MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – JANUARY 2026

(HELD ON WEDNESDAY 28th JANUARY 2026) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

1. A bag contains 'k' red balls and (10 – k) black balls. If 3 balls are drawn at random and they are found to be black then the probability that bag has 9 black balls & 1 red ball is

(1) $\frac{7}{11}$ (2) $\frac{14}{55}$

(3) $\frac{21}{55}$ (4) $\frac{6}{11}$

Ans. **(2)**

Sol. Probability $= \dfrac{{}^1C_0 \cdot {}^9C_3}{\sum\limits_{k=0}^{10} {}^kC_0 \cdot {}^{10-k}C_3}$

$$= \frac{{}^9C_3}{{}^{10}C_3 + {}^9C_3 + {}^8C_3 + \ldots {}^3C_3}$$

$$= \frac{{}^9C_3}{{}^{11}C_4} = \frac{14}{55}$$

2. Area bounded by

$\{(x, y) : xy \le 8 ; y \le x^2, y \ge 1\}$ is :

(1) $16 \log_e 2 - \frac{14}{3}$

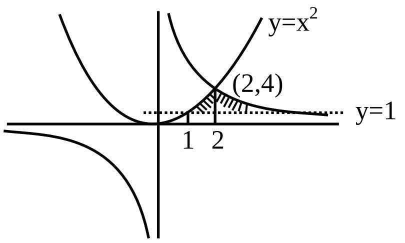
(2) $16 \log_e 2 - \frac{13}{3}$

(3) $16 \log_e 2 - \frac{17}{3}$

(4) $16 \log_e 2 - \frac{19}{3}$

Ans. **(1)**

Sol.



$$A = \int_1^2 \left(x^2 - 1\right)dx + \int_2^8 \left(\frac{8}{x} - 1\right)dx$$

$$A = 8 \log_e 4 - \frac{14}{3} = 16 \log_e 2 - \frac{14}{3}$$

3. Value of : $\sum\limits_{k=1}^{\infty} \dfrac{(-1)^k . k(k+1)}{k!}$

(1) $-\frac{1}{e}$ (2) $-\frac{2}{e}$

(3) $-\frac{3}{e}$ (4) $-\frac{4}{e}$

Ans. **(1)**

Sol. $\sum\limits_{k=1}^{\infty} \dfrac{(-1)^k}{k!}\left(k(k-1)+2k\right)$

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{(k-2)!} + 2\sum_{k=1}^{\infty} \frac{(-1)^k}{(k-1)!}$$

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-2}}{(k-2)!} - 2\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{(k-1)!}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} - 2\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} = -\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}$$

$$= -\left(\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots\right)$$

$$= -\frac{1}{e}$$

4. In ΔABC if $\dfrac{\tan(A-B)}{\tan A} + \dfrac{\sin^2 C}{\sin^2 A} = 1$ where $A, B, C \in \left(0, \frac{\pi}{2}\right)$ then

(1) tan A, tan B, tan C are in A.P.

(2) tan A, tan C, tan B are in A.P.

(3) tan A, tan B, tan C are in G.P.

(4) tan A, tan C, tan B are in G.P.

Ans. **(4)**

Sol. $\frac{\tan A-\tan B}{\tan A\left(1+\tan A\tan B\right)}+\frac{1+\cot^2 A}{1+\cot^2 C}=1$

Let $\tan A = x$

$\tan B = y$

$\tan C = z$

$$\frac{x-y}{x(1+xy)}+\frac{1+\frac{1}{x^2}}{1+\frac{1}{z^2}}=1$$

$$\frac{(x-y)}{x(1+xy)}+\frac{z^2(x^2+1)}{x^2(z^2+1)}=1$$

$x(x-y)(z^2+1)+z^2(x^2+1)(1+xy)=x^2(1+xy)(z^2+1)$

After solving

$z^2 = xy$

$\Rightarrow \tan^2 C = \tan A \tan B$

5. Let x be the number of 9 digit numbers formed by taking digits from first 9 natural numbers, where only one digit is repeated twice & y be the number of 9 digit numbers formed from first 9 natural numbers, such that exactly 2 digits repeated twice then

