PART – A (MATHEMATICS)

SECTION - A

(One Options Correct Type)

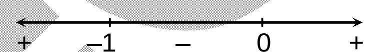
This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

1. The function $f(x) = 2x + 3(x)^{2/3}$, $x \in R$, has
   (1) exactly one point of local maxima and exactly one point of local minima
   (2) exactly one point of local minima and no point of local maxima
   (3) exactly two point of local maxima and exactly one point of local minima
   (4) exactly one point of local maxima and no point of local minima

Ans. **(1)**

Sol. $f(x) = 2x + 3x^{\frac{2}{3}}$

$f'(x) = 2 + 3 \times \frac{2}{3}x^{\frac{2}{3}-1} = 2 + \frac{2}{x^{\frac{1}{3}}} = 0 \Rightarrow \frac{2\left(x^{\frac{1}{3}}+1\right)}{x^{\frac{1}{3}}} = 0$



x = –1 is point of maxima, x = 0 is point of minima

*2. Let A be the point of intersection of the lines 3x + 2y = 14, 5x – y = 6 and B be the point of intersection of the lines 4x + 3y = 8, 6x + y = 5. The distance of the point P(5, –2) from the line AB is

(1) 8 (2) $\frac{13}{2}$

(3) $\frac{5}{2}$ (4) 6

Ans. **(4)**

Sol. On solving, we get A(2, 4), $B\left(\frac{1}{2}, 2\right)$

Equation of AB is $\frac{y-4}{x-2} = \frac{4-2}{2-\frac{1}{2}} = \frac{2(2)}{3} = \frac{4}{3}$

3y – 12 = 4x – 8 ⇒ 3y – 4x – 4 = 0

Distance from P(5, –2)

$d = \frac{|-6-20-4|}{5} = \frac{30}{5} = 6$

*3. If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of the first four observations is $\frac{7}{2}$, then the variance of the first four observations in equal to

(1) $\frac{5}{4}$ (2) $\frac{4}{5}$

(3) $\frac{105}{4}$ (4) $\frac{77}{12}$

Ans. **(1)**

Sol. $\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{24}{5} \Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 24$

$\frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{7}{2} \Rightarrow x_1 + x_2 + x_3 + x_4 = 14$

Also, $\frac{x_1^2 + x_2^2 + ... + x_5^2}{5} - \frac{576}{25} = \frac{194}{25}$

$x_1^2 + x_2^2 + x_3^2 + x_4^2 = 54$

Var. = $\frac{\sum_{i=1}^{4} x_i^2}{4} - \left(\frac{\sum x_i}{4}\right)^2 = \frac{54}{4} - \frac{49}{4} = \frac{5}{4}$

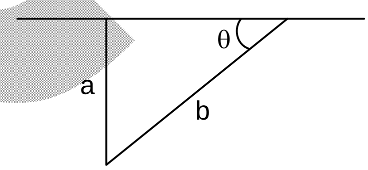
*4. The distance of the point (2, 3) from the line 2x – 3y + 28 = 0 measured parallel to the line $\sqrt{3}x - y + 1 = 0$, is equal to

(1) $6\sqrt{3}$
(2) $4\sqrt{2}$
(3) $3 + 4\sqrt{2}$
(4) $4 + 6\sqrt{3}$

Ans. **(4)**

Sol. $m_1 = \frac{2}{3}, m_2 = \sqrt{3} \Rightarrow a = \frac{14 - 9 + 281}{\sqrt{13}} = \frac{23}{\sqrt{13}}$

(Where a is perpendicular distance)



$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$; $\tan\theta = \frac{\left|\sqrt{3} - \frac{2}{3}\right|}{\left|1 + \sqrt{3} \times \frac{2}{3}\right|}$

$\Rightarrow \sin\theta = \frac{(3\sqrt{3} - 2)}{\sqrt{52}} = \frac{a}{b} = \frac{23}{\sqrt{13}} \times \frac{1}{b}$ (Put value of a)

$\frac{(3\sqrt{3} - 2)}{2\sqrt{13}} = \frac{23}{\sqrt{13}\,b} \Rightarrow (3\sqrt{3} - 2)b = 46 \Rightarrow b = 4 + 6\sqrt{3}$ (After rationalisation)

*5. If $\log_e a$, $\log_e b$, $\log_e c$ are in an A.P. and $\log_e a - \log_e 2b$, $\log_e 2b - \log_e 3c$, $\log_e 3c - \log_e a$ are also in an A.P., then a : b : c is equal to

(1) 6 : 3 : 2
(2) 16 : 4 : 1
(3) 9 : 6 : 4
(4) 25 : 10 : 4

Ans. **(3)**

Sol. ln a, ln b, ln c $\rightarrow$ A.P.

$\Rightarrow$ a, b, c are in G.P.

Let a = d, b = dr, c = dr^2 (On putting values)

ln d – ln (2 dr), ln (2 dr) – ln (3 dr^2), ln (3 dr^2) – ln (d) $\rightarrow$ A.P.

$\ln\left(\frac{1}{2r}\right), \ln\left(\frac{2}{3r}\right), \ln(3r^2)$ are in A.P.

$\frac{1}{2r}, \frac{2}{3r}, 3r^2$ are in G.P.

$\left(\frac{2}{3r}\right)^2 = \frac{1}{2r}(3r^2) = \frac{3}{2}r \Rightarrow \frac{4}{9r^2} = \frac{3}{2}r \Rightarrow \frac{8}{27} = r^3 \Rightarrow r = \frac{2}{3}$

Then, a : b : c $\equiv$ d : dr : dr^2 $\equiv 1 : \frac{2}{3} : \frac{4}{9} \Rightarrow$ a : b : c $\equiv$ 9 : 6 : 4

6. Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$. The sum of the prime factors of $|P^{-1} AP - 2I|$ is equal to

(1) 27 (2) 26
(3) 23 (4) 66

Ans. **(2)**

Sol. Let $X = P^{-1}AP - 2I$

PX = AP – 2P (Multiply with P)

$|PX P^{-1}| = |A - 2I|$

$$|P||X||P^{-1}| = \begin{vmatrix} 2-2 & 1 & 2 \\ 6 & 2-2 & 11 \\ 3 & 3 & 2-2 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{vmatrix}$$

$|X| = -1[-33] + 2[18] = 33 + 36 = 69 = 23 \times 3$

Sum = 23 + 3 = 26

*7. If each term of a geometric progression a_1, a_2, a_3 …… with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \ldots\ldots a_n$, then $S_{20} - S_{18}$ is equal to

(1) 2^{18} (2) 2^{15}
(3) -2^{18} (4) -2^{15}

Ans. **(4)**

Sol. a_1, a_2, a_3, ….. G.P.

$\Rightarrow \frac{1}{8}, \frac{(r)}{8}, \frac{r^2}{8}, \frac{r^3}{8}$.....

According to question $\frac{1}{8} = \frac{\frac{r}{8} + \frac{r^2}{8}}{2} = \frac{2 + r^2}{2(8)}$

$r^2 + r = 2 \Rightarrow r = -2$

$$S_{20} - S_{18} = a_1\frac{(r^{20}-1)}{r-1} - a_1\frac{(r^{18}-1)}{r-1} = \frac{a_1}{r-1}[r^{20} - 1 - r^{18} + 1] = a_1(2^{18})(-2+1) = -2^{15}$$

8. An integer is chosen at random from the integers 1, 2, 3, …… 50. The probability that the chosen integer is a multiple of atleast one of 4, 6 and 7 is

(1) $\frac{21}{50}$ (2) $\frac{9}{50}$
(3) $\frac{14}{25}$ (4) $\frac{8}{25}$

Ans. **(1)**

Sol. S = {1, 2, 3, ….., 50}

A = {4, 6, 8, ….. } ; common = {12, 24, 36, 48}{28, 42}

m(A) = multiplies of {4, 6, 7} – common multiplies

(12 + 8 + 7) – (6) = (27) – 6 = 21

$P = \frac{m(A)}{n(S)} = \frac{21}{50}$

9. Let P(3, 2, 3), Q(4, 6, 2) and R(7, 3, 2) be the vertices of ΔPQR. Then, the angle $\angle QPR$ is

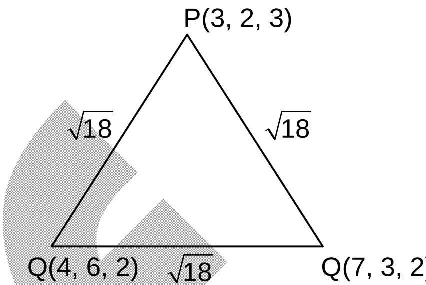
(1) $\cos^{-1}\left(\frac{7}{18}\right)$ (2) $\frac{\pi}{3}$

