MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – JANUARY 2026

(HELD ON WEDNESDAY 21st JANUARY 2026) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. If α & β are roots of the quadratic equation $x^2 + x + 1 = 0$, then the value of $(\alpha + \beta)^4 + (\alpha^2 + \beta^2)^4 + (\alpha^3 + \beta^3)^4 + \ldots\ldots + (\alpha^{25} + \beta^{25})^4$ is -

(1) 145 (2) 140

(3) 415 (4) 541

Ans. **(1)**

Sol. $x^2 + x + 1 = 0$

$\Rightarrow x = \omega$ or ω^2

$\therefore \alpha = \omega$ & $\beta = \omega^2$

Now, $(\alpha + \beta)^4 + (\alpha^2 + \beta^2)^4 + (\alpha^3 + \beta^3)^4 + \ldots + (\alpha^{25} + \beta^{25})^4$

$=(\omega + \omega^2)^4 + (\omega^2 + \omega^4)^4 + (\omega^3 + \omega^6)^4 + \ldots + (\omega^{25} + \omega^{50})^4$

$=[(\omega + \omega^2)^4 + (\omega^2 + \omega^4)^4 + (\omega^4 + \omega^8)^4 + \ldots + (\omega^{25} + \omega^{50})^4] + [(\omega^3 + \omega^6)^4 + (\omega^6 + \omega^{12})^4 + (\omega^9 + \omega^{18})^4 + \ldots + (\omega^{24} + \omega^{48})^4]$

$= \underbrace{[1+1+1+\ldots\ldots+1]}_{17\text{ times}} + \underbrace{[(1+1)^4+(1+1)^4+\ldots\ldots(1+1)^4]}_{8\text{ times}}$

$= 17 + 128$

$= 145$

2. If $A = \begin{bmatrix} \alpha & 2 \\ 1 & 2 \end{bmatrix}$ and B = $A = \begin{bmatrix} 1 & 1 \\ 1 & \beta \end{bmatrix}$ and $A^2 - 4A + 2I = \mathbf{O}$ and $B^2 = 3B + I = \mathbf{O}$ then the value of $\left|\text{adj}(A^3 - B^3)\right|$ is

(1) – 5 (2) –15

(3) 9 (4) –7

Ans. **(2)**

Sol. $Tr(A) = 4 \Rightarrow \alpha + 2 = 4 \Rightarrow \alpha = 2$

$Tr(B) = 3 \Rightarrow \beta + 1 = 3 \Rightarrow \beta = 2$

$A^2 - 4A + 2I = \mathbf{O}$

$\Rightarrow A^2 = 4A - 2I$

$A^3 = 4A^2 - 2A = 16A - 8I - 2A$

$= 14A - 8I$

$= \begin{bmatrix} 28 & 28 \\ 14 & 28 \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ 0 & -8 \end{bmatrix}$

$= A^3 = \begin{bmatrix} 20 & 28 \\ 14 & 20 \end{bmatrix}$

$B^2 - 3B + I = \mathbf{O}$

$B^2 = 3B - I \Rightarrow B^3 = 3B^2 - B = 3(3B - I) - B = 8B - 3I$

$B^3 = \begin{pmatrix} 8 & 8 \\ 8 & 16 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix}$

$A^3 - B^3 = \begin{pmatrix} 20 & 28 \\ 14 & 20 \end{pmatrix} - \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix} = \begin{pmatrix} 15 & 20 \\ 6 & 7 \end{pmatrix}$

$\Rightarrow \left|A^3 - B^3\right| = 105 - 120 = -15$

$\Rightarrow \left|\text{adj}(A^3 - B^3)\right| = \left|A^3 - B^3\right| = -15$

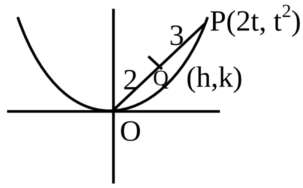
3. Let the parabola $x^2 = 4y$ and consider its vertex O and a variable point P on the parabola and consider another point Q which divides OP is the ratio 2 : 3 internally. Then the equation of chord of locus Q whose mid point is (1, 2):

