CAT · QUANTITATIVE APTITUDE · PRACTICE SERIES

Number Systems

A 20-Question Practice Paper with Detailed Solutions & Shortcuts

SECTION	Quantitative Ability — Number Systems
QUESTION TYPES	Multiple Choice (Single Correct)
DIFFICULTY MIX	Easy · Medium · Hard
INCLUDES	Answer Key, Step-by-Step Solutions, CAT Shortcuts

20	5	10	5
QUESTIONS	EASY	MEDIUM	HARD

DIFFICULTY LEGEND

Easy · Medium · Hard

Topics: Factors & Multiples · Remainders · Divisibility · Cyclicity · HCF-LCM · Base Systems · Modular Arithmetic

SECTION — NUMBER SYSTEMS

1. EASY *Factors & Multiples / Perfect Squares*

What is the smallest positive integer that has exactly 15 factors?

A) 1296 **B)** 324
C) 144 **D)** 400

2. EASY *Remainders*

What is the remainder when 7^{84} is divided by 342?

A) 1 **B)** 7
C) 49 **D)** 341

3. EASY *Divisibility Rules*

A 6-digit number is formed by repeating a 3-digit block twice (for example, 456456). Which of the following numbers will this 6-digit number always be divisible by?

A) 7 only **B)** 11 only
C) 13 only **D)** 1001

4. EASY *Cyclicity*

What is the units digit of the expression $3^{1024} + 7^{1025}$?

A) 0 **B)** 4
C) 6 **D)** 8

5. EASY *Prime Numbers*

How many prime numbers p exist such that $p^2 + 8$ is also a prime number?

A) 0 **B)** 1
C) 2 **D)** Infinitely many

6. MEDIUM *Number of Factors & Digit Properties*

In a 3-digit number N, the digits are non-zero and distinct such that none of the digits is a perfect square, and exactly one of the digits is a prime number. What is the number of factors of the maximum possible value of N?

A) 8 **B)** 10
C) 12 **D)** 16

7. MEDIUM *Base Systems*

A 3-digit number in base 7 is numerically equal to the reverse of the same number in base 9. What is the decimal (base 10) value of this number?

A) 146 **B)** 248
C) 305 **D)** 503

8. MEDIUM *Factorials & Remainders*

What is the remainder when 40! is divided by 43?

A) 1 **B)** 21
C) 22 **D)** 42

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9. MEDIUM *HCF & LCM*

The LCM of two positive integers is 1260, and their HCF is 14. How many such unordered pairs of integers are possible?

A) 2 **B)** 3
C) 4 **D)** 8

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10. MEDIUM *Modular Arithmetic*

What is the remainder when 13^{100} is divided by 15?

A) 1 **B)** 2
C) 4 **D)** 13

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11. MEDIUM *Digit Properties & Divisibility*

For a 4-digit number, the sum of its digits in the thousands, hundreds, and tens places is 14, and the sum of its digits in the hundreds, tens, and units places is 15. If the tens place digit is 4 more than the units place digit, and the entire number is divisible by 9, what is the number?

A) 2484 **B)** 3384
C) 3834 **D)** 4284

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12. MEDIUM *Sum of Factors*

What is the sum of the reciprocals of all the factors of 360?

A) 2.75 **B)** 3.25
C) 3.50 **D)** 3.75

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13. MEDIUM *Factorials*

How many trailing zeros are there at the end of the product $1^1 \times 2^2 \times 3^3 \times \ldots \times 50^{50}$?

A) 250 **B)** 325
C) 350 **D)** 375

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14. MEDIUM *Base Systems & Factorials*

What is the number of trailing zeros of 100! when it is written in base 12?

A) 24 **B)** 33
C) 48 **D)** 97

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15. MEDIUM *Last Two Digits*

Find the last two digits of 7^{2026}.

A) 01 **B)** 07

C) 43 **D)** 49

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16. HARD *Combinatorics & Number of Factors*

How many factors of the number $N = 2^5 \times 3^4 \times 5^3 \times 7^2$ are of the form $4k + 1$, where k is a non-negative integer?

A) 24 **B)** 32

C) 36 **D)** 60

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17. HARD *Remainders*

What is the remainder when 3^{1000} is divided by 143?