(3) $\cos^{-1}\left(\frac{1}{18}\right)$ (4) $\frac{\pi}{6}$

Ans. **(2)**

Sol. $\angle QPR = \frac{\pi}{3}$

Δ is equilateral



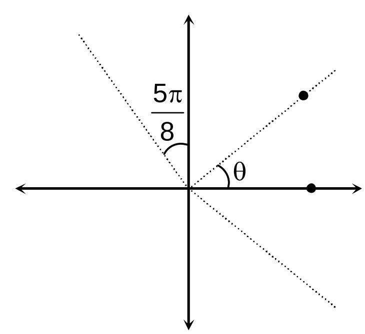
*10. Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$, then (r, θ) is equal to

(1) $\left(2\sec\frac{11\pi}{8}, \frac{11\pi}{8}\right)$ (2) $\left(2\sec\frac{5\pi}{8}, \frac{3\pi}{8}\right)$

(3) $\left(2\sec\frac{3\pi}{8}, \frac{5\pi}{8}\right)$ (4) $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$

Ans. **(4)**

Sol. $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$



$|z| = \sqrt{4 + 4\tan^2\frac{5\pi}{8}}$

$= 2\sqrt{1 + \tan^2\frac{5\pi}{8}} = 2\left|\sec\left(\frac{5\pi}{8}\right)\right| = 2\sec\left(\frac{3\pi}{8}\right)$

Amplitude $\tan\theta = \left|\frac{2\tan\frac{5\pi}{8}}{2}\right| \Rightarrow \theta = \frac{5\pi}{8}$

AMP $= \pi - \frac{5\pi}{8} = \frac{3\pi}{8}$

11. If $\sin\left(\frac{y}{x}\right) = \log_e|x| + \frac{\alpha}{2}$ is the solution of the differential equation $\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ and $y(1) = \frac{\pi}{3}$, then α^2 is equal to

(1) 4 (2) 9

(3) 3 (4) 12

Ans. **(3)**

Sol. For answer put $y = \frac{\pi}{3}$ at x = 1

$\sin\left(\frac{\pi}{3}\right) = 0 + \frac{\alpha}{2} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \sqrt{3}$

$\therefore\ \alpha^2 = 3$

12. Let $y = \log_e\left(\frac{1-x^2}{1+x^2}\right) - 1 < x < 1$. Then at $x = \frac{1}{2}$, the value of 225(y' – y") is equal to

(1) 736
(2) 732
(3) 742
(4) 746

Ans. **(1)**

Sol. $y = \ln(1 - x^2) - \ln(1 + x^2)$

$$\frac{dy}{dx} = -\frac{4x}{1-x^4}$$

$$\frac{d^2y}{dx^2} = \frac{-4\left[1+3x^2\right]}{\left(1-x^4\right)^2}$$

$$\left[\frac{dy}{dx} - \frac{d^2y}{dx^2}\right]_{\text{at } x=\frac{1}{2}} = \frac{-2}{1-\frac{1}{16}} + \frac{4\left[1+\frac{3}{16}\right]}{\left(1-\frac{1}{10}\right)^2} = \frac{16(46)}{(15)(15)} = \frac{736}{225}$$

Then, $\left(\frac{736}{225}\right)(225) = 736$

13. If $\int \frac{\sin^{3/2}x + \cos^{3/2}x}{\sqrt{\sin^3 x \cos^3 x \sin(x-\theta)}}dx = A\sqrt{\cos\theta\tan x - \sin\theta} + B\sqrt{\cos\theta - \sin\theta\cot x} + C$, where C is the integration constant, then AB is equal to

(1) 4 cosec(2θ)
(2) 8 cosec(2θ)
(3) 4 secθ
(4) 2 secθ

Ans. **(2)**

Sol. After separation of terms, we will get

$$\Rightarrow \int \frac{dx}{\cos^{3/2}x\sqrt{\sin x\cos\theta - \cos x\sin\theta}} + \int \frac{dx}{\sin^{3/2}x\sqrt{\sin x\cos\theta - \cos x\sin\theta}}$$

$$\Rightarrow \int \frac{\sec^2 x\,dx}{\sqrt{\tan x\cos\theta - \sin\theta}} + \int \frac{\text{cosec}^2 x\,dx}{\sqrt{\cos\theta - \cos x\sin\theta}} \quad \text{(Integrate it)}$$

$$= \frac{2}{\cos\theta}\sqrt{(\tan x\cos\theta - \sin\theta)} + \frac{2}{\sin\theta}\sqrt{(\cos\theta - \cot x\sin\theta)} + c$$

AB = 8 cosec 2θ

*14. Let $x = \frac{m}{n}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos\left(2\sin^{-1}x\right) = \frac{1}{9}$ and let α, β(α > β) be the roots of the equation $mx^2 - nx - m + n = 0$. Then the point (α, β) lies on the line

(1) 5x – 8y = –9
(2) 3x – 2y = –2
(3) 5x + 8y = 9
(4) 3x + 2y = 2

Ans. **(3)**

Sol. $\sin^{-1}x = \theta$ (1st quadrant)

$$\cos 2\theta = \frac{1}{9} \Rightarrow 1 - 2\sin^2\theta = \frac{1}{9} \Rightarrow \sin^2\theta = \frac{4}{9}$$

$$\Rightarrow x^2 = \frac{4}{9} \Rightarrow x = \frac{2}{3} = \frac{m}{n} \text{ then, } mx^2 - mx - m + n = 0$$

$2x^2 - 3x + 1 = 0$ roots are 1, $\frac{1}{2} \Rightarrow \alpha = 1, \beta = \frac{1}{2}$

So, $(\alpha, \beta) \equiv \left(1, \frac{1}{2}\right)$ lies on $5x + 8y = 9$

*15. Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to

(1) 15 (2) 16
(3) 18 (4) 12

Ans. **(1)**

Sol. Since all are identical, we have to distribute 8 into 4 parts

(1) 8 0 0 0	(9) 4 2 2 0
(2) 7 1 0 0	(10) 3 3 2 0
(3) 6 2 0 0	(11) 5 1 1 1
(4) 5 3 0 0	(12) 4 2 1 1
(5) 4 4 0 0	(13) 3 3 1 1
(6) 6 1 1 0	(14) 3 2 2 1
(7) 5 2 1 0	(15) 2 2 2 2
(8) 4 3 1 0	

16. If R is the smallest equivalence relation on the set {1, 2, 3, 4} such that $\{(1, 2), (1, 3)\} \subset R$, then the number of elements in R is

(1) 15 (2) 8
(3) 12 (4) 10

Ans. **(4)**

Sol. {1, 2, 3, 4}

R = {(1, 2), (1, 3), (2, 1), (3, 1), (1, 1), (2, 2), (3, 3), (4, 4), (2, 3), (3, 2)}

$\Rightarrow$ Total 8 elements

17. Let a unit vector $\hat{u} = x\hat{i} + y\hat{j} + \hat{k}$ make angles $\frac{\pi}{2}, \frac{\pi}{3}$ and $\frac{2\pi}{3}$ with the vectors $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k} \cdot \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ respectively. If $\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$, then $|\vec{u} - \vec{v}|^2$ is equal to

(1) $\frac{11}{2}$ (2) 7
(3) $\frac{5}{2}$ (4) 9

Ans. **(3)**

Sol. Using dot product of vectors

$$\cos\left(\frac{\pi}{2}\right) = \frac{x+z}{\sqrt{2}}, \cos\left(\frac{\pi}{3}\right) = \frac{y+z}{\sqrt{2}}, \cos\left(\frac{2\pi}{3}\right) = \frac{x+y}{\sqrt{2}}$$

$$x + z = 0,\ z + y = \frac{1}{\sqrt{2}},\ x + y = \frac{-1}{\sqrt{2}}$$

After solving

$$\vec{u} = -\frac{1}{\sqrt{2}}(\hat{i}) + \frac{1}{\sqrt{2}}(\hat{k})$$

$$|\vec{u} - \vec{v}|^2 = \left|-\sqrt{2}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right|^2 = \left(2 + \frac{1}{2}\right) = \frac{5}{2}.$$

18. Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = 12\vec{a} + 4\vec{b}$ and $\overrightarrow{OC} = \vec{b}$ where O is the origin. If S is the parallelogram with adjacent sides OA and OC, then $\dfrac{\text{area of the quadrilateral OA BC}}{\text{area of S}}$ is equal to

