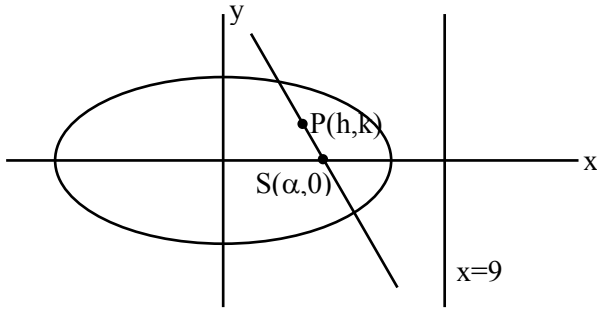


Sol.



$$\frac{a}{e} = 9 \Rightarrow a = 3$$

$$b^2 = a^2(1 - e^2) = 9\left(1 - \frac{1}{9}\right) = 8$$

$$\frac{x^2}{9} + \frac{y^2}{8} = 1$$

$$S(\alpha, 0) \equiv S(1, 0)$$

$$T = S_1 \Rightarrow \frac{hx}{9} + \frac{ky}{8} = \frac{h^2}{9} + \frac{k^2}{8}$$

$$(1, 0) \Rightarrow \frac{h}{9} + 0 = \frac{h^2}{9} + \frac{k^2}{8}$$

$$h = h^2 + \frac{9}{8}k^2$$

$$9y^2 = 8x(1 - x)$$

7. The value of x which satisfies the equation

$$\sin^{-1}\left(\frac{2}{3}\sqrt{1-x^2}\right) = \cot^{-1}(2\sqrt{x}), \text{ is}$$

(1) $\frac{1}{2}$

(2) $\frac{1}{4}$

(3) $\frac{1}{8}$

(4) $\frac{1}{9}$

Ans. (1)

Sol. Taking sine on both the sides,

$$\sin\left(\sin^{-1}\frac{2}{3}\sqrt{1-x^2}\right) = \sin\left(\cot^{-1}2\sqrt{x}\right)$$

$$\Rightarrow \frac{2}{3}\sqrt{1-x^2} = \frac{1}{\sqrt{1+4x}}$$

$$\Rightarrow (1-x^2)(1+4x) = \frac{9}{4}$$

$$\Rightarrow x = \frac{1}{2}$$

8. Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \frac{2x^2 - 3x + 2}{3x^2 + x + 3}$, then $f(x)$ is

- (1) one-one and onto (2) one-one and into
 (3) many-one and into (4) many-one and onto

Ans. (3)

Sol. $\frac{2x^2 - 3x + 2}{3x^2 + x + 3} = y$

$$(3y-2)x^2 + (y+3)x + (3y-2) = 0$$

$$D \geq 0$$

$$(7y-1)(5y-7) \leq 0$$

$$y \in \left[\frac{1}{7}, \frac{7}{5}\right]$$

Hence $f(x)$ is into

obviously $f(x)$ is many-one because $f(x)$ is non-monotonic

9. An ellipse has directrix $x = 9$ & eccentricity = $\frac{1}{3}$.

If one of its focus is $(\alpha, 0)$, $\alpha > 0$, then locus of the mid-point of the chord passing through $P(\alpha, 0)$ is

(1) $\frac{x^2}{9} + \frac{y^2}{8} = \frac{x}{9}$

(2) $\frac{x^2}{9} + \frac{y^2}{2} = \frac{x}{9}$

(3) $\frac{x^2}{8} + \frac{y^2}{2} = \frac{x}{8}$

(4) $\frac{x^2}{9} + \frac{y^2}{8} = \frac{x}{4}$

Ans. (1)

Sol. $e = 1/3$

$x = 9$ is the directrix

$$\therefore \frac{a}{e} = 9$$

$$a = 3$$

focus $(\alpha, 0) \equiv (ae, 0) \equiv (1, 0)$

let mid-point of the chord is $m(h, k)$

$$\text{Also } b^2 = a^2(1 - e^2) = 9\left(1 - \frac{1}{9}\right) = 8$$

$$\text{Eqn. of ellipse is } \frac{x^2}{9} + \frac{y^2}{8} = 1$$

By $T = S_1$



$$\frac{hk}{9} + \frac{ky}{8} - 1 = \frac{h^2}{9} + \frac{k^2}{8} - 1$$

$$\Rightarrow 1 \frac{hk}{9} + \frac{ky}{8} = \frac{h^2}{9} + \frac{k^2}{8}$$

It is passing through P(1, 0)

$$\therefore \frac{h}{9} + 0 = \frac{h^2}{9} + \frac{k^2}{8}$$

$$\therefore \text{Req. locus is } \frac{h^2}{9} + \frac{y^2}{8} = \frac{x}{9}$$

10. A lift of a 10 floor building contains 9 persons and group of 4 and 5 leave the lift on different floor and there is no stoppage of lift at 1st and 2nd floor, then find number of ways this can be done.

