CAT QUANTITATIVE APTITUDE STUDENT REFERENCE

CAT QUANTITATIVE APTITUDE | ALGEBRA

COMPLETE STUDENT GUIDE

EXAM	CAT (Common Admission Test)	**SECTION**	Quantitative Aptitude
TOPIC WEIGHTAGE	5–7 direct questions per paper (~15–21 marks); structurally impacts ~30% of the entire QA section (Coordinate Geometry, Functions, modern math setups)		
GUIDE LENGTH	8 Sections — Formulas + Solved Examples + Practice	**BEST USED AS**	Reference during study + quick revision

TREND HISTORICAL QUESTION TRENDS

CAT YEAR	ALGEBRA QUESTIONS	MARKS	DIFFICULTY
CAT 2021	5	15	Moderate–Hard
CAT 2022	6	18	Hard
CAT 2023	7	21	Moderate–Hard
CAT 2024	6	18	Moderate–Hard
CAT 2025	6	18	Hard

1 INTRODUCTION & PURPOSE

Algebra is the foundational structural backbone of the CAT Quantitative Aptitude section. Unlike arithmetic, which tests intuitive logic and scenario scaling, Algebra tests structural manipulation, boundary conditions, and systemic constraints. CAT rarely tests pure mechanical expansions; instead, it embeds algebraic structures inside hidden constraint problems — demanding integer roots, using absolute values to limit boundaries, or masking progressions within sequence equations. Master this chapter to unlock high-scoring systemic shortcuts.

ROADMAP HOW TO USE THIS GUIDE

- Read **Section 2 & 3** first to master algebraic notation, core definitions, and identity rules.
- Study **Section 4** topic-by-topic to map specific equation models directly to their solved proofs.
- Internalize **Section 5** to avoid walking into algebraic constraint traps and range errors.
- Drill **Section 6** to build raw factorization, substitution, and graph-scaling speed.
- Review **Section 7** once you start attempting mocks to isolate common operational blindspots.
- Use **Section 8** as your final 10-minute abstract formula summary sheet before the exam.

2 CORE CONCEPTS

TERM	FULL FORM / MEANING	DEFINITION & EXAMPLE
Expression	A collection of terms	A mathematical phrase combining variables, coefficients, and operators without an equivalence relation. Example: $3x^2 - 5x + 2$.
Equation	An equivalence statement	A mathematical statement asserting that two expressions are equal. Example: $3x^2 - 5x + 2 = 0$.
Identity	An absolute equivalence	An equation structurally true for every possible value of its variables. Example: $(a+b)^2 = a^2 + 2ab + b^2$.
Degree	Highest variable exponent	The highest power of the variable present within a polynomial. Example: the degree of $x^3 - x + 1$ is 3.
Root / Zero	Solution value	The value(s) of a variable that satisfy an equation or make a polynomial equal zero. Example: $x = 2$ is a root of $x - 2 = 0$.
Discriminant	Nature-of-roots indicator	The algebraic term $\Delta = b^2 - 4ac$ within a quadratic that determines the structural nature of its roots.
Constraint	Boundary condition	External limiting conditions placed on a variable (e.g. x must be a positive integer, $y \neq 0$).

KEY INSIGHT THE SEQUENTIAL MECHANIC

$$\text{Algebraic Expression} \rightarrow \text{Apply Equations / Equivalences} \rightarrow \text{Locate Roots within Specified Constraints}$$

FOUNDATIONAL AXIOM

In every advanced CAT algebra problem, the algebraic structure is **secondary** to the specified variable domain constraint (Real, Integer, Positive). The single biggest source of error is solving an equation correctly but reporting a root that violates the hidden boundary or integer constraint.

ASK THE GOLDEN QUESTION

“What domain restrictions apply to this variable right now?”

3 FORMULA SHEET

3A — BASIC ALGEBRAIC IDENTITIES

- Linear Expansion: $(a \pm b)^2 = a^2 + b^2 \pm 2ab$
- Difference of Squares: $a^2 - b^2 = (a - b)(a + b)$
- Cubic Expansions: $(a \pm b)^3 = a^3 \pm b^3 \pm 3ab(a \pm b)$
- Sum & Difference of Cubes: $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$
- Symmetric Trio Identity: $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

SPECIAL BOUNDARY CONDITION

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

3B — QUADRATIC & HIGHER-DEGREE EQUATIONS

For $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

DISCRIMINANT CONDITION	NATURE OF ROOTS
$\Delta > 0$, **perfect square**	Real, rational, distinct
$\Delta = 0$	Real and equal ($x = -b/2a$)
$\Delta < 0$	Complex / imaginary conjugates

