MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – APRIL 2026

(HELD ON SATURDAY 04th APRIL 2026) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

1. If $\sum_{i=1}^{10}(x_i+2)^2=180$ and $\sum_{i=1}^{10}(x_i-1)^2=90$, then the Standard Deviation is equal to

(1) 3 (2) 2

(3) 4 (4) 5

Ans. **(1)**

Sol. $\sum_{i=1}^{10}(x_i+2)^2=180$

$$\sum_{i=1}^{10}x_i^2+4\sum_{i=1}^{10}x_i+\sum_{i=1}^{10}4=180$$

$$\sum_{i=1}^{10}x_i^2+4\sum_{i=1}^{10}x_i=180-40 \quad \ldots(1)$$

Also $\sum_{i=1}^{10}(x_i-1)^2=90$

$$\sum_{i=1}^{10}x_i^2-2\sum_{i=1}^{10}x_i+\sum_{i=1}^{10}1=90$$

$$\sum_{i=1}^{10}x_i^2-2\sum_{i=1}^{10}x_i=90-10 \quad \ldots(2)$$

$$\sum_{i=1}^{10}x_i^2+4\sum_{i=1}^{10}x_i=140$$

$$\sum_{i=1}^{10}x_i^2-2\sum_{i=1}^{10}x_i=80$$

$$\sum_{i=1}^{10}x_i^2=100 \text{ and } \sum_{i=1}^{10}x_i=10$$

$$\sigma^2=\frac{\sum_{i=1}^{10}x_i^2}{N}-\left(\frac{\sum_{i=1}^{10}x_i}{N}\right)^2$$

$$=\frac{100}{10}-\left(\frac{10}{10}\right)^2$$

$\sigma^2 = 10 - 1 = 9$

$\Rightarrow \sigma = 3$

2. If the system of equations

$x + y + z = 5$

$x + 2y + 3z = 9$

$x + 3y + \lambda z = \mu$

has infinitely many solutions, then value of $\lambda + \mu$ is

(1) 13 (2) 20

(3) 18 (4) 26

Ans. **(3)**

Sol. $D = \lambda - 5$

$D_1 = \lambda + \mu - 18$

$D_2 = 4\lambda - 2\mu + 6$

$D_3 = \mu - 13$

Here $x=\frac{D_1}{D}$; $y=\frac{D_2}{D}$; $z=\frac{D_3}{D}$

For infinitely many solutions

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow \lambda = 5$; $\mu = 13$

$\therefore \lambda + \mu = 18$

3. Let $f(x)=\begin{cases} e^{x-1} & x<0 \\ x^2-5x+6 & x\geq 0 \end{cases}$

and $g(x) = f(|x|) + |f(x)|$.

If α = number of points of discontinuity of g(x) and

β = number of points of non-differentiability of g(x),

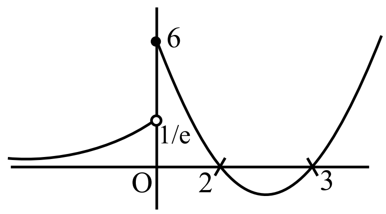
then $\alpha + \beta =$

(1) 2 (2) 4

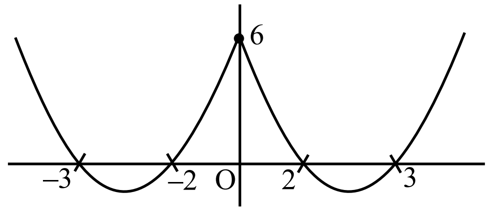
(3) 3 (4) 5

Ans. **(2)**

Sol. Graph of f(x)