(1) $x = 27y$ (2) $21x = 4y$

(3) $5x = 27y$ (4) $7x = 27y$

Ans. **(2)**

Sol. $S = \{1, 2, 3, \ldots, 9\}$

$$x={}^9C_1.{}^8C_7\times\frac{9!}{2}=\frac{9\times8\times9!}{2}$$

$$y={}^9C_2.{}^7C_5\times\frac{9!}{2!\times2!}=\frac{9\times8}{2}\times\frac{7\times6}{2}\times\frac{9!}{2!\times2!}$$

$$\Rightarrow \frac{x}{y}=\frac{4}{21}$$

$21x = 4y$

6. Consider the 10 observations 2, 3, 5, 10, 11, 13, 15, 21, a and b such that mean of observation is a and variance is 34.2. Then the mean deviation about median, is :

(1) 5 (2) 6

(3) 7 (4) 8

Ans. **(1)**

Sol. $\frac{2+3+5+10+11+13+15+21+a+b}{10}=9$

$\frac{80+a+b}{10}=9 \Rightarrow a+b=10$ ….. (1)

$$\frac{\Sigma x_i^2}{10}-\left(\frac{\Sigma x_i}{10}\right)^2=34.2$$

$$\frac{2^2+3^2+5^2+10^2+11^2+13^2+15^2+21^2+a^2+b^2}{10}-(9)^2=34.2$$

$1094 + a^2 + b^2 - 810 = 342$

$a^2 + b^2 = 58$ ….(2)

$a = 7, b = 3$

or $a = 3, b = 7$

Number $\rightarrow$ 2, 3, 5, 7, 10, 11, 13, 15, 21

$\text{Mean}=\frac{7+10}{2}=8.5$

$$\text{M.D}=\frac{6.5+5.5+5.5+3.5+1.5+1.5+2.5+4.5+6.5+12.5}{10}$$

$=\frac{50}{5}$

$= 5$

7. Let $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{2}{3}\right)+\tan\left(\frac{1}{2}\sin^{-1}\frac{2}{3}\right)=k$. Then number of solution of the equation $\sin^{-1}(kx-1)=\sin x-\cos^{-1}x$ is/are :

(1) No solution (2) exactly one solution

(3) Two solutions (4) infinite solutions

Ans. **(2)**

Sol. $k=\tan\theta+\cot\theta=\frac{1}{\sin\theta\cos\theta}=\frac{2}{\sin2\theta}$

where $\theta=\frac{1}{2}\sin^{-1}\frac{2}{3}$

$$k=\frac{2}{\frac{2}{3}}=3$$

$\sin^{-1}(3x-1)=\sin^{-1}x-\cos^{-1}x$

$\sin^{-1}(3x-1)=\frac{\pi}{2}-2\cos^{-1}x$

$3x-1=\sin\left(\frac{\pi}{2}-2\cos^{-1}x\right)$

$3x-1=2x^2-1 \Rightarrow x=0, \frac{3}{2}$

No. of solution = 1

8. If $f(x^2+1) = x^4 + 5x^2 + 1$, then find $\int_0^3 f(x)\,dx$:

(1) 13.5 (2) 15.3

(3) 13 (4) 15.5

Ans. (1)

Sol. $\because\ f(x^2+1) = x^4 + 5x^2 + 1$

$\{\text{put } x^2 + 1 = t\}$

$\Rightarrow f(t) = (t-1)^2 + 5(t-1) + 1$

$\Rightarrow f(t) = t^2 + 3t - 3$

Now, $\int_0^3 f(t)dt = \int_0^3 (t^2+3t-3)dt$

$$\left[\frac{t^3}{3}+\frac{3t^2}{2}-3t\right]_0^3$$

$$\left[\frac{27}{3}+\frac{27}{2}-9\right]$$

$$=\frac{27}{2}=13.5$$

9. The value of $\lim\limits_{x\to 0}\dfrac{\ln(\sec(ex).\sec(e^2x)......\sec(e^{10}x))}{e^2-e^{2\cos x}}$ is :

(1) $\dfrac{1}{2}\dfrac{(e^2-1)}{(e^{20}-1)}$ (2) $\dfrac{1}{2}\dfrac{(e^{20}-1)}{(e^2-1)}$

