

variance $(\sigma^2) = \frac{\Sigma x_i^2}{n} - \bar{x}^2$

$\Rightarrow a^2 + b^2 = 6344$

*25. If $\frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + ... + \frac{^{11}C_9}{10} = \frac{n}{m}$ with gcd(n, m) = 1, then n + m is equal to _____ .

Ans. **2041**

Sol. $\frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + ... + \frac{^{11}C_9}{10} = \frac{n}{m}$, gcd(n, m) = 1

$\because \frac{^{n}C_r}{(r+1)} = \frac{^{n+1}C_{r+1}}{(n+1)}$

$\Rightarrow \sum_{r=1}^{9}\left(\frac{^{11}C_r}{r+1}\right) = \frac{1}{12}\left(\sum_{r=1}^{9} {^{12}C_{r+1}}\right) = \frac{2^{12}-26}{12} = \frac{2035}{6} = \frac{n}{m}$

n + m = 2041

*26. If the points of intersection of two distinct conics $x^2 + y^2 = 4b$ and $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ lie on the curve $y^2 = 3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is _____ .

Ans. **432**

Sol. $x^2 + y^2 = 4b$ …(1)

$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ …(2)

and $y2 = 3x^2$ …(3)

Solving equation (1), (2), (3)
for point of intersection, we get
$x^2 = b$ and $b^2 - 16b + 48 = 0$
$b = 4$ or 12

$\because$ $b = 4$ is rejected as conics are distinct
$x = \pm 2\sqrt{3}$, $y = \pm 6$ (vertices of rectangle)

$\therefore$ $\text{Area}(A) = 4\sqrt{3} \times 12$
now $3\sqrt{3} \times A = 48 \times 9 = 432$

*27. Let α, β be the roots of the equation $x^2 - x + 2 = 0$ with $\text{Im}(\alpha) > \text{Im}(\beta)$. Then $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$ is equal to _______ .

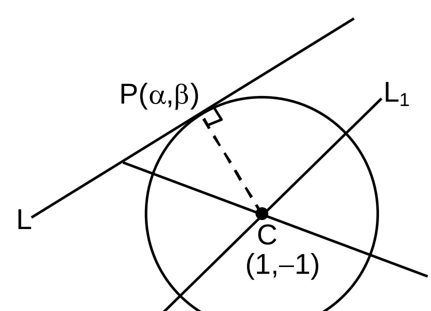
Ans. **13**

Sol. $x^2 - x + 2 = 0$, $\text{Im}(\alpha) > \text{Im}(\beta)$
now, $\alpha^2 - \alpha + 2 = 0 \Rightarrow \alpha^2 = 2 - \alpha$ and $\beta^2 = 2 - \beta$ …(1)
and $\alpha + \beta = 1$, $\alpha\beta = 2$ …(2)
$\therefore$ $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2 = -3(\alpha + \beta) + 16$ (using equation (1) and (2))
$= 16 - 3 = 13$

*28. Equations of two diameters of a circle are $2x - 3y = 5$ and $3x - 4y = 7$. The line joining the points $\left(-\frac{22}{7}, -4\right)$ and $\left(-\frac{1}{7}, 3\right)$ intersects the circle at only one point $P(\alpha, \beta)$. Then, $17\beta - \alpha$ is equal to _______ .

Ans. **2**

Sol. Given $L_1 : 2x - 3y = 5$
$L^2 : 3x - 4y = 7$
$\Rightarrow$ $c \equiv (1, -1)$
Equation of line L passing through
$\left(-\frac{22}{7}, -4\right)$ and $\left(-\frac{1}{7}, 3\right)$ is $3y - 7x = 10$
Now point $P(\alpha, \beta)$ is the point of contact
$\therefore$ $3\beta - 7\alpha = 10$ …(1)
and $\left(\frac{\beta + 1}{\alpha - 1}\right) = -\frac{3}{7} \Rightarrow 7\beta + 3\alpha = -4$ …(2)
Equation (1) + 2 × Equation (2)
$\Rightarrow$ $17\beta - \alpha = 2 \times -4 + 10$
$17\beta - \alpha = 2$



*29. All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" is______ .

Ans. **553**

Sol. Rank of GTWENTY
$= \frac{6!}{2!} + \frac{5!}{2!} \times (1 + 1) + 4! \times (1 + 1 + 1) + 1 = 553$

30. Let $f(x) = 2^x - x^2$, $x \in R$. If m and n are respectively the number the number of points at which the curves $y = f(x)$ and $y = f'(x)$ intersect the x-axis, then the value of m + n is ______ .

Ans. **5**

Sol. $f(x) = 2^x - x^2$, $x \in R$

$y = f(x)$ cuts x-axis at 3 points $\therefore m = 3$

and $y = f'(x) = 2^x \ln 2 - 2x$ cuts x-axis at 2 points

$\therefore n = 2 \quad \therefore m + n = 5$

FIITJEE

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

31. The de-Broglie wavelength of an electron is the same as that of a photon. If velocity of electron is 25% of the velocity of light, then the ratio of K.E. of electron and K.E. of photon will be:

(1) $\frac{1}{4}$ (2) $\frac{1}{1}$

(3) $\frac{8}{1}$ (4) $\frac{1}{8}$

Ans. (4)

Sol. Given that ($v_e = 0.25c$)

For photon,

$$\lambda_P = \frac{hc}{E_P}$$

For electron,

$$\lambda_e = \frac{h}{m_e v_e} = \frac{hv_e}{2K_e}$$

As $\lambda_P = \lambda_e$

$$\Rightarrow \frac{hc}{E_P} = \frac{hv_e}{2K_e}$$

$$\frac{hc}{E_P} = \frac{h(0.25c)}{2K_e}$$

$$\frac{K_e}{E_P} = \frac{1}{8}$$

*32. If the radius of curvature of the path of two particles of same mass are in the ratio 3 : 4, then in order to have constant centripetal force, their velocities will be in the ratio of:

(1) $1:\sqrt{3}$ (2) $\sqrt{3}:1$

(3) $\sqrt{3}:2$ (4) $2:\sqrt{3}$

Ans. (3)

Sol. Given that,

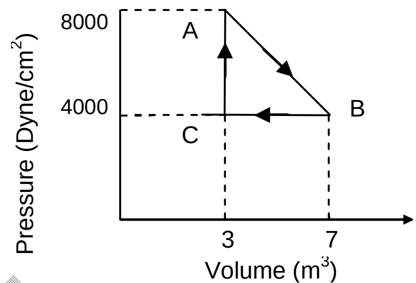
$$\frac{mv^2}{r} = \text{constant and } m_1 = m_2 = m$$

$$\therefore \frac{mv_1^2}{r_1} = \frac{mv_2^2}{r_2}$$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{3}{4}}$$

*33. A thermodynamic system is taken from an original state A to an intermediate state B by a linear process as shown in the figure. It's volume is then reduced to the original value from B to C by an isobaric process. The total work done by the gas from A to B and B to C would be:

(1) 2200 J (2) 1200 J
(3) 33800 J (4) 600 J



Ans. 800 J (No option is matching)

Sol. Work done by the gas from A to B and B to C = Area of triangle ABC

$$W = \frac{1}{2}(800 - 400) \times (7 - 3) = 800\,J$$

*34. A body starts moving from rest with constant acceleration covers displacement S_1 in first (p – 1) seconds and S_2 in first p seconds. The displacement $S_1 + S_2$ will be made in time.