- (1) 7056 (2) 7656
(3) 7066 (4) 7057

Ans. (1)

Sol. Form group of 4 and 5 person $\frac{9!}{4!5!} = 126$

choose two floor out of 8 floor ${}^8C_2 = 28$

Total ways in which 4 & 5 persons can leave at different floor $126 \times 28 \times 2 = 7056$

11. Let R be a relation such that

$$R = \{ \ell n(x+y) \leq 2, (x,y) \in \mathbb{N} \times \mathbb{N} \}.$$

Minimum no. of elements to be added in R so that it becomes transitive is _____

- (1) 10 (2) 15
(3) 14 (4) 16

Ans. (2)

Sol. $x + y \leq e^2$ ($e^2 \approx 7.29$)

$$R = \{(1, 1), (1,2), (1, 4), (1,5), (1,6)\}$$

$$(2, 1), (2,2), (2,3), (2,4), (2,5)$$

$$(3,1), (3,2), (3,3), (3,4)$$

$$(4,1), (4,2), (4,3)$$

$$(5,1), (5,2)$$

$$(6,1)$$

For R to be transitive we have to add

$$(6,2), (6,3), (6,4), (6,5), (6,6)$$

$$(5,3), (5,4), (5,5), (5,6)$$

$$(4,4), (4,5), (4,6)$$

$$(3,5), (3,6)$$

$$(2,3)$$

= 15 elements

12. The shortest distance between the lines

$$\frac{x-3}{-1} = \frac{y-2}{4} = \frac{z-1}{2} \quad \& \quad \frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{5}$$

$$(1) \sqrt{6} \quad (2) 6\sqrt{6}$$

$$(3) \sqrt{5} \quad (4) 5\sqrt{5}$$

Ans. (1)

Sol. $\vec{a} = 3i + 2j + k$ & $\vec{b} = i + j + 2k$

$$\vec{r}_1 = -\hat{i} + 4j + 2k$$

$$\vec{r}_2 = 2i + j + 5k$$

$$\vec{r}_1 \times \vec{r}_2 = \begin{vmatrix} i & j & k \\ -1 & 4 & 2 \\ 2 & 1 & 5 \end{vmatrix} = 18i + 9j - 9k$$

$$\Rightarrow |\vec{r}_1 \times \vec{r}_2| = 9\sqrt{6}$$

Now

$$[\vec{a} - \vec{b} \quad \vec{r}_1 \quad \vec{r}_2] = \begin{vmatrix} 2 & 1 & -1 \\ -1 & 4 & 2 \\ 2 & 1 & 5 \end{vmatrix} = 54$$

$$\text{shortest distance} = \frac{[\vec{a} - \vec{b} \quad \vec{r}_1 \quad \vec{r}_2]}{|\vec{r}_1 \times \vec{r}_2|} = \frac{54}{9\sqrt{6}} = \sqrt{6}$$

13. A bag contains 6 Red and 6 black balls. 6 pair of balls are selected one by one without replacement then the probability that each of the 6 pairs contains 1 red and 1 black ball.

$$(1) \frac{15}{231} \quad (2) \frac{14}{231}$$

$$(3) \frac{13}{231} \quad (4) \frac{16}{231}$$

Ans. (4)

$$\text{Sol. probability} = \frac{|6 \times 6|}{\binom{12}{2} \times 6} = \frac{16}{231}$$



14. The area (in square units) of the region

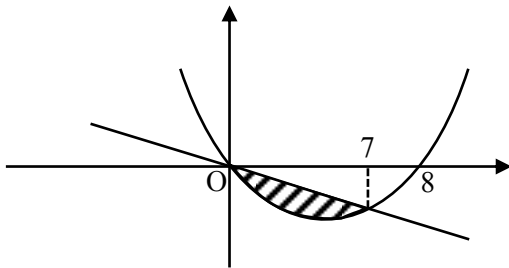
$$\{(x, y) : x^2 - 8x \leq y \leq -x\}, \text{ is}$$

- (1) $\frac{343}{6}$ (2) $\frac{343}{2}$
 (3) $\frac{1715}{6}$ (4) $\frac{340}{3}$

Ans. (1)