A) 1 **B)** 12

C) 133 **D)** 142

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18. HARD *Modular Arithmetic & Perfect Squares*

Let S be the set of all positive integers n such that $n^2 + 7n + 12$ is a multiple of 17. What is the sum of all two-digit numbers in set S?

A) 655 **B)** 672

C) 689 **D)** 690

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19. HARD *HCF & LCM Pairs*

Let $N = 2^{10} \times 3^6 \times 5^4$. How many ordered pairs (x, y) of positive integers exist such that their LCM is equal to N?

A) 1228 **B)** 2457

C) 2730 **D)** 4914

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20. HARD *Factorial Series*

Find the remainder when the sum of the series $1 \times 1! + 2 \times 2! + 3 \times 3! + \ldots + 50 \times 50!$ is divided by 51.

A) 0 **B)** 1

C) 49 **D)** 50

ANSWER KEY

Q1 **C**	Q2 **A**	Q3 **D**	Q4 **D**	Q5 **B**
Q6 **C**	Q7 **B**	Q8 **B**	Q9 **C**	Q10 **A**
Q11 **B**	Q12 **B**	Q13 **C**	Q14 **C**	Q15 **D**
Q16 **B**	Q17 **C**	Q18 **B**	Q19 **B**	Q20 **D**

DETAILED SOLUTIONS

Q1. Correct Answer: **C (144)**

The formula for the number of factors of a number expressed as $p^a \times q^b$ is $(a+1)(b+1)$. For a number to have exactly 15 factors, its prime factorization structure can either be p^{14} or $p^4 \times q^2$.

To minimize the value of the integer, we assign the smallest prime bases to the largest exponents:

$$Case\ 1: 2^{14} = 16384$$

$$Case\ 2: 2^4 \times 3^2 = 16 \times 9 = 144$$

Comparing the values, 144 is the smallest possible positive integer.

> **Axiom:** Any integer that possesses an odd number of unique factors must be a perfect square.

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Since 15 is odd, the target integer must be a perfect square. Testing the smallest squares given in the options: $144 = 12^2 = 2^4 \times 3^2$. The number of factors is $(4+1)(2+1) = 15$. This confirms 144 is correct.

Q2. Correct Answer: **A (1)**

We need to find $7^{84} \pmod{342}$. Notice that $7^3 = 343$, which is close to the divisor 342. Rewrite the expression using the rules of exponents:

$$7^{84} = (7^3)^{28} = (343)^{28}$$

Applying modular arithmetic:

$$343 \equiv 1 \ (mod\ 342)$$

$$(343)^{28} \equiv 1^{28} \equiv 1 \ (mod\ 342)$$

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Recognize nearby power relationships. Since $7^3 = 342 + 1$, the binomial expansion of $(342 + 1)^{28}$ divided by 342 leaves a final remainder of $1^{28} = 1$.

Q3. Correct Answer: **D (1001)**

Let the repeated 3-digit block be represented by digits A, B, C. The resulting 6-digit number takes the form ABCABC. Using base-10 place value expansion:

$$ABCABC = A\times100000 + B\times10000 + C\times1000 + A\times100 + B\times10 + C\times1$$

$$ABCABC = 1000 \times (100A + 10B + C) + 1 \times (100A + 10B + C)$$

$$ABCABC = (100A + 10B + C) \times (1000 + 1) = ABC \times 1001$$

Since $1001 = 7 \times 11 \times 13$, any number of this form is always divisible by 7, 11, 13, and 1001. Option D gives the most complete factor encompassing the structural divisibility.

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Any block repetition structure of type XYZXYZ factors directly into XYZ × 1001. Hence it is always a multiple of 1001.

Q4. Correct Answer: **D (8)**

The units digit of base 3 operates on a cycle of 4: [3, 9, 7, 1]. The exponent 1024 is a perfect multiple of 4 (1024 mod 4 = 0), so 3^{1024} has the same unit digit as 3^4, which is 1.

The units digit of base 7 also operates on a cycle of 4: [7, 9, 3, 1]. The exponent 1025 leaves a remainder of 1 when divided by 4, so 7^{1025} has the same unit digit as 7^1, which is 7.

Summing the units digits: 1 + 7 = 8.

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1024 mod 4 = 0 $\Longrightarrow$ units digit = 1.   1025 mod 4 = 1 $\Longrightarrow$ units digit = 7.   Total = 1 + 7 = 8.