(1) $\sqrt{(2p^2 - 2p + 1)}\,s$ (2) $(2p^2 - 2p + 1)\,s$
(3) $(2p + 1)\,s$ (4) $(2p - 1)\,s$

Ans. (1)

Sol. $S_1 = \frac{1}{2}a(p-1)^2$ …(i)

$S_2 = \frac{1}{2}ap^2$ …(ii)

and $S_1 + S_2 = \frac{1}{2}at^2$ …(iii)

Putting S_1 and S_2 in equation (iii), we get

$t = \sqrt{2p^2 - 2p + 1}$

*35. Given below are two statements:

Statement (I): If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in hot water.

Statement (II): If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in cold water.

In the light of the above statements, choose the most appropriate from the options given below:

(1) **Statement I** is false but **Statement II** is true
(2) Both **Statement I** and **Statement II** are false
(3) **Statement I** is true but **Statement II** is false
(4) Both **Statement I** and **Statement II** are true

Ans. (3)

Sol. $h = \frac{2T\cos\theta}{\rho g r}$

and surface tension decreases with the increase of temperature

36. Match List I with List II

	List-I		List-II
(A)	$\oint \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$	(I)	Gauss' law for electricity
(B)	$\oint \vec{E}.\vec{dl} = \frac{d\phi_B}{dt}$	(II)	Gauss law for magnetism
(C)	$\oint \vec{E}.\vec{dA} = \frac{Q}{\varepsilon_0}$	(III)	Faraday law
(D)	$\oint \vec{B}.\vec{dA} = 0$	(IV)	Ampere – Maxwell law

Choose the correct answer from the options given below:
(1) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)
(2) (A) – (IV), (B) – (I), (C) – (III), (D) – (II)
(3) (A) – (IV), (B) – (III), (C) – (I), (D) – (II)
(4) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

Ans. **(3)**
Sol. Maxwell's equations of electromagnetism

37. The explosive in a Hydrogen bomb is a mixture of ${}_1H^2, {}_1H^3$ and ${}_3Li^6$ in some condensed form. The chain reaction is given by:

$${}_3Li^6 + {}_0n^1 \rightarrow {}_2He^4 + {}_1H^3$$

$${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1$$

During the explosion the energy released is approximately
[Given: M (Li) = 6.01690 amu, M $({}_1H^2)$ = 2.01471 amu, M$({}_2He^4)$ = 4.00388 amu, and 1 amu = 931.5 MeV]
(1) 28.12 MeV
(2) 16.48 MeV
(3) 12.64 MeV
(4) 22.22 MeV

Ans. **(4)**
Sol. Q = $(\Delta M)c^2$

Q = $[M(Li) + M({}_1H^2) - 2M({}_2H^4)] \times 931.5$ MeV

*38. Two vessels A and B are of the same size and are at same temperature. A contains 1g of hydrogen and B contains 1g of oxygen. P_A and P_B are the pressures of the gases in A and B respectively, then $\frac{P_A}{P_B}$ is:
(1) 8
(2) 16
(3) 32
(4) 4

Ans. **(2)**
Sol. Given that
$V_A = V_B$ and $T_A = T_B$
PV = nRT

$$\therefore \frac{P}{n} = \text{constant}$$

$$\frac{P_A}{P_B} = \frac{n_A}{n_B} = \frac{1/2}{1/32} = 16$$

39. A galvanometer having coil resistance 10Ω shows a full scale deflection for a current of 3mA. For it to measure a current of 8A, the value of the shunt should be:
(1) $4.85 \times 10^{-3}\Omega$
(2) $3 \times 10^{-3}\Omega$
(3) $3.75 \times 10^{-3}\Omega$
(4) $2.75 \times 10^{-3}\Omega$

Ans. **(3)**

Sol. Given that

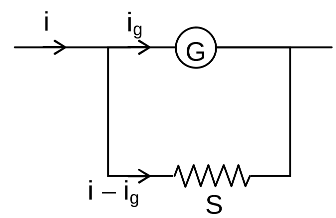
G = 10 Ω

i_g = 3mA

i = 8A

$i_g G = (i - i_g)S$

$\Rightarrow S = 3.75 \times 10^{-3}\ \Omega$



40. A convex mirror of radius of curvature 30 cm forms an image that is half the size of the object. The object distance is:

(1) –15 cm (2) – 45 cm

(3) 45 cm (4) 15 cm

Ans. **(1)**

Sol. Given that

$f = \frac{R}{2} = +15$ cm

$m = +1/2 = -\frac{v}{u}$

$v = -\frac{u}{2}$

using mirror formula

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$\frac{1}{15} = \frac{1}{u} + \frac{1}{-u/2}$

$\Rightarrow u = -15$ cm

41. A biconvex lens of refractive index 1.5 has a focal length of 20 cm in air. Its focal length when immersed in a liquid of refractive index 1.6 will be:

(1) – 16 cm (2) + 16 cm

(3) + 160 cm (4) – 160 cm

Ans. **(4)**

Sol. $\frac{1}{f_a} = (1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{20}$

$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{10}$

$\frac{1}{f_m} = \left(\frac{\mu_L}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f_m} = \left(\frac{1.5-1.6}{1.6}\right) \times \frac{1}{10}$

$f_m = -160$ cm

42. The electric current through a wire varies with time as $I = I_0 + \beta t$, where $I_0 = 20A$ and $\beta = 3A/s$. The amount of electric charge crossed through a section of the wire in 20 s is:

(1) 1600 C (2) 800 C

(3) 80 C (4) 1000 C

Ans. **(4)**

Sol. $I = \frac{dQ}{dt}$

Given that

$I = I_0 + \beta t = (20 + 3t)$

$\Rightarrow \int_0^Q dQ = \int_0^{20} (20+3t)dt$

$Q = \left[20t\right]_0^{20} + \left[\frac{3t^2}{2}\right]_0^{20}$

$Q = (20\times20) + \left(\frac{3\times20\times20}{2}\right)$

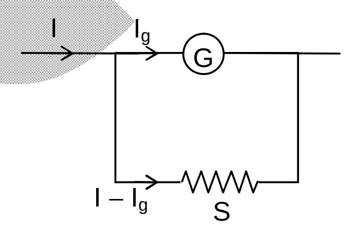
Q = 1000 C

43. The deflection in moving coil galvanometer falls from 25 divisions to 5 division when a shunt of 24Ω is applied. The resistance of galvanometer coil will be:

