General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question paper contains **38** questions. All questions are **compulsory**.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.*

(iii) *In Section **A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In Section **B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In Section **C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In Section **D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In Section **E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in few questions in all the Sections except Section – **A**.*

(ix) *Use of calculator is **NOT** allowed.*

SECTION – A

This section comprises **20** Multiple Choice Questions (MCQs) of **1** mark each.

1. A relation R on set A = {1, 2, 3} defined as R = {(1, 1), (2, 2), (1, 2)} is **1**

   (A) Reflexive only
   (B) Reflexive and Transitive
   (C) Symmetric and Transitive
   (D) Transitive only

2. If A and B are square matrices of same order, then which of the following statements is/are always true ? **1**

   (i) $(A + B)(A - B) = A^2 - B^2$

   (ii) $AB = BA$

   (iii) $(A + B)^2 = A^2 + AB + BA + B^2$

   (iv) $AB = 0 \Rightarrow A = 0$ or $B = 0$

   (A) Only (i) and (iii)
   (B) Only (ii) and (iii)
   (C) Only (iii)
   (D) Only (iv)...

3. If $A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $3a + b + c$ is 1

(A) 2

(B) 6

(C) 4

(D) 0

4. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ and $A + A' = I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is 1

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

5. For a square matrix A, $(3A)^{-1} =$ 1

(A) $3A^{-1}$

(B) $9A^{-1}$

(C) $\frac{1}{3}A^{-1}$

(D) $\frac{1}{9}A^{-1}$

6. If $\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = -86$, then the sum of all possible values of a is 1

(A) 4

(B) 5

(C) –4

(D) 9

7. If $e^{-x} + e^{-y} = 2$, then $\frac{dy}{dx}$ is 1

(A) e^{x-y}

(B) e^{y-x}

(C) $-e^{x-y}$

(D) $-e^{y-x}$

8. For $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ 1

(A) local maximum value is 2

(B) local minimum value is –2

(C) local maximum value is –2

(D) local minimum value < local maximum value

9. If $\int_0^{2a} \frac{1}{1+4x^2} dx = \frac{\pi}{6}$, then the value of a is 1

(A) $\frac{\sqrt{3}}{4}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$

(D) $2\sqrt{3}$

10. Which of the following expressions will give the area of region bounded by the curve $y = x^2$ and line $y = 16$? 1

(A) $\int_0^4 x^2 \, dx$

(B) $2\int_0^4 x^2 \, dx$

(C) $\int_0^{16} \sqrt{y} \, dy$

(D) $2\int_0^{16} \sqrt{y} \, dy$

11. The general solution of the differential equation $\frac{dy}{dx} = \frac{\sqrt{y}}{\sqrt{x}}$ is 1

(A) $\log \sqrt{y} = \log \sqrt{x} + C$

(B) $\sqrt{y} + \sqrt{x} = C$

(C) $\sqrt{y} - \sqrt{x} = C$

(D) $\log \sqrt{y} + \log \sqrt{x} = C$

12. The integrating factor of the differential equation $2x\frac{dy}{dx} - y = 3$ is 1

(A) $\sqrt{x}$

(B) $\frac{1}{\sqrt{x}}$

(C) e^x

(D) e^{-x}

13. If $|\vec{a}| = 5$ and $-2 \le \lambda \le 1$, then the sum of greatest and the smallest value of $|\lambda \vec{a}|$ is 1