Q5. Correct Answer: **B (1)**

Test the properties of prime p under modulo 3:

$$\textit{If } p = 3\textit{: } p^2 + 8 = 9 + 8 = 17\textit{, which is prime.}$$

If p ≠ 3, since p is prime it cannot be a multiple of 3, so p ≡ 1 or 2 (mod 3). In both cases:

$$p^2 \equiv 1 \pmod 3$$

$$p^2 + 8 \equiv 1 + 8 = 9 \equiv 0 \pmod 3$$

For any prime p > 3, the expression $p^2 + 8$ is a multiple of 3 greater than 3, hence composite. Only 1 such prime exists (p = 3).

Q6. Correct Answer: **C (12)**

The available non-zero single digits are {1, 2, 3, 4, 5, 6, 7, 8, 9}.

1. Eliminate all perfect squares {1, 4, 9}. Remaining valid set: {2, 3, 5, 6, 7, 8}.

2. Exactly one digit must be prime. Primes available: {2, 3, 5, 7}. Non-primes: {6, 8}.

3. To satisfy the condition, choose exactly one digit from the prime group and both digits from the non-prime group {6, 8}.

4. To maximize N, pick the largest available prime digit, 7. Digit pool is now {6, 8, 7}. Descending arrangement gives the maximum value: N = 876.

Prime factorization of 876:

$$876 = 4 \times 219 = 2^2 \times 3^1 \times 73^1$$

$$\textit{Number of factors} = (2+1)(1+1)(1+1) = 3 \times 2 \times 2 = 12$$

Q7. Correct Answer: **B (248)**

Let the 3-digit number in base 7 be abc_7. Its value in base 9 is its reverse, cba_9. Converting both to decimal:

$$49a + 7b + c = 81c + 9b + a$$

$$48a - 2b - 80c = 0 \implies 24a - b - 40c = 0 \implies b = 24a - 40c$$

Since b is a base-7 digit, 0 ≤ b ≤ 6. Searching integer values of a, c (with a, c < 7, a, c ≠ 0): with a = 5, c = 3:

$$b = 24(5) - 40(3) = 120 - 120 = 0$$

This yields a valid digit b = 0. The number in base 7 is 503_7. Converting to base 10:

$$5 \times 49 + 0 \times 7 + 3 = 245 + 3 = 248$$

Q8. Correct Answer: **B (21)**

> **Wilson's Theorem:** For any prime number p, $(p - 1)! \equiv -1 \pmod{p}$.

Since 43 is prime, Wilson's Theorem gives:

$$42! \equiv -1 \pmod{43}$$

Expand 42! to isolate 40!:

$$42 \times 41 \times 40! \equiv -1 \pmod{43}$$

Substitute modular equivalents: $42 \equiv -1$, $41 \equiv -2 \pmod{43}$:

$$(-1)(-2) \times 40! \equiv -1 \pmod{43} \implies 2 \times 40! \equiv -1 \equiv 42 \pmod{43}$$

Dividing by 2 (since gcd(2, 43) = 1):

$$40! \equiv 21 \pmod{43}$$

Q9. Correct Answer: **C (4)**

Let the two integers be 14a and 14b. Since HCF is 14, a and b must be coprime (gcd(a,b) = 1). Using LCM × HCF = product:

$$1260 \times 14 = (14a)(14b) \implies 1260 = 14ab \implies ab = 90$$

Prime factorization of 90:

$$90 = 2^1 \times 3^2 \times 5^1$$

To ensure gcd(a,b) = 1, each distinct prime cluster (2^1, 3^2, 5^1) must move entirely to either a or b. With 3 distinct prime clusters, the number of ordered distributions is $2^3 = 8$. For unordered pairs, divide by 2:

$$\text{Unordered pairs} = 2^3 / 2 = 2^2 = 4$$

Q10. Correct Answer: **A (1)**

Evaluate $13^{100} \pmod{15}$ using Euler's Totient Theorem. First compute φ(15):

$$\varphi(15) = 15 \times (1 - 1/3) \times (1 - 1/5) = 15 \times 2/3 \times 4/5 = 8$$

Since gcd(13, 15) = 1, Euler's theorem gives $13^8 \equiv 1 \pmod{15}$. Dividing 100 by 8: 100 = 8×12 + 4.

$$13^{100} = (13^8)^{12} \times 13^4 \equiv 13^4 \pmod{15}$$

Using negative remainders, $13 \equiv -2 \pmod{15}$:

$$13^4 \equiv (-2)^4 = 16 \equiv 1 \pmod{15}$$

Q11. Correct Answer: **B (3384)**

Let the 4-digit number be ABCD. Construct the system of equations:

$$(1)\ A + B + C = 14 \quad (2)\ B + C + D = 15 \quad (3)\ C = D + 4$$

Substitute (3) into (2): B + 2C = 19. Subtract (2) from (1): $A - D = -1 \implies A = D - 1$.