(1) 8
(2) 7
(3) 6
(4) 10

Ans. **(1)**

Sol. Area of S = Ar(S) = $\vec{a} \times \vec{b}$

Area(OABC) = $\dfrac{1}{2}\left(\left(12\vec{a} + 4\vec{b}\right) \times \vec{b}\right) + \dfrac{1}{2}\left(\left(12\vec{a} + 4\vec{b}\right) \times \vec{a}\right) = 8\left|\vec{a} \times \vec{b}\right|$

Ratio = $\dfrac{8\left|\vec{a} \times \vec{b}\right|}{\left|\vec{a} \times \vec{b}\right|} = 8$

19. The function $f(x) = \dfrac{x}{x^2 - 6x - 16}$, $x \in R - \{-2, 8\}$

(1) decrease in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$
(2) decrease in $(-\infty, -2)$ and increases in $(8, \infty)$
(3) decrease in $(-2, 8)$ and increases in $(-\infty, -2) \cup (8, \infty)$
(4) increase in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

Ans. **(1)**

Sol. $f(x) = \dfrac{x}{x^2 - 6x - 16}$

$f'(x) = \dfrac{-\left(x^2 + 16\right)}{(x-8)^2 (x+2)^2}$

As $f'(x) < 0 \ \forall \ x \in R$

$\Rightarrow$ f(x) always decreases.

*20. The sum of the solutions $x \in R$ of the equation $\dfrac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$ is

(1) 0
(2) 3
(3) –1
(4) 1

Ans. **(4)**

Sol. $\dfrac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$

After simplification we will get

$$\dfrac{12 + 4\cos^2 2x}{3 + \cos^2 2x} = x^3 - x^2 + 6$$

$x^3 - x^2 + 6 = 4 \Rightarrow x^3 - x^2 + 2 = 0$

sum of roots = 1.

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

*21. Let α, β be the roots of the equation $x^2 - \sqrt{6}x + 3 = 0$ such that Im(α) > Im(β). Let a, b be integers not divisible by 3 and n be a natural number such that $\frac{\alpha^{99}}{\beta} + \alpha^{98} = 3^n(a + ib), i = \sqrt{-1}$. Then n + a + b is equal to _______ .

Ans. **49**

Sol. $x^2 - \sqrt{6}x + 3 = 0$

Then $\alpha = \frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}}i, \beta = \frac{\sqrt{3}}{\sqrt{2}} - \frac{\sqrt{3}}{\sqrt{2}}i$

$$\alpha = \sqrt{3}\,e^{i\frac{\pi}{4}}, \beta = \sqrt{3}e^{-i\frac{\pi}{4}}$$

$$\frac{\alpha^{99}}{\beta} + \alpha^{98}$$

$$3^{49}e^{i\left(\frac{\pi}{2}\right)}[i+1] = 3^{49}[i-1] = 3^n(a+ib)$$

n + a + b = 49 – 1 + 1 = 49.

22. Let the slope of the line 45x + 5y + 3 = 0 be $27r_1 + \frac{9r_2}{2}$ for some $r_1.r_2 \in R$.

Then $\lim_{x\to3}\left(\int_3^x \frac{8t^2}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x}dt\right)$ is equal to ______ .

Ans. **12**

Sol. 45x + 5y + 3 = 0

$6r_1 + r_2 + 2 = 0$ (on putting value of slope)

then $\lim_{x\to3}\int_3^x\left(\frac{8t^2dt}{\frac{3r_2}{2}x - r_2x^2 - r_1x^3 - 3x}\right)$

$$= \lim_{x\to3}\frac{8\left[x^3 - 27\right]}{3\left[\frac{3r_2}{2}x - r_2x^2 - r_1x^3 - 3x\right]}$$

After simplification putting value of r_2

$$= \lim_{x\to3}\frac{16(x-3)\left(x^2+3x+9\right)}{(3x)(x-3)\left[-2r_1(x-3)+4\right]} = \frac{16(9+9+9)}{9(4)} = 12$$

*23. Let P(α, β) be a point on the parabola $y^2 = 4x$. If P also lies on the chord of the parabola $x^2 = 8y$ whose mid point is $\left(1, \frac{5}{4}\right)$, then ($\alpha$ – 28) (β – 8) is equal to _______ .

Ans. **192**

Sol. Using T = S_1 we will get equation of chord with given middle point as

$x - 4y + 4 = 0$ & P lies on it $P(t^2, 2t)$

$t^2 - 4(2t) + 4 = 0 \Rightarrow t^2 - 8t + 4 = 0$

$t = 4 \pm 2\sqrt{3} \Rightarrow t^2 = 28 \pm 16\sqrt{3}$

take t with any one sign

then $(\alpha - 28)(\beta - 8) = \left(16\sqrt{3}\right)\left(4\sqrt{3}\right) = 192$.

24. Let O be the origin and M and N be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\overrightarrow{OM} \cdot \overrightarrow{ON}$ is equal to ______ .

Ans. **9**

Sol. Any point can be

$M(4\lambda + 5, \lambda + 4, 3\lambda + 5)$, $N(12\mu - 8, 5\mu - 2, 9\mu - 11)$

Since MN is shortest MN is perpendicular to both lines.

Using dot product

$40\mu - 13\lambda - 53 = 0$ … (i)

$25\mu - 8\lambda - 33 = 0$ … (ii)

Solving (i) & (ii) we get $\lambda = -1$, $\mu = 1$

$\overrightarrow{OM} = \hat{i} + 3\hat{j} + 2\hat{k}$, $\overrightarrow{ON} = 4\hat{i} + 3\hat{j} - 2\hat{k}$

$\Rightarrow \left(\overrightarrow{OM}\right)\left(\overrightarrow{ON}\right) = 9$.

25. Let for any three distinct consecutive terms a, b, c of an A.P. the lines ax + by + c = 0 be concurrent at the point P and $Q(\alpha, \beta)$ be a point such that the system of equations $x + y + z = 6$, $2x + 5y + \alpha z = \beta$ and $x + 2y + 3z = 4$, has infinitely many solutions. Then $(PQ)^2$ is equal to _____

Ans. **113**

Sol. $ax + by + c = 0$ now a, b, c = a_0, $a_0 + d$, $a_0 + 2d$

$a_0(x + y + 1) + d(y + 2) = 0$

$P(1, -2)$

For infinite number of solutions

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow \alpha = 8$

And $$\begin{vmatrix} 1 & 1 & 6 \\ 2 & 5 & \beta \\ 1 & 2 & 4 \end{vmatrix} = 0$$

$\Rightarrow \beta = 6$

Q(8, 6) & P(1, – 2)

$PQ^2 = (8 - 1)^2 + (6 + 2)^2 = 113$

*26. Remainder when $64^{32^{32}}$ is divided by 9 is equal to _____ .

Ans. **1**

Sol. $(64)^{32^{32}}$ let $t = 32^{32}$

$(64)^t = (8)^{2t} = (9-1)^{2t} = {}^{2t}C_0(9)^{2t} - {}^{2t}C_1(9)^{2t-1} + {}^{2t}C_2(9)^{2t-2} + \ldots + {}^{2t}C_{2t-1}(9)^t + {}^{2t}C_{2t}$
$= 9^{2t} - 2t(9)^{2t-1} + \ldots + 9 + 1$
$= 9\lambda + 1$
$\Rightarrow$ remainder = 1.

27. Let $f(x) = \sqrt{\lim\limits_{r \to x}\left\{\frac{2r^2\left[(f(r))^2 - f(x)f(r)\right]}{r^2 - x^2} - r^3 e^{\frac{f(r)}{r}}\right\}}$ be differentiable in $(-\infty, 0) \cup (0, \infty)$ and $f(1) = 1$. Then the value of ea, such that f(a) = 0, is equal to _____

Ans. **2**

Sol. $f(x) = \sqrt{xf(x)f'(x) - x^3 e^{(f(x))/x}}$

$y^2 = xy\frac{dy}{dx} - x^3 e^{y/x}$

$-e^{-y/x}\left(1 + \frac{y}{x}\right) = x + c$

Put x = 1, 4 = 1 $\Rightarrow -2e^{-1} = 1 + c$
$C = -2/e - 1$
$\Rightarrow$ ea = 2

*28. Let the set $C = \{(x,y) \mid x^2 - 2^y = 2023, x, y \in N\}$. Then $\sum\limits_{(x,y)\in C}(x+y)$ is equal to ______ .