(1) $5x - 4y + 3 = 0$ (2) $4x - 5y + 3 = 0$

(3) $5x + 4y + 3 = 0$ (4) $4x + 5y + 3 = 0$

Ans. **(1)**

Sol. $h = \dfrac{4t}{5}$



$k = \dfrac{2t^2}{5} = \dfrac{2}{5}\cdot\left(\dfrac{5h}{4}\right)^2$

$8k = 5h^2$

$\Rightarrow 5x^2 = 8y$

$T = S_1$

$5(xx_1) - 4(y + y_1) = 5x_1^2 - 8y_1$

$5(x) - 4(y + 2) = 5 - 8.2$

$5x - 4y + 3 = 0$

4. Let $f : R \to R$ be twice differentiable function such that $f(x)m^2 - 2m.f'(x) + f''(x) = 0$ for all $m \in R$ has equal roots where $f(0) = 1$, $f'(0) = 2$ and $f(\ln x - x)$ is increasing in the interval $(\alpha, \beta]$, then find $(\alpha + \beta)$.

(1) 1 (2) 2

(3) 3 (4) 4

Ans. **(1)**

Sol. Given quadratic equation has equal roots, thus

$D = 0 \Rightarrow (f'(x))^2 = f''(x) \cdot f(x)$

$$\frac{f'(x)}{f(x)} = \frac{f''(x)}{f'(x)}$$

Integrate

$\ln(f(x)) = \ln(f'(x)) + \ln C \Rightarrow f(x) = c.f'(x)$

Put = x = 0

$1 = c.2 \Rightarrow c = \frac{1}{2}$

Now $2f(x) = f'(x)$

$$\Rightarrow \frac{f'(x)}{f(x)} = 2$$

Integrate

$\ln(f(x)) = 2x + d$

Put x = 0

$\Rightarrow d = 0$

$\Rightarrow \ln(f(x)) = 2x \Rightarrow f(x) = e^{2x}$

Now Let $g(x) = f(\ln x - x) = e^{2(\ln x - x)}$

$$\therefore g'(x) = e^{2(\ln x - x)}\left(\frac{1}{x} - 1\right) \geq 0$$

$$\Rightarrow \frac{1-x}{x} \geq 0$$

$\Rightarrow x \in (0, 1]$

$\Rightarrow \alpha = 0, \beta = 1$

$\alpha + \beta = 1.$

5. Given two set A = {1, 2, 3, ..., 6} and B = {1, 2, 3, ..., 9}, the find the number of increasing funcitons $f : A \to B$, such that $f(i) \neq i \ \forall i \in \{1, 2, 3, ..., 6\}$:

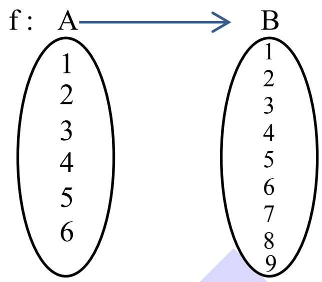
(1) 26 (2) 27

(3) 28 (4) 29

Ans. **(3)**

Sol. $f(i) \neq i$, f(x) is strictly increasing function $f : A \to B$, where A {1, 2,3 ......6}

B {1, 2, 3, ...9}, then number of function $f : A \to B$ is equal to .



$f(i) \neq i$ Case-i $f(1) = 2 \Rightarrow {}^7C_5 = 21$

Case-ii $f(1) = 3 \Rightarrow {}^6C_5 = 6$

Case-iii $f(1) = 4 \Rightarrow {}^5C_5 = 1$

No of function A to B = 21 + 6 + 1 = 28

6. Tangent are drawn from P to the circle $x^2 + y^2 - 4x - 6y + 12 = 0$ touching at A & B such that $\angle AOB = 60°$ (0 is centre of circle) then locus of P is :

