MATHEMATICS

61. If $z=(\lambda+3)+i\sqrt{5-\lambda^2}$, then the locus of z is

   (a) $x^2+y^2=25$ (b) $x^2+y^2=9$
   (c) $x^2+y^2-6x+4=0$ (d) $x^2+y^2-6x+25=0$

62. If $\cos\theta+i\sin\theta$ is a root of the equation $a_0x^n+a_1x^{n-1}+............+a_n=0$, then the value of $a_0+a_1\cos\theta+a_2\cos 2\theta+.........+a_n\cos n\theta$ is

   (a) 0 (b) n (c) $\cos(n+1)\theta$ (d) $\sin(n+1)\theta$

63. If $\cos A+\cos B+\cos C=0=\sin A+\sin B+\sin C$, then the value of $\cos(A-B)+\cos(B-C)+\cos(C-A)$ is

   (a) $1/2$ (b) -3 (c) $3/2$ (d) $-3/2$

64. Given that $\sin A$, $\cos A$ and $\tan A$ are in G.P., the value of $\cot^6 A-\cot^2 A$ is

   (a) -1 (b) 0 (c) 1 (d) 2

65. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right)-\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then 'x' is

   (a) $\frac{a+b}{1+ab}$ (b) $\frac{a-b}{1+ab}$ (c) $\frac{a-b}{1-ab}$ (d) $\frac{a+b}{1-ab}$

66. The value of $\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix}$ is

   (a) $xyz(x+y+z)$ (b) xyz
   (c) $(x+y+z)^3$ (d) $x^2y^2z^2(x+y+z)$

67. The positive solution of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix}=0$ is

   (a) 3 (b) 9 (c) 12 (d) 6

Rough work

68. If $A^{-1} = \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$, then Adj(A) is

$(a)\begin{pmatrix} -2 & 0 & -1 \\ 9 & -2 & 3 \\ 6 & -1 & 2 \end{pmatrix}$ $(b)\begin{pmatrix} 2 & 0 & 1 \\ -9 & -2 & -3 \\ -6 & -1 & -2 \end{pmatrix}$

$(c)\begin{pmatrix} 2 & 0 & -1 \\ -9 & 2 & 3 \\ -6 & 1 & 2 \end{pmatrix}$ $(d)\begin{pmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{pmatrix}$

69. The system of equations $x+2y-z=2;\ 5y-5z=3;\ 2x-y+\lambda z=\mu$ has infinitely many solutions if the pair $\{\lambda, \mu\}$ is

(a) $\{3, 1\}$ (b) $\{1, 3\}$ (c) $\{-3, 1\}$ (d) $\{-1, 3\}$

70. It is given that x, y, z not all zero satisfy the equations $x = cy + bz$, $y = az + cx$ and $z = bx + ay$, then $a^2 + b^2 + c^2$ is

(a) abc (b) $abc - 1$ (c) $1 - 2abc$ (d) $1 + 2abc$

71. In a plane, a set of 15 parallel lines intersect another set of 20 parallel lines to form parallelograms. The number of such parallelograms formed is

(a) 19850 (b) 19750 (c) 19000 (d) 19950

72. If $|\bar{a}| = 5$, $|\bar{b}| = 7$ and $|\bar{a} - \bar{b}| = 12$, then $|\bar{a} + \bar{b}|$ is equal to

(a) 2 (b) 4 (c) 12 (d) $\sqrt{74}$

73. If $\bar{a}$ and $\bar{b}$ are non collinear vectors, then $\frac{\bar{a}}{|\bar{a}|} + \frac{\bar{b}}{|\bar{b}|}$ is

(a) *a unit vector* (b) *in the plane of* $\bar{a}$ *and* $\bar{b}$

(c) *perpendicular to* $\bar{a}$ *and* $\bar{b}$ (d) *parallel to* $\bar{a}$ *and* $\bar{b}$

Rough work

74. Forces acting on a particle having magnitudes 3, 2, 1 units act in the directions of the vectors $2\hat{i}+4\hat{j}+4\hat{k}$, $4\hat{i}-4\hat{j}+2\hat{k}$ and $4\hat{i}-4\hat{j}-2\hat{k}$ respectively. The work done by the forces in displacing the particle from the point $A(2,-1,6)$ to the point $B(5,-1,3)$ is

(a) 2 *units* (b) 4 *units* (c) 6 *units* (d) 3 *units*

75. $\underset{n\to\infty}{Lt}\, 4^{n-1}\sin\left(\frac{a}{4^n}\right)$ is equal to