Ans. **46**

Sol. $x^2 - 2^y = 2023$, $x, y \in N$
$2^y = x^2 - 2023$
$2^{y-1} = \left(\frac{x^2}{2} - 2023\right)$ always odd
As R.H.S. is odd
$\Rightarrow$ $y - 1 = 0$, $y = 1$
$x = 45$
$\sum(x+y) = 45 + 1 = 46$.

29. If $\int\limits_{\pi/6}^{\pi/3}\sqrt{1-\sin 2x}\,dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$, where α, β and γ are rational numbers, then $3\alpha + 4\beta - \gamma$ is equal to _______ .

Ans. **6**

Sol. $\int\limits_{\pi/6}^{\pi/3}\sqrt{1-\sin 2x}\,dx$

$\int\limits_{\pi/6}^{\pi/4}(\cos x - \sin x)dx + \int\limits_{\pi/4}^{\pi/3}(\sin x - \cos x)dx$

$= \sqrt{2}\int\limits_{\pi/6}^{\pi/4}\cos\left(x + \frac{\pi}{4}\right)dx - \sqrt{2}\int\limits_{\pi/4}^{\pi/3}\cos\left(x + \frac{\pi}{4}\right)dx$

$= \sqrt{2}\left[1 - \sin 75^o - \sin 75^o + 1\right] = 2\sqrt{2}\left[1 - \sin 75^o\right]$

$$2\sqrt{2}\left(1-\frac{\left(\sqrt{3}+1\right)}{2\sqrt{2}}\right)=2\sqrt{2}-\sqrt{3}-1$$

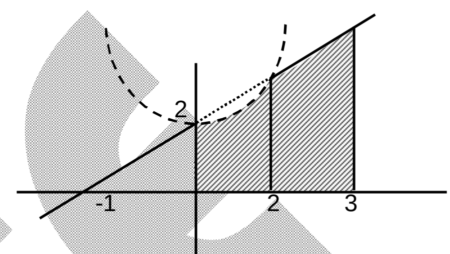
$\Rightarrow$ $\alpha = -1, \beta = 2, \gamma = -1$

Then the value of $3\alpha + 4\beta - \gamma = 6$

30. Let the area of the region $\{(x, y) : 0 \le x \le 3, 0 \le y \le \min\{x^2 + 2, 2x + 2\}\}$ be A. Then 12A is equal to ________ .

Ans. **164**

Sol. $A = \int_0^2 \left(x^2 + 2\right)dx + \int_2^3 \left(2x + 2\right)dx$



$= \left[\frac{x^3}{3} + 2x\right]_0^2 + \left[x^2 + 2x\right]_2^3$

$= \frac{8}{3} + 4 + 9 + 6 - 4 - 4$

$A = \frac{8}{3} + 11 = \frac{41}{3}$

then 12A = 164

PART – B (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

31. If the distance between object and its two times magnified virtual image produced by a curved mirror is 15 cm, the focal length of the mirror must be:
(1) 10/3 cm (2) – 12 cm
(3) – 10 cm (4) 15 cm

Ans. **(3)**

Sol. $m_t = -\frac{v}{u} = \frac{I}{O} = 2$
$|v| = 2u$ … .(i)
$u + v = 15$ … (ii)
from (i) and (ii) u = 5 cm, v = 10 cm
$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$
$\frac{1}{f} = \frac{1}{10} + \frac{1}{-5} \Rightarrow f = -10$ cm

32. Two particles X and Y having equal charges are being accelerated through the same potential difference. Thereafter they enter normally in a region of uniform magnetic field and describes circular paths of radii R_1 and R_2 respectively. The mass ratio of X and Y is:
(1) $\left(\frac{R_1}{R_2}\right)$ (2) $\left(\frac{R_2}{R_1}\right)$
(3) $\left(\frac{R_2}{R_1}\right)^2$ (4) $\left(\frac{R_1}{R_2}\right)^2$

Ans. **(4)**

Sol. $R = \frac{mv}{qB} = \sqrt{\frac{2mV}{qB^2}}$
$\frac{R_1}{R_2} = \sqrt{\frac{m_1}{m_2}} \Rightarrow \frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$

*33. The temperature of a gas having 2.0×10^{25} molecules per cubic meter at 1.38 atm (Given, $k = 1.38 \times 10^{-23}$ JK^{-1})is:
(1) 300 K (2) 500 K
(3) 100 K (4) 200 K

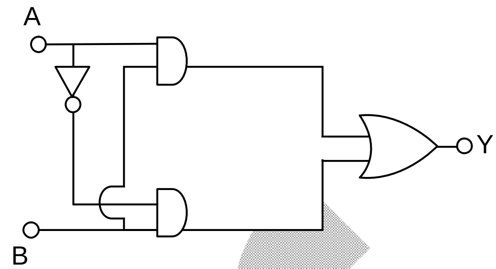
Ans. **(2)**

Sol. PV = nRT

$PV = NkT$

$T = \frac{PV}{Nk} = 500K$.

34. The truth table for this given circuit is:



(1)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(3)

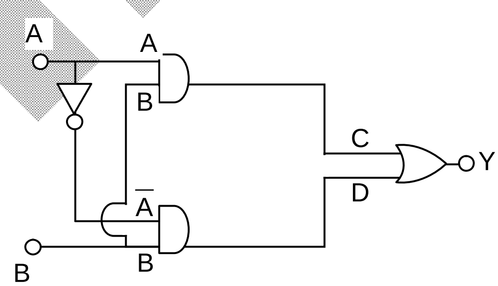
A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(4)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Ans. **(3)**

Sol. The truth table for this given circuit is:



A	$\overline{A}$	B	C	D	Y
0	1	0	0	0	0
0	1	1	0	1	1
1	0	0	0	0	0
1	0	1	1	0	1

35. In an a.c. circuit, voltage and current are given by:
V = 100 sin (100 t) V and

$I = 100\sin(100t + \frac{\pi}{3})$mA respectively.

The average power dissipated in one cycle is:

(1) 10 W
(2) 2.5 W
(3) 25 W
(4) 5 W

Ans. **(2)**

Sol. $P_{av} = \frac{V_0 I_0 \cos\phi}{2} = 2.5$ watt

*36. A stone of mass 900 g is tied to a string and moved in a vertical circle of radius 1 m making 10 rpm. The tension in the string, when the stone is at the lowest point is (if $\pi^2 = 9.8$ and $g = 9.8\,m/s^2$):

(1) 17.8 N
(2) 8.82 N
(3) 97 N
(4) 9.8 N

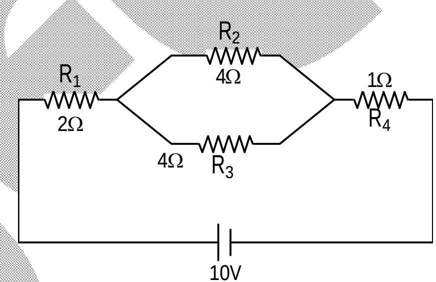
Ans. **(4)**

Sol. $T - mg = m\ell\omega^2$

$T = mg + m\ell\omega^2$

$T = 9.8$ N.

37. In the given circuit, the current in resistance R_3 is:



(1) 2.5 A
(2) 1 A
(3) 1.5 A
(4) 2 A

Ans. **(2)**

Sol. $R_{eq.} = 2 + 2 + 1 = 5\Omega$
$i = 10/5 = 2$ A
$i_{R_3} = 2\left(\frac{4}{8}\right) = 1\,A$

*38. A particle is moving in a straight line. The variation of position 'x' as a function of time 't' is given as $x = (t^3 - 6t^2 + 20t + 15)$ m. The velocity of the body when its acceleration becomes zero is:

(1) 6 m/s
(2) 10 m/s
(3) 8 m/s
(4) 4 m/s

Ans. **(3)**

Sol. $x = t^3 - 6t^2 + 20t + 15$
$v = 3t^2 - 12t + 20$
$a = 6t - 12 = 0$
$\Rightarrow t = 2$ sec.
$\Rightarrow v(t = 2) = 3 \times 4 - 12 \times 2 + 20$
$\Rightarrow v(t = 2) = 8$ m/s.

39. In Young's double slit experiment, light from two identical sources are superimposing on a screen. The path difference between the two lights reaching at a point on the screen is $7\lambda/4$. The ratio of intensity of fringe at this point with respect to the maximum intensity of the fringe is:

(1) $\frac{1}{4}$ (2) $\frac{1}{3}$

(3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Ans. **(4)**

Sol. $\Delta x = 7\lambda/4$

$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\left(\frac{7\lambda}{4}\right) = \frac{7\pi}{2}$.