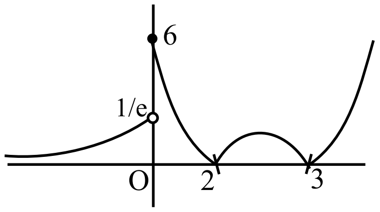


Now graph of f(|x|)



f(|x|) is continuous function and it is non diff. at x = 0

Graph of |f(x)|



|f(x)| is discontinuous at x = 0

|f(x)| is non-diff. at x = 0, 2, 3

g(x) = f(|x|) + |f(x)|

g(x) will be discontinuous at x = 0

g(x) will be non diff at x = 0, 2, 3

$\alpha = 1, \beta = 3$

$\alpha + \beta = 1 + 3 = 4$

4. Area bounded between the curves $x = -2y^2$ and $x = 1 - 4y^2$ is

(1) $\frac{\sqrt{2}}{3}$ (2) $\frac{2\sqrt{2}}{3}$

(3) $\frac{2}{3}$ (4) $\frac{3\sqrt{3}}{3}$

Ans. **(2)**

Sol. Solving both the curves

$-2y^2 = 1 - 4y^2$

$y = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

Required area $= \left|\int_{-1/\sqrt{2}}^{1/\sqrt{2}} (x_2 - x_1)dx\right|$

$= \left|\int_{-1/\sqrt{2}}^{1/\sqrt{2}} (1 - 4y^2 + 2y^2)dy\right|$

$= \left|2\int_{0}^{1/\sqrt{2}} (1 - 2y^2)dy\right|$

$= \left|2\left[\left(y - \frac{2y^3}{3}\right)\right]_0^{\frac{1}{\sqrt{2}}}\right|$

$= \left|2\left(\frac{1}{\sqrt{2}} - \frac{2}{3}\cdot\frac{1}{2\sqrt{2}}\right)\right| = \frac{2\sqrt{2}}{3}$

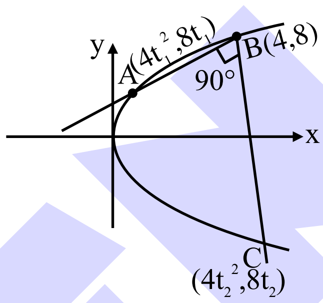
5. From point B(4,8) on the parabola $y^2 = 16x$, two perpendicular chords BA and BC are drawn. Given that the locus of centroid of triangle BAC is another parabola with length of the latus rectum equal to ℓ, then 3ℓ is equal to

(1) 14 (2) 15

(3) 12 (4) 16

Ans. **(4)**

Sol.



Variable point A & C are shown in the figure

$\because m_{AB}.m_{BC} = -1$

$\Rightarrow t_1 + t_2 + t_1t_2 = -5$ ...(1)

Suppose locus of centroid of ΔABC is (h,k)

$\therefore 3h = 4 + 4t_1^2 + 4t_2^2$ & $3k = 8 + 8t_1 + 8t_2$

by eliminating t_1 & t_2 using equation (1) also

we get $h = \frac{9}{48}k^2 + \frac{40}{3}$

$\therefore$ locus of centroid of parabola is

$x = \frac{9}{48}y^2 + \frac{40}{3}$

$\therefore \ell(L.R.) = \frac{48}{9}$

$\therefore 3\ell(L.R.) = \frac{48}{3}$

6. If the quadratic equation $(\lambda + 2)x^2 - 3\lambda x + 4\lambda = 0$ (where $\lambda \neq -2$) has two positive roots then the numbers of possible integral values of λ is