(3) $\dfrac{1}{2}\dfrac{(e-1)}{(e^{20}-1)}$ (4) None of these

Ans. (2)

Sol. $\Rightarrow \lim\limits_{x\to 0}\dfrac{\ln(\sec(ex))+\ln(\sec(e^2x))+.....\ell n(\sec(e^{10}x))}{e^{2\cos x}\left(\dfrac{e^{2-2\cos x}-1}{2-2\cos x}\right)\times\dfrac{2-2\cos x}{x^2}\times x^2}$

$$\Rightarrow \lim_{x\to 10}\frac{\ell n(\sec(ex))+\ell n(\sec(e^2x))+.....\ell n(\sec(e^{10}x))}{e^2\ x^2}$$

Using L'H rule

$$\Rightarrow \lim_{x\to 10}\frac{e\tan ex+e^2\tan e^2x+.....+e^{10}\tan e^{10}x}{2e^2x}$$

$$\Rightarrow \frac{1}{2e^2}[e^2+e^4+e^6+....+e^{20}]$$

$$\Rightarrow \frac{1}{2}\frac{e^2((e^2)^{10}-1)}{e^2(e^2-1)}$$

$$\Rightarrow \frac{1}{2}\frac{(e^{20}-1)}{(e^2-1)}$$

10. Let $x\dfrac{dy}{dx}-\sin 2y = x^3(2-x^3)\cos^2 y$; $y(2) = 0$, then find $\tan(y(1))$:

(1) 5/4 (2) 3/4

(3) 7/4 (4) 9/4

Ans. (3)

Sol. $\sec^2 y\dfrac{dy}{dx}-\dfrac{2\tan y}{x}=x^3(2-x^3)$

$$\tan y = t \Rightarrow \sec^2 y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dt}{dx}-\frac{2t}{x}=x^3(2-x^3)$$

$$\text{I.F} = e^{\int -\frac{2}{x}dx} = e^{-2\ln x}=\frac{1}{x^2}$$

$$t\left(\frac{1}{x^2}\right)=\int(2-x^3)dx$$

$$\frac{t}{x^2}=2x-\frac{x^4}{4}+C$$

$$\frac{\tan y}{x^2}=2x-\frac{x^4}{4}+C$$

$y(2) = 0$

$$\Rightarrow 0 = 4-\frac{16}{4}+C \Rightarrow \boxed{C=0}$$

Now put $\boxed{x=1}$

$$\tan y = 2-\frac{1}{4}=\frac{7}{4}$$

$$\tan y(1)=\frac{7}{4}$$

11. Consider a circle C_1, passing through origin and lying in region $0 \le x$ only, with diameter = 10. Consider a chord PQ of C_1 with equation $y = x$ and another Circle C_2 which has PQ as diameter. A chord is drawn to C_2 passing through (2, 3) such that distance of chord from centre of C_2 is maximum has equation $x + ay + b = 0$ then $|b - a|$ is equal to :

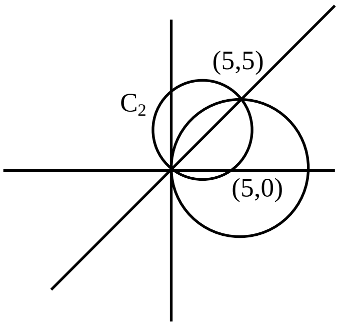
(1) 1 (2) 2

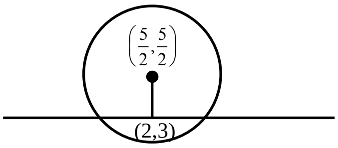
(3) 3 (4) 4

Ans. (2)