(1) 100Ω
(2) 48Ω
(3) 12Ω
(4) 96Ω

Ans. **(4)**

Sol. $\frac{I_g}{I} = \frac{5}{25} = \frac{1}{5} \Rightarrow I_g = \frac{1}{5}I$



In case of shunted galvanometer (S = 24 Ω)

$(I - I_g)S = I_g G$

$\left(I - \frac{1}{5}I\right)\times 24 = \frac{1}{5}I\cdot G$

$\left(\frac{5-1}{5}\right)\times 24 = \frac{1}{5}\times G$

G = 96 Ω

*44. A block of mass 100 kg slides over a distance of 10 m on a horizontal surface. If the co-efficient of friction between the surfaces is 0.4, then the work done against friction (in J) is:

(1) 4000
(2) 3900
(3) 4200
(4) 4500

Ans. **(1)**

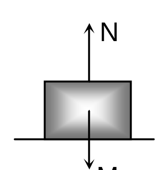
Sol. $F = \mu N$

N = mg

$W = F \times x$

$= 0.4 \times 100 \times 10 \times 10$

W = 4000 J



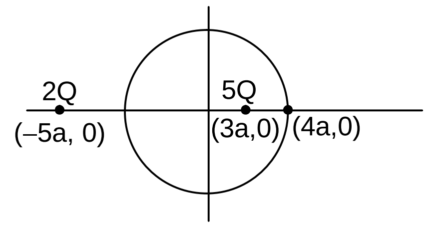
45. Two charges of 5Q and –2Q are situated at the points (3a, 0) and (–5a, 0) respectively. The electric flux through a sphere of radius '4a' having center at origin is:

(1) $\frac{3Q}{\varepsilon_0}$
(2) $\frac{5Q}{\varepsilon_0}$
(3) $\frac{7Q}{\varepsilon_0}$
(4) $\frac{2Q}{\varepsilon_0}$

Ans. **(2)**

Sol. $\phi = \frac{q_{enclosed}}{\varepsilon_0}$

$\phi = \frac{5Q}{\varepsilon_0}$



46. A capacitor of capacitance $100\,\mu F$ is charged to a potential of 12 V and connected to a 6.4 mH inductor to produce oscillations. The maximum current in the circuit would be:

(1) 1.2 A
(2) 1.5 A
(3) 3.2 A
(4) 2.0 A

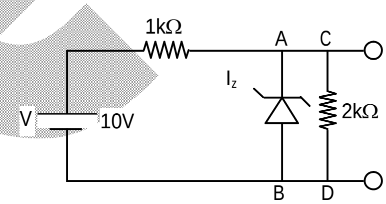
Ans. **(2)**

Sol. $\frac{1}{2}Li_{max}^2 = \frac{1}{2}CV^2$

$i_{max} = \sqrt{\frac{C}{L}}V$

$i_{max} = 1.5$ A

47. In the given circuit, the breakdown voltage of the Zener diode is 3.0 V. What is the value of I_z ?



(1) 7 mA
(2) 5.5 mA
(3) 10 mA
(4) 3.3 mA

Ans. **(2)**

Sol. $I_{battery} = \frac{10-3}{1000} = 7\ mA$

$I_{2k\Omega} = \frac{3}{2000} = 1.5 mA$

$I_z = (7 - 1.5)$ mA

$I_z = 5.5$ mA

*48. At what distance above and below the surface of the earth a body will have same weight. (take radius of earth as R.)

(1) $\frac{R}{2}$
(2) $\sqrt{5}R - R$
(3) $\frac{\sqrt{3}R - R}{2}$
(4) $\frac{\sqrt{5}R - R}{2}$

Ans. **(4)**

Sol. Given, d = h

$$\frac{g}{\left(1+\frac{h}{R}\right)^2} = g\left(1-\frac{d}{R}\right)$$

$$\frac{g}{\left(1+\frac{h}{R}\right)^2} = g\left(1-\frac{h}{R}\right)$$

$\Rightarrow h^2 + Rh - R^2 = 0$

$h = \frac{\sqrt{5}R - R}{2}$

49. The resistance $R = \frac{V}{I}$ where $V = (200 \pm 5)$ V and $I = (20 \pm 0.2)$ A, the percentage error in the measurement of R is:
(1) 3.5 % (2) 5.5%
(3) 7% (4) 3%

Ans. (1)

Sol. R = V/I

$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$

$= \frac{5}{200} \times 100 + \frac{0.2}{20} \times 100$

$= 3.5\%$

*50. The potential energy function (in J) of a particle in a region of space is given as $U = (2x^2 + 3y^3 + 2z)$. Here x, y and z are in meter. The magnitude of x-component of force (in N) acting on the particle at point P(1, 2, 3) m is:
(1) 6 (2) 2
(3) 8 (4) 4

Ans. (4)

Sol. $U = (2x^2 + 3y^3 + 2z)$

$F_x = -\frac{\partial U}{\partial x} = -4x$

at x = 1

$|F_x| = 4$ N

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

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*51. When the displacement of a simple harmonic oscillator is one third of its amplitude, the ratio of total energy to the kinetic energy is $\frac{x}{8}$, where x = __________ .

Ans. 9

Sol. $x = \frac{1}{3}A$

$\frac{T.E.}{K.E.} = \frac{\frac{1}{2}m\omega^2 A^2}{\frac{1}{2}m\omega^2(A^2 - x^2)} = \frac{A^2}{A^2 - x^2} = \frac{A^2}{A^2 - \frac{A^2}{9}} = \frac{9}{8}$

52. A square loop of side 10 cm and resistance 0.7Ω is placed vertically in east-west plane. A uniform magnetic field of 0.20 T is set up across the plane in north east direction. The magnetic field is decreased to zero in 1 s at a steady rate. Then, magnitude of induced emf is $\sqrt{x}\times10^{-3}$ V. The value of x is __________ .