Sol. Required area = $\int_0^7 (7x - x^2) dx$

$$= \left[\frac{7x^2}{2} - \frac{x^3}{3} \right]_0^7 = \frac{343}{6}$$



15. If system of equations

$$x \cos 3\theta - 8y - 12z = 0$$

$$x \cos 2\theta + y + 3z = 0$$

$x + y + 3z = 0$ has non-trivial solution, then find sum of values of θ (where $\theta \in [0, 2\pi]$)

- (1) π (2) 2π (3) 3π (4) 4π

Ans. (3)

Sol. For Non-trivial solution

$$\begin{vmatrix} \cos 3\theta & -8 & -12 \\ \cos 2\theta & 1 & 3 \\ 1 & 1 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \cos 3\theta & -8 & 12 \\ \cos 2\theta & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

$$\cos 2\theta = 1$$

$$\theta = 0, \pi, 2\pi$$

\therefore sum of values of θ is 3π

16. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 3\hat{k}$, $\vec{b} = 6\hat{i} + 3\hat{j} + 3\hat{k}$. If $2\vec{a} + 3\vec{b}$ and

$\vec{a} - \vec{b}$ are two adjacent sides of a triangle then square of area of the triangle is

- (1) 1800 (2) 1600
 (3) 2000 (4) 2200

Ans. (1)

Sol. $2\vec{a} + 3\vec{b} = 2(2\hat{i} + 3\hat{j} + 3\hat{k}) + 3(6\hat{i} + 3\hat{j} + 3\hat{k})$

$$\Rightarrow 22\hat{i} + 15\hat{j} + 15\hat{k}$$

$$\vec{a} - \vec{b} = -4\hat{i}$$

$$\text{Area} = \frac{1}{2} |(2\vec{a} + 3\vec{b}) \times (\vec{a} - \vec{b})|$$

$$= \frac{1}{2} |-60\hat{j} + 60\hat{k}| = \frac{1}{2} \sqrt{(60)^2 \times 2}$$

$$A = \frac{60}{\sqrt{2}}$$

Square of area is $A^2 = 1800$

17. Consider the observations : 2, 4, α , β , 6, 12, 14. If their mean is 8 and variance = 16, then the quadratic equation whose roots are $3\alpha + 2$ & $2\beta + 1$, is

- (1) $x^2 - 49x + 544 = 0$ (2) $x^2 - 49x - 544 = 0$
 (3) $x^2 - 23x - 512 = 0$ (4) $x^2 + 23x - 512 = 0$

Ans. (1)

Sol. $S = 2 + 4 + \alpha + \beta + 6 + 12 + 14$

$$= \alpha + 38 + \beta$$

$$\text{Mean} = \frac{\alpha + \beta + 38}{7} = 8$$

$$\alpha + \beta = 18$$

$$\text{Variance} = \sum \frac{x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2$$

$$\Rightarrow \frac{1}{7}(4 + 16 + \alpha^2 + \beta^2 + 36 + 144 + 196) - 64 = 16$$

$$\Rightarrow \frac{1}{7}(\alpha^2 + \beta^2 + 396) = 80$$

$$\alpha^2 + \beta^2 + 396 = 560$$

$$\alpha^2 + \beta^2 = 164$$

$$(\alpha + \beta)^2 - 2\alpha\beta = 164$$

$$324 - 164 = 2\alpha\beta \Rightarrow \alpha\beta = 80$$

$$\alpha = 10, \beta = 8$$

$$\text{Sum} = (3\alpha + 2) + (2\beta + 1) = 32 + 17 = 49$$

$$\text{Product} = (3\alpha + 2)(2\beta + 1) = 32 \times 17 = 544$$

$$\text{Required quadratic equation is } x^2 - 49x + 544 = 0$$

18. Let a hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & ellipse be

$$\frac{x^2}{9} + \frac{y^2}{8} = 1. \text{ If length of latus rectum of hyperbola}$$

is equal to minor axis of ellipse & eccentricity of hyperbola is equal to semi-major axis of ellipse, then $2ae$ is equal to

(where 'e' is the eccentricity of hyperbola)

- (1) $3\sqrt{2}$ (2) $\frac{3\sqrt{2}}{2}$ (3) $2\sqrt{2}$ (4) $\frac{\sqrt{2}}{3}$

Ans. (2)

