CAT QUANTITATIVE APTITUDE | NUMBER SYSTEMS STUDENT REFERENCE GUIDE

Topic Weightage Table

PARAMETER	DESCRIPTION
Exam / Section Name	Common Admission Test (CAT) | Quantitative Aptitude (QA)
Historical Weightage	3 to 5 Direct Questions (9 to 15 Marks per slot)
Core Influence	Forms the bedrock of QA. Directly influences Algebra (Inequalities, Quadratic Equations), Permutations & Combinations (Counting principles), and Data Interpretation (Structure of data sets).
Guide Length / Best Used As	8 Comprehensive Sections | Complete reference during core study + high-density quick revision tool.

Historical Trends Table

CAT CYCLE	DIRECT QUESTIONS	MARKS CONTRIBUTION	AVERAGE DIFFICULTY LEVEL
CAT 2022	4 Questions	12 Marks	Moderate to High
CAT 2023	3 Questions	9 Marks	High
CAT 2024	5 Questions	15 Marks	Moderate
CAT 2025	4 Questions	12 Marks	High
CAT 2026 (Predictive)	4 Questions	12 Marks	High (Conceptual/Application-heavy)

1 | INTRODUCTION & PURPOSE

Number Systems is rarely tested by the IIMs as straightforward computation. Instead, core properties like cyclicity, divisibility rules, and prime factorization are heavily disguised inside complex word problems, logical puzzles, or algebraic constraints. Mastering this chapter is not about memorizing multiplication tables — it is about developing an intuitive comfort with the DNA of integers, so hidden constraints in data-heavy scenarios can be decoded on sight.

2 | CORE CONCEPTS

Term & Definition Table

TERM NAME	FULL FORM / LITERAL MEANING	DEFINITION & MATHEMATICAL/PRACTICAL EXAMPLE
Prime Factorization	Breaking an integer into its fundamental building blocks.	Expressing a composite number strictly as a product of prime powers: $N = p_1^a p_2^b p_3^c$. Example: $360 = 2^3 \times 3^2 \times 5^1$.
Highest Common Factor (HCF)	The greatest structural divisor shared between integers.	The largest integer dividing two or more integers with zero remainder. Example: $HCF(24, 36) = 12$.
Least Common Multiple (LCM)	The smallest common ground where cycles synchronize.	The smallest positive integer perfectly divisible by two or more integers. Example: $LCM(12, 18) = 36$.
Euler's Totient, $\phi(N)$	Count of co-prime elements within a numerical boundary.	The number of positive integers up to N that are relatively prime to N. Example: $\phi(12) = 12\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right) = 4$.

KEY INSIGHT — THE PRIME FACTORIZATION CORE ENGINE

Every composite integer possesses a completely unique prime signature. When a problem mentions "number of factors," "trailing zeros," or "perfect squares," the first step is always to decompose: $N = p_1^a p_2^b p_3^c \dots$. Every downstream property is driven entirely by the behavior of the exponents a, b, c.

THE GOLDEN QUESTION

Before writing a single equation or plugging in numbers, ask: *"What constraints do the properties of integers place on these variables right now?"*

---

3 | FORMULA SHEET

3A | Basic Formulas

- **Total Number of Factors of N:**

$$F(N) = (a+1)(b+1)(c+1)\dots$$

- **Sum of Factors of N:**

$$S(N) = \left(\frac{p_1^{a+1}-1}{p_1-1}\right)\left(\frac{p_2^{b+1}-1}{p_2-1}\right)\left(\frac{p_3^{c+1}-1}{p_3-1}\right)\dots$$

- **Product of Factors of N:**

$$P(N) = N^{F(N)/2}$$

3B | Advanced / Sub-Topic Formulas

- **Number of ways to express N as a product of two factors:**
  - If N is **not** a perfect square: $\frac{F(N)}{2}$
  - If N **is** a perfect square: $\frac{F(N)+1}{2}$ (including $\sqrt{N} \times \sqrt{N}$), or $\frac{F(N)-1}{2}$ (excluding it)
- **Number of ways to express N as a product of two co-prime factors:** 2^{k-1}, where k = number of distinct prime factors of N.
- **Euler's Theorem for Remainders:** If $HCF(a, M) = 1$, then

$$a^{\phi(M)} \equiv 1 \pmod{M}$$

- **Highest power of a prime p in $n!$ (Legendre's Formula):**

$$E_p(n!) = \left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \dots$$

3C | Special / Boundary Cases

- **Perfect Square Constraint:** For any integer to be a perfect square, every exponent $(a, b, c, \dots)$ in its prime factorization must be **even**. Consequently, $F(N)$ for a perfect square is always **odd**.
- **HCF and LCM of Fractions:**

$$HCF_{\text{fractions}} = \frac{HCF(\text{Numerators})}{LCM(\text{Denominators})}, \qquad LCM_{\text{fractions}} = \frac{LCM(\text{Numerators})}{HCF(\text{Denominators})}$$