(a) $-a$ (b) $a/2$ (c) $a/4$ (d) $-a/4$

76. If $f(x)$ is a continuous function satisfying $f(x)f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$ and $f(1)>0$, then $\underset{x\to 1}{Lt}\, f(x)$ is equal to

(a) 2 (b) 1 (c) 3 (d) 3/2

77. If $y=\log\sqrt{e\log x}$, $\frac{dy}{dx}$ at $x=e$ is

(a) $\frac{1}{\sqrt{e\log e}}$ (b) $\frac{1}{2e\log e}$ (c) $\frac{e\log e}{2}$ (d) $\sqrt{e\log e}$

78. The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ is

(a) 1/3 (b) $-1/3$ (c) $-2/3$ (d) 2/3

79. If α,β are the roots of the equation $x^2-3x+7=0$, then the equation whose roots are α^2+2, β^2+2 is

(a) $y^2+y+43=0$ (b) $y^2-y+43=0$ (c) $y^2+y-43=0$ (d) $y^2-y-43=0$

80. If one of the roots of $ax^2+bx+c=0$ is the 4^{th} power of the other, then the value of $(ac^4)^{1/5}+(a^4c)^{1/5}$ is

(a) 0 (b) 1 (c) $-b$ (d) b

Rough work

81. If $\log 2$, $\log(2^x - 1)$, $\log(2^x + 3)$ are in arithmetic progression, then the value of 'x' is

(a) $\log_5 2$ (b) 2 (c) $\log_e 2$ (d) $\log_2 5$

82. If a, b, c are in harmonic progression and if, $x = \frac{c}{a+b}$, $y = \frac{b}{a+c}$, $z = \frac{a}{c+b}$, then $\frac{1}{x} + \frac{1}{z}$ is

(a) $\frac{2}{y}$ (b) $\frac{1}{y}$ (c) $\frac{3}{y}$ (d) $-\frac{1}{y}$

83. If $\sum_{r=1}^{n}(3r+2)(r-5) = an^3 + bn^2 + cn$, then the values of a, b, c are

(a) 1, −5, −16 (b) −5, 1, −16 (c) −16, 1, 5 (d) −5, −16, 1

84. If $z = (\cos 2 + i\sin 2 + 1)^n$, then $|z|$ is

(a) $2^n \cos 1$ (b) $2^n \cos n$ (c) $2^n \sin n$ (d) $2^n \cos^n 1$

85. The number of times the digit '5' will be written when listing the numbers from 1 to 1000 (assuming that a single digit number is written as 00x and a double digit number as 0xy) is

(a) 109 (b) 300 (c) 271 (d) 250

86. If $\int \frac{dx}{\sqrt{x(1-4x)}} = K\sin^{-1}(8x-1) + C$, then K is equal to

(a) $1/\sqrt{2}$ (b) $-1/\sqrt{2}$ (c) $-1/2$ (d) $1/2$

87. Let $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}, x > 0$. If $\int_a^b \frac{2e^{\sin x^2}}{x}dx = F(k) - F(l)$, then one of the possible set of values of 'k' and 'l' is respectively

(a) $\frac{b-a}{2}, \frac{b+a}{2}$ (b) $\frac{b+a}{2}, \frac{b-a}{2}$ (c) b^2, a^2 (d) a^2, b^2

Rough work

88. The solution of the differential equation $e^{\log\frac{dy}{dx}} = e^{2x} + y - 1,\ y(0) = 1$ is

$(a)\ y = e^{2x} + e^{x} + 1$ $(b)\ y = e^{2x} - e^{x}$

$(c)\ y = e^{2x} - e^{x} + 1$ $(d)\ y = e^{2x} + e^{-2x} + 1$

89. The arithmetic mean of n observations is 'm'. If two observations 0 and m are added, then the new mean is

$(a)\ m$ $(b)\ \frac{n}{m+1}$ $(c)\ \frac{mn}{n+2}$ $(d)\ \frac{m(n+1)}{n+2}$

90. Two events A and B have probabilities 0.20 and 0.40 respectively. The probability that both A and B occur simultaneously is 0.15. Then the probability that neither A nor B occurs is