(A) –5

(B) 5

(C) 10

(D) 15

14. Vector of magnitude 3 making equal angles with x and y axes and perpendicular to z axis is 1

(A) $\hat{i} + 2\sqrt{2}\,\hat{j}$

(B) $3\hat{k}$

(C) $\frac{3\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}$

(D) $\sqrt{3}\,\hat{i} + \sqrt{3}\,\hat{j} + \sqrt{3}\,\hat{k}$

15. Direction cosines of the line given by equations : $\frac{2x-1}{4} = \frac{1-y}{3} = \frac{-z}{6}$ are 1

(A) 2, –3, –6

(B) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$

(C) $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$

(D) $\frac{4}{\sqrt{61}}, \frac{-3}{\sqrt{61}}, \frac{-6}{\sqrt{61}}$

16. In a linear programming problem, the linear function which has to be maximized or minimized is called 1

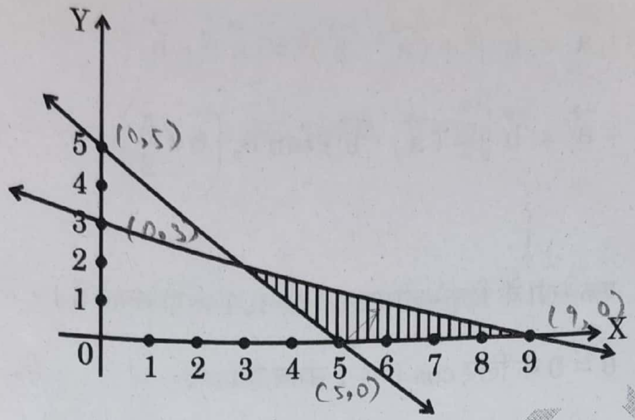
(A) a feasible function

(B) an objective function

(C) an optimal function

(D) a constraint

17. For the feasible region shown below, the non-trivial constraints of the linear programming problem are 1



(A) $x + y \leq 5,\ x + 3y \leq 9$ ×

(B) $x + y \leq 5,\ x + 3y \geq 9$ ×

(C) $x + y \geq 5,\ x + 3y \leq 9$

(D) $x + y \geq 5,\ 3x + y \leq 9$

18. For two events A and B such that $P(A) \neq 0$ and $P(B) \neq 1$, $P(A'/B') =$ 1

(A) $1 - P(A/B)$

(B) $1 - P(A'/B)$

(C) $\dfrac{1 - P(A \cap B)}{P(B')}$

(D) $\dfrac{1 - P(A \cup B)}{P(B')}$

Assertion – Reason Based Questions

Direction : Question numbers **19** and **20** are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. For two vectors $\vec{a}$ and $\vec{b}$

Assertion (A) : $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$ 1

Reason (R) : $|\vec{a} \times \vec{b}| = (\vec{a} \cdot \vec{b}) \tan\theta, \left(\theta \neq \frac{\pi}{2}\right)$.

20. **Assertion (A)** : A line can have direction cosines $< 1, 1, 1 >$ 1

Reason (R) : $\cos\theta = 1$ is possible for $\theta = 0$.

SECTION – B

This section comprises Very Short Answer (VSA) type questions carrying 2 marks each.

21. (a) Check whether $f : R - \{3\} \to R$ defined as $f(x) = \dfrac{x-2}{x-3}$ is onto or not. 2

OR

(b) Check whether $f : Z \times Z \to Z \times Z$ (where Z is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not. 2

22. If $x = a\sin^3 t$, $y = b\cos^3 t$, then find $\dfrac{dy}{dx}$ at $t = \dfrac{\pi}{4}$. 2

23. (a) Find the absolute maximum value of $f(x) = \cos x + \sin^2 x$, $x \in [0, \pi]$ 2

OR

(b) If the volume of a solid hemisphere increases at a uniform rate, prove that its surface area varies inversely as its radius. 2

24. If $\overrightarrow{AB} = \hat{j} + \hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$ represent the two vectors along the sides AB and AC of ΔABC, prove that the median $\overrightarrow{AD} = \dfrac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$, where D is midpoint of BC.
Hence, find the length of median AD. 2

25. Find the co-ordinates of the point on the line $\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ such that the sum of co-ordinates is 3. 2

SECTION – C

This section comprises Short Answer (SA) type questions of 3 marks each.

26. Find : $\int \dfrac{x+2}{\sqrt{9x - x^2}}\, dx$ 3

27. (a) Evaluate : $\displaystyle\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1 + \sqrt{\cot x}}$ 3

OR

(b) Evaluate : $\displaystyle\int_{\frac{-\pi}{6}}^{\frac{\pi}{2}} (\sin|x| + \cos|x|)\, dx$ 3

28. If $\dfrac{d}{dx}(F(x)) = \dfrac{1}{e^x + 1}$, then find F(x) given that $F(0) = \log\dfrac{1}{2}$. 3

29. (a) Solve the following differential equation : 3

$$x\frac{dy}{dx} = y - x\sin^2\left(\frac{y}{x}\right), \text{ given that } y(1) = \frac{\pi}{6}$$

OR

(b) Find the general solution of the differential equation : $y \log y \dfrac{dx}{dy} + x = \dfrac{2}{y}$. 3