- **Linear Congruence Boundary (LCM + constant remainder):** If N leaves the same remainder r when divided by

d_1, d_2, d_3:

$$N = LCM(d_1, d_2, d_3) \cdot k + r$$

> **DERIVE-ON-THE-SPOT TRICK**
>
> If you blank out on the "number of factors" formula under pressure, rebuild it live using a minimal example:
>
> 1. Take a small known number: 12.
> 2. List its factors manually: $1, 2, 3, 4, 6, 12$ — total $= 6$.
> 3. Prime-factorize it: $12 = 2^2 \times 3^1$.
> 4. Observe the exponents 2 and 1: $(2+1)(1+1) = 3 \times 2 = 6$. Matches.
> 5. Rule recovered: **add 1 to every exponent, then multiply.**

---

4 | TOPIC-WISE CONCEPT SUMMARIES & SOLVED EXAMPLES

4A | Factor Theory & Factor Counting Mechanics

Core Concepts

- **Exponent Multipliers:** Treat factor counting as independent choices. For p^a, you may pick $p^0, p^1, \ldots, p^a$ — that's $(a+1)$ choices.
- **Condition Filtering:** For "even factors only," eliminate the 2^0 choice. The multiplier for base 2 drops from $(a+1)$ to exactly a.
- **Perfect Square Tracking:** Factors that are themselves perfect squares can only use even-power positions of each prime — $\left\lfloor \frac{a}{2} \right\rfloor + 1$ choices per base.

> **CAT TIP**
>
> For "factors greater than $\sqrt{N}$" questions, remember factors occur in symmetric pairs $(d, N/d)$. If N is not a perfect square, exactly half its factors lie below $\sqrt{N}$ and half lie above.

Solved Example

Question: Find the total number of factors of $N = 37800$ that are perfectly divisible by 12, but are not divisible by 24.

1. **Prime factorize N:**

$$37800 = 378 \times 100 = (2 \times 3^3 \times 7)(2^2 \times 5^2) = 2^3 \times 3^3 \times 5^2 \times 7^1$$

2. **Set the structural condition:** valid factors must carry **exactly** 2^2, never 2^3, and at least 3^1.
3. **Isolate choices per prime base:**
   - Base 2: only 2^2 allowed $\rightarrow 1$ option
   - Base 3: $3^1, 3^2, 3^3 \rightarrow 3$ options
   - Base 5: $5^0, 5^1, 5^2 \rightarrow 3$ options
   - Base 7: $7^0, 7^1 \rightarrow 2$ options
4. **Apply the product-rule multiplier:**

$$\rightarrow 1 \times 3 \times 3 \times 2 = \boxed{18 \text{ factors}}$$

4B | Divisibility Rules & Cryptarithmetic Constraints

Core Concepts

- **Composite Divisibility Splits:** To check divisibility by composite C, split into co-prime factors A, B (e.g., $72 \rightarrow$ check 8 and 9 independently).
- **Last-Digit Truncation:** Divisibility by 2^k or 5^k depends **only** on the last k digits.
- **Digital Sum Cyclicity:** Rules for 3 and 9 depend purely on the digit sum, ignoring positional place value entirely.

> **CAT TIP**
>
> In missing-digit problems, resolve the trailing-digit rule first (e.g., last 3 digits for divisibility by 8) to lock down variables
> **BEFORE**
>
> applying the global digit-sum rule (e.g., divisibility by 9).

Solved Example

Question: If the 9-digit number $875X32Y08$ is perfectly divisible by 36, find the maximum possible value of $(2X + Y)$, given X, Y are single-digit natural numbers.

