(ENGLISH VERSION)

1. (a) Answer any ***one*** question : $2 \times 1 = 2$

(i) $f(x) = \frac{3x+4}{5x-7}\left(x \neq \frac{7}{5}\right)$ and $g(x) = \frac{7x+4}{5x-3}\left(x \neq \frac{3}{5}\right)$, show that $f(g(x)) = g(f(x))$.

(ii) Let, $A = \{1, 2\}$, $B = \{1, 8\}$ and $f : A \to B$, $g : A \to B$ are two mappings defined as $f(x) = x^3$ and $g(x) = 6x^2 - 11x + 6$, then prove that $f = g$.

(iii) Find the value of $\sec^2\left(\cot^{-1}\frac{1}{3}\right) + \operatorname{cosec}^2\left(\tan^{-1}\frac{1}{2}\right)$.

(b) Answer any ***one*** question : $2 \times 1 = 2$

(i) If $A = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix}$, then show that $A - A^T$ is a skew symmetric matrix.

(ii) If $\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, then find the values of x and y.

(iii) If $\begin{vmatrix} 5 & 4 \\ 3 & 2 \end{vmatrix} = \begin{vmatrix} 2x & 7 \\ x & 3 \end{vmatrix}$, find the value of x.

(c) Answer any ***three*** questions : $2 \times 3 = 6$

(i) If $f(x) = x$ for $x \geq 0$

$= 2$ for $x < 0$

show that $f(x)$ is discontinuous at $x = 0$.

(ii) If $ye^y = x$, then show that $\frac{dy}{dx} = \frac{y}{x(1+y)}$.

(iii) Evaluate $\int_{-1}^{1} x|x|\,dx$.

(iv) Solve : $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$.

(v) $x > 0$, $y > 0$ and $xy = 1$, find the least value of $x + y$.

(vi) Find the equation of tangent of $x^2 + y^2 = 32$ at $(4, 4)$.

(d) Answer any ***one*** question : $2 \times 1 = 2$

(i) Show that the line joining $(1, -1, 2)$, $(3, 4, -2)$ is perpendicular to the line through $(0, 3, 2)$ and $(3, 5, 6)$.

(ii) Find the equation of the plane passing through $(1, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 4)$.

(iii) If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular to each other.

(e) Answer any **one** question : $2 \times 1 = 2$

(i) If $P(A) = \frac{6}{13}$, $P(B) = \frac{5}{13}$ and $P(A \cup B) = \frac{7}{13}$, find $P(A/B)$.

(ii) A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even ; then show that E and F are independent.

2. (a) Answer any **one** question : $4 \times 1 = 4$

(i) Find the range of the function $f(x) = \frac{1}{1-x^2}$, x is real and $x \neq \pm 1$.

(ii) $f : R \to R$ is a mapping where $f(x) = x^3 - 6$, for all $x \in R$, R = set of real numbers. Prove that f is a bijective mapping.

(b) Answer any **two** from the following questions : $4 \times 2 = 8$

(i) If $A = \begin{pmatrix} x & -2 \\ 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 4 \\ 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} -1 & -2 \\ y & 2 \end{pmatrix}$ and $A + B = BC$ then find the values of x and y.

OR

If $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$, then find the matrices A and B.

(ii) Show that $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ matrix satisfies the equation

$A^2 - 4A - 5I_3 = 0$. Hence find A^{-1}. $\left[\quad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right]$

OR

If $A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ then show that B^TAB is a diagonal matrix.

(iii) Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$.

OR

Show that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$.

(c) Answer the following questions : $4 \times 3 = 12$

(i) $f(x) = \frac{|x|}{x}$ for $x \neq 0$

$= 0$ for $x = 0$

Find the point of discontinuity of $f(x)$.

OR

If $y = \sin(2\sin^{-1} x)$, then show that $(1 - x^2)\frac{d^2y}{dx^2} = x\frac{dy}{dx} - 4y$.

(ii) Evaluate $\int \frac{dx}{x(x^2+1)}$.

OR

Evaluate $\int \frac{dx}{1+\tan x}$.

(iii) Solve $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$.

OR

Find the equation of the curve passing through the point $(-2, 3)$ given the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.

(d) Answer any ***one*** question : $4 \times 1 = 4$

(i) Find the equation of the line that passes through the origin and $(5, 2, 4)$.

(ii) Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$.

(iii) If sum of two unit vectors be a unit vector, then show that difference of those two vectors is $\sqrt{3}$.

$4 \times 1 = 4$

(e) Answer any ***one*** question :

(i) Find the area of the circle $x^2 + y^2 = 16$ using integral calculus.

(ii) From the definition of definite integral find the value of $\int_0^1 (2x+1)dx$.

$4 \times 1 = 4$

(f) Answer any ***one*** question :

(i) Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number ?

(ii) A die is thrown twice and the sum of the numbers appearing is observed to be 6, what is the probability that the number 4 has appeared at least once ?

3. (a) Answer any ***one*** question : $5 \times 1 = 5$

(i) A dietician wishes to mix two types of foods in such a way that the mixture contain at least 8 units of Vitamin *A* and 10 units of Vitamin *C*. Food-I contains 2 units/kg of Vitamin *A* and 1 unit/kg of Vitamin *C*, while Food-II contains 1 unit/kg of Vitamin *A* and 2 units/kg of Vitamin *C*. It costs Rs. 50 per kg to purchase Food-I and Rs. 70 per kg to purchase Food-II. Formulate the above as a LPP to minimise the cost of such a mixture.

(ii) Solve the linear programming problem graphically :

Maximise $Z = 4x + y$

where $x + y \leq 50$, $3x + y \leq 90$

$x \geq 0$ and $y \geq 0$.

(Graph sheet is not required).

(b) Answer any ***two*** questions : $5 \times 2 = 10$

(i) Using calculus, show that the straight line $lx + my + n = 0$ touches the circle $x^2 + y^2 = a^2$ if $a^2(l^2 + m^2) = n^2$.

(ii) x is real. Using differential calculus find the maximum and minimum values of $\frac{x^2 - x + 1}{x^2 + x + 1}$.

(iii) Find differential equation by eliminating a and b from $y = e^x(a\cos x + b\sin x)$.

(iv) Evaluate $\lim_{n \to \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{3n}\right]$.

(c) Answer any ***one*** question : $5 \times 1 = 5$

(i) Find the cartesian equation of the line which passes through (1,2,3) and parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

(ii) Find the coordinates of the foot of the perpendicular from origin to the plane $x + y + z = 3$.

(iii) A plane has the intercepts on axes are a, b, c respectively and p be the perpendicular distance from origin to the plane. Show that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$.

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