(1) $x^2 + y^2 - 4x - 6y + \frac{35}{3} = 0$

(2) $x^2 + y^2 - 6x - 4y + \frac{35}{3} = 0$

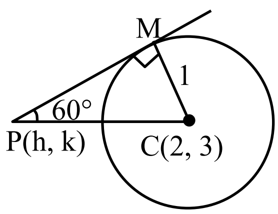
(3) $x^2 + y^2 - 4x - 6y + 35 = 0$

(4) $x^2 + y^2 - 4x - 6y + 40 = 0$

Ans. **(1)**

Sol. $\tan 60° = \sqrt{3} = \frac{OM}{PM}$

$$\therefore PM = \frac{1}{\sqrt{3}}$$



& equation to circle is

$S \equiv x^2 + y^2 - 4x - 6y + 12 = 0$

$\therefore\ PM = \sqrt{S(h,k)} = \dfrac{1}{\sqrt{3}}$

$\therefore$ locus to P(h, k) is

$x^2 + y^2 - 4x - 6y + 12 = \dfrac{1}{3}$

i.e $x^2 + y^2 - 4x - 6y + \dfrac{35}{3} = 0$

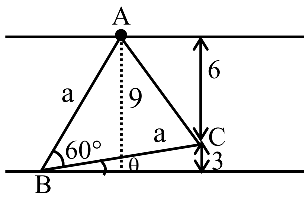
7. L_1 & L_2 are two parallel lines and ABC is an equilateral triangle , such that A & B lie on L_1 & L_2 respectively. Vertex C is at a distance of 6 unit from L_1 & 3 unit from L_2. Then Area ΔABC is :

(1) $21\dfrac{\sqrt{3}}{3}$ (2) $21\sqrt{3}$

(3) $23\dfrac{\sqrt{3}}{3}$ (4) $23\sqrt{3}$

Ans. (2)

Sol.



$\sin\theta = \dfrac{3}{a}$

$\sin(60° + \theta) = \dfrac{9}{a}$

$\dfrac{\sqrt{3}}{2}\cos\theta + \dfrac{1}{2}\sin\theta = \dfrac{9}{a}$

$\sqrt{3}\sqrt{1 - \dfrac{9}{a^2}} + \dfrac{3}{a} = \dfrac{18}{a}$

$a = \sqrt{84}$

Area of $\Delta ABC = \dfrac{\sqrt{3}}{4}a^2$

$= \dfrac{\sqrt{3}}{4} \times 84$

$= 21\sqrt{3}$

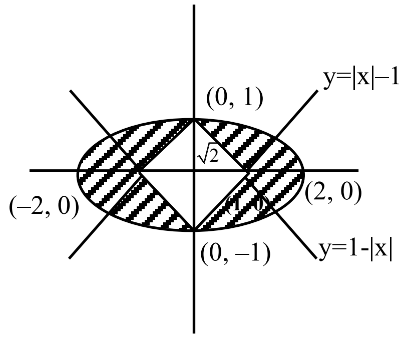
8. Area bounded inside ellipse $x^2 + 4y^2 = 4$ and outside area bounded by curve $y = |x| - 1$ and $y = 1 - |x|$:

(1) $2\pi + 1$ (2) $2\pi - 1$

(3) $2\pi + 2$ (4) $2\pi - 2$

Ans. (4)

Sol. $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$



to find Area of shaded portion

= Total Area – Area of square

$= \pi(2)(1) - \left(\sqrt{2}\right)\left(\sqrt{2}\right)$

$= 2\pi - 2$

9. If $y = y(x)$ and $(1 + x^2)\,dy + (1 - \tan^{-1}x)\,dx = 0$ and $y(0) = 1$ then $y(1)$ is :

(1) $\dfrac{\pi^2}{32} + \dfrac{\pi}{4} + 1$ (2) $\dfrac{\pi^2}{32} - \dfrac{\pi}{4} + 1$

(3) $\dfrac{\pi^2}{32} - \dfrac{\pi}{2} - 1$ (4) $\dfrac{\pi^2}{32} - \dfrac{\pi}{2} + 1$

Ans. (2)

Sol. $(1 + x^2)dy + (1 - \tan^{-1}x)\,dx = 0$

$\Rightarrow \int dy + \int \dfrac{1 - \tan^{-1}(x)}{1 + x^2}dx = 0$

$\left\{\text{put}\ \begin{array}{c} 1 - \tan^{-1}x = t \\ \Rightarrow \dfrac{dx}{1 + x^2} = -dt \end{array}\right\}$

$\Rightarrow y - \int t\,dt = 0$

$\Rightarrow y = \dfrac{t^2}{2} + c \Rightarrow y = \dfrac{\left(1 - \tan^{-1}x\right)^2}{2} + c$

$\left\{\text{put}\ \begin{array}{c} x = 0\ \&\ y = 1 \\ \therefore 1 = \dfrac{1}{2} + c \Rightarrow c = \dfrac{1}{2} \end{array}\right\}$