30. Solve the following linear programming problem graphically : 3

Maximize $Z = 10500x + 9000y$

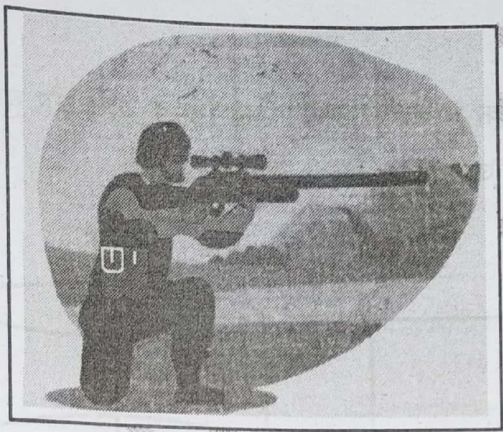
Subject to constraints

$$x + y \leq 50$$

$$2x + y \leq 80$$

$$x, y \geq 0$$

31. (a) The probability of hitting the target by a trained sniper is three times the probability of not hitting the target on a stormy day due to high wind speed. 3



The sniper fired two shots on the target on a stormy day when wind speed was very high. Find the probability that

(i) target is hit

(ii) atleast one shot misses the target.

OR

(b) Mother, Father and Son line up at random for a family picture. Let events E : Son on one end and F : Father in the middle. Find P(E/F). 3

SECTION – D

This section comprises Long Answer (LA) type questions of 5 marks each.

32. (a) If $P = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find (QP) and hence solve the following system of equations using matrices :

$x - y = 3,\ 2x + 3y + 4z = 17,\ y + 2z = 7$ 5

OR

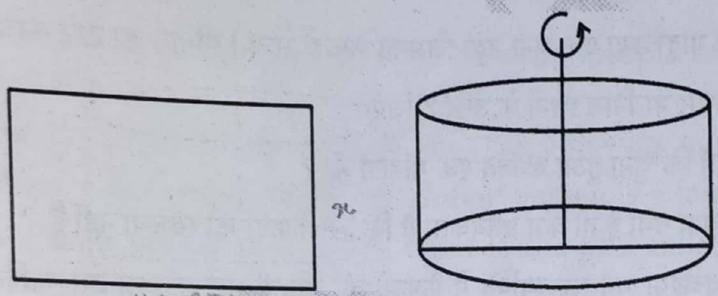
(b) Obtain the value of $\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ in terms of x, y and z. 5

Further, if $\Delta = 0$ and x, y, z are non-zero real numbers, prove that $x^{-1} + y^{-1} + z^{-1} = -1$.

33. (a) Find the sub intervals in which $f(x) = \cot^{-1} (\sin x + \cos x)$, $x \in (0, \pi)$ is increasing and decreasing. 5

OR

(b) A rectangle of perimeter 36 cm is revolved around one of its sides to sweep out a cylinder of maximum volume. 5



Find the dimensions of the rectangle.

34. Find the domain of $g(x) = \cos^{-1} (x^2 - 1)$. Hence, find the value of x for which $g(x) = \frac{\pi}{3}$. 5

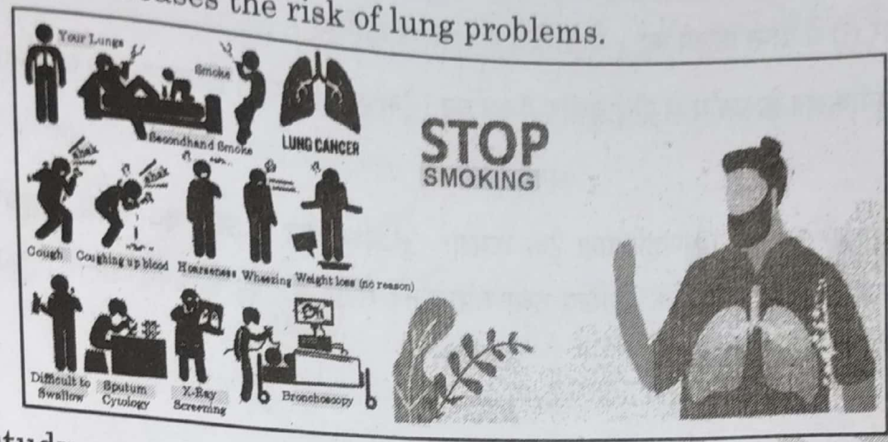
Also, write the range of $\cos^{-1} x$ other than its principal branch.