Ans. **2**

Sol. $\phi_1 = BA\cos\theta$

$= 0.20\times\left(\frac{10}{100}\right)^2\cos45° = \sqrt{2}\times10^{-3}$ Wb

$\phi_2 = 0$

$e = \frac{\Delta\phi}{\Delta t} = \sqrt{2}\times10^{-3}$ V , x = 2

*53. In a test experiment on a model aeroplane in wind tunnel, the flow speeds on the upper and lower surfaces of the wings are 70 ms^{-1} and 65 ms^{-1} respectively. If the wing area is 2 m^2, the lift of the wing is _______ N.
(Given density of air = 1.2 kg m^{-3})

Ans. **810**

Sol. Using Bernoulli's equation

$P_1 + \frac{1}{2}\rho_1 v_1^2 = P_2 + \frac{1}{2}\rho_2 v_2^2$

$\Rightarrow P_2 - P_1 = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$

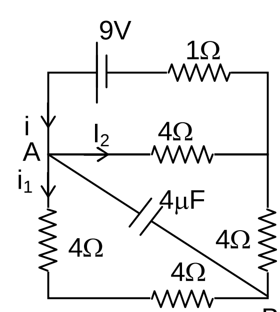
Lift on wing = $(P_2 - P_1)A$

$= \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)A = \frac{1}{2}\times1.2\times(70^2 - 65^2)2 = 810N$

54. A 16Ω wire is bend to form a square loop. A 9V battery with internal resistance 1Ω is connected across one of its sides. If a $4\mu F$ capacitor is connected across one of its diagonals, the energy stored by the capacitor will be $\frac{x}{2}\mu J$, where x = _________ .

Ans. **81**

Sol. $i = \frac{V}{R_{eq}} = \frac{9}{1+\frac{12\times4}{12+4}} = \frac{9}{4}A$



$4i_2 = 12i_1$ …(i)

and $i_1 + i_2 = \frac{9}{4}$ …(ii)

Solving (i) and (ii)

$i_1 = \frac{9}{16}A$

$V_A - V_B = i_1\times8 = \frac{9}{2}V$

$U = \frac{1}{2}CV^2$

$U = \frac{1}{2}\times4\times\left(\frac{9}{2}\right)^2 = \frac{81}{2}\mu J$

55. The magnetic potential due to a magnetic dipole at a point on its axis situated at a distance of 20 cm from its center is 1.5×10^{-5} T m. The magnetic moment of the dipole is ______ Am^2

(Given: $\frac{\mu_0}{4\pi} = 10^{-7}\,T\,m\,A^{-1}$)

Ans. **6**

Sol. $V = \frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}$

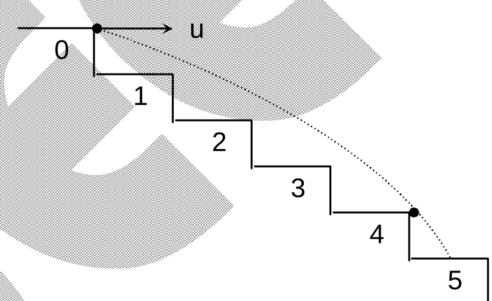
$1.5\times10^{-5} = \frac{10^{-7}\times M\times \cos 0°}{(0.2)^2}$

M = 6 Am^2

*56. A ball rolls off the top of a stairway with horizontal velocity u. The steps are 0.1 m high and 0.1 m wide. The minimum velocity u with which that ball just hits the step 5 of the stairway will be $\sqrt{x}\,ms^{-1}$ where x = ________ [use g = 10 m/s^2]

Ans. **2**

Sol. For minimum velocity ball needs to just cross 4th step to just hit 5th step



In vertical

$0.4 = \frac{1}{2}gt^2$ ...(i)

In horizontal

0.4 = ut ...(ii)

Solving (i) and (ii)

$u = \sqrt{2}$ m/s

57. An electron is moving under the influence of the electric field of a uniformly charged infinite plane sheet S having surface charge density $+\sigma$. The electron at t = 0 is at a distance of 1 m from S and has a speed of 1 m/s. The maximum value of σ if the electron strikes S at t = 1 s is $\alpha\left[\frac{m\epsilon_0}{e}\right]\frac{C}{m^2}$, the value of α is ________ .

Ans. **8**

Sol. $E = \frac{\sigma}{2\varepsilon_0},\ a = \frac{-e\sigma}{2m\varepsilon_0}$

given u = 1 m/s and t = 1 sec

using

$S = ut + \frac{1}{2}at^2$

$-1 = 1(1) - \frac{1}{2}\left(\frac{e\sigma}{2m\varepsilon_0}\right)1^2$

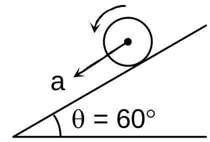
$\Rightarrow \sigma = 8\left(\frac{m\varepsilon_0}{e}\right)$ C/m^2

*58. A cylinder is rolling down on an inclined plane of inclination 60°. It's acceleration during rolling down will be $\frac{x}{\sqrt{3}}$ m/s^2, where x = _________ (use g = 10 m/s^2)

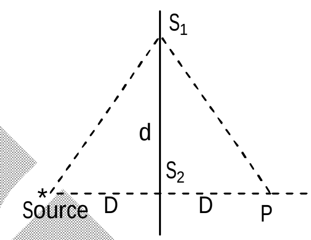
Ans. **10**

Sol. $a = \dfrac{g\sin\theta}{1+\dfrac{I_{cm}}{mR^2}}$

$a = \dfrac{2}{3}g\sin 60^\circ = \dfrac{10}{\sqrt{3}}\ m/s^2$



59. In a double slit experiment shown in figure, when light of wavelength 400 nm is used, dark fringe is observed at P. If D = 0.2 m, the minimum distance between the slits S_1 and S_2 is ______ mm.



Ans. **0.2 (Not matching with answer key)**

Sol. $2\left[(D^2+d^2)^{1/2} - D\right] = \Delta x$

$2\left[D\left(1+\dfrac{d^2}{D^2}\right)^{1/2} - D\right] = \Delta x$

$2D\left[1+\dfrac{d^2}{2D^2} - 1\right] = \Delta x$

For dark fringe

$\dfrac{d^2}{D} = \Delta x = (2n+1)\dfrac{\lambda}{2}$

For minimum distance n = 0

$\dfrac{d^2}{D} = \dfrac{\lambda}{2}$

$d^2 = \dfrac{D\lambda}{2} \Rightarrow d = \sqrt{\dfrac{0.2\times 400\times 10^{-9}}{2}} = 0.2\,mm$

60. When a hydrogen atom going from n = 2 to n = 1 emits a photon, its recoil speed is $\dfrac{x}{5}m/s$. Where x = __________. (Use, mass of hydrogen atom $= 1.6\times 10^{-27}kg$)

Ans. **17**

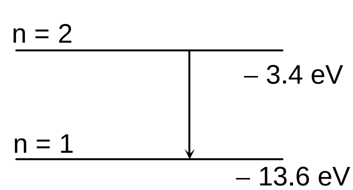
Sol. $\therefore$ ΔE = 10.2 eV

Recoil speed

$v = \dfrac{\Delta E}{mc} = \dfrac{10.2\times 1.6\times 10^{-19}}{1.6\times 10^{-27}\times 3\times 10^{8}}$