(1) 2 (2) 4

(3) 1 (4) 3

Ans. **(1)**

Sol. $f(x) = (\lambda + 2)x^2 - 3\lambda x + 4\lambda$

C-1 $af(0) > 0$

$(\lambda + 2)4\lambda > 0$

$\Rightarrow \lambda < -2 \quad \text{or} \quad \lambda > 0$

C-2 $-\dfrac{b}{2a} > 0$

$\dfrac{3\lambda}{2(\lambda+2)} > 0$

$\Rightarrow \lambda < -2 \ \text{or} \ \lambda > 0$

C-3 $D \geq 0$

$(-3\lambda)^2 - 4(\lambda + 2) \times 4\lambda \geq 0$

$\lambda(7\lambda + 32) \leq 0$

$\Rightarrow \lambda \in \left[\dfrac{-32}{7}, 0\right]$

C-1 and C-2 and C-3

$\Rightarrow \lambda \in \left[\dfrac{-32}{7}, -2\right)$

$\lambda \in [-4.57, -2)$

$\Rightarrow \lambda = -4, -3$

$\therefore$ Number of values of $\lambda = 2$

7. If function $y(x)$ satisfies the differential equation $\dfrac{dy}{dx} + \left[\dfrac{6x^2 + e^{-2x}(3x^2 + 2x^3 + 4)}{(x^3 + 2)(2 + e^{-2x})}\right]y = e^{-2x} + 2$ such that $y(0) = \dfrac{3}{2}$ & $y(1) = \alpha(e^{-2} + 2)$, then α is equal to

(1) $\dfrac{13}{12}$ (2) $\dfrac{12}{13}$

(3) $\dfrac{4}{13}$ (4) $\dfrac{17}{12}$

Ans. **(1)**

Sol. I.F. $= e^{\int \frac{6x^2 + e^{-2x}(3x^2 + 2x^3 + 4)}{(x^3+2)(2+e^{-2x})} dx}$

$= e^{\int \frac{6x^2 + e^{-2x}(3x^2 - 2x^3 - 4) + (4x^3 + 8)e^{-2x}}{(x^3+2)(2+e^{-2x})} dx}$

Let $(x^3 + 2)(2 + e^{-2x}) = t$

$(6x^2 + e^{-2x}(3x^2 - 2x^3 - 4))dx = dt$

I.F. $= e^{\ell n|(x^3+2)(2+e^{-2x})| - \int \frac{4e^{-2x}dx}{2+e^{-2x}}}$

$= e^{\ell n|(x^3+2)(2+e^{-2x})| - 2\,\ell n|2+e^{-2x}|}$

$= e^{\ell n\left|\frac{x^3+2}{2+e^{-2x}}\right|} = \dfrac{x^3 + 2}{2 + e^{-2x}}$

$\Rightarrow y.\left(\dfrac{x^3+2}{2+e^{-2x}}\right) = \int\left(\dfrac{x^3+2}{2+e^{-2x}}\right)(e^{-2x} + 2)\,dx + C$

$y.\left(\dfrac{x^3+2}{2+e^{-2x}}\right) = \dfrac{x^4}{4} + 2x + C$ at $y(0) = \dfrac{3}{2} \Rightarrow C = 1$

So $y.\left(\dfrac{x^3+2}{2+e^{-2x}}\right) = \dfrac{x^4}{4} + 2x + 1$

at $x = 1$

$y.\left(\dfrac{3}{2+e^{-2}}\right) = \dfrac{1}{4} + 2 + 1$

$\Rightarrow y = \dfrac{13}{12}(e^{-2} + 2)$

So $\alpha = \dfrac{13}{12}$

8. 3 numbers are selected randomly from numbers 1, 2, 3...., 31. The probability that they are in A.P. is

(1) $\dfrac{15}{31}$ (2) $\dfrac{7}{31}$

(3) $\dfrac{8}{17}$ (4) $\dfrac{45}{899}$

Ans. **(4)**

Sol. Total numbers of ways of selecting

3 numbers $= {}^{31}C_3 = \dfrac{31 \times 30 \times 29}{3 \times 2 \times 1} = 4495$

a, b, c are in A.P.

$\Rightarrow$ either a & c both odd numbers

or both even numbers

$= {}^{16}C_2 + {}^{15}C_2$

$= 120 + 105$

$= 225$

Probability $= \dfrac{225}{4495} = \dfrac{45}{899}$

9. The maximum value of $E = 16\sin\dfrac{x}{2}\cos^3\dfrac{x}{2}$ where $x \in [0, \pi]$, is

(1) 3 (2) $3\sqrt{3}$

(3) $6\sqrt{3}$ (4) 6

Ans. (2)

Sol. $E = 16\sin\frac{x}{2}\cos^3\frac{x}{2}$

$E = 4\sin x\,[1 + \cos x]$

$\frac{dE}{dx} = 4\,[\cos x + \cos 2x]$

$= 8\cos\frac{3x}{2}\cos\frac{x}{2} = 0$

$\Rightarrow \cos\frac{3x}{2} = 0$ or $\cos\frac{x}{2} = 0$

$\Rightarrow x = \left\{\frac{\pi}{3}, \pi\right\}$ are critical points of the function