1. **Factor the divisor:** $36 = 4 \times 9$, with $HCF(4, 9) = 1$. Must satisfy both.
2. **Check divisibility by 4:** Last two digits are 08, divisible by 4 for any Y.
3. **Check divisibility by 9:**

$$8 + 7 + 5 + X + 3 + 2 + Y + 0 + 8 = 33 + X + Y$$

   Valid targets: 36 or $45 \rightarrow$ Case 1: $X + Y = 3$; Case 2: $X + Y = 12$.
4. **Maximize** $(2X + Y)$**:** maximize X first using Case 2:

$$\rightarrow X = 9 \Rightarrow Y = 12 - 9 = 3$$

5. **Final value:**

$$\rightarrow 2(9) + 3 = \boxed{21}$$

4C | Advanced Remainder Theorems (Cyclicity & Modular Arithmetic)

Core Concepts

- **Negative Remainder Normalization:** Use negative remainders to reduce large bases to -1 or $+1$ relative to the divisor, converting back to positive at the end.
- **Coprime Power Reductions:** Apply Euler's Totient to shrink massive exponents when base and divisor share no common factor.
- **Base Splitting:** For sums in the numerator, compute each term's remainder individually, then add.

> **CAT TIP**
>
> If the divisor is prime p, immediately invoke Fermat's Little Theorem ($a^{p-1} \equiv 1 \pmod{p}$), reducing the exponent modulo $(p - 1)$ instantly.

Solved Example

Question: What is the remainder when $32^{(32^{32})}$ is divided by 7?

1. **Simplify the base:**

$$32 \equiv 4 \pmod{7} \Rightarrow 32^{(32^{32})} \equiv 4^{(32^{32})} \pmod{7}$$

2. **Apply Fermat's Little Theorem:** $\phi(7) = 6$, so $4^6 \equiv 1 \pmod{7}$. Reduce the exponent modulo 6:

$$\text{Target Exponent} = 32^{32} \bmod 6$$

3. **Evaluate the exponent power modulo 6:**

$$32 \equiv 2 \pmod{6} \Rightarrow 32^{32} \equiv 2^{32} \pmod{6}$$

Cycle of $2^n \bmod 6$: $2, 4, 2, 4, \ldots$ — odd $n \to 2$, even $n \to 4$. Since 32 is even:

$$\to 2^{32} \equiv 4 \pmod 6$$

4. **Substitute back:**

$$4^{(32^{32})} \equiv 4^4 \pmod 7$$

5. **Compute the final remainder:**

$$4^4 = 256 = 7 \times 36 + 4 \Rightarrow \to \boxed{\text{Remainder} = 4}$$

---

5 | CAT TRAP IDENTIFIER

The Trap Matrix

TRAP CATEGORY / NAME	WRONG APPROACH (THE INSTINCTIVE MISTAKE)	RIGHT APPROACH (THE CORRECTION)
The "1 is Prime" Fallacy	Treating 1 as prime when counting prime factors or applying $\phi(N)$.	1 is neither prime nor composite; exclude it from all prime-factor counts and totient calculations.
Factor Count Off-by-One	Forgetting that 1 and N itself are always counted as factors.	$F(N) = (a+1)(b+1)(c+1)\ldots$ inherently includes 1 and N — do not add/subtract separately.
Ignoring Sign in Remainders	Treating a negative remainder like -2 as final.	Normalize: Remainder = Divisor + (negative remainder), e.g. $-2 \bmod 7 = 5$.
Euler's Theorem Misfire	Applying $a^{\phi(M)} \equiv 1 \pmod M$ even when $HCF(a, M) \neq 1$.	Requires co-primality. If not, factor out common terms first or use direct cyclicity.
HCF/LCM of Fractions Inversion	Using $\frac{LCM(\text{Num})}{LCM(\text{Den})}$ for HCF of fractions.	HCF of fractions uses denominators' **LCM**; LCM of fractions uses denominators' **HCF** — the rule inverts.
Perfect Square Exponent Slip	Assuming a "balanced-looking" number is a perfect square.	Check strictly that **every** exponent in the prime factorization is even.

PRE-ATTEMPT MENTAL CHECKLIST

1. Have I fully prime-factorized the number(s) before attempting factor/divisibility logic?
2. Is the divisor prime, composite, or splittable into co-primes?
3. Am I using positive remainders throughout?
4. Does Euler's Theorem actually apply — are base and modulus genuinely co-prime?
5. Does the question want factors, prime factors, or co-prime factor pairs?
6. Should 1 and N itself be included or excluded per the question's wording?