VIETA'S RELATION	VALUE
Quadratic: $\alpha + \beta$	$-b/a$
Quadratic: $\alpha\beta$	c/a
Cubic: $\alpha + \beta + \gamma$	$-b/a$
Cubic: $\alpha\beta + \beta\gamma + \gamma\alpha$	c/a
Cubic: $\alpha\beta\gamma$	$-d/a$

3C — INEQUATIONS, PROGRESSIONS & LOGARITHMS

- **AM-GM-HM Inequality** (positive reals): $AM \geq GM \geq HM$, i.e. $\frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2}{\frac{1}{a} + \frac{1}{b}}$
- **Arithmetic Progression (AP):** $T_n = a + (n-1)d$; $S_n = \frac{n}{2}[2a + (n-1)d]$
- **Geometric Progression (GP):** $T_n = ar^{n-1}$; $S_n = \frac{a(r^n - 1)}{r - 1}$; $S_\infty = \frac{a}{1 - r}$ (where $|r| < 1$)
- **Logarithmic Fundamentals:** $\log_a(xy) = \log_a x + \log_a y$; $\log_a(x/y) = \log_a x - \log_a y$; $\log_a b = \frac{\log_c b}{\log_c a}$ (Base Change Rule)

DERIVE-ON-THE-SPOT TRICKS

- Stuck on a complex polynomial expansion? Substitute small values like $a = 1, b = 1, c = 0$ directly into the abstract identity to verify symmetry on the fly.
- Forgot the max/min formula for a quadratic? Differentiate $ax^2 + bx + c$, set to zero ($2ax + b = 0 \Rightarrow x = -b/2a$), and plug back in.
- Cannot remember log base operations? Convert instantly back to exponential form ($y = \log_a x \Rightarrow a^y = x$) to check correctness.

4 TOPIC-WISE CONCEPT SUMMARIES & SOLVED EXAMPLES

4A — LINEAR & SIMULTANEOUS EQUATIONS

- Ensure the number of independent equations matches the number of variables to determine a unique solution state.
- Watch for infinite-solution frameworks (dependent equations) and zero-solution frameworks (parallel constraints).

CAT TIP

When CAT presents three variables but only two equations, the question is not unsolvable — it is a hidden Diophantine equation or an optimization problem anchored to integer constraints.

SOLVED EXAMPLE 4A

Q: If $3x + 4y + 5z = 120$ and $x + y + z = 30$, find $2x + y$.

→ Two equations: (1) $3x + 4y + 5z = 120$, (2) $x + y + z = 30$

→ Multiply (2) by 5: (3) $5x + 5y + 5z = 150$

→ Subtract (1) from (3): $(5x + 5y + 5z) - (3x + 4y + 5z) = 150 - 120$

→ $2x + y = 30$

Answer: 30

4B — QUADRATIC EQUATIONS & ROOT NATURE

- Anchor your focus on the discriminant (Δ) to reveal root properties without plotting coordinates.
- Maximize use of symmetric expressions ($\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$) to save computation time.

CAT TIP

If a quadratic has rational coefficients and one root is $2 + \sqrt{3}$, instantly realize the other root must be its irrational conjugate, $2 - \sqrt{3}$.

SOLVED EXAMPLE 4B

Q: Find k for which $x^2 - 2kx + (k + 2) = 0$ has equal roots.

→ For equal roots, $\Delta = b^2 - 4ac = 0$

→ Here $a = 1$, $b = -2k$, $c = k + 2$: $(-2k)^2 - 4(1)(k + 2) = 0$

→ $4k^2 - 4k - 8 = 0 \Rightarrow k^2 - k - 2 = 0$

→ Factorize: $(k - 2)(k + 1) = 0 \Rightarrow k = 2$ or $k = -1$

Answer: k = 2, -1

4C — HIGHER-DEGREE POLYNOMIALS & FACTOR THEOREMS

- If $P(a) = 0$, then $(x - a)$ is structurally a factor of $P(x)$.
- Always extract sum, product, and pairwise sum of roots via Vieta's formulas before running division.

CAT TIP

High-degree polynomials in CAT (degree 4, 5+) are almost always symmetric or palindromic. Group terms with matching coefficients from the outside inward to simplify quickly.

SOLVED EXAMPLE 4C

Q: If $x^3 - 6x^2 + 11x - 6 = 0$ has roots α, β, γ, find $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.

→ Common denominator: $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

→ Vieta's: $\alpha\beta + \beta\gamma + \gamma\alpha = c/a = 11$; $\alpha\beta\gamma = -d/a = 6$

→ Substitute: $11/6$

Answer: 11/6

4D — INEQUALITIES & ABSOLUTE VALUES (MODULUS)

- $|x - a| \leq b$ maps geometrically to a zone where the distance from a does not exceed b: $a - b \leq x \leq a + b$.
- Squaring an inequality is dangerous unless both sides are confirmed positive.