$E(0) = 0$

$E(\pi) = 0$

$E\left(\frac{\pi}{3}\right) = 3\sqrt{3}$

$\therefore$ maximum value of $E = 3\sqrt{3}$

10. $\int_0^1 \cot^{-1}(1 + x + x^2)dx$ is equal to

(1) $2\tan^{-1}2 - \frac{1}{2}\log_e\left(\frac{5}{4}\right) - \frac{\pi}{2}$

(2) $2\tan^{-1}2 - \frac{1}{2}\log_e\left(\frac{5}{4}\right) + \frac{\pi}{2}$

(3) $2\tan^{-1}2 + \frac{1}{2}\log_e\left(\frac{5}{4}\right) - \frac{\pi}{2}$

(4) $2\tan^{-1}2 + \frac{1}{2}\log_e\left(\frac{5}{4}\right) + \frac{\pi}{2}$

Ans. (1)

Sol. $= \int_0^1 \tan^{-1}\left(\frac{1}{1 + x + x^2}\right)dx$

$= \int_0^1 \tan^{-1}\left(\frac{(x+1) - x}{1 + x(x+1)}\right)dx$

$= \int_0^1 (\tan^{-1}(x+1) - \tan^{-1}x)dx$

$= \left(x\tan^{-1}(x+1) - \frac{1}{2}\ell n|1 + (1+x)^2| + \tan^{-1}(x+1)\right)_0^1$

$- \left(x\tan^{-1}x - \frac{1}{2}\ell n|1 + x^2|\right)_0^1$

$= \left(2\tan^{-1}2 - \frac{\pi}{4} - \frac{1}{2}\ell n\frac{5}{2}\right) - \left(\frac{\pi}{4} - \frac{1}{2}\ell n 2\right)$

$= 2\tan^{-1}2 - \frac{1}{2}\ell n\left(\frac{5}{4}\right) - \frac{\pi}{2}$

11. Let $S = \{z : z^2 + 4z + 16 = 0,\ z \in C\}$, then value of $\sum_{z\in S}|z + \sqrt{3}i|^2$ is equal to

(1) 34 (2) 35

(3) 38 (4) 41

Ans. (3)

Sol. $z^2 + 4z + 16 = 0$

$\Rightarrow (z + 2)^2 = -12$

$\Rightarrow z = -2 \pm 2\sqrt{3}i$

$\Rightarrow S = \{-2 + 2\sqrt{3}i, -2 - 2\sqrt{3}i\}$

$\therefore \sum_{z\in S}\left|z + \sqrt{3}i\right|^2$

$= \left|-2 + 2\sqrt{3}i + \sqrt{3}i\right|^2 + \left|-2 - 2\sqrt{3}i + \sqrt{3}i\right|^2$

$= \left|-2 + 3\sqrt{3}i\right|^2 + \left|-2 - \sqrt{3}i\right|^2$

$= 38$

12. If f(x) satisfies the functional equation $f(x+y) = f(x) + 2y^2 + y + \alpha xy$ (where x, y are whole numbers), such that $f(0) = -1$ & $f(1) = 2$, then the value of $\sum_{i=1}^{5}\left(f(i)+\alpha\right)$ is

(1) 130 (2) 145

(3) 120 (4) 140

Ans. (4)

Sol. Given $f(x+y) = f(x) + 2y^2 + y + \alpha\, x\, y$ ......(1)

put $x = 1, y = 1 \Rightarrow f(2) = 5 + \alpha$

put $x = 2, y = 1 \Rightarrow f(3) = 8 + 3\alpha$ ......(2)

put $x = 1, y = 2 \Rightarrow f(3) = 12 + 2\alpha$ .....(3)

from (2) & (3) $\Rightarrow \alpha = 4$

put $x = 3, y = 1 \Rightarrow f(4) = 35$

put $x = 4, y = 1 \Rightarrow f(5) = 54$

$\therefore \Rightarrow$ value of $\sum_{i=1}^{5}\left(f(i)+\alpha\right) = \sum_{i=1}^{5} f(i) + 5\alpha$