35. A line passing through the points A(1, 2, 3) and B(5, 8, 11) intersects the line $\vec{r} = 4\hat{i} + \hat{j} + \lambda(5\hat{i} + 2\hat{j} + \hat{k})$. Find the co-ordinates of the point of intersection. Hence, write the equation of a line passing through the point of intersection and perpendicular to both the lines. 5

SECTION – E

This section comprises of 3 case study based questions of 4 marks each.

36. Smoking increases the risk of lung problems.



A study revealed that 170 in 1000 males who smoke develop lung complications, while 120 out of 1000 females who smoke develop lung related problems. In a colony, 50 people were found to be smokers of which 30 are males.

A person is selected at random from these 50 people and tested for lung related problems.

Based on the given information, answer the following questions :

(i) What is the probability that selected person is a female ? **1**

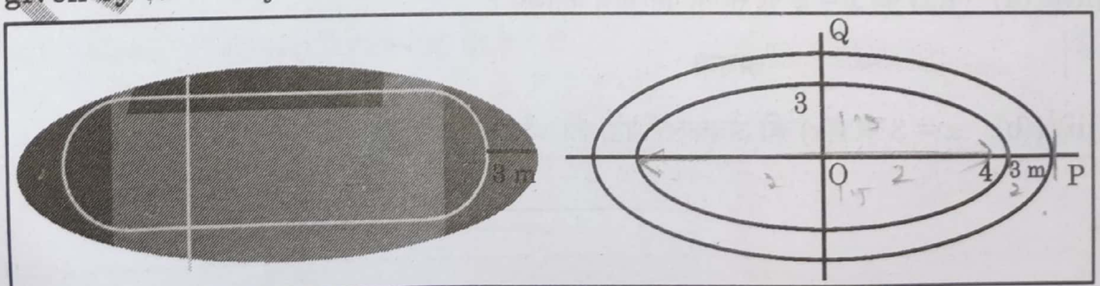
(ii) If a male person is selected, what is the probability that he will not be suffering from lung problems ? **1**

(iii) (a) A person selected at random is detected with lung complications. Find the probability that selected person is a female. **2**

OR

(iii) (b) A person selected at random is not having lung problems, find the probability that the person is a male. **2**

37. A racing track is build around an elliptical ground whose equation is given by $9x^2 + 16y^2 = 144$. The width of the track is 3 m as shown below :



Based on given information, answer the following questions :

(i) Express y as a function of x from the given equation of ellipse. 1

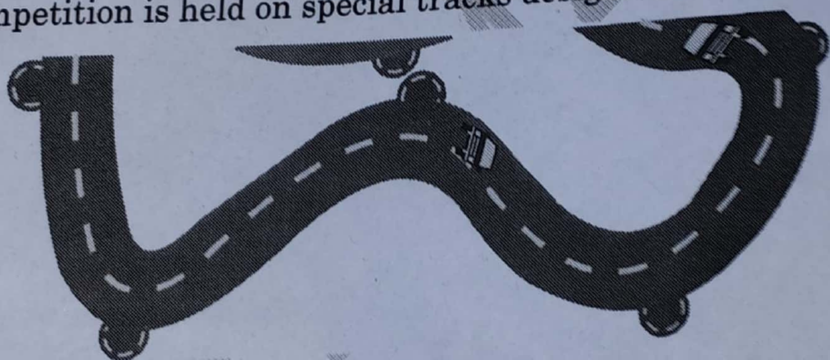
(ii) Integrate the function obtained in (i) with respect to x. 1

(iii) (a) Find the area of the region enclosed within the elliptical ground excluding the track using integration.

OR

(iii) (b) Write the co-ordinates of the points P and Q where the outer edge of the track cuts x axis and y axis in first quadrant and find the area of the triangle formed by points P, O, Q using integration. 2

38. Sports car racing is a form of motorsport which uses sports car prototypes. The competition is held on special tracks designed in various shapes.



The equation of one such track is given as follows :

$$f(x) = \begin{cases} x^4 - 4x^2 + 4, & 0 \le x < 3 \\ x^2 + 40 \quad , & x \ge 3 \end{cases}$$

Based on given information, answer the following questions :

(i) Find $f'(x)$ for $0 < x < 3$. 1

(ii) Find $f'(4)$ 1

(iii) (a) Test for continuity of $f(x)$ at $x = 3$. 2

OR

(iii) (b) Test for differentiability of $f(x)$ at $x = 3$. 2