Sol. Equation of C_2 is $x(x-5) + y(y-5) = 0$

$x^2 + y^2 - 5x - 5y = 0$





equation of chard is

$T = S_1$

Slope of chard is 1

Equation of line

$y - 3 = 1(x-2)$

$x - y = -1$

$$\Rightarrow \left.\begin{matrix} x-y+1=0 \\ x+ay+b=0 \end{matrix}\right\} \begin{matrix} b=1, a=-1 \\ |b-a|=2 \end{matrix}$$

12. Given $\frac{1}{\alpha}-\frac{1}{\beta}=\frac{1}{3}$ such that roots of the quadratic equation $\lambda x^2+(\lambda+1)x+3=0$ are α & β, then sum of values of λ is equal to :

(1) 8 (2) 9

(3) 10 (4) 11

Ans. **(3)**

Sol. $\frac{\beta-\alpha}{\alpha\beta}=\frac{1}{3}$, $\alpha+\beta=-\frac{(\lambda+1)}{\lambda}$, $\alpha\beta=\frac{3}{\lambda}$,

$$\beta-\alpha=\frac{\alpha\beta}{3}=\frac{3}{\lambda.3}=\frac{1}{\lambda}$$

$$\text{sq. } \alpha^2+\beta^2-2\alpha\beta=\frac{1}{\lambda^2} \quad \ldots\ldots..(1)$$

$$\text{sq. } \alpha^2+\beta^2+2\alpha\beta=\frac{(\lambda+1)^2}{\lambda^2} \quad \ldots\ldots\ldots..(2)$$

$$(2)-(1) \qquad 4\alpha\beta=\frac{(\lambda+1)^2}{\lambda^2}-\frac{1}{\lambda^2}$$

$$\frac{12}{\lambda}=\frac{\lambda^2+2\lambda+1-1}{\lambda^2}$$

$$\lambda^2-10\lambda=0$$

$$\boxed{\lambda=0,10}$$

$$\boxed{\lambda=10}$$

13. The value of $S=\sum_{r=1}^{20}\sqrt{\pi\int_0^r x|\sin\pi x|dx}$ is :

(1) 210 (2) 201

(3) 120 (4) 102

Ans. **(1)**

Sol. $I_r=\int_0^r x|\sin\pi x|dx$

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$$=\int_0^r (r-x)|\sin\pi x|dx$$

$$2I_r = \int_0^r r|\sin \pi x|dx \Rightarrow I_r = \frac{r}{2}\int_0^r |\sin \pi x|dx$$

$$I_1 = \frac{1}{2}\int_0^1 |\sin \pi x|dx = \frac{1}{2\pi}\int_0^{\pi} |\sin t|dt = \frac{1}{2\pi}(2)$$

$$I_2 = \frac{2}{2}\int_0^2 |\sin \pi x|dx = \frac{2}{2\pi}\int_0^{2\pi} |\sin t|dt = \frac{2}{2\pi}(4)$$

$$S = \sqrt{\pi.\frac{1}{2\pi}.2} + \sqrt{\pi.\frac{2}{2\pi}.4} + \sqrt{\pi.\frac{3}{2\pi}.6} + ... + \sqrt{\pi.\frac{20}{2\pi}(2.20)}$$

$= 1 + 2 + 3 + ... + 20$

$$= \frac{20 \times 21}{2} = 210$$

14. Let $\int \frac{1-5\cos^2 x}{\sin^5 x \cos^2 x}dx = f(x) + c$

then find $f\left(\frac{\pi}{4}\right) - f\left(\frac{\pi}{6}\right)$:

(1) $4\sqrt{2} - \frac{32}{\sqrt{3}}$

(2) $4\sqrt{3} - \frac{32}{\sqrt{3}}$

(3) $4\sqrt{3} - \frac{32}{\sqrt{2}}$

(4) $4\sqrt{2} - \frac{32}{\sqrt{2}}$

Ans. **(1)**

Sol. $\int \frac{dx}{\sin^5 \cos^2 x} - 5\int \frac{dx}{\sin^5 x}$

$\int \frac{\sec^2 x dx}{\sin^5 x} - 5\int \frac{dx}{\sin^5 x}$

IBP

$$= \frac{\tan x}{\sin^5 x} - \int -\frac{5}{\sin^6 x}.\cos x.\tan x dx - 5\int \frac{dx}{\sin^5 x}$$

$$= \frac{\tan x}{\sin^5 x} + c$$

$$f(x) = \frac{\tan x}{\sin^5 x}$$

$$f\left(\frac{\pi}{4}\right) - f\left(\frac{\pi}{6}\right) = \left(\sqrt{2}\right)^5 - \frac{2^2}{\sqrt{3}} = 4\sqrt{2} - \frac{32}{\sqrt{3}}$$

15. If $\vec{a}, \vec{b}$ & $\vec{c}$ are unit vectors such that $\left(\vec{a}-\vec{b}\right)^2 + \left(\vec{b}-\vec{c}\right)^2 + \left(\vec{c}-\vec{a}\right)^2 = 9$ .Find positive k if $\left|2\vec{a} + k\vec{b} + k\vec{c}\right| = 3$:

(1) 2 (2) 4

(3) 5 (4) 7

Ans. **(3)**

Sol. $\left(\vec{a}-\vec{b}\right)^2 + \left(\vec{b}-\vec{c}\right)^2 + \left(\vec{c}-\vec{a}\right) = 9$

$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} - = -\frac{3}{2}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 0 \quad \Rightarrow \vec{b} + \vec{c} = -\vec{a}$

$\left|2\vec{a} + k\left(\vec{b} + \vec{c}\right)\right| = 3$

$\left|\vec{a}(2-k)\right| = 3$

K = 5 or – 1

16. Product of first 3 terms of a G.P. is 27 and sum is $R-\{a,b\}$, then $a^2 + b^2$ is equal to :

(1) 90 (2) 81

(3) 9 (4) 18

Ans. **(1)**

Sol. $\frac{A}{r}.A.Ar = 27$

A = 3

$$3\left(\frac{1}{r} + 1 + r\right) = 3 + 3\left(r + \frac{1}{r}\right)$$

We know, $r + \frac{1}{r} \geq 2$ or $r + \frac{1}{r} \leq -2$

$S \in R - (-3, 9)$

$a^2 + b^2 = 9 + 81 = 90$

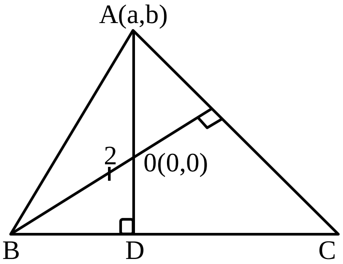
17. Orthocentre of equilateral ΔABC is at the origin. If side BC lies along $x + 2\sqrt{2}y = 4$. If coordinates of vertex A are (a,b). Find the value of $\left[\left|a + \sqrt{2}b\right|\right]$, where $[\cdot]$ denotes G.I.F. :

(1) 1 (2) 2

(3) 3 (4) 4

Ans. **(4)**

Sol.



$\because m_{BC}.m_{AD} = -1$

$\Rightarrow \left(-\frac{1}{2\sqrt{2}}\right)\left(\frac{b}{a}\right) = -1$

$\Rightarrow b = 2\sqrt{2}a \qquad ....(1)$

$\because \; OD = \left|\frac{-4}{\sqrt{1+8}}\right| = \frac{4}{3} \Rightarrow AO = \frac{8}{3}$

So $AD = \frac{8}{3} + \frac{4}{3} = 4$

$\Rightarrow \frac{|a + 2\sqrt{2}b - 4|}{3} = 4 \Rightarrow a = \frac{16}{9}$ or $-\frac{8}{9}$

Now, $(a,b) = \left(\frac{16}{9}, \frac{32\sqrt{2}}{9}\right)$;

$\left\{\begin{matrix}\because A(a,b) \,\&\, (0,0) \text{ lies same} \\ \text{side of given line}\end{matrix}\right\}$

so $(a,b) = \left(-\frac{8}{9}, \frac{-16\sqrt{2}}{9}\right)$

$= \left[\left|a + \sqrt{2}b\right|\right] = \left[\left|\frac{-8-32}{9}\right|\right] = 4$

