Series : PQSR5

SET-1

प्रश्न-पत्र कोड
Q.P. Code **65/5/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे
Time allowed : 3 hours

अधिकतम अंक : 80
Maximum Marks : 80

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains **23** printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **38** questions.
- **Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A**, Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B**, Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C**, Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D**, Questions no. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises Multiple Choice Questions (MCQs) of 1 mark each.

1. If matrix $A = \begin{bmatrix} -p & q \\ r & p \end{bmatrix}$ is such that $A^2 = I$, then :

(A) $1 + p^2 + qr = 0$ (B) $1 - p^2 - qr = 0$

(C) $1 - p^2 + qr = 0$ (D) $1 + p^2 - qr = 0$

2. If A is a square matrix such that $A^2 = A$, then $(A - I)^3 - A$ is equal to :

(A) I (B) – I

(C) A (D) A^2

3. For the inverse trigonometric functions, which of the following Principal Value Branch is *not* correctly defined ?

(A) $\tan^{-1} : R \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(B) $\sec^{-1} : R - (-1, 1) \to [0, \pi] - \left\{\frac{\pi}{2}\right\}$

(C) $\cot^{-1} : R \to (0, \pi)$

(D) $\operatorname{cosec}^{-1} : R - (-1, 1) \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

4. Let $A = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix}$. If A + B + C = O, then matrix C is :

(A) $\begin{bmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 3 & 5 \\ -3 & -4 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} -3 & -3 & -5 \\ 3 & 4 & 2 \end{bmatrix}$

5. If A is a non-singular matrix, then which of the following is *not* true ?

(A) adj A is singular

(B) $(\text{adj } A)^{-1} = (\text{adj } A^{-1})$

(C) $|A| \neq 0$

(D) A^{-1} exists

. If $f(x) = \begin{cases} \dfrac{x^2 - 4x - 5}{x + 1}, & x \neq -1 \\ k, & x = -1 \end{cases}$

is continuous at $x = -1$, then the value of k is :

(A) Any real value

(B) 6

(C) -1

(D) -6

7. If the area of Δ ABC with vertices A(3, 1), B(– 2, 1) and C(0, k) is 5 sq. units, then values of k are :

(A) 3, 1

(B) – 1, 3

(C) – 1, 2

(D) 0, 2

8. Derivative of $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$ with respect to x is :

(A) – 1

(B) 1

(C) $\frac{\pi}{4}$

(D) $-\frac{\pi}{4}$

9. Absolute minimum value of $f(x) = (x-2)^2 + 5$ in the interval [–3, 2] is :

(A) – 3

(B) 2

(C) 5

(D) 30

10. $\int \frac{1}{\sqrt{1+\cos 2x}} dx$ is equal to :

(A) log cos x + C

(B) $\frac{1}{\sqrt{2}} \log |\sec x + \tan x| + C$

(C) $\frac{1}{\sqrt{2}} \log |\sec x - \tan x| + C$

(D) log sin 2x + C

11. The value of $\int_{-5}^{-1} \frac{1}{x} dx$ is equal to :

(A) – log 5

(B) x^6

(C) log (– 5)

(D) x^{-6}

12. An ant is observed crawling on a sheet of paper along a straight line given by equation y = 2x – 4. Area of the surface covered by the ant bounded by y-axis, x-axis and x = 1 is :

(A) 1 sq. unit

(B) 3 sq. units

(C) 2 sq. units

(D) 4 sq. units

13. The order and degree of the differential equation

$$1 + \left(\frac{d^3y}{dx^3}\right)^3 = \lambda \frac{d^2y}{dx^2} \text{ is :}$$

(A) Order = 3, Degree = 3
(B) Order = 2, Degree = 2
(C) Order = 3, Degree = 1
(D) Order = 2, Degree = 1

14. The general solution for the differential equation $\frac{dy}{dx} = e^{3x-y}$ is :

(A) $3e^y = e^{3x} + C$
(B) $\log(3x - y) = C$
(C) $e^{3x-y} = C$
(D) $-e^y + 3e^{3x} = C$

15. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40) (60, 20) and (60, 0). If the objective function of an LPP is Z = 4x + 3y, then the maximum value is :

(A) 200
(B) 300
(C) 240
(D) 120

16. If position vector $\vec{p}$ of a point (24, n) is such that $|\vec{p}| = 25$, then the value of n is :

(A) ± 49
(B) ± 5
(C) ± 1
(D) ± 7

17. If vectors $\vec{a} = 3\hat{i} + 2\hat{j} + \lambda\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$, represent the two strips of the Red Cross sign placed outside a doctor's clinic, then the value of λ is :

(A) 1
(B) $\frac{5}{2}$
(C) $\frac{2}{5}$
(D) 0

18. If $3P(A) = P(B) = \frac{3}{5}$ and $P(A|B) = \frac{2}{3}$, then $P(A \cup B)$ is :

(A) $\frac{3}{5}$

(B) $\frac{1}{5}$

(C) $\frac{2}{15}$

(D) $\frac{2}{5}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* A relation R on the set {1, 2, 3} defined as R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)} is an equivalence relation.

Reason (R) : A relation that is reflexive, symmetric and transitive is an equivalence relation.

20. *Assertion (A) :* Consider a Linear Programming Problem with minimise $Z = x + 2y$ subject to constraints $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$ which gives minimum Z at infinitely many points. The corner points of feasible region are (0, 3) and (6, 0).

Reason (R) : If two corner points produce the same minimum value of the objective function, then every point on the line segment joining the points will give the same minimum value.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Evaluate $\sin\left[\tan^{-1}\tan\left(\frac{3\pi}{4}\right)\right]$.

22. (a) Differentiate x^x with respect to $x \log x$.

OR

(b) If $y = P \cos ux + Q \sin ux$, show that $\frac{d^2y}{dx^2} + u^2y = 0$.

23. Determine the values of x for which $f(x) = \frac{x-3}{x+1}$, $x \neq -1$ is an increasing function.

24. (a) Three honey bees were found flying along the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 4\hat{j} - 2\hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{k}$ respectively. Find the value of λ such that the path for $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$.

OR

(b) If A, B and C be three non-collinear points such that $\overrightarrow{AB} = \hat{i} + 2\hat{j} - \hat{k}$ and $\overrightarrow{AC} = 2\hat{i} - 3\hat{j}$, then find the area of Δ ABC.

25. Find the angle between the following pair of lines :

$$\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6} \text{ and}$$

$$\frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}$$

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. A spherical balloon loses its volume due to escape of air from it in such a way that decrease of volume at any instant is proportional to its surface area. Show that the radius is decreasing at a constant rate.

27. (a) Find :

$$\int \frac{x - \sin x}{1 - \cos x} \, dx$$

OR

(b) Evaluate :

$$\int_0^2 \frac{1}{\sqrt{x^2 + 2x + 3}} \, dx$$

28. Solve the differential equation $(x + 2y^3) \, dy = y \, dx$.

29. Solve the following Linear Programming Problem graphically :

Maximize $Z = \frac{2x}{5} + \frac{3y}{10}$

subject to constraints

$$2x + y \leq 1000$$
$$x + y \leq 800$$
$$x, y \geq 0.$$

30. (a) Let three toys A, B and C be placed in the same straight line. If the position vectors of A, B and C are $55\hat{i} - 2\hat{j}$, $5\hat{i} + 8\hat{j}$ and $a\hat{i} - 52\hat{j}$ respectively, find the value of 'a'.

OR

(b) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then prove that

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 9.$$

31. (a) A die is rolled. Consider events :

$A = \{1, 2, 5\}$, $B = \{3, 5\}$, $C = \{2, 3, 4, 5\}$

and hence find :

(i) $P(A \mid C)$ and $P(C \mid A)$

(ii) $P(A \cap B \mid C)$ and $P(A \cup B \mid C)$

OR

(b) A box contains 6 cards numbered 1 to 6. A student is asked to pick up two cards, one by one after replacement and note down the numbers on the cards. Let A be the event of getting sum of the numbers on two cards as 10, and B, the event of a number other than 4 on the first card selected.