(a) 0.60 (b) 0.40 (c) 0.45 (d) 0.55

91. The equations $ax + by + c = 0$ and $dx + ey - f = 0$ represent the same straight line if and only if

$(a)\ a = d,\quad b = e$ $(b)\ \frac{a}{d} = -\frac{c}{f}$

$(c)\ \frac{a}{d} = \frac{b}{e}$ $(d)\ \frac{a}{d} = -\frac{c}{f} = \frac{b}{e}$

92. The straight line $y = mx + c$ cuts the circle $x^2 + y^2 = a^2$ in real points if

$(a)\ \sqrt{a^2(1+m^2)} < c$ $(b)\ \sqrt{a^2(1-m^2)} < c$

$(c)\ \sqrt{a^2(1+m^2)} \geq c$ $(d)\ \sqrt{a^2(1-m^2)} \geq c$

93. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is

(a) 1 (b) 5 (c) 9 (d) 7

94. If ω is the cube root of unity, then the value of $(1-\omega)(1-\omega^2) + (2-\omega)(2-\omega^2) + \ldots\ldots\ldots\ldots + (n-\omega)(n-\omega^2)$ is

$(a)\ \frac{n}{3}(n^2 + 3n - 5)$ $(b)\ \frac{n}{3}(n^2 - 3n + 5)$

$(c)\ \frac{n}{3}(n^2 + 3n + 5)$ $(d)\ \frac{n}{3}(n^2 - 3n - 5)$

Rough work

95. In a triangle ABC, $\tan(A/2)$, $\tan(B/2)$, $\tan(C/2)$ are in H.P. and the sides 'a' and 'c' are given by 5 and 9 units, then the side 'b' is

(a) 6 (b) 8 (c) 7 (d) 11

96. The bases of two towers subtend an angle of $120°$ at a point on the ground which is at 10m distance from each of the bases. A bird sitting at the top of the higher tower starts flying at a constant speed along a straight path inclined at an angle of $45°$ to the tower and reaches the other top in 5 sec. The speed of the flight (in m/s) is

(a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) $3\sqrt{6}$ (d) $4\sqrt{6}$

97. The area bounded by the curve $y^2 = 4ax$ and the line $y = 2a$ and the y-axis is (in square units)

(a) $\frac{1}{3}a^2$ (b) $\frac{2}{3}a^2$ (c) $\frac{4}{3}a^2$ (d) $\frac{3}{4}a^2$

98. A solution of the equation $y\frac{dx}{dy} = x(\log x - \log y + 1)$ is

(a) $y = xe^{cx}$ (b) $x^2 = cy\log y$ (c) $x = ye^{cy}$ (d) $\log x = cy$

99. The solution of $\frac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y)$ is

(a) $\sec y = C - 2\cos x$ (b) $y = C - 2\cos x$

(c) $\tan y = C - \sin x$ (d) $\cos y = C + 2\cos x$

100. The integrating factor of the linear differential equation $(\sin^2 y + x\cot y)\frac{dy}{dx} = 1$ is

(a) $\operatorname{cosec} y$ (b) $\sin y$ (c) $\tan y$ (d) $\cos y$

101. The variance of the first n natural numbers is

(a) $\frac{n(n+1)(2n+1)}{12}$ (b) $\frac{n^2-1}{12}$ (c) $\sqrt{\frac{n^2-1}{12}}$ (d) $\sqrt{\frac{n^2+1}{12}}$

Rough work

102. For a distribution, the coefficient of variation is 22.5% and the value of the arithmetic average is 7.5. Then the value of the standard deviation is

(a) 2 (b) 1.68 (c) 2.5 (d) 1

103. The mean of 10 numbers is 6 and their standard deviation is 2. Then the sum of the squares of these numbers is

(a) 600 (b) 300 (c) 100 (d) 400

104. A class has 10 boys and 5 girls. Three students are selected at random one after the other. The probability that first two are boys and the third is a girl is

(a) $\frac{2}{45}$ (b) $\frac{5}{91}$ (c) $\frac{15}{91}$ (d) $\frac{21}{91}$

105. A fair coin is tossed repeatedly. If the tail appears on first three tosses, then the probability of the head appearing on the fourth toss is

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{1}{16}$

106. If $P(X \le 4) = 0.8$ and $P(X = 4) = 0.2$, then $P(X \ge 4)$ is

(a) 0.2 (b) 0.4 (c) 0.5 (d) 0.6

107. If in a Binomial distribution $n = 4$, $P(X = 0) = \frac{81}{625}$, then $P(X = 4)$ is

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{32}{625}$ (d) $\frac{16}{625}$