$= f(1) + f(2) + f(3) + f(4) + f(5) + 5\alpha$

$= 140$

13. If $f(x) = \int_0^x \tan(t-x)dt + \int_0^x f(t)\tan t\, dt$, then the value of $f''\left(\frac{\pi}{6}\right) - 12f'\left(-\frac{\pi}{6}\right) + f\left(\frac{\pi}{6}\right)$ is equal to

(1) $-7-\frac{16}{3\sqrt{3}}$ (2) $7+\frac{5}{3\sqrt{3}}$

(3) $7-\frac{16}{3\sqrt{3}}$ (4) $\frac{1}{3\sqrt{3}}$

Ans. (1)

Sol. $f(x) = \int_0^x \tan\left(x-t-x\right)dt + \int_0^x f(t)\tan t\, dt$

$f(x) = -\int_0^x \tan t dt + \int_0^x f(t)\tan t\, dt$

$f'(x) = -\tan x + f(x)\tan x$

$\frac{dy}{y-1} = \tan x\, dx$

$y - 1 = \sec x . c$

$y = 1 + c \sec x$

$f(0) = 0$

$0 = 1 + c \sec 0$

$c = -1$

$y = 1 - \sec x$

$f(x) = 1 - \sec x$

$f\left(\frac{\pi}{6}\right) = 1 - \frac{2}{\sqrt{3}}$

$f'(x) = -\sec x \tan x$

$f'\left(\frac{-\pi}{6}\right) = +\frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{2}{3}$

$f''(x) = -\left(\sec x \tan^2 x + \sec^3 x\right)$

$f''\left(\frac{\pi}{6}\right) = -\left(\frac{2}{\sqrt{3}} \times \frac{1}{3} + \frac{8}{3\sqrt{3}}\right) = \frac{-10}{3\sqrt{3}}$

$f''\left(\frac{\pi}{6}\right) - 12f'\left(\frac{-\pi}{6}\right) + f\left(\frac{\pi}{6}\right)$

$= \frac{-10}{3\sqrt{3}} - 12 \times \left(\frac{2}{3}\right) + 1 - \frac{2}{\sqrt{3}}$

$= -7 - \frac{16}{3\sqrt{3}}$

14. Number of non-negative integral solutions of the equation $a + b + 2c = 22$ is

(1) 144 (2) 121 (3) 168 (4) 99

Ans. (1)

Sol. $c = 0 \Rightarrow a + b = 22 \rightarrow {}^{22+2-1}C_{2-1} = {}^{23}C_1$

$c = 1 \Rightarrow a + b = 20 \rightarrow {}^{20+2-1}C_{2-1} = {}^{21}C_1$

⋮ ⋮ ⋮

$c = 10 \Rightarrow a + b = 2 \rightarrow {}^{2+2-1}C_{2-1} = {}^{3}C_1$

$c = 11 \Rightarrow a + b = 0 \rightarrow {}^{0+2-1}C_{2-1} = {}^{1}C_1$

Number of non-negative integral solutions

${}^{1}C_1 + {}^{3}C_1 + {}^{5}C_1 + \ldots + {}^{23}C_1$

$1 + 3 + 5 \ldots\ldots + 23$

$= 144$

15. If $\alpha = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots..$ up to infinity

$\beta = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots..$ up to infinity

Then value of $(0.2)^{\log_{\sqrt{5}}\alpha} + (0.04)^{\log_5 \beta}$ is

(1) $\frac{1}{2}$ (2) 8 (3) 3 (4) $\frac{3}{4}$

Ans. (2)

Sol. $\alpha = \frac{1/4}{1-\frac{1}{2}} = \frac{1}{2}$

$\beta = \frac{1/3}{1-\frac{1}{3}} = \frac{1}{2}$

$(0.2)^{\log_{\sqrt{5}}\alpha} = \alpha^{\log_{\sqrt{5}}\frac{1}{5}} = \left(\frac{1}{2}\right)^{-2} = 4$