Find P(A and B) and find whether the events A and B are independent events or not.

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. A man goes to buy fruits from the market. The shopkeeper informs him that 4 apples, 3 oranges and 2 bananas cost ₹ 60; 2 apples, 4 oranges and 6 bananas cost ₹ 90; whereas 6 apples, 2 oranges and 3 bananas cost ₹ 70. Using matrix method, find the cost of one fruit of each kind.

33. (a) If $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$, show that

$$(x^2 + 1)\frac{dy}{dx} + xy + 1 = 0.$$

OR

(b) Find the differential of $x^{\cot x} + \dfrac{2x^2-3}{2x^2-x+2}$ with respect to x.

34. Sketch the curve $\{(x, y) : 100x^2 + 25y^2 = 2500\}$ and find the area of the region enclosed by it, using integration.

35. (a) Find the foot of the perpendicular from the point (0, 2, 3) on the line $\frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$ and hence find the length of the perpendicular.

OR

(b) Find the value of p if the shortest distance between the lines

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and

$\vec{r} = (p\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

is $\frac{3}{\sqrt{2}}$ units.

SECTION E

This section comprises 3 Case Study based questions of 4 marks each.

Case Study – 1

36. A school wants the students of class XII to do a project on 'Sustainability' keeping the world environment in mind. They select the student participants on the basis of an essay writing competition.

7 students out of 80 are selected for the project and are categorized into two sets such that :

Girl students belong to Set A = $\{G_1, G_2, G_3, G_4\}$,

Boy students belong to Set B = $\{B_1, B_2, B_3\}$.

Based on the above information, answer the following questions :

(i) How many relations are possible from Set A → Set B ? *1*

(ii) Let R be a relation from $A \to B$ such that

$R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$.

Is R an injective function ? Justify your answer. *1*

(iii) (a) Let the relation R from $A \to A$ be such that

$R = \{(x, y), x, y \in A$, x and y are students from the same colony in the city$\}$

Verify if R is an equivalence relation. *2*

OR

(iii) (b) Verify if any function $f : B \to A$ is bijective. Give reason to support your answer. *2*

Case Study – 2

37. There are three types of vaccines A_1, A_2, A_3, available in the market to protect the population of the country from spread of certain infection. According to a survey conducted, it was found that 25% of the population was given Vaccine A_1, 35% of the population was given Vaccine A_2 and 40% of the population was given Vaccine A_3. The survey also stated that the probabilities that Vaccines A_1, A_2 and A_3 would protect against the infection were 60%, 55% and 50% respectively.

Based on the above information, answer the following questions :

Find the probability that :

(i) The person taking vaccine A_2 will get infected. *1*

(ii) If a person is chosen randomly, he/she will be protected from the infection. *1*

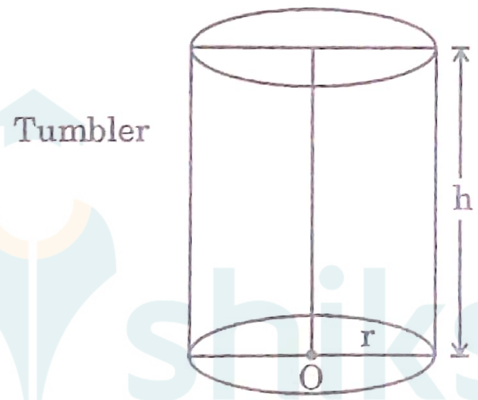
(iii) (a) The person was given Vaccine A_1, given that the randomly chosen person is infected. 2

OR

(iii) (b) The person was given Vaccine A_3, given that the randomly chosen person is not infected. 2

Case Study – 3

38. A company produces cylindrical tumblers, open from the top. Since they want uniformity in the product, they fix the surface area of the tumblers produced.



Based on the above information, answer the following questions :

If for a tumbler, V is its volume, h the height and r the radius of the circular base, then :

(i) Differentiate its volume with respect to radius of the base, where the surface area is constant. 2

(ii) If the company wants to maximize the volume of each tumbler, then establish a relation between its height and the radius of the base. 2