$I = 2I_0 + 2I_0 \cos(7\pi/2)$

$\frac{I}{I_{max}} = \frac{2I_0}{4I_0} = \frac{1}{2}$

40. Two sources of light emit with a power of 200 W. The ratio of number of photons of visible light emitted by each source having wavelengths 300nm and 500nm respectively, will be:

(1) 3 : 5 (2) 1 : 5

(3) 5 : 3 (4) 1 : 3

Ans. **(1)**

Sol. $200 = n_1\left(\frac{hc}{\lambda_1}\right) = n_2\left(\frac{hc}{\lambda_2}\right)$

$\frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{300}{500} = \frac{3}{5}$.

*41. A small liquid drop of radius R is divided into 27 identical liquid drops. If the surface tension is T, then the work done in the process will be:

(1) $4\pi R^2T$ (2) $8\pi R^2T$

(3) $3\pi R^2T$ (4) $\frac{1}{8}\pi R^2T$

Ans. **(2)**

Sol. $\frac{4}{3}\pi R^3 = 27\left(\frac{4}{3}\pi R'^3\right)$

$R' = R/3$

$U_i = 4\pi R^2T$

$U_f = \left(4\pi\left(\frac{R}{3}\right)^2 .T\right)27$

$= 12\pi R^2 T$

$\Delta U = 8\pi R^2T$

*42. The bob of a pendulum was released from a horizontal position. The length of the pendulum is 10 m. If it dissipates 10% of its initial energy against air resistance, the speed with which the bob arrives at the lowest point is:

[Use, g : 10 ms^{-2}]

(1) $6\sqrt{5}\,ms^{-1}$ (2) $5\sqrt{5}\,ms^{-1}$

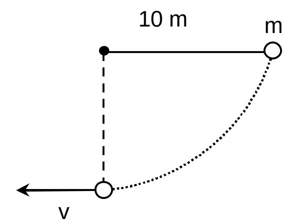
(3) $2\sqrt{5}\,ms^{-1}$ (4) $5\sqrt{6}\,ms^{-1}$

Ans. (1)

Sol. $\frac{1}{2}mv^2 = \frac{90}{100}mgh$

v = $\sqrt{1.8gh}$ (where g = 10, h = 10)

v = $6\sqrt{5}$ m/s.



43. A physical quantity Q is found to depend on quantities a, b, c by the relation $Q = \frac{a^4b^3}{c^2}$. The percentage error in a, b and c are 3%, 4% and 5% respectively. Then, the percentage error in Q is:

(1) 66%
(2) 14%
(3) 34%
(4) 43%

Ans. (3)

Sol. $\frac{dQ}{Q} = 4.\frac{da}{a} + 3\frac{db}{b} + 2\frac{dc}{c}$

= (4)(0.03) + 3(0.04)+2(0.05)

= 0.12+0.12+0.10

$\Rightarrow \frac{dQ}{Q} = 0.34$

% error = 34 %.

44. A plane electromagnetic wave of frequency 35 MHz travels in free space along the X-direction. At a particular point (in space and time) $\vec{E} = 9.6\hat{j}$V/m. The value of magnetic field at this point is:

(1) $9.6\hat{j}$T
(2) $3.2\times10^{-8}\hat{i}$T
(3) $3.2\times10^{-8}\hat{k}$T
(4) $9.6\times10^{-8}\hat{k}$T

Ans. (3)

Sol. $\frac{E}{B} = C$

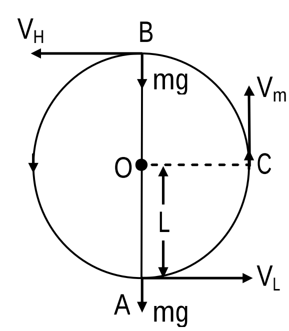
B = $\frac{E}{C}$

= $\frac{9.6}{3\times10^8} = 3.2\times10^{-8}$ T

$\vec{B} = 3.2\times10^{-8}\hat{k}$ T.

*45. A bob of mass 'm' is suspended by a light string of length 'L'. It is imparted a minimum horizontal velocity at the lowest point A such that it just completes half circle reaching the top most position B. The ratio of kinetic energies $\frac{(K.E.)_A}{(K.E.)_B}$ is:

(1) 3 : 2
(2) 5 : 1
(3) 2 : 5
(4) 1 : 5



Ans. (2)

Sol. $K.E_A = \frac{1}{2}m\left(\sqrt{5g\ell}\right)^2$

$K.E_B = \frac{1}{2}m\left(\sqrt{g\ell}\right)^2$

$\frac{K.E_A}{K.E_B} = \frac{5}{1}$

46. Given below are two statements:

Statement (I): Most of the mass of the atom and all its positive charge are concentrated in a tiny nucleus and the electrons revolve around it, is Rutherford's model.

Statement (II): An atom is a spherical cloud of positive charges with electrons embedded in it, is a special case of Rutherford's model.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is true but Statement II is false
(2) Both Statement I and Statement II are true
(3) Both Statement I and Statement II are false
(4) Statement I is false but Statement II is true

Ans. **(1)**

*47. N mole of a polyatomic gas (f = 6) must be mixed with two moles of a monoatomic gas so that the mixture behaves as a diatomic gas. The value of N is:

(1) 4 (2) 3
(3) 6 (4) 2

Ans. **(1)**

Sol. $NCv_1T + 2Cv_2T = (N + 2)\, C_{max}T$

$\frac{6}{2}NR + 2\left(\frac{3}{2}R\right) = (N+2)\left(\frac{5}{2}R\right)$

$3N + 3 = \frac{5N+10}{2}$

$6N + 6 = 5N + 10$

$N = 10 - 6 = 4$

*48. A planet takes 200 days to complete one revolution around the Sun. If the distance of the planet from Sun is reduced to one fourth of the original distance, how many days will it take to complete one revolution?

(1) 20 (2) 25
(3) 50 (4) 100

Ans. **(2)**

Sol. $T^2 \propto r^3$

$(200)^2 = kr^3$

$(T)^2 = k(r/4)^3$

$\left(\frac{T}{200}\right)^2 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$

$T = \frac{200}{8} = 25$

49. An electric field is given by $(6\hat{i}+5\hat{j}+3\hat{k})$N/C. The electric flux through a surface area $30\hat{i}\ m^2$ lying in YZ-plane (in SI unit) is:
(1) 180 (2) 90
(3) 150 (4) 60

Ans. **(1)**

Sol. $\vec{E} = 6\hat{i}+5\hat{j}+3\hat{k}$
$\phi = \int \vec{E}.d\vec{s}$
$= ((6\hat{i}+5\hat{j}+3\hat{k}).(30\hat{i})$
$= 180$

*50. A wire of length L and radius r is clamped at one end. If its other end is pulled by a force F, its length increases by ℓ. If the radius of the wire and the applied force both are reduced to half of their original values keeping original length constant, the increase in length will become:
(1) 3 times (2) 4 times
(3) $\frac{3}{2}$ times (4) 2 times

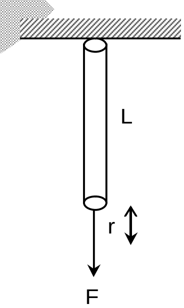
Ans. **(4)**

Sol. $F = \left(\frac{YA}{L}\right)\ell$

$F = Y\left(\frac{4\pi r^2}{L}\right)\ell$ …(i)

$\frac{F}{2} = \frac{Y(4)(\pi(r/2)^2)}{L}\ell'$ …(ii)

$\frac{1}{2} = \frac{\ell'}{4\ell} \Rightarrow \frac{\ell'}{4\ell} \Rightarrow \ell' = 2\ell$



SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

51. In a single slit diffraction pattern, a light of wavelength $6000\,\mathring{A}$ is used. The distance between the first and third minima in the diffraction pattern is found to be 3 mm when the screen in placed 50 cm away from slits. The width of the slit is ________ × 10^{-4} m.

Ans. 2

Sol. $d \sin\theta = n\lambda$
$d.(y/D) = n\lambda$
$y_1 = \frac{D\lambda}{d}$

$y_3 = 3\frac{D\lambda}{d}$

$\Delta y = 2\frac{D\lambda}{d}$

$d = \frac{2\times 50\times 10^{-2}\times 6000\times 10^{-10}}{3\times 10^{-3}}$

$d = 2 \times 10^{-4}$ m.

*52. A simple harmonic oscillator has an amplitude A and time period 6π second. Assuming the oscillation starts from its mean position, the time required by it to travel from x = A to $x = \frac{\sqrt{3}}{2}A$ will be $\frac{\pi}{x}$s, where x = _________ .