$(0.04)^{\log_5\beta} = 5^{-2\log_5\beta} = \left(\frac{1}{2}\right)^{-2} = 4$

$\Rightarrow 4+4 = 8$

16. If z_1, z_2 and z_3 are roots of $x^3 + ax^2 + bx + c = 0$. Let $z_1 = 1$, $z_2 = 1+i\sqrt{2}$ and a, b, c $\in R$ then the value of $\int_{-1}^{1}(x^3+ax^2+bx+c)\,dx$ is

(1) –8 (2) 8 (3) 6 (4) –4

Ans. (1)

Sol. $\int_{-1}^{1}(x^3+ax^2+bx+c)\,dx$

$\Rightarrow \int_{-1}^{1}(ax^2+c)\,dx = 2\int_{0}^{1}(ax^2+c)\,dx \quad \ldots(1)$

Roots are $1, 1+i\sqrt{2}$ and $1-i\sqrt{2}$ because a, b, c $\in R$

$1+1+i\sqrt{2}+1-i\sqrt{2} = -a$

$a = -3$

$1(1+i\sqrt{3})(1-i\sqrt{3}) = -c$

$c = -4$

$2\int_{0}^{1}(-3x^2-3)\,dx \Rightarrow -2[x^3+3x]_0^1$

$= -2[4]$

$= -8$

17. The shortest distance between the lines is :

$\vec{r} = \frac{1}{3}\hat{i} + 2\hat{j} + \frac{8}{3}\hat{k} + \lambda(2\hat{i}-5\hat{j}+6\hat{k})$ and

$\vec{r} = \left(-\frac{2}{3}\hat{i} - \frac{1}{3}\hat{k}\right) + \mu(\hat{j}-\hat{k})$, $\lambda, \mu \in R$

(1) $2\sqrt{3}$ (2) 3

(3) $\sqrt{15}$ (4) $\sqrt{5}$

Ans. (2)

Sol. $\vec{a} = \frac{1}{3}\hat{i} + 2\hat{j} + \frac{8}{3}\hat{k}$

$\vec{a}_2 = \frac{-2}{3}\hat{i} - \frac{1}{3}\hat{k}$

$\vec{b}_1 = 2\hat{i} - 5\hat{j} + 6\hat{k}, \quad \vec{b}_2 = \hat{j} - \hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 6 \\ 0 & 1 & -1 \end{vmatrix} = -\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{a}_1 - \vec{b}_2 = \hat{i} + 2\hat{j} + 3\hat{k}$

$S.D = \left|\frac{(\vec{a}_1-\vec{a}_2)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$S.D = \left|\frac{(\hat{i}+2\hat{j}+3\hat{k})\cdot(-\hat{i}+2\hat{j}+2\hat{k})}{3}\right|$

$= \frac{9}{3} = 3$

18. If $\hat{u}, \hat{v}$ are unit vectors and $|\hat{u}\times\hat{v}| = \frac{\sqrt{3}}{2}$ and $\hat{A} = \lambda\hat{u} + \hat{v} + \hat{u}\times\hat{v}$ then find λ [angle between $\hat{u}$ & $\hat{v}$ is acute]

(1) $\lambda = \frac{1}{3}\hat{A}.\hat{u} - \frac{1}{3}\hat{A}.\hat{v}$

(2) $\lambda = \frac{4}{3}\hat{A}.\hat{u} - \frac{2}{3}\hat{A}.\hat{v}$

(3) $\lambda = \frac{8}{3}\hat{A}.\hat{u} - \frac{2}{3}\hat{A}.\hat{v}$

(4) $\lambda = \frac{2}{3}\hat{A}.\hat{u} - \frac{4}{3}\hat{A}.\hat{v}$

Ans. (2)