Sol. $\frac{2b^2}{a} = 4\sqrt{2} = b^2 = 2\sqrt{2}a$

$$e_H = \sqrt{1 + \frac{b^2}{a^2}} = 3 \Rightarrow 1 + \frac{b^2}{a^2} = 9 \Rightarrow b^2 = 8a^2$$

$$a = \frac{\sqrt{2}}{4}, b^2 = 1 \Rightarrow b = 1$$

$$\therefore 2ae = 2 \times \frac{\sqrt{2}}{4} \times 3 = \frac{3\sqrt{2}}{2}$$

19. Let $f(x)$ be a function satisfying the functional rule $f(xy) = f(x) f(y)$, $f(0) \neq 0$.

If $x^2 g(x) = \int_1^x (t^2 f(t) + t g(t)) dt$, then $g(2)$ is equal to

- (1) $\frac{15}{32}$ (2) $\frac{3}{4}$ (3) $\frac{4}{3}$ (4) $\frac{32}{15}$

Ans. (2)

Sol. $\therefore f(xy) = f(x) f(y)$

$$\therefore f(x) = x^n \text{ or } f(x) = k$$

$$\text{But } f(0) \neq 0 \Rightarrow f(x) \neq x^n$$

$$\text{For } f(x) = k \Rightarrow k = k.k \Rightarrow k = 1 (\because f(0) \neq 0)$$

$$f(x) = 1$$

$$\text{Now, } x^2 \cdot g(x) = \int_1^x (t^2 \cdot 1 + t \cdot g(t)) dt$$

Differentiating both the sides

$$x^2 \cdot g'(x) + g(x) \cdot 2x = x^2 + xg(x)$$

$$x^2 g'(x) + xg(x) = x^2$$

$$xg'(x) + g(x) = x$$

$$x \cdot g(x) = \frac{x^2}{2} + c$$

$$\therefore g(1) = 0 \Rightarrow c = -\frac{1}{2}$$

$$\Rightarrow x \cdot g(x) = \frac{x^2 - 1}{2}$$

$$\Rightarrow g(2) = \frac{3}{4}$$

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SECTION-B

1. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$. If $B = [b_{ij}]_{3 \times 3}$ and $B = A^{99} - I$,

then find $\frac{b_{31} - b_{21}}{b_{32}}$.

Ans. (149)

Sol. $A = P + I$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A = P + I$ where $P^3 = 0$

$$A^{99} = I + 99P + {}^{99}C_2 P^2$$

$$A^{99} - I = 99 \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + {}^{99}C_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 99 \times 3 & 0 & 0 \\ 44550 & 99 \times 3 & 0 \end{bmatrix}$$

$$b_{31} = 44550, \quad b_{21} = 99 \times 3, \quad b_{32} = 99 \times 3$$

$$\frac{44550 - 297}{297} = 149$$

2. If $(1 - x^3)^{10} = \sum_{r=0}^{10} a_r x^r \cdot (1 - x)^{30-2r}$, then find $\frac{9a_9}{a_{10}}$.

Ans. (30)

Sol. $(1 - x)^3 = 1 - x^3 - 3x(1 - x)$

$$(1 - x)^3 = 1 - x^3 - 3x + 3x^2$$

$$(1 - x)^3 + 3x - 3x^2 = 1 - x^3$$

$$(1 - x^3)^{10} = [(1 - x)^3 + 3x(1 - x)]^{10}$$

$$\Rightarrow {}^{10}C_r (3x(1 - x))^r \cdot (1 - x)^{30-2r}$$

$$(1 - x^3)^{10} = {}^{10}C_r \cdot 3^r \cdot x^r \cdot (1 - x)^{30-2r}$$

$$\therefore a_r = {}^{10}C_r \cdot 3^r$$

$$\frac{a_9}{a_{10}} = \frac{{}^{10}C_9 \cdot 3^9}{{}^{10}C_{10} \cdot 3^{10}}$$

$$\frac{a_9}{a_{10}} = \frac{10}{3}$$

$$\therefore \frac{9a_9}{a_{10}} = 30$$

3. If α & β are the roots of the equation $z^2 - \sqrt{6}i z - 3 = 0$, then find $\alpha^8 + \beta^8$.

Ans. (162)

$$\text{Sol. } z = \frac{\sqrt{6}i \pm \sqrt{-6 + 12}}{2}$$

$$= \frac{\sqrt{6}i \pm \sqrt{6}}{2}$$

$$= \frac{\sqrt{6}}{2} (\pm 1 + i)$$

$$\alpha = \frac{\sqrt{6}}{2} (1 + i), \beta = \frac{-\sqrt{6}}{2} (1 - i)$$

$$\alpha = \frac{\sqrt{6}}{2} \times \sqrt{2} e^{i\pi/4}, \beta = -\frac{\sqrt{6}}{2} \times \sqrt{2} e^{-i\pi/4}$$