Ans. **2**

Sol. $X = A\sin\left(\frac{2\pi}{T}t\right)$

$\frac{\sqrt{3}}{2}A = A\sin\left(\frac{2\pi}{T}t\right)$

$\frac{2\pi}{T}t = \frac{\pi}{3} \Rightarrow t = T/6$

The time to move the particle from mean position to extreme position is T/4

So, $\Delta t = T/4 - T/6$

$= \frac{3T}{12} - \frac{2T}{12} = \frac{T}{12} = \frac{6\pi}{12} = \left(\frac{\pi}{2}\right)$

*53. A particle is moving in a circle of radius 50 cm in such a way that at any instant the normal and tangential components of it's acceleration are equal. If its speed at t = 0 is 4 m/s, the time taken to complete the first revolution will be $\frac{1}{\alpha}[1-e^{-2\pi}]$s. Where $\alpha =$ _________ .

Ans. **8**

Sol. $\frac{dv}{dt} = \frac{v^2}{R}$

$\int_4^v \frac{dv}{v^2} = \int_0^t \frac{dt}{R}$

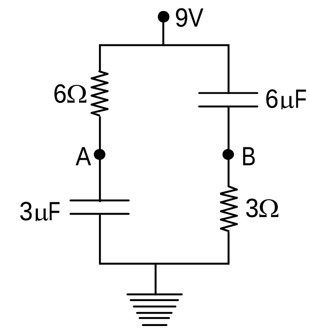
$v = \frac{4}{1-8t}$

$\int_0^{2\pi R} ds = \int_0^t \frac{4dt}{1-8t}$

$\Rightarrow\ 2\pi R = -\frac{1}{2}\ln(1-8t)$

$\Rightarrow\ t = \frac{1}{8}(1-e^{-2\pi})$

54. In the given figure, the charge stored in $6\mu F$ capacitor, when points A and B are joined by a connecting wire is _________ μC.



Ans. **36**

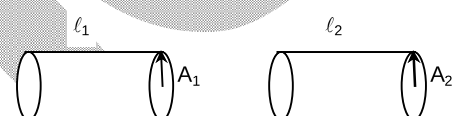
Sol. i = 9/9 = 1 amp.
ΔV (across 6 μF) = $6 \times 1 = 6V$
$q = 6 \times 6 = 36\ \mu C$.

*55. Two metallic wires P and Q have same volume and are made up of same material. If their area of cross sections are in the ratio 4 : 1 and force F_1 is applied to P, an extension of $\Delta\ell$ is produced. The force which is required to produce same extension in Q is F_2

The value of $\frac{F_1}{F_2}$ is ________ .

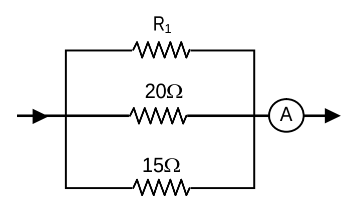
Ans. **16**

Sol. $$\frac{F_1}{F_2} = \frac{\left(\frac{yA_1}{\ell_1}\right)\Delta\ell_1}{\left(\frac{yA_2}{\ell_2}\right)\Delta\ell_2} \qquad (\Delta\ell_1 = \Delta\ell_2)$$



$$\frac{F_1}{F_2} = \left(\frac{A_1}{A_2}\right)\left(\frac{\ell_2}{\ell_1}\right) = \frac{4}{1}\left(\frac{4}{1}\right) = \frac{16}{1} \qquad (A_1\ell_1 = A_2\ell_2)$$

56. In the given circuit, the current flowing through the resistance 20Ω is 0.3 A, while the ammeter reads 0.9 A. The value of R_1 is _________ Ω.



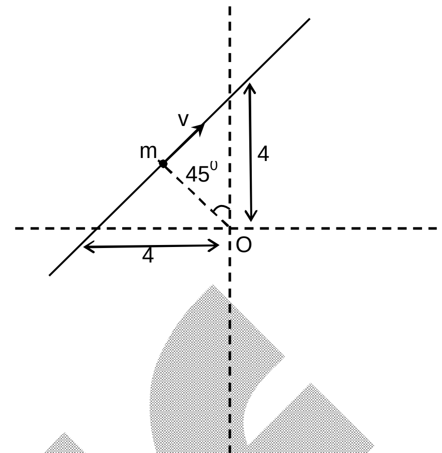
Ans. **30**

Sol. $20 \times (0.3) = 15\, i_1$
$i_1 = \frac{6}{15} = 0.4$ amp
$i_{R_1} = 0.9 - 0.4 - 0.3 = 0.2$ amp.
$20 \times 0.3 = 0.2\, R_1$
$R_1 = 30\ \Omega$.

*57. A body of mass 5 kg moving with a uniform speed $3\sqrt{2}\,ms^{-1}$ in X-Y plane along the line $y = x + 4$. The angular momentum of the particle about the origin will be _________ kg m^2s^{-1}

Ans. **60**

Sol. $P = 5 \times 3\sqrt{2} = 15\sqrt{2} m/s$
$y = x + 4$
$y - x = 4$
$\frac{x}{-4} + \frac{y}{4} = 1.$
$L_0 = (15\sqrt{2})\left(\frac{4}{\sqrt{2}}\right) = 60$



58. A charge of $4.0\mu C$ is moving with a velocity of $4.0\times10^6 ms^{-1}$ along the positive y-axis under a magnetic field $\vec{B}$ of strength $(2\hat{k})T$. The force acting on the charge is $x\hat{i}N$. The value of x is __________ .

Ans. **32**

Sol. $\vec{F} = q\vec{v}\times\vec{B}$
$= (4 \times 10^{-6})(4 \times 10^{6})\hat{j} \times (2\hat{k})$

$\vec{F} = 32\hat{i}$.

59. A horizontal straight wire 5 m long extending from east to west falling freely at right angle to horizontal component of earths magnetic field $0.60\times10^{-4} Wbm^{-2}$. The instantaneous value of emf induced in the wire when its velocity is 10 ms^{-1} is ________ $\times 10^{-3}$ V.

Ans. **3**

Sol. $\varepsilon = B\ell v$
$= 0.6 \times 10^{-4} \times 5 \times 10$
$= 30 \times 10^{-4} = 3 \times 10^{-3}$

60. Hydrogen atom is bombarded with electrons accelerated through a potential difference of V, which causes excitation of hydrogen atoms. If the experiment is being performed at T = 0 K, the minimum potential difference needed to observe any Balmer series lines in the emission spectra will be $\frac{\alpha}{10}$V, where $\alpha =$ ________.

Ans. **121**

Sol. For emission of Balmer Series the electron in ground state should at least be excited to n = 3 energy level.
$\Delta E = 13.6\left(\frac{1}{1^2} - \frac{1}{3^2}\right)eV = \frac{120.88}{10}eV = \frac{121}{10}eV$
$\Rightarrow \alpha = 121.$

PART – C (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (1), (2), (3) and (4), out of which **ONLY ONE** option is correct.

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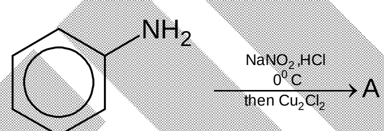
61. Which of the following acts as a strong reducing agent? (Atomic number: Ce = 58, Eu = 63, Gd=64, Lu= 71)
(1) Lu^{3+} (2) Ce^{4+}
(3) Gd^{3+} (4) Eu^{2+}

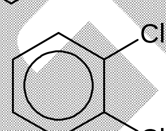
Ans. (4)
Sol. Eu^{2+} act as strong reducing agent and the general group oxidation state is +3.

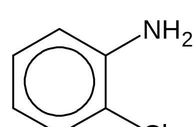
62. Which of the following statements are about Zn, Cd and Hg?
(A) They exhibit high enthalpy of atomization as the d-subshell is full.
(B) Zn and Cd do not show variable oxidation state while Hg shows +I and +II.
(C) Compounds of Zn, Cd and Hg are paramagnetic in nature.
(D) Zn, Cd and Hg are called soft metals.
Choose the most appropriate from the options is given below:
(1) B, C only (2) A, D only
(3) C, D only (4) B, D only

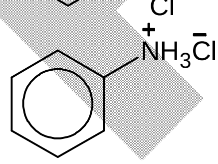
Ans. (4)
Sol. Zn and Cd have fully filled d-orbitals.