Sol. $|\hat{u}\times\hat{v}| = \frac{\sqrt{3}}{2}$

$|\hat{u}||\hat{v}|\sin\theta = \frac{\sqrt{3}}{2} \quad \therefore \theta = \frac{\pi}{3}$

and $\hat{u}\cdot\hat{v} = |\hat{u}||\hat{v}|\cos\frac{A}{3} = \frac{1}{2}$

$\hat{A} = \lambda\hat{u} + \hat{v} + \hat{u}\times\hat{v} \quad \ldots(A)$

Dot with $\hat{u}$

$\hat{A}\cdot\hat{u} = \lambda(1) + \hat{u}.\hat{v} + \hat{u}\cdot(\hat{u}\times\hat{v})$

$\hat{A}\cdot\hat{u}=\lambda+\frac{1}{2}$

Or $2\hat{A}\cdot\hat{u}=2\lambda+1$ ...(1)

Dot equation (A) with $\hat{v}$

$\hat{A}\cdot\hat{v}=\lambda(\hat{u}\cdot\hat{v})+\hat{v}\cdot\hat{v}+\hat{v}\cdot(\hat{u}\times\hat{v})$

$\hat{A}\cdot\hat{v}=\frac{\lambda}{2}+1$

$\hat{A}\cdot\hat{v}-\frac{\lambda}{2}=1$ ...(2)

From (1) and (2)

$2\hat{A}\cdot\hat{u}-2\lambda=\hat{A}.\hat{v}-\frac{\lambda}{2}$

$\therefore\ \lambda=\frac{4}{3}\hat{A}\cdot\hat{u}-\frac{2}{3}\hat{A}.\hat{v}$

19. In the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$, if coefficient of term independent of x is 221k, then value of k is

Ans. **(84)**

Sol. General term in above expression is

$T_{r+1}={}^{18}C_r(9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r$

$=\left(-\frac{1}{3}\right)^r\ {}^{18}C_r 9^{18-r}\cdot x^{18-\frac{3r}{2}}$

For term independent of x, $18-\frac{3r}{2}=0\Rightarrow r=12$

$\therefore$ Coefficient of term independent of x

$=\left(-\frac{1}{3}\right)^{12}\ {}^{18}C_{12}9^{18-12}$

= 18564

= 221 k (given)

$\Rightarrow$ k = 84

20. If $f(x)=(x-1)^4+1\ \forall x\in[1,\infty)$.

Statement-1 : $f(x)=f^{-1}(x)$ has only two solutions.

Statement-2 : $f^{-1}(x+1)=f(x)$ has no solution.

(1) Statement 1 and Statement 2 both are true

(2) Statement 1 is false and Statement 2 is true

(3) Statement 1 is true and Statement 2 is false

(4) Statement 1 and Statement 2 both are false

Ans. **(3)**

Sol. $f(x)=(x-1)^4+1$

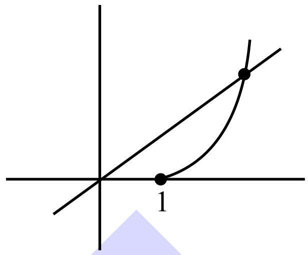
$f'(x)=4(x-1)^3$; $f'(x)\geq 0$

$\therefore$ f(x) is increasing

$\therefore (x-1)^4+1=x$

$(x-1)[(x-1)^3-1]=0$

x = 1, x = 2 are two solution



Now

$f^{-1}(x)=(x-1)^{1/4}+1$

$f^{-1}(x+1)=x^{1/4}+1$

$f^{-1}(x+1)=f(x)$

$x^{1/4}+1=(x-1)^4+1$

$x=(x-1)^{16}$

Above equation has one solution using graph.

Hence option (3) is correct.