18. If $g(x) = 3x^2 + 2x - 3$, $f(0) = -3$, $4g(f(x)) = 3x^2 - 32x + 72$ then find $f(g(2))$ where $f(x) > 0$ for all valid x:

(1) $\frac{7}{2}$ (2) $\frac{5}{2}$

(3) $\frac{3}{2}$ (4) $\frac{1}{2}$

Ans. **(1)**

Sol. $g(2) = 3(2)^2 + 4 - 3 = 13$

$f(g(2)) = f(13)$

$4g(f(x)) = 3x^2 - 32x + 72$

$4(3f^2(x) + 2f(x) - 3) = 3x^2 - 32x + 72$

$12f^2(x) + 8f(x) - 12 = 3x^2 - 32x + 72$

Put $x = 13$

$12f^2(13) + 8f(13) - 12 = 3(13)^2 - 32(13) + 72$

$12f^2(13) + 8f(13) - 175 = 0$

$f(13) = \frac{-8 \pm 92}{24} \Rightarrow \boxed{f(13) = \frac{7}{2}}$ or

$f(13) = -\frac{15}{4}$ (rejected)

19. Given conic $x^2 - y^2\sec^2\theta = 8$ whose eccentricity is 'e_1' & length of latus rectum 'ℓ_1' and for conic $x^2 + y^2\sec^2\theta = 6$, eccentricity is '$e_2$' & length of latus rectum 'ℓ_2'. If $e_1^2 = e_2^2(1 + \sec^2\theta)$ then value of

$\frac{e_1\ell_1}{e_2\ell_2}\tan\theta$

Ans. **(2)**

Sol. $\frac{x^2}{8} - \frac{y^2}{8\cos^2\theta} = 1, \; e_1 = \sqrt{1 + \frac{8\cos^2\theta}{8}}$

$\ell_1 = \frac{2b^2}{a} = \frac{2.(8\cos^2\theta)}{2\sqrt{2}}$

$\frac{x^6}{6} + \frac{y^2}{6\cos^2\theta} = 1; \; e_2 = \sqrt{1 - \frac{6\cos^2\theta}{6}} = \sin\theta$

$\ell_2 = \frac{2b^2}{a} = \frac{2.6\cos^2\theta}{\sqrt{6}}$

$e_1^2 = e_2^2\left(1 + \sin^2\theta\right)$

$1 + \cos^2\theta = \sin^2\theta\left(1 + \frac{1}{\cos^2\theta}\right)$

$1 + \cos^2\theta = \sin^2\theta + \tan^2\theta$

Solving we get $\theta = \frac{\pi}{4}$

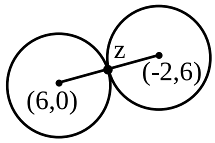
$\frac{e_1\ell_1}{e_2\ell_2}\tan\theta$

= 2 by putting values.

20. A complex number 'z' satisfy both $|z-6|=5$ & $|z+2-6i|=5$ simultaneously. Find the value of $z^3 + 3z^2 - 15z + 141$.

Ans. (53)

Sol. $|z-6|=5$ & $|z+2-6i|=5$



Locus of z is circle both equation

$z = 2 + 3i$

$z - 2 = 3i$

$\Rightarrow z^2 - 4z + 4 = -9$

$z^2 - 4z + 13 = 0$

$z^3 + 3z^2 - 15z + 141 = (z^2 - 4z + 13)(z + 7) + 53$

$= 53$

21. Find possible no. of triplets (δ, c, d), such that $x^2 + 2$ is divisor of $x^3 + bx^2 + cx + d$ & δ, c, d ≤ 20 & δ, c, d ∈ N :

Ans. (10)