Now $\boxed{y = \dfrac{\left(1 - \tan^{-1}(x)\right)^2 + 1}{2}}$

Find $y(1) - \dfrac{\left(1 - \dfrac{\pi}{4}\right)^2 + 1}{2} = \dfrac{\pi^2}{32} - \dfrac{\pi}{4} + 1$

10. The value of $\operatorname{cosec}10^\circ - \sqrt{3}\sec 10^\circ$ is :

(1) 1 (2) 2

(3) 3 (4) 4

Ans. **(4)**

Sol. $= \dfrac{1}{\sin 10^\circ} - \dfrac{\sqrt{3}}{\cos 10^\circ}$

$= \dfrac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cos 10^\circ}$

$= 4.\left[\dfrac{\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ}{2\sin 10^\circ \cos 10^\circ}\right]$

$= 4.\left[\dfrac{\sin(30^\circ - 10^\circ)}{\sin 20^\circ}\right]$

$= 4$

11. Let H be a Hyperbola confocal with ellipse $\dfrac{x^2}{36} + \dfrac{y^2}{16} = 1$ having eccentricity '5', then length of latus rectum of H is :

(1) $\dfrac{96}{\sqrt{5}}$ (2) $\dfrac{9}{\sqrt{5}}$

(3) 96 (4) $\sqrt{5}$

Ans. **(1)**

Sol. Let e_1 be eccentricity of ellipse

$\Rightarrow e_1 = \sqrt{1 - \dfrac{16}{36}} = \sqrt{1 - \dfrac{4}{9}} = \dfrac{\sqrt{5}}{3}$

So $ae_1 = 6.\dfrac{\sqrt{5}}{3} = 2\sqrt{5}$

Now $H : \dfrac{x^2}{p^2} - \dfrac{y^2}{q^2} = 1$

$p.e = ae_1$

$p.5 = 2\sqrt{5}$

$p = \dfrac{2}{\sqrt{5}} \Rightarrow e^2 = 1 + \dfrac{q^2}{p^2} \Rightarrow 25 = 1 + \dfrac{5q^2}{4} \Rightarrow q^2 = \dfrac{96}{5}$

So length of LR $\dfrac{2q^2}{p} = \dfrac{96}{\sqrt{5}}$

12. The value of integral $I = \displaystyle\int_{-\pi/6}^{\pi/6} \dfrac{2\pi + 4x^{11}}{1 - \sin\left(|x| + \pi/6\right)} dx$ is

(1) π (2) 2π

(3) 4π (4) 8π

Ans. **(4)**

Sol. $= 4\pi \displaystyle\int_0^{\pi/6} \dfrac{1}{1 - \sin\left(x + \frac{\pi}{6}\right)} dx$ let $x + \dfrac{\pi}{6} = t$ $dx = dt$

$= 4\pi \displaystyle\int_{\pi/6}^{\pi/3} \dfrac{dt}{1 - \sin t} = 4\pi \int_{\pi/6}^{\pi/3} \dfrac{1 + \sin t}{\cos^2 t} dt$

$= 4\pi \left[\displaystyle\int_{\pi/6}^{\pi/3} \sec^2 t\,dt + \int_{\pi/6}^{\pi/3} \sec t \tan t\,dt\right]$

$= 4\pi \left[(\tan t)_{\pi/6}^{\pi/3} + (\sec t)_{\pi/6}^{\pi/3}\right]$

$= 4\pi \left[\left(\sqrt{3} - \dfrac{1}{\sqrt{3}}\right) + \left(2 - \dfrac{2}{\sqrt{3}}\right)\right]$

$= 4\pi\left[\sqrt{3} + 2 - \sqrt{3}\right] = 8\pi$

SECTION-B

13. Consider the expansion $(ax^2 + bx + c)(1 - 2x)^{26}$. If the coefficient of x^2 and x^3 are zero and coefficient of x is – 56 then (a + b + c) is