$v = \dfrac{17}{5}\ m/s$

$\therefore$ x = 17



PART – C (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

---

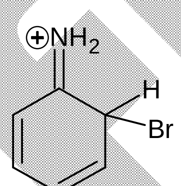
*61. Given below are two statements: One is labeled as Assertion A and the other is labeled as Reason R.
Assertion A: The first ionization enthalpy decreases across a period
Reason R : The increasing nuclear charge outweighs the shielding across the period.
In the light of the above statements, choose the most appropriate from the options given below:
(1) A is false but R is true
(2) A is true but R is false
(3) Both A and R are true and R is the correct explanation of A
(4) Both A and R are true but R is NOT the correct explanation of A

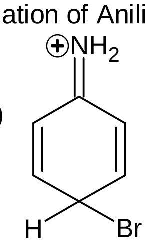
Ans. **(1)**
Sol. **Assertion (A)** is false
The first ionization enthalpy increases across a period.
Reason is correct.

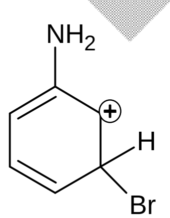
*62. The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is
(1) electromeric energy
(2) ionization energy
(3) hyperconjugation energy
(4) resonance energy

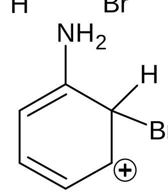
Ans. **(4)**
Sol. Resonance Energy: A compound with delocalized electron is more stable than if it would be if all its electron were localized. The extra stability of compound gains from having delocalized electrons is called Resonance Energy.

*63. The arenium ion which is not involved in the bromination of Aniline is _______.

(1)


(2)


(3)


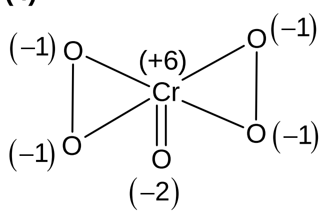
(4)


Ans. **(3)**
Sol. NH_2 group being ortho and para directing. Bromination is not possible at meta position.

64. In chromyl chloride test for confirmation of Cl^- ion, a yellow solution is obtained. Acidification of the solution and addition of amyl alcohol and 10% H_2O_2 turns organic layer blue indicating formation of chromium pentoxide. The oxidation state of chromium in that is
(1) +5 (2) +3
(3) +10 (4) +6

Ans. **(4)**

Sol.



65. Match List I with List II

LIST I (Substances)		LIST II (Element Present)	
A.	Ziegler catalyst	I.	Rhodium
B.	Blood Pigment	II.	Cobalt
C.	Wilkinson catalyst	III.	Iron
D.	Vitamin B_{12}	IV.	Titanium

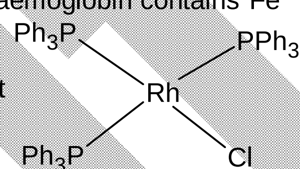
Choose the correct answer from the options given below
(1) A-II, B-IV, C-I, D-III (2) A-IV, B-III, C-I, D-II
(3) A-II, B-III, C-IV, D-I (4) A-III, B-II, C-IV, D-I

Ans. **(2)**

Sol. Ziegler catalyst - $TiCl_4 + Al(C_2H_5)_3$

Blood pigment – Haemoglobin contains Fe^{2+}

Wilkinson's catalyst



Vitamin B_{12} – contain Co^{2+}

66. Type of amino acids obtained by hydrolysis of proteins is :
(1) γ (2) δ
(3) β (4) α

Ans. **(4)**

Sol. $\text{Proteins} \xrightarrow{\text{Hydrolysis}} \text{Peptides} \xrightarrow{\text{Hydrolysis}} \alpha - \text{amino acids}$

*67. The interaction between π bond and lone pair of electrons present on an adjacent atom is responsible for
(1) Resonance effect (2) Electromeric effect
(3) Inductive effect (4) Hyperconjugation

Ans. **(1)**

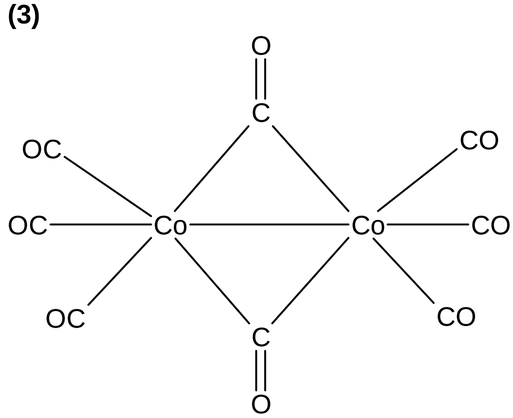
Sol. Resonance – The delocalization of electrons is known as resonance. For a compound to show resonance, compound with unsaturated system would be in conjugation with $\pi -$bond, –ve charge, positive charge, lone pair or odd electron.

68. In which one of the following metal carbonyls, CO forms a bridge between metal atoms?

(1) $\left[Os_3(CO)_{12}\right]$ (2) $\left[Ru_3(CO)_{12}\right]$
(3) $\left[Co_2(CO)_8\right]$ (4) $\left[Mn_2(CO)_{10}\right]$

Ans. **(3)**

Sol.



69. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Aryl halides cannot be prepared by replacement of hydroxyl group of phenol by halogen atom.

Reason R : Phenols react with halogen acids violently.

In the light of the above statements, choose the most appropriate from the options given below:

(1) A is false but R is true
(2) Both A and R are true and R is the correct explanation of A
(3) A is true but R is false
(4) Both A and R are true but R is NOT the correct explanation of A

Ans. **(3)**

Sol. Aryl halides cannot be prepared by replacement of hydroxyl group of phenol by halogen atom.

*70. The correct set of four quantum numbers for the valence electron of rubidium atom (Z = 37) is:

(1) $5, 1, 0, +\frac{1}{2}$ (2) $5, 1, 1, +\frac{1}{2}$
(3) $5, 0, 1, +\frac{1}{2}$ (4) $5, 0, 0, +\frac{1}{2}$

Ans. **(4)**

Sol. $\text{Rubidium} = [Kr]5s^1$

[At. No. = 37]

$n = 5, \ell = 0, m = 0, s = +\frac{1}{2}$

71. Identify the incorrect pair from the following:

(1) Carnallite- $KCl.MgCl_2.6H_2O$ (2) Cryolite –Na_3AlF_6
(3) Fluoroapatite-3 $Ca_3(PO_4).CaF_2$ (4) Fluorspar –BF_3

Ans. **(4)**

*72. Appearance of blood red colour, on treatment of the sodium fusion extract of an organic compound with $FeSO_4$ in presence of concentrated H_2SO_4 indicates the presence of element/s

(1) N and S (2) N
(3) S (4) Br

Ans. **(1)**

Sol. Lassaigne's filtrate (sodium extract) consist of NaCN and NaOH and when organic compound contains both 'N' and 'S'. Sodium thiocyanate is formed, which gives red coloration (of ferric sulphocyanide) with ferric ion.

$$3NaCNS + FeCl_3 \longrightarrow \underset{(\text{Blood red})}{Fe(CNS)_3} + 3NaCl$$

73. In alkaline medium, MnO_4^- oxidizes I^- to

(1) IO^- (2) IO_4^-

(3) I_2 (4) IO_3^-

Ans. **(4)**

Sol. In alkaline medium $KMnO_4$ oxidises I^- to IO_3^-

$$6MnO_4^- + I^- + 6OH^- \longrightarrow IO_3^- + 6MnO_4^{2-} + 3H_2O$$

*74. Which of the following is not correct?