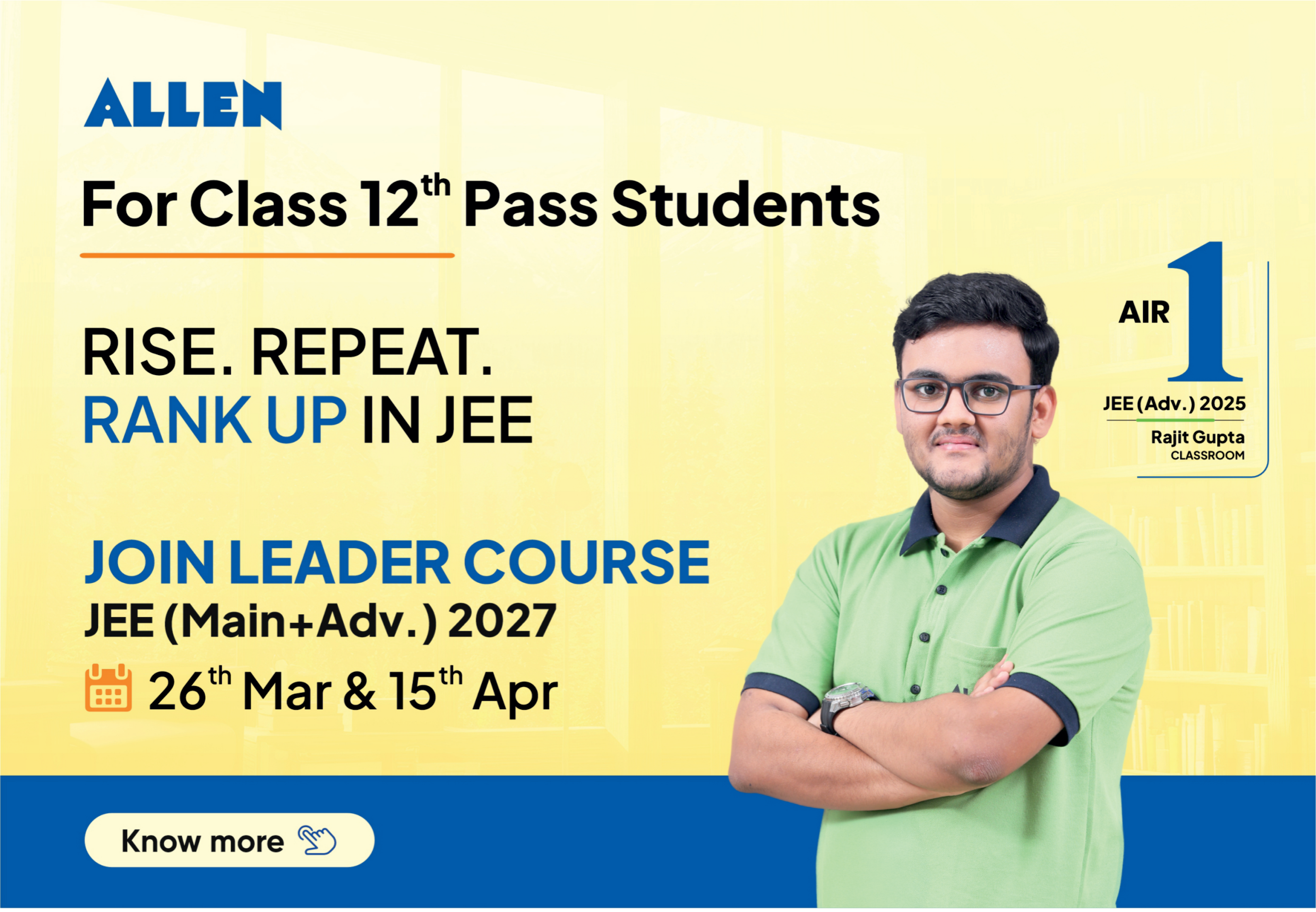
Sol. $x^3 + bx^2 + (x + d = (x^2 + 2)\left(x + \frac{d}{2}\right)$

$x^2 : b = \frac{d}{2}$

$x : c = 2$

$c = 2, \ b = \frac{d}{2} \quad d \in \{2, 4, \ldots\ldots., 20\}$

No. of triplet = 10



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