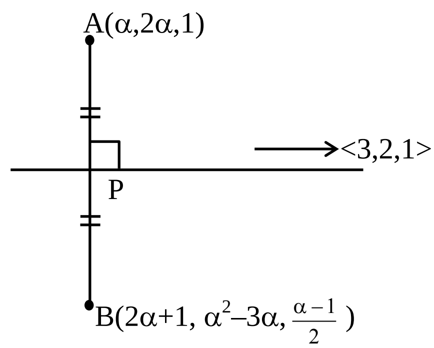
SECTION-B

1. If $(2\alpha+1,\ \alpha^2-3\alpha,\ \frac{\alpha-1}{2})$ is image of $(\alpha, 2\alpha, 1)$ in the line $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z}{1}$, then the value of α is

Ans. **(3)**

Sol. $\therefore$ P is mid point of AB and lies on the given line

$\Rightarrow P\left(\frac{3\alpha+1}{2},\frac{\alpha^2-\alpha}{2},\frac{\alpha+1}{4}\right)$



Now put coordinates of P in the line

$$\Rightarrow \frac{\frac{3\alpha+1}{2}-2}{3}=\frac{\frac{\alpha^2-\alpha}{2}-1}{2}=\frac{\alpha+1}{4}$$

$$\Rightarrow \frac{\alpha-1}{2}=\frac{\alpha+1}{4}=\frac{\alpha^2-\alpha-2}{4}$$

$\Rightarrow \alpha = 3$

Now for $\alpha = 3$, clearly AB is perpendicular to $3\hat{i}+2\hat{j}+k$

for $\alpha = 3$ point B is image of point A

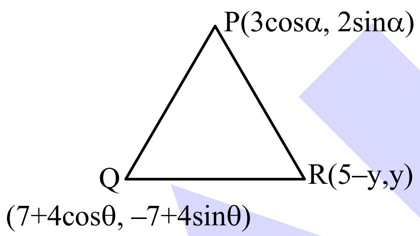
2. P is a point on $\frac{x^2}{9}+\frac{y^2}{4}=1$ as $(3\cos\alpha, 2\sin\alpha)$ Q is a point on $x^2 + y^2 - 14x + 14y + 82 = 0$ R is a point on line $x + y = 5$ their centroid is on $\left(\cos\alpha+2, \frac{2\sin\alpha}{3}+3\right)$. Find the sum of possible ordinates of R.

Ans. (22)

Sol. Centroid :

$$\left(\frac{3\cos\alpha+7+4\cos\theta+5-y}{3}, \frac{2\sin\alpha-7+4\sin\theta+y}{3}\right)$$

$$=\left(\cos\alpha+2, \frac{2}{3}\sin\alpha+3\right)$$



On comparison

$$\cos\theta=\frac{y-6}{4} \ \& \ \sin\theta=\frac{16-y}{4}$$

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow (y-6)^2 + (y-16)^2 = 16$

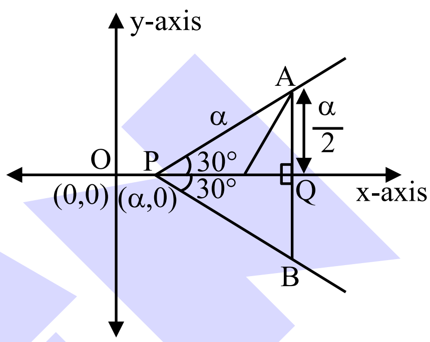
$y^2 - 22y + 138 = 0$

Sum of possible values of y = 22

3. P is point of intersection of these two set of half lines $x-\sqrt{3}y=\alpha, \alpha>0$. A and B are two points on these lines at distance 'α' from 'P'. If length of perpendicular from P on AB is $\frac{9}{2}$ and radius of circumcircle of ΔPAB is R, then $\frac{\alpha^2}{R}$ is

Ans. (9.00)

Sol. ΔPAB is equilateral



$$PQ=\frac{\sqrt{3}\alpha}{2}=\frac{9}{2} \qquad \therefore \ \alpha=\frac{9}{\sqrt{3}}$$

$$\alpha=3\sqrt{3}$$

$$\frac{\alpha}{2r}=\cos 30^\circ$$

$$\alpha=\frac{\sqrt{3}}{2}\cdot 2R$$

$$\alpha=\sqrt{3}R$$

$$\boxed{R=3}$$

$$\frac{\alpha^2}{R}=\frac{(3\sqrt{3})^2}{3}=9$$


PHYSICS