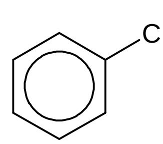
63. The product A formed in the following reaction is



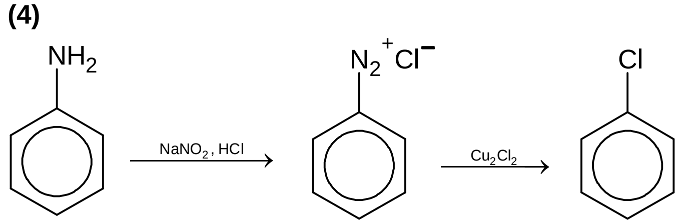
(1) 

(2) 

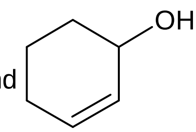
(3) 

(4) 

Ans. (4)

Sol. 

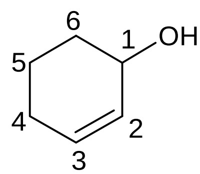
Sandmeyer reaction

*64. According to IUPAC system, the compound  is named as

(1) Cyclohex-1-en-3-ol
(2) Cyclohex-2-en-1-ol
(3) Cyclohex-1-en-2-ol
(4) 1-Hydroxyhex-2-ene

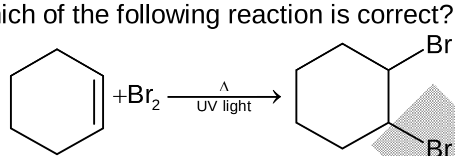
Ans. **(2)**

Sol.



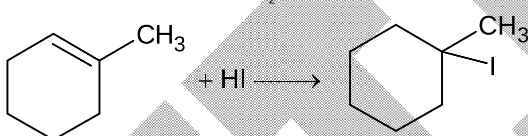
Cyclohex-2-en-1-ol

65. Which of the following reaction is correct?

(1) 

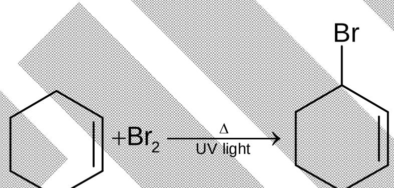
(2) $C_2H_5CONH_2 + Br_2 + NaOH \rightarrow C_2H_5CH_2NH_2 + Na_2CO_3 + NaBr + H_2O$

(3) $CH_3CH_2CH_2NH_2 \xrightarrow[H_2O]{HNO_2,\ 0^0C} CH_3CH_2OH + N_2 + HCl$

(4) 

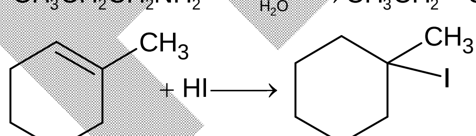
Ans. **(4)**

Sol.

1. 

2. $C_2H_5CONH_2 + Br_2 + NaOH \longrightarrow CH_3CH_2NH_2 + Na_2CO_3 + NaBr + H_2O$

3. $CH_3CH_2CH_2NH_2 \xrightarrow[H_2O]{HNO_2,0^0C} CH_3CH_2 - CH_2 - OH + N_2 + HCl$

4. 

*66. The element having the highest first ionization enthalpy is

(1) C
(2) Al
(3) Si
(4) N

Ans. **(4)**

Sol. Due to half-filled electronic configuration N has highest ionization enthalpy.

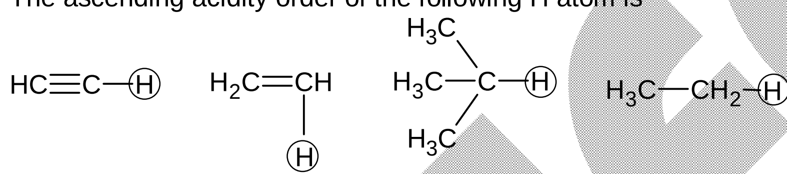
*67. Given below are two statements
Statement I: Fluorine has most negative electron gain enthalpy in its group.
Statement II: Oxygen has least negative electron gain enthalpy in its group.
In the light of the above statements, choose the most appropriate from the options given below
(1) Statement I is false but statement II is true
(2) Both Statement I and Statement II are true
(3) Both Statement I and Statement II are false
(4) Statement I is true but statement II is false

Ans. **(1)**
Sol. $O(-141.4\ kJ)$

$S(-208\ kJ)$

$Po(-174\ kJ)$

*68. The ascending acidity order of the following H atom is



A B C D

(1) A < B < C < D
(2) D < C < B < A
(3) A < B < D < C
(4) C < D < B < A

Ans. **(4)**
Sol. Electronegativity of hybridized carbon
$sp > sp^2 > sp^3$
Hydrogen attached with more electronegative carbon is more acidic.

69. Match List I with List II

LIST I (Bio Polymer)		LIST II (Monomer)	
A.	Starch	I.	Nucleotide
B.	Cellulose	II.	α-glucose
C.	Nucleic acid	III.	β-glucose
D.	Protein	IV.	α-amino acid

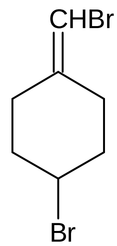
Choose the correct answer from the options given below
(1) A-II, B-III, C-I, D-IV
(2) A-II, B-I, C-III, D-IV
(3) A-IV, B-II, C-I, D-III
(4) A-I, B-III, C-IV, D-II

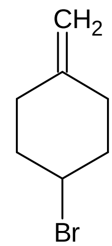
Ans. **(1)**
Sol. Starch is polymer of $\alpha-$Glucose
Cellulose is polymer of $\beta-$Glucose
Protein is polymer of $\alpha-$Amino acid
Nucleic acid form nucleotide.

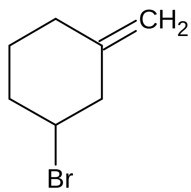
70. The correct IUPAC name of K_2MnO_4 is
(1) Potassium tetraoxidomanganese (VI)
(2) Potassium tetraoxidomanganate (VI)
(3) Dipotassium tetraoxidomanganate (VII)
(4) Potassium tetraoxomanganate (VI)

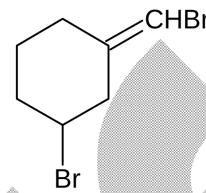
Ans. **(2)**
Sol. IUPAC name of K_2MnO_4 is potassium tetraoxidomanganate.

*71. Which one of the following will show geometrical isomerism?

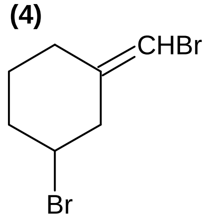
(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol.  will show geometrical isomer

*72. Anomalous behavior of oxygen is due to its

(1) small size and high electronegativity
(2) small size and low electronegativity
(3) large size and high electronegativity
(4) large size and low electronegativity

Ans. (1)

Sol. Due to small size, high electronegativity and absence of d-orbital oxygen shows anomalous behaviour.

73. Alkyl halide is converted into alkyl isocyanide by reaction with

(1) NaCN
(2) AgCN
(3) NH_4CN
(4) KCN

Ans. (2)

Sol.
$$R-X+AgCN \longrightarrow R-NC+AgX$$

74. A reagent which gives brilliant red precipitate with Nickel ions in basic medium is

(1) dimethyl glyoxime
(2) meta-dinitrobenzene
(3) sodium nitroprusside
(4) neutral $FeCl_3$

Ans. (1)

Sol.
$$Ni^{2+}+2C_4H_8O_2N_2 \longrightarrow \underset{\text{(Red ppt.)}}{\left[Ni\left(C_4H_7O_2N_2\right)_2\right]}+2H^+$$

75. On passing a gas, 'X', through Nessler's reagent, a brown precipitate is obtained. The gas 'X' is

(1) Cl_2
(2) H_2S
(3) CO_2
(4) NH_3

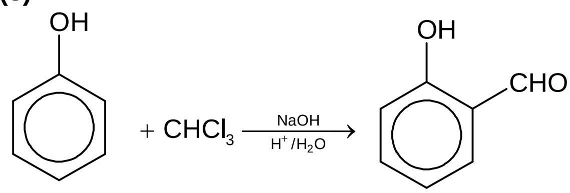
Ans. (4)

Sol.
$$2K_2HgI_4+3KOH+NH_3 \longrightarrow \underset{\text{(Brown ppt.)}}{\left[OHg_2NH_2\right]I}+7KI+2H_2O$$

76. Phenol treated with chloroform in presence of sodium hydroxide, which is further hydrolyzed in presence of an acid results
(1) Benzene -1, 2-diol
(2) Benzene -1, 3-diol
(3) 2-Hydroxybenzaldehyde
(4) Salicylic acid

Ans. **(3)**

Sol.