Using D = C − 4: $A = C - 5 \implies C = A + 5$.

The total digit sum A + B + C + D equals A + 15 (from equation 2). Since the number is divisible by 9, A + 15 must be a multiple of 9. As $1 \le A \le 9$, the only valid value is A = 3 (digit sum = 18).

Solving the remaining digits: C = 3 + 5 = 8, D = 8 − 4 = 4, B = 19 − 2(8) = 3.

The number is **3384**.

Q12. Correct Answer: **B (3.25)**

The sum of the reciprocals of all factors of N equals (sum of all factors of N) / N. Prime factorization of 360:

$$360 = 2^3 \times 3^2 \times 5^1$$

Sum of factors:

$$(1+2+4+8) \times (1+3+9) \times (1+5) = 15 \times 13 \times 6 = 1170$$

Sum of reciprocals:

$$1170 / 360 = 117 / 36 = 3.25$$

Q13. Correct Answer: **C (350)**

Trailing zeros come from pairs of prime factors 2 and 5. Since factor 2 occurs in abundance, the count of prime factor 5 is the limiting constraint.

Collect powers of 5 from terms that are multiples of 5:

$$5^5 \times 10^{10} \times 15^{15} \times \ldots \times 50^{50}$$

Exponents form an arithmetic progression:

$$S_1 = 5 + 10 + 15 + \ldots + 50 = 5 \times (1+2+\ldots+10) = 5 \times 55 = 275$$

Multiples of 25 (25^{25} and 50^{50}) contain an extra factor of 5 each, so count their exponents again:

$$S_2 = 25 + 50 = 75$$

$$Total\ trailing\ zeros = 275 + 75 = 350$$

Q14. Correct Answer: **C (48)**

To find trailing zeros in base 12, find the highest power of 12 dividing 100!. Since $12 = 2^2 \times 3$, use Legendre's formula for primes 2 and 3.

Highest power of 3 in 100!:

$$E_3(100!) = \lfloor 100/3 \rfloor + \lfloor 100/9 \rfloor + \lfloor 100/27 \rfloor + \lfloor 100/81 \rfloor = 33 + 11 + 3 + 1 = 48$$

Highest power of 2 in 100!:

$$E_2(100!) = \lfloor 100/2 \rfloor+\lfloor 100/4 \rfloor+\lfloor 100/8 \rfloor+\lfloor 100/16 \rfloor+\lfloor 100/32 \rfloor+\lfloor 100/64 \rfloor = 50+25+12+6+3+1 = 97$$

Each base-12 unit needs a 2^2 component, so maximum pairs of $2^2 = \lfloor 97/2 \rfloor = 48$. This matches the 48 available units of 3, giving exactly 48 instances of base 12.

Q15. Correct Answer: **D (49)**

The last two digits of a number equal the value modulo 100. Observe the powers of 7:

$$7^4 = 2401 \equiv 1 \pmod{100}$$

This shows a cycle length of 4. Dividing 2026 by 4: $2026 = 4 \times 506 + 2$.

$$7^{2026} = (7^4)^{506} \times 7^2 \equiv 1 \times 49 \equiv 49 \pmod{100}$$

The last two digits are **49**.

Q16. Correct Answer: **B (32)**

We want factors of $N = 2^5 \times 3^4 \times 5^3 \times 7^2$ satisfying $F \equiv 1 \pmod 4$.

1. Any factor with a power of $2 \geq 1$ is even, and cannot satisfy 4k+1. So the power of 2 must be $2^0 = 1$.

2. Analyze the remaining primes modulo 4: $3 \equiv -1$, $5 \equiv 1$, $7 \equiv -1 \pmod 4$.

A general odd factor $3^a \times 5^b \times 7^c$, evaluated modulo 4, becomes:

$$F \equiv (-1)^a \times 1^b \times (-1)^c \equiv (-1)^{a+c} \pmod 4$$

For $F \equiv 1 \pmod 4$, a + c must be even. This occurs in two scenarios:

Scenario 1 — both a and c even: $a \in \{0,2,4\}$ (3 choices), $c \in \{0,2\}$ (2 choices) $\Longrightarrow$ 6 pairs.