CAT TIP

For nested modulus inequality chains, find the critical points where each mod block hits zero, then evaluate behavior across the resulting intervals.

SOLVED EXAMPLE 4D

Q: Solve for real x: $|2x - 5| \leq 7$.

→ Open the modulus: $-7 \leq 2x - 5 \leq 7$

→ Add 5 across the chain: $-2 \leq 2x \leq 12$

→ Divide by 2: $-1 \leq x \leq 6$

Answer: [-1, 6]

4E — PROGRESSIONS, SEQUENCES & SERIES

- Look for telescoping terms in non-standard summation problems so middle terms cancel.
- Use AM–GM whenever optimizing a product or sum of positive variables.

CAT TIP

If a sequence doesn't fit standard AP/GP, compute the first 4 terms explicitly — it will almost always reveal a cyclical loop or clean recursive pattern.

SOLVED EXAMPLE 4E

Q: Find the minimum value of $4x + \frac{9}{x}$ for positive real x.

→ Apply AM–GM: $\frac{4x + \frac{9}{x}}{2} \geq \sqrt{4x \cdot \frac{9}{x}}$

→ Simplify inside the root: $\frac{4x + \frac{9}{x}}{2} \geq \sqrt{36} = 6$

→ $4x + \frac{9}{x} \geq 12$

Answer: 12

4F — FUNCTIONS & DOMAIN/RANGE MAPPING

- The domain is the set of all valid inputs — values under square roots must stay non-negative, denominators must never hit zero.
- Composite functions require tracking how the range of the inner function feeds the domain of the outer function.

CAT TIP

If $f(x) \cdot f(y) = f(x + y)$, assume a standard exponential base structure $f(x) = k^x$ to solve instantly.

SOLVED EXAMPLE 4F

Q: Find the domain of $f(x) = \frac{1}{\sqrt{x^2 - 9}}$.

→ The expression under the root must be strictly positive (it sits in the denominator): $x^2 - 9 > 0$

→ Factor: $(x - 3)(x + 3) > 0$

→ Wavy curve method: $x > 3$ or $x < -3$

Answer: (-∞, -3) ∪ (3, ∞)

4G — LOGARITHMS & EXPONENTIAL EQUATIONS

- Logarithmic bases must always satisfy $a > 0$ and $a \neq 1$.
- Convert log terms with different bases into a single uniform base before combining.

CAT TIP

Look for hidden quadratic setups where k^x is substituted as a placeholder t, turning an intimidating exponential equation into a simple quadratic.

SOLVED EXAMPLE 4G

Q: Solve for x: $\log_2 x + \log_2(x - 2) = 3$.

→ Combine via product rule: $\log_2(x(x-2)) = 3$

→ Convert to exponential form: $x(x-2) = 2^3 \Rightarrow x^2 - 2x - 8 = 0$

→ Factorize: $(x-4)(x+2) = 0 \Rightarrow x = 4$ or $x = -2$

→ Check domain: logs cannot process negative inputs, so $x = -2$ is invalid.

Answer: x = 4

5 CAT TRAP IDENTIFIER

TRAP CATEGORY	WRONG APPROACH (INSTINCTIVE MISTAKE)	RIGHT APPROACH (CORRECT MECHANICS)
Negative Log Inputs	Keeping negative roots solved from quadratic log equations	Verify roots against original log domain constraints ($x > 0$)
Inequality Flipping	Multiplying an inequality chain by an unverified variable expression	Never cross-multiply by variables unless their sign is proven
Constraint Violations	Tracking fractional roots when constraints demand integer states	Check for integer constraints before listing total solution sets
Asymmetric Modulus	Splitting mod equations into basic positive terms while ignoring negative states	Map absolute boundaries to double-sided intervals ($\pm$)
Extraneous Roots	Squaring both sides of an equation blindly and keeping all answers	Substitute final answers back into the original equation to catch invalid results

PRE-ATTEMPT MENTAL CHECKLIST

- What specific domain constraint limits the variables in this problem?
- Am I structurally allowed to cross-multiply here, or is the sign of the denominator unknown?
- Does this non-standard sequence loop back or form a telescoping pattern if I write out the first few terms?
- Are the logarithmic bases perfectly identical before I attempt to combine these terms?
- Did I plug my final solution values back into the original constraints to check for extraneous roots?