*77. Match List I with List II

LIST I (Spectral series for Hydrogen)		LIST II (Spectral Region/ Higher Energy State)	
A.	Lyman	I.	Infrared region
B.	Balmer	II.	UV region
C.	Paschen	III.	Infrared region
D.	Pfund	IV.	Visible region

Choose the correct answer from the options given below
(1) A-I, B-III, C-II, D-IV
(2) A-II, B-III, C-I, D-IV
(3) A-II, B-IV, C-III, D-I
(4) A-I, B-II, C-III, D-IV

Ans. **(3)**

Sol. Lyman – U. V. Region.
Balmer – Visible region.
Paschen, Pfund – Infrared region.

78. Match List I with List II

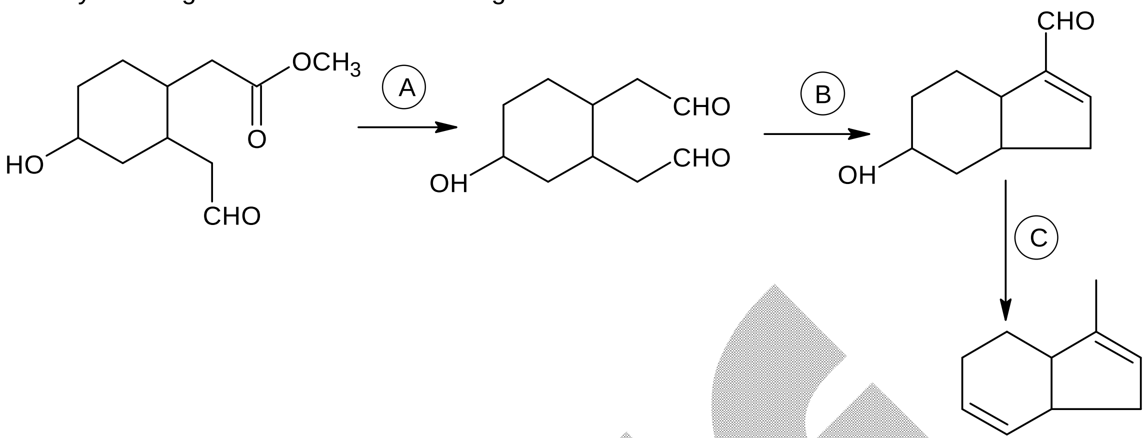
LIST I (Compound)		LIST II (pK_a value)	
A.	Ethanol	I.	10.0
B.	Phenol	II.	15.9
C.	m-Nitrophenol	III.	7.1
D.	p-Nitrophenol	IV.	8.3

Choose the correct answer from the options given below
(1) A-I, B-II, C-III, D-IV
(2) A-II, B-I, C-IV, D-III
(3) A-III, B-IV, C-I, D-II
(4) A-IV, B-I, C-II, D-III

Ans. **(2)**

Sol. Acidity order : p-Nitrophenol > m-Nitrophenol > Phenol > Ethanol

79. Identify the reagents used for the following conversion



(1) $LiAlH_4$, $B = NaOH(aq)$, $C = NH_2 - NH_2$ / KOH, ethylene glycol

(2) DIABAL – H, $B = NaOH(aq)$, $C = NH_2 - NH_2$ / KOH, ethylene glycol

(3) A = DIBAL – H, $B = NaOH(alc)$, $C = Zn / HCl$

(4) $A = LiAlH_4$, $B = NaOH(alc)$, $C = Zn / HCl$

Ans. **(2)**
Sol. DIBAL-H reduces ester into aldehyde.

80. Chromatographic techniques based on the principle of differential adsorption is/are
A. Column chromatography
B. Thin layer chromatography
C. Paper chromatography
Choose the most appropriate answer from the options given below

(1) B only
(2) A only
(3) A & B only
(4) C only

Ans. **(3)**
Sol. These three types of adsorption chromatography
1. Column chromatography.
2. Thin layer chromatography.
3. Gas/Solid chromatography.

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

81. The total number of anti bonding molecular orbitals, formed from 2s and 2p atomic orbitals in a diatomic molecule is ……………….

Ans. **4**
Sol. 3 antibonding with 2p-orbital and 4^{th} antibonding with 2s-orbital.

*82. Standard enthalpy of vapourisation for CCl_4 is 30.5 kJ mol^{-1}. Heat required for vapourisation of 284 g of CCl_4 at constant temperature is ……………. kJ.
(Given molar mass in g mol^{-1} : C = 12, Cl = 35.5)

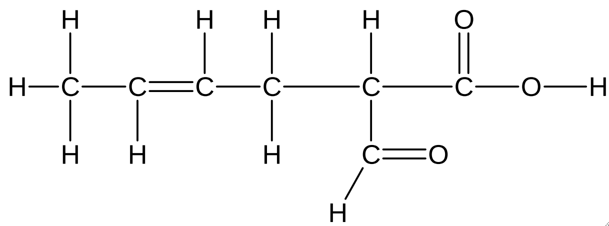
Ans. **56**

Sol. $\Delta H_{vap} = \frac{30.5}{154} \times 1284 = 56.24 \approx 56$

*83. The total number of 'Sigma' and 'Pi' bonds in 2-formylhex-4-enoic acid is …………………

Ans. **22**

Sol.



= 25

*84. If 50 mL of 0.5 M oxalic acid is required to neutralize 25 mL of NaOH solution, the amount of NaOH in 50 mL of given NaOH solutions is ……………….g

Ans. **4**

Sol. $M \times V \times n-\text{factor of oxalic acid} = M \times V \times n-\text{factor of NaOH}$

$50 \times 0.5 \times 2 = 25 \times 1 \times M$

M = 2

$M = \frac{W}{M} \times \frac{1}{V} \times 1000 = 2 = \frac{W}{40} \times \frac{1}{50} \times 1000 = 4$

85. The oxidation number of iron in the compound formed during brown ring test for NO_3^- ion is ………

Ans. **+1**

Sol. $\left[Fe^{+1}(NO)(H_2O)_5\right]^{+2}$

*86. The half-life of radioisotope bromine -82 is 36 hours. The fraction which remains after one day is ………… $\times 10^{-2}$.
(Given antilog 0.2006 = 1.587)

Ans. **63**

Sol. $K = \frac{0.693}{36}$

$\frac{0.693}{36} = \frac{2.303}{24} \log \frac{100}{100-x} (\text{amount left})$

$\frac{0.693 \times 24}{36 \times 2.303} = \log \frac{100}{(100-x)}, \qquad 0.2006 = \log \frac{100}{(100-x)}$

$\frac{100}{100-x} = 1.587$

(100 – x) = 63
(amount left)

87. A constant current was passed through a solution of $AuCl_4^-$ ion between gold electrodes. After a period of 10.0 minutes, the increase in mass of cathode was 1.314 g. The total charge passed through the solution is……………………… $\times 10^{-2}$F.
(Given atomic mass Au = 197)

Ans. **2**

Sol. $AuCl_4^- \longrightarrow Au^{3+} + 4Cl^-$

$Au^{3+} + 3e^- \longrightarrow Au$

197 g obtained by – 3F

1 g obtained by $\frac{3}{197}F$

1.314 g obtained by $\frac{3\times 1.314}{197}F = 2\times 10^{-2}$ F

*88. Molality of 0.8 M H_2SO_4 solution (density 1.06 g cm^{-3}) is …………..× 10^{-3} m.

Ans. **815**

Sol. $m = \frac{1000M}{1000d - M.M.}$

$= \frac{1000\times 0.8}{1000\times 1.06 - 98\times 0.8}$

$= \frac{800}{1060 - 78.4} = 815\times 10^{-3}$

*89. The total number of molecules with zero dipole moment among CH_4, BF_3, H_2O, HF, NH_3, CO_2 and SO_2 is ……….

Ans. **3**

Sol. CO_2, BF_3, CH_4 have symmetrical structure.

*90. The following concentrations were observed at 500K for the formation of NH_3 from N_2 and H_2. At equilibrium: $[N_2] = 2\times 10^{-2}$M, $[H_2] = 3\times 10^{-2}$ and $[NH_3] = 1.5\times 10^{-2}$M. Equilibrium constant for the reaction is ………………

Ans. **417**

Sol. $N_2 + 3H_2 \rightleftharpoons 2NH_3$

$K = \frac{[NH_3]^2}{[N_2][H_2]^3} = \frac{(1.5\times 10^{-2})^2}{2\times 10^{-2}\times (3\times 10^{-2})^3}$

$K = 0.04166\times 10^4$

$K = 417$