$$\alpha = \sqrt{3} e^{i\pi/4}, \beta = -\sqrt{3} e^{-i\pi/4}$$

$$\alpha^8 + \beta^8 = 81 \cdot e^{i2\pi} + 81 \cdot e^{i(-2\pi)}$$

$$= 81 (1 + 1) = 162$$

4. The line $x - y = 4$ is a chord of the circle $(x - 4)^2 + (y + 3)^2 = 9$, which cuts the circle at points Q & R. If P(α , β) lies on the circle such that PQ = PR, then find $(6\alpha + 8\beta)^2$.

Ans. (18)

Sol. Solving $x - y = 4$ & $(x - 4)^2 + (y + 3)^2 = 9$

We get Q(4,0) & R(1,-3)

$$PQ^2 = PR^2$$

$$\Rightarrow (\alpha - 4)^2 + \beta^2 = (\alpha - 1)^2 + (\beta + 3)^2$$

$$\Rightarrow \alpha + \beta = 1 \quad \dots(1)$$

$$\text{Also } (\alpha - 4)^2 + (\beta + 3)^2 = 9 \quad \dots(2)$$

Solving (1) & (2), we get

$$\alpha = 4 + \frac{3}{\sqrt{2}}, \beta = -3 - \frac{3}{\sqrt{2}}$$



and $\alpha = 4 - \frac{3}{\sqrt{2}}, \beta = -3 + \frac{3}{\sqrt{2}}$

For both pairs we get $(6\alpha + 8\beta)^2 = 18$

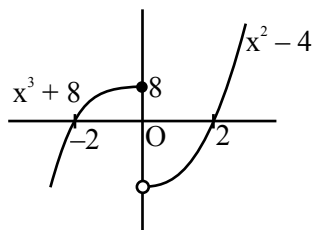
5. Let $f(x) = \begin{cases} x^3 + 8 & x < 0 \\ x^2 - 4 & x \geq 0 \end{cases}$

and $g(x) = \begin{cases} (x-8)^{1/3} & x < 0 \\ (x+4)^{1/2} & x \geq 0 \end{cases}$

then find number of points of discontinuity of $g(f(x))$.

Ans. (3)

Sol. $g(f(x)) = \begin{cases} (f(x)-8)^{1/3} & f(x) < 0 \\ (f(x)+4)^{1/2} & f(x) \geq 0 \end{cases}$



$$g(f(x)) = \begin{cases} (x^3)^{1/3}, & x < -2 \\ (x^2 - 12)^{1/3}, & 0 < x < 2 \\ (x^3 + 12)^{1/2}, & -2 \leq x \leq 0 \\ (x^2)^{1/2}, & x \geq 2 \end{cases}$$

$$g(f(x)) = \begin{cases} x, & x < -2 \\ (x^3 + 12)^{1/2}, & -2 \leq x \leq 0 \\ (x^2 - 12)^{1/3}, & 0 < x < 2 \\ x, & x \geq 2 \end{cases}$$

Number of points of discontinuity is equal to 3.

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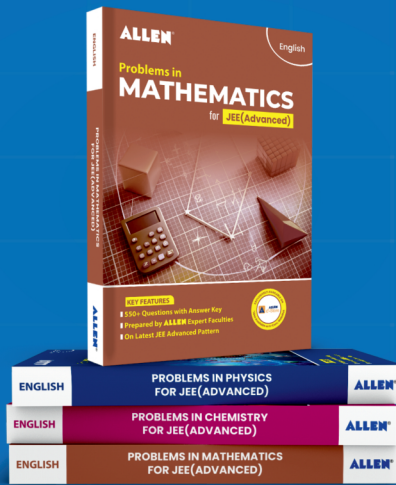
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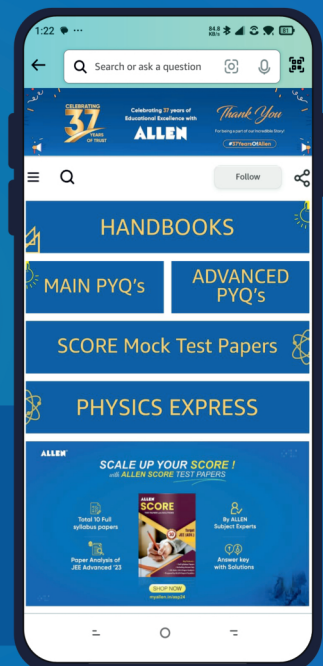
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