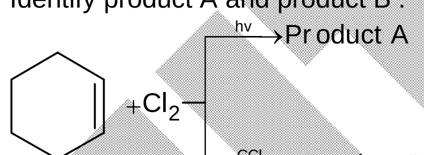
(1) ΔG is zero for a reversible reaction
(2) ΔG is negative for a spontaneous reaction
(3) ΔG is positive for a spontaneous reaction
(4) ΔG is positive for a non-spontaneous reaction

Ans. **(3)**

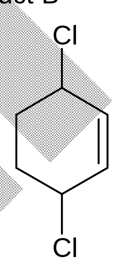
Sol. At constant temperature and pressure

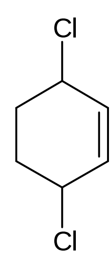
$\Delta G = -ve$ for spontaneous process
$\Delta G = 0$ for process at equilibrium
$\Delta G = +ve$ for non-spontaneous process.

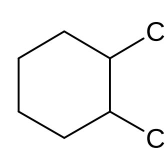
*75. Identify product A and product B :

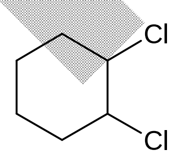


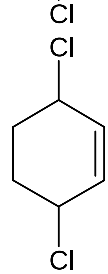
(1) A : 

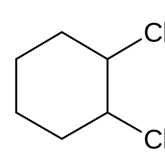
B : 

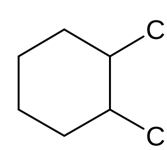
(2) A : 

B : 

(3) A : 

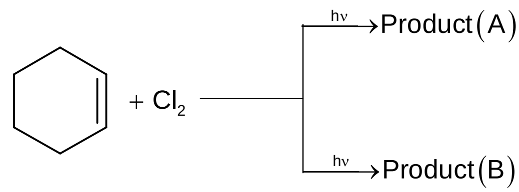
B : 

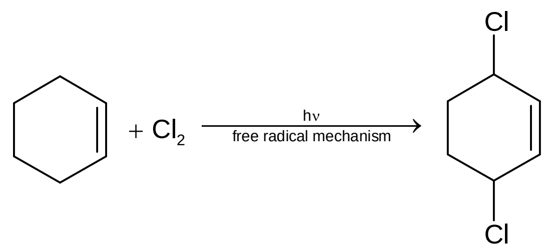
(4) A : 

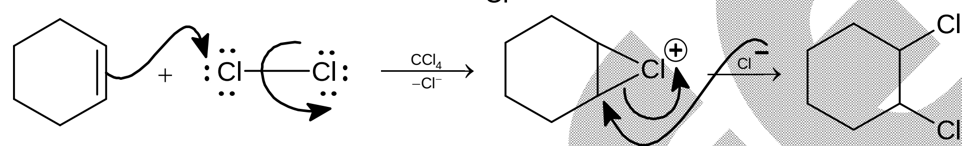
B : 

Ans. **(2)**

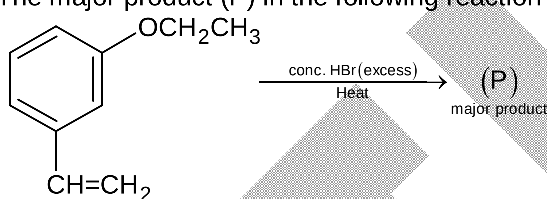
Sol.



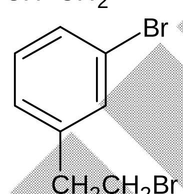




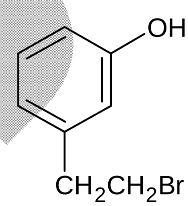
76. The major product (P) in the following reaction is



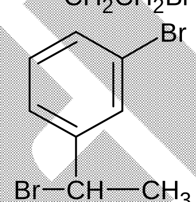
(1)



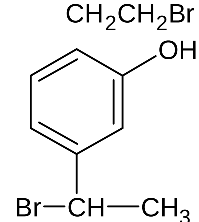
(2)



(3)

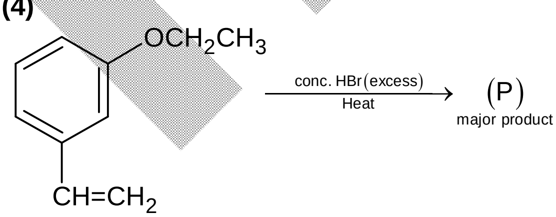


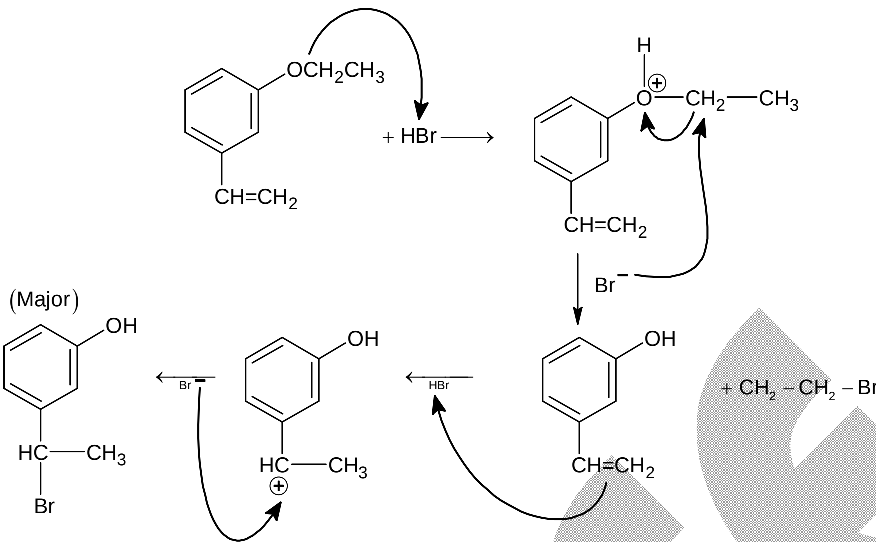
(4)



Ans. **(4)**

Sol.