Ans. **(1403)**

Sol. $(ax^2 + bc + c)\sum_{r=0}^{26} {}^{26}C_r(-2x)^r$

Coeff. of x^2 : $a.{}^{26}C_0(-2)^0 + b.{}^{26}C_1(-2) + c.{}^{26}C_2(-2)^2 = 0$

$\Rightarrow a - 52b + 1300c = 0 \quad \ldots (1)$

Coeff. of x^3 : $a.{}^{26}C_1(-2) + b.{}^{26}C_2(-2)^2 + c.{}^{26}C_3(-2)^3 = 0$

$\Rightarrow -52a + 1300b - 20800c = 0 \quad \ldots (2)$

Coeff. of x = – 56

$\Rightarrow b.{}^{26}C_0(-2)^0 + c.{}^{26}C_1(-2)^1 = -56$

$b - 52c = -56 \quad \ldots (3)$

After solving (1), (2) & (3)

a = 1300, b = 100, c = 3

$\Rightarrow a + b + c = 1403$

14. Sum of roots of equation $|x-1|^2 - 5|x-1| + 6 = 0$ is

Ans. **(4)**

Sol. Let $|x-1| = t$

$t^2 - 5x + 6 = 0$

$t = 2$ & $t = 3$

$|x-1| = 2$ & $|x-1| = 3$

$x - 1 = \pm 2$ & $x - 1 = \pm 3$

$x = 1 \pm 2$ & $x = 1 \pm 3$

$\therefore$ root = 3, – 1, 4, – 2

$\therefore$ Sum of root = 3 + (–1) + 4 + (– 2) = 4.

15. If Domain of

$f(x) = \cos^{-1}\left(\frac{2x-5}{11-3x}\right) + \sin^{-1}(2x^2 - 3x + 1)$ is [a, b],

find value of (a + 2b):

Sol. $-1 \le \frac{2x-5}{11-3x} \le 1$ & $-1 \le 2x^2 - 3x + 1 \le 1$

$\Rightarrow \frac{2x-5}{11-3x} - 1 \le 0,\ \frac{2x-5}{11-3x} + 1 \ge 0,\ 2x^2 - 3x \le 0$

$\frac{2x-5-11+3x}{11-3x} \le 0,\ \frac{-x+6}{11-3x} \ge 0,\ 2x^2 - 3x \le 0$

$\frac{5x-16}{11-3x} \le 0$ & $\frac{x-6}{3x-11} \ge 0$ & $2x^2 - 3x \le 0$

$x \le \frac{16}{5}$ or $\frac{11}{3} < x$ & $x < \frac{11}{3}$ or $6 \le x$ & $0 \le x \le \frac{3}{2}$

taking intersection

$\Rightarrow x \in \left[0, \frac{3}{2}\right] \Rightarrow a = 0, b = \frac{3}{2}$

$\Rightarrow a + 2b = 0 + 2\left(\frac{3}{2}\right)$

= 3

16. Given

$a_{n+1} - \frac{1}{2}a_n = \frac{n^2 - 2n - 1}{n^2(n+1)^2}, n \ge 1$ such that $a_1 = 1$ then

evaluate $\left|\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right)\right|$

Sol. $a_{n+1} - \frac{1}{2}a_n = \frac{n^2 - 2n - 1}{n^2(n+1)^2} = \frac{2n^2 - (n+1)^2}{n^2(n+1)^2}$

$\Rightarrow a_{n+1} - \frac{1}{2}a_n = \frac{2}{(n+1)^2} - \frac{1}{n^2}$

$n = 1 \quad a_2 - \frac{1}{2}a_1 = \frac{2}{2^2} - \frac{1}{1^2}$

$2\left[a_3 - \frac{1}{2}a_2 = \frac{2}{3^2} - \frac{1}{2^2}\right]$

$2^2\left[a_4 - \frac{1}{2}a_3 = \frac{2}{4^2} - \frac{1}{3^2}\right]$

$\vdots$

$2^{n-2}\left[a_n - \frac{1}{2}a_{n-1} = \frac{2}{n^2} - \frac{1}{(n-1)^2}\right]$

$2^{n-1}\left[a_{n+1} - \frac{1}{2}a_n = \frac{2}{(n+1)^2} - \frac{1}{n^2}\right]$

Adding

$a_{n+1} = \frac{2}{(n+1)^2} - \frac{1}{2^n} \quad \Rightarrow a_n = \frac{2}{n^2} - \frac{1}{2^{n-1}}$

$\Rightarrow \left|\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right)\right| = \left|\sum_{n=1}^{\infty} -\frac{1}{2^{n-1}}\right| = 2$

17. If a_1, a_2, a_3.......are increasing geometric progression such that $a_1+a_3+a_5 = 21$ and $a_1a_3a_5 = 64$, then value of $a_1 + a_2 + a_3$ is

Ans. **(7)**

Sol. Let the G.P. a, ar, ar^2, ar^3..........