108. The points $(3, 3)$, $(-h, 0)$, $(0, k)$ are collinear if

(a) $\frac{1}{h}+\frac{1}{k}=\frac{1}{2}$ (b) $\frac{1}{h}+\frac{1}{k}=\frac{1}{3}$ (c) $\frac{1}{h}-\frac{1}{k}=\frac{1}{3}$ (d) $\frac{1}{h}+\frac{1}{k}=\frac{-1}{3}$

109. The foot of the perpendicular from the point $(1, 2)$ upon $x + y = 1$ is

(a) $(0, 1/2)$ (b) $(0, 1)$ (c) $(1, 0)$ (d) $(1/2, 1/2)$

110. If a, b, c are in H.P., then the line $\frac{x}{a}+\frac{y}{b}-\frac{1}{c}=0$ always passes through the point

(a) $(-1, -2)$ (b) $(-1, 2)$ (c) $(1, -2)$ (d) $(1, -1)$

Rough work

111. If the sum of the slopes of the lines given by $x^2 - 2kxy + 9y^2 = 0$ is 6 times their product, then k has the value

$(a)\,2$ $(b)\,-2$ $(c)\,-3$ $(d)\,3$

112. The equation of the circle, if its centre is $(4, 5)$ and the circumference passes through the centre of the circle $x^2 + y^2 + 4x - 6y = 12$ is

$(a)\,x^2 + y^2 - 8x - 10y + 1 = 0$ $(b)\,x^2 + y^2 + 8x - 10y + 1 = 0$

$(c)\,x^2 + y^2 - 8x + 10y + 1 = 0$ $(d)\,x^2 + y^2 - 8x - 10y - 1 = 0$

113. The extremities of the diameter of a circle have coordinates $(-4, 3)$ and $(6, -2)$. Then the length of the intercept which the circle makes on the y-axis is

$(a)\,\sqrt{124}$ $(b)\,12$ $(c)\,11$ $(d)\,\sqrt{136}$

114. The eccentricity of the hyperbola $\frac{x^2}{5} - \frac{y^2}{5} = \frac{1}{\sqrt{1999}}$ is

$(a)\,2$ $(b)\,\sqrt{2}$ $(c)\,4$ $(d)\,2\sqrt{2}$

115. Let $\bar{a} = 2\hat{i} + \hat{k}$, $\bar{b} = \hat{i} + \hat{j} + \hat{k}$ and $\bar{c} = 4\hat{i} - 2\hat{j} + 7\hat{k}$, then the vector $\bar{r}$ such that $\bar{r} \times \bar{b} = \bar{c} \times \bar{b}$ and $\bar{r} \cdot \bar{a} = 0$ is

$(a)\ \hat{i} + 7\hat{j} + 2\hat{k}$ $(b)\ -\hat{i} - 7\hat{j} + 2\hat{k}$

$(c)\ 7\hat{i} + \hat{j} + 2\hat{k}$ $(d)\ \hat{i} + \hat{j} + 7\hat{k}$

116. If $f(x) = |\cos x|$, then $f'\left(\frac{3\pi}{4}\right)$ is equal to

$(a)\ -1/\sqrt{2}$ $(b)\,1/\sqrt{2}$ $(c)\,1$ $(d)\ -1$

Rough work

117. Let $f(x)=\sin^4 x+\cos^4 x,\ 0<x<\frac{\pi}{2}$. Then the minimum value of $f(x)$ is

(a) 1/2 (b) −1/2 (c) 1/4 (d) *does not exist*

118. $\int\sqrt{x}\left(\sqrt[4]{1+x^{3/2}}\right)dx$ equals

$(a)\ \frac{4}{15}\left(1+x^{3/2}\right)^{5/4}+C$

$(b)\ \frac{8}{15}\left(1+x^{3/2}\right)^{5/4}+C$

$(c)\ \frac{8}{15}\left(1+x^{3/2}\right)^{5/2}+C$

$(d)\ \frac{15}{4}\left(1+x^{3/2}\right)^{5/4}+C$

119. $\int e^{\tan^{-1}x}\left(1+\frac{x}{1+x^2}\right)dx$ is equal to

$(a)\ xe^{\tan^{-1}x}+C$ $(b)\ \frac{x}{2}e^{\tan^{-1}x}+C$ $(c)\ \frac{1}{2}e^{\tan^{-1}x}+C$ $(d)\ e^{\tan^{-1}x}+C$

120. The value of $\int_0^{\pi/2}\frac{\sin x}{\sin x+\cos x}dx$ is

$(a)\ \pi$ $(b)\ \pi/2$ $(c)\ \pi/4$ $(d)\ 2\pi$

Rough work