*77. Chlorine undergoes disproportionation in alkaline medium as shown below :

$$a\ Cl_{2(g)} + b\ OH^-_{(aq)} \rightarrow c\ ClO^-_{(aq)} + d\ Cl^-_{(aq)} + e\ H_2O_{(l)}$$

The values of a, b, c and d in a balanced redox reaction are respectively:

(1) 2, 2, 1 and 3
(2) 1, 2, 1 and 1
(3) 2, 4, 1 and 3
(4) 3, 4, 4 and 2

Ans. **(2)**

Sol.

lost

$$\overset{0}{a\,Cl_2}(g) + b\,\overline{OH}(aq) \rightleftharpoons \overset{+1}{c\,ClO^-}(aq) + d\,Cl^-(aq) + e\,H_2O(\ell)$$

gain

Lost part

$$2\overset{0}{Cl} \longrightarrow 2Cl^+ + 2e^- \quad \ldots (1)$$

Gain part

$$2e^- + 2\overset{0}{Cl} \longrightarrow 2Cl^- \quad \ldots (2)$$

Add Eqn. (1) and (2)

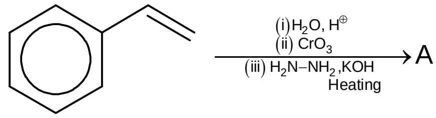
$$2\overset{0}{Cl} \longrightarrow +2Cl^+ + 2Cl^-$$

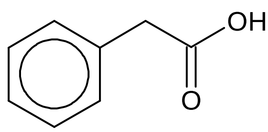
$$4\overline{OH} + 2Cl_2 \longrightarrow 2ClO^- + 2Cl^- + 2H_2O$$

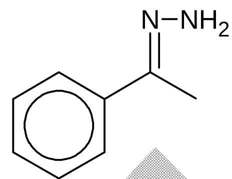
$$2\overline{OH} + Cl_2 \longrightarrow ClO^- + Cl^- + H_2O$$

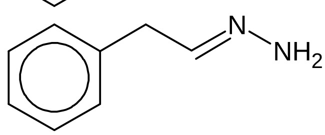
Hence, a = 1, b = 2, c = 1, d = 1, e = 1

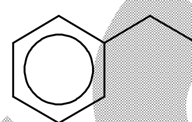
78. The final product A formed in the following multistep reaction sequence is



(1) 

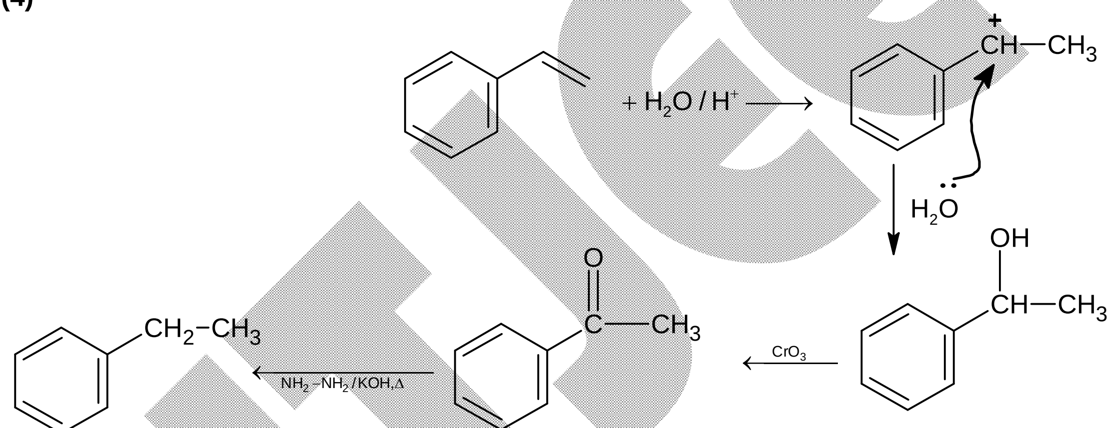
(2) 

(3) 

(4) 

Ans. **(4)**

Sol.



*79. Given below are two statements:
Statement I: The electronegativity of group 14 elements from Si to Pb gradually decreases.
Statement II: Group 14 contains non-metallic, metallic, as well as metalloid elements.

In the light of the above statements, choose the most appropriate from the options given below:
(1) Statement I is true but Statement II is false
(2) Statement I is false but Statement II is true
(3) Both Statement I and Statement II are false
(4) Both Statement I and Statement II are true

Ans. **(2)**

Sol. Order of E. N.
C > Pb > Ge > Si $\simeq$ Sn

80. $KMnO_4$ decomposes on heating at 513 K to form O_2 along with
(1) Mn & KO_2
(2) K_2MnO_4 & MnO_2
(3) K_2MnO_4 & Mn
(4) MnO_2 & K_2O_2

Ans. **(2)**
Sol. $KMnO_4 \xrightarrow[513\ K]{\Delta} K_2MnO_4 + MnO_2 + O_2$

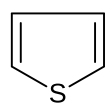
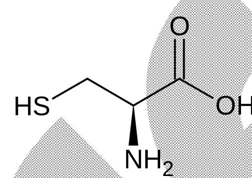
SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

*81. Number of compounds among the following which contains sulphur as heteroatom is________ .
Furan, Thiophene, Pyridine, Pyrrole, Cysteine, Tyrosine

Ans. **2**

Sol. Thiophene ≡  , Cysteine ≡ 

and furan, pyridine, pyrrole and tyrosine does not contain sulphur (heteroatom).

82. The osmotic pressure of a dilute solution is 7×10^5 Pa at 273 K. Osmotic pressure of the same solution at 283 K is __________ $\times10^4\ Nm^{-2}$

Ans. **73**

Sol. Osmotic pressure $=7\times10^5$ Pa, T = 273 K

$$\pi = CRT \Rightarrow \frac{\pi}{T} = CR = \text{constant}$$

$$\therefore \frac{7\times10^5}{273} = \frac{\pi_{req}}{283}$$

$$\pi_{req} = \frac{7\times10^5}{273}\times283 = 72.56$$

*83. The number of species from the following which are paramagnetic and with bond order equal to one is __________ .
H_2, He_2^+, O_2^+, N_2^{2-}, O_2^{2-}, F_2, Ne_2^+, B_2

Ans. **1**

Sol. $H_2 \equiv \sigma1s^2$, Bond order $=\frac{2}{2}=1$, M. B. = Diamagnetic

$He_2^- \equiv \sigma1s^2\sigma^*1s^2\sigma2s^1$, Bond order $=\frac{1}{2}[3-2]=\frac{1}{2}=0.5$, M. B. = P. M.