$a_1+a_3+a_5=21 \Rightarrow a+ar^2+ar^4=21$ ..... (1)

$a_1.a_3.a_5=64 \Rightarrow a.ar^2.ar^4=64 \Rightarrow a^3r^6=64$ .... (2)

$ar^2 = 4$

$$\frac{4}{r^2}+4+4r^2=21$$

$$\Rightarrow 4r^4-17r^2+4=0$$

$$r^2=4 \text{ or } r^2=\frac{1}{4}\ (\text{Reject})$$

$$\boxed{r=2}$$

$$\Rightarrow \boxed{a=1}$$

$a_1 + a_2 + a_3 = a + ar + ar^2$

$= a(1 + r + r^2) = 1.(1 + 2 + 2^2) = 7$

18. The value of $6\int_0^{\pi}|\sin x+\sin 3x+\sin 2x|dx$ is :

Ans. **(17)**

Sol. $$6\int_0^{\pi}|2\sin 2x\cos x+\sin 2x|dx$$

$$6\int_0^{\pi}|4\sin x\cos^2 x+2\sin x\cos x|dx$$

$$I=12\int_0^{\pi}\sin x|2\cos^2 x+\cos x|dx$$

Put cosx = t, –sinx dx = dt

$$I=-12\int_1^{-1}|2t^2+t|dt=12\int_{-1}^{1}|2t^2+t|dt$$

$$I=12\left(\int_{-1}^{-1/2}(2t^2+t)dt+\int_{-1/2}^{0}-(2t^2+t)dt+\int_0^1(2t^2+t)dt\right)$$

I = 17

19. Let a twice differentiable function f(x) on R such that f(3) = 18, f'(3) = 0 and $f''(3)=4$, then the value of $\log_e\left(\lim_{x\to1}\left(\frac{f(x+2)}{f(3)}\right)^{\frac{18}{(x-1)^2}}\right)$ is :

Ans. **(2)**

Sol. Let $T=\lim_{x\to1}\left(\frac{f(x+2)}{f(3)}\right)^{\frac{18}{(x-1)^2}}$;1^∞ form

$$\Rightarrow T=e^{\lim_{x\to1}\frac{18}{(x-1)^2}\left(\frac{f(x+2)-f(3)}{f(3)}\right)}$$

$$\Rightarrow T=e^{\lim_{x\to1}\frac{18}{(x-1)^2}\left(\frac{f(x+2)-f(3)}{18}\right)}$$

$$\Rightarrow T=e^{\lim_{x\to1}\left(\frac{f(x+2)-f(3)}{(x-1)^2}\right);\ \frac{0}{0}\text{ form}}$$ apply L'pital

$$\Rightarrow T=e^{\lim_{x\to1}\frac{f'(x+2)}{2(x-1)};\ \frac{0}{0}\text{ form}}$$ apply L'pital

$$\Rightarrow T=e^{\lim_{x\to1}\frac{f''(x+2)}{2}};\ =e^{\frac{4}{2}}=e^2$$

$$\Rightarrow \log_e(T)=2$$

20. Consider set $S=\{1,2,3,......50\}$ where m, n, $\in$ S. If P is the no. of ways such that $6^m + 9^n$ is divisible by 5 and Q is the no. of ways in which (m + n) is the square of prime number. Find (P + Q)

Ans. **(1333)**

Sol. $S=\{1,2,3,......50\}$

P = $(6^m + 9^n)$ is divisible by 5

No. of ways

$6^m = (5\lambda + 1)^m = 5k + 1$

$9^n = (10–1)^n = 10\mu–1$ if n is odd $\Rightarrow$ n must be odd

$10\mu + 1$ if n is even

$\Rightarrow$ No. of ways = 50 × 25 = 1250

Q $\Rightarrow$ (m + n) is square of a prime

	m + n = 4	m + n = 9	m + n = 25
	↓ ,	↓ ,	↓ ,
No. of ways	3	8	24

m + n = 49
↓ ,
48

Q = 3 + 8 + 24 + 48 = 83

= P + Q = 1250 + 83 = 1333