6 SPEED TECHNIQUES & SHORTCUTS

ALGEBRAIC IDENTITY SHORTCUTS

SCENARIO	EQUIVALENT MULTIPLIER STRUCTURE / FORM
If $x + \frac{1}{x} = k$	Then $x^2 + \frac{1}{x^2} = k^2 - 2$
If $x + \frac{1}{x} = k$	Then $x^3 + \frac{1}{x^3} = k^3 - 3k$
Symmetric Quadratic Root Form ($ax^2 + bx + a = 0$**)**	Roots are reciprocals $(\alpha, \frac{1}{\alpha})$
Linear Equation Matrix with Equal Ratios	Infinite solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

WHEN TO USE: ALGEBRAIC SUBSTITUTION VS. ALGEBRAIC FACTORING

USE SUBSTITUTION WHEN...	USE FACTORING WHEN...
The problem is framed entirely with abstract variables, and answer choices are variables too	The problem demands a count of unique integer roots or precise Diophantine combinations
You need to quickly eliminate options by plugging in simple test numbers like 0, 1, -1	The equations match standard forms (difference of squares, symmetric cubic structures)
You want to find boundary conditions or sign changes of an inequality expression	The question asks for exact analytical proofs or minimum/maximum values

APPROXIMATION & ELIMINATION FRAMEWORK

- Substitute small numbers into abstract identities to instantly eliminate options that fail basic numeric checks.
- Test $x = 0$ to collapse a long, intimidating polynomial into its constant term — eliminate options instantly.
- Use the AM-GM limit to establish quick maximum/minimum boundaries, letting you drop options outside those ranges.

7 COMMON MISTAKES TO AVOID

Mistake 1 — Squaring Inequalities Blindly

WRONG: Assuming that if $x > y$, then $x^2 > y^2$ always holds (e.g. $2 > -5$ is true, but $4 > 25$ is false).

CORRECT: Only square an inequality if both sides are confirmed positive real numbers, or split into separate positive/negative interval cases.

Mistake 2 — Forgetting the Logarithmic Base Condition

WRONG: Solving $\log_{(x-2)} 16 = 2$ and keeping $x = 2$ or $x = 1$ as valid.

CORRECT: The base must be strictly positive and $\neq 1$. Since $x = 2$ makes the base zero and $x = 3$ makes it 1, both must be discarded.

Mistake 3 — Missing the Negative Conjugate in Roots

WRONG: Assuming that if a cubic has root $3 + \sqrt{2}$, the other roots are unrelated real numbers.

CORRECT: Irrational and complex roots always appear in conjugate pairs for polynomials with rational coefficients — if $3 + \sqrt{2}$ is a root, $3 - \sqrt{2}$ must also be a root.

Mistake 4 — Treating Modulus Openings as Single-Sided

WRONG: Simplifying $\sqrt{x^2}$ to just x for unconstrained real ranges.

CORRECT: Always expand $\sqrt{x^2}$ to its proper modulus form $|x|$, tracking both positive and negative solution states.

Mistake 5 — Blindly Relying on Formulas for Infinite Series

WRONG: Applying $S_\infty = \frac{a}{1-r}$ to a series without checking the common ratio.

CORRECT: Verify $|r| < 1$ before using the infinite summation formula.

8 QUICK REVISION CARD

MASTER FORMULA CHECKLIST

- Symmetric Trio Identity: $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
- Quadratic Root Form: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Vieta's Quadratic: $\alpha + \beta = -b/a \mid \alpha\beta = c/a$
- Vieta's Cubic: $\sum \alpha = -b/a \mid \sum \alpha\beta = c/a \mid \alpha\beta\gamma = -d/a$
- Log Base Change Rule: $\log_a b = \dfrac{\log_c b}{\log_c a}$
- AM-GM Bound: $\dfrac{a + b}{2} \geq \sqrt{ab}$

CRITICAL RATIOS & VALUES TO REMEMBER

CONDITION	RESULT
$x + \frac{1}{x} = 2$	$x = 1$
$x + \frac{1}{x} = -2$	$x = -1$
$x + \frac{1}{x} = 3$	$x^2 + \frac{1}{x^2} = 7$; $x^3 + \frac{1}{x^3} = 18$
$\Delta = 0$	Equal Roots
$\Delta < 0$	Imaginary Roots

THE TOP 5 ALGEBRA TRAPS

- **T1:** Log domain rules — always double-check that inputs and bases stay positive.
- **T2:** Variable inequality changes — never cross-multiply by variables with unknown signs.
- **T3:** Hidden boundaries — watch for integer constraints that limit solution sets.
- **T4:** Extraneous roots — always test answers back in the original equation after squaring.
- **T5:** Single-sided mod expansions — remember both positive and negative cases.

PRE-SOLVE CHECKLIST

- ☐ What domain constraint applies to the variable?
- ☐ Is the sign of the denominator known before cross-multiplying?
- ☐ Do log inputs and bases satisfy all core domain constraints?
- ☐ Are irrational and complex roots properly paired as conjugates?
- ☐ Can I substitute small values to quickly check the algebraic options?

SHORTCUT / TIMING TARGETS

QUESTION TYPE	TARGET TIME
Direct questions	Under 45–60 sec
Complex / Nested Function sets	Under 75–90 sec
Accuracy goal	**90%+**