Scenario 2 — both a and c odd: $a \in \{1,3\}$ (2 choices), $c \in \{1\}$ (1 choice) $\Longrightarrow$ 2 pairs.

Total valid (a,c) pairs = 6 + 2 = 8. The exponent b (base 5) has no effect on sign since $1^b \equiv 1$, and can take any of 4 values (0 to 3).

$$\text{Total valid factors} = 1 \times 8 \times 4 = 32$$

Q17. Correct Answer: **C (133)**

Since 143 = 11 × 13, compute remainders modulo each prime factor and combine using the Chinese Remainder Theorem.

Modulo 11: By Fermat's Little Theorem, $3^{10} \equiv 1 \pmod{11}$. Since 1000 is a multiple of 10:

$$3^{1000} \equiv 1 \pmod{11}$$

Modulo 13: By Fermat's Little Theorem, $3^{12} \equiv 1 \pmod{13}$. Dividing 1000 by 12 leaves remainder 4 (1000 = 12×83 + 4):

$$3^{1000} \equiv 3^4 = 81 \equiv 3 \pmod{13}$$

Combining via congruences: Let $R \equiv 1 \pmod{11}$, so R = 11k + 1, and $R \equiv 3 \pmod{13}$:

$$11k + 1 \equiv 3 \pmod{13} \implies 11k \equiv 2 \pmod{13}$$

Using $11 \equiv -2 \pmod{13}$:

$$-2k \equiv 2 \pmod{13} \implies k \equiv -1 \equiv 12 \pmod{13}$$

Substituting k = 12:

$$R = 11(12) + 1 = 133$$

Q18. Correct Answer: **B (672)**

Factor the quadratic expression:

$$n^2 + 7n + 12 = (n + 3)(n + 4)$$

For this to be a multiple of prime 17, either linear term must be a multiple of 17:

$$\text{Case 1: } n + 3 \equiv 0 \pmod{17} \Longrightarrow n \equiv 14 \pmod{17}$$

$$\text{Case 2: } n + 4 \equiv 0 \pmod{17} \Longrightarrow n \equiv 13 \pmod{17}$$

Listing valid 2-digit values ($10 \leq n \leq 99$):

For $n \equiv 13 \pmod{17}$: {13, 30, 47, 64, 81, 98} — 6 terms. $\text{Sum}_1 = (6/2)(13 + 98) = 333$.

For $n \equiv 14 \pmod{17}$: {14, 31, 48, 65, 82, 99} — 6 terms. $\text{Sum}_2 = (6/2)(14 + 99) = 339$.

$$\text{Total sum} = 333 + 339 = 672$$

Q19. Correct Answer: **B (2457)**

For the LCM of x and y to equal a prime power p^k, the maximum exponent of p between x and y must equal k exactly. Let $x = p^a$, $y = p^b$; we require max(a,b) = k. Valid ordered pairs (a,b) occur in three configurations:

1. a = k, $0 \le b < k$ (k values)   2. b = k, $0 \le a < k$ (k values)   3. a = k, b = k (1 combination)

$$\text{Total ordered pairs for one prime component} = k + k + 1 = 2k + 1$$

Apply this across all prime components of $N = 2^{10} \times 3^6 \times 5^4$:

$$\text{Total ordered pairs} = (2\times10+1) \times (2\times6+1) \times (2\times4+1) = 21 \times 13 \times 9 = 2457$$

Q20. Correct Answer: **D (50)**

Consider the general term k × k!. Rewrite it to form a telescoping series:

$$k \times k! = (k+1-1) \times k! = (k+1)! - k!$$

Expanding the summation:

$$S = (2!-1!) + (3!-2!) + (4!-3!) + \ldots + (51!-50!)$$

After cancellation, only the first and last components remain:

$$S = 51! - 1! = 51! - 1$$

Evaluate modulo 51. Since 51 is a factor within 51!, $51! \equiv 0 \pmod{51}$:

$$S \equiv 0 - 1 = -1 \equiv 50 \pmod{51}$$

The final remainder is **50**.