$O_2^+ \Rightarrow$ Bond order = 2.5, M. B. = P. M.

$N_2^{2-} \Rightarrow$ Bond order = 2.0, M. B. = P. M.

Bond order $=\frac{1}{2}[10-6]=\frac{4}{2}=2$

$O_2^{2-} \rightarrow$ Bond order = 1, M. B. = Diamagnetic

$F_2 \rightarrow$ Bond order = 1.0, M. B. = Diamagnetic

$Ne_2^+ \rightarrow$ Bond order = 0.5, M. B. = Paramagnetic

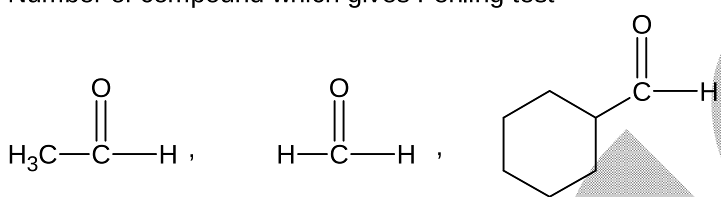
$B_2 \rightarrow$ Bond order $=\frac{1}{2}[6-4]=\frac{2}{2}=1$, M. B. = P. M.

Hence, the number of species from the following which are paramagnetic and with bond order equal to one = 1.

84. From the compounds given below, number of compounds which give positive Fehling's test is _____ .
Benzaldehyde, Acetaldehyde, Acetone, Acetophenone, Methanal, 4-nitrobenzaldehyde, cyclohexane carbaldehyde.

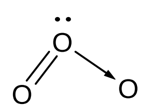
Ans. **3**

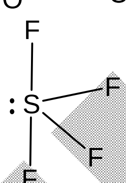
Sol. Number of compound which gives Fehling test

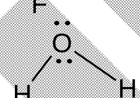


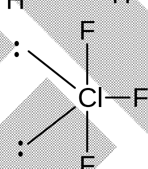
*85. Number of compounds with one lone pair of electron on central atom amongst following is ________ .
O_3, H_2O, SF_4, ClF_3, NH_3, BrF_5, XeF_4

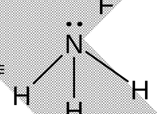
Ans. **4**

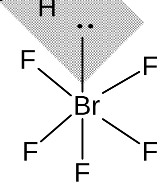
Sol. $O_3 \equiv$ 

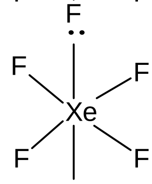
$SF_4 \equiv$ 

$H_2O \equiv$ 

$ClF_3 \equiv$ 

$NH_3 \equiv$ 

$BrF_5 \equiv$ 

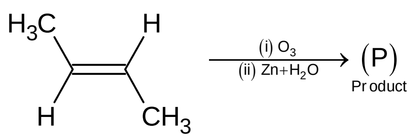
$XeF_4 \equiv$ 

$\therefore$ Number of compound with one lone pair of electron on central atom = 4 (which are O_3, NH_3, BrF_5 and SF_4)

*86. For the reaction $N_2O_{4(g)} \rightleftarrows 2NO_{2(g)}, K_P = 0.492\,atm$ at 300 K. K_C for the reaction at same temperature is _________ $\times 10^{-2}$.
(Given : R = 0.082 L atom mol^{-1} K^{-1})

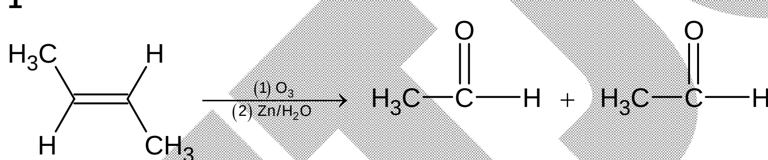
Ans. **2**

Sol. $N_2O_4(g) \rightleftharpoons 2NO_2(g), K_P = 0.492$ atm, $T = 300$ K
$\Delta n_g = 2 - 1 = 1$
$\therefore K_P = K_c \times (RT)^{\Delta n_g} \Rightarrow 0.492 = K_c \times (0.082 \times 300)^1$
$K_c = \dfrac{0.492}{300 \times 0.082} = \dfrac{0.492}{24.6} = 2 \times 10^{-2}$

*87. 

Consider the given reaction. The total number of oxygen atom/s present per molecule of the product (P) __________ .

Ans. **1**

Sol. 

*88. A solution of H_2SO_4 is 31.4 % H_2SO_4 by mass and has a density of 1.25 g/mL.
The molarity of the H_2SO_4 solution is __________ M (nearest integer)
[Given molar mass of H_2SO_4 = 98 g mol^{-1}]

Ans. **4**

Sol. 31.4% by mass of H_2SO_4, d_{soln} = 1.25 g/mL
$[M]_{H_2SO_4} = ?$
Let mass of solution = 100 g
Mass of $H_2SO_4 = 31.4$
$\therefore V_{soln} = \dfrac{100}{1.25} = 80$ mL
$(mole)_{H_2SO_4} = \dfrac{31.4}{98} = 0.32$
$[M]_{H_2SO_4} = \dfrac{mole}{V_{soln\ in\ mL}} \times 1000 = 4$

89. The mass of zinc produced by the electrolysis of zinc sulphate solution with a steady current of 0.015 A for 15 minutes is __________ $\times 10^{-4}$ g.
(Atomic mass of zinc = 65.4 amu)

Ans. **46**

Sol. $Zn^{2+} + 2e^- \longrightarrow Zn$
$W \propto Q$

$W = zit$

$= \frac{65.4}{2 \times 96500} \times 0.015 \times 15 \times 60 = \frac{882.9}{193000} = 45.75 \times 10^{-4}$ g

*90. For a reaction taking place in three steps at same temperature, overall rate constant $K = \frac{K_1 K_2}{K_3}$. If Ea_1. Ea_2 and Ea_3 are 40, 50 and 60 kJ/ mol respectively, the overall Ea is _____ kJ/mole

Ans. **30**

Sol. $E_{a_1} = 40$, $E_{a_2} = 50$ and $E_{a_3} = 60$

$k_{eff} = \frac{k_1 k_2}{k_3}$

$*A_{eff.}\ e^{-\frac{E}{RT}} = \frac{A_1 \times e^{-\frac{E_1}{RT}} \times A_2 e^{-\frac{E_2}{RT}}}{A_3 \times e^{-\frac{E_3}{RT}}}$

On differentiating w. r. t. to temperature,

$E = E_{a_1} + E_{a_2} - E_{a_3}$

= 40 + 50 – 60 = 30