---

6 | SPEED TECHNIQUES & SHORTCUTS

Table A — Last-Digit Cyclicity Chart

BASE'S LAST DIGIT	CYCLE LENGTH	CYCLE PATTERN (UNITS DIGIT)
$0, 1, 5, 6$	1	Same digit repeats every time
$4, 9$	2	e.g. $4 \to 4, 6, 4, 6\ldots$
$2, 3, 7, 8$	4	e.g. $2 \to 2, 4, 8, 6, 2\ldots$

Table B — Quick Prime-Power Reference

EXPONENT VALUE	"+1" MULTIPLIER CONTRIBUTION
p^1	$\times 2$
p^2	$\times 3$
p^3	$\times 4$
p^4	$\times 5$

Strategic Directives — When to Use X vs. Y

SITUATION	USE THIS TOOL	REASONING
Divisor is prime, huge exponent	**Fermat's Little Theorem**	Instantly reduces exponent modulo $(p-1)$.
Divisor composite, base co-prime to it	**Euler's Totient Theorem**	Generalizes Fermat to any co-prime modulus.
Base and divisor share common factors	**Direct Cyclicity / Pattern Method**	Euler's Theorem invalid — build the remainder cycle manually.
Multiple divisors, common remainder	**LCM + Constant Remainder Framework**	Avoids solving each congruence separately.
Only last few digits matter	**Last-Digit Truncation Rule**	Skip full division for $2^k/5^k$ divisibility.

APPROXIMATION & ELIMINATION FRAMEWORK

- **Digit-Sum First Pass:** Run a quick digit-sum check for 3/9 divisibility to eliminate 1–2 options instantly.
- **Order-of-Magnitude Scan:** For "how many factors/multiples up to N," estimate $\frac{N}{d}$ roughly to sanity-check option ranges.
- **Parity Boundary Check:** For perfect square/cube checks, scan option parity before computing exactly.

---

7 | COMMON MISTAKES TO AVOID

Mistake 1 — Miscounting Factors by Ignoring Exponent Boundaries

WRONG	Assuming the factor count equals the count of prime factors, or forgetting to add 1 to each exponent.
CORRECT	$$F(N) = (a+1)(b+1)(c+1)\dots$$

Mistake 2 — Applying Euler's Theorem Without Verifying Co-primality

WRONG	Using $a^{\phi(M)} \equiv 1 \pmod{M}$ even when a and M share a common factor.
CORRECT	Verify $HCF(a, M) = 1$ first; otherwise factor out the common term or use direct cyclicity.

Mistake 3 — Confusing HCF/LCM Rules for Fractions

WRONG	Applying the integer-style formula structure without inverting the roles.
CORRECT	$$HCF_{\text{fractions}} = \frac{HCF(\text{Num})}{LCM(\text{Den})}, \qquad LCM_{\text{fractions}} = \frac{LCM(\text{Num})}{HCF(\text{Den})}$$

Mistake 4 — Leaving Negative Remainders Unconverted

WRONG	Reporting a computed remainder of -3 as the final answer.
CORRECT	$$\text{Remainder} = \text{Divisor} - \lvert\text{negative remainder}\rvert \;\Rightarrow\; -3 \bmod 11 = 8$$

Mistake 5 — Treating "Factors" and "Prime Factors" as Interchangeable

WRONG	Answering a "how many factors" question with the count of distinct prime factors, or vice versa.
CORRECT	Distinct prime factor count is simply k; total factors requires the full $(a+1)(b+1)(c+1)\ldots$ computation.

---

8 | QUICK REVISION CARD

Master Formula Checklist

- $F(N) = (a+1)(b+1)(c+1)\ldots$
- $S(N) = \left(\frac{p_1^{a+1}-1}{p_1-1}\right)\left(\frac{p_2^{b+1}-1}{p_2-1}\right)\ldots$
- $P(N) = N^{F(N)/2}$
- Euler's Theorem: $a^{\phi(M)} \equiv 1 \pmod{M}$ (only if $HCF(a, M) = 1$)
- Fermat's Little Theorem: $a^{p-1} \equiv 1 \pmod{p}$ (for prime p)
- Legendre's Formula: $E_p(n!) = \left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \ldots$
- $N = LCM(d_1, d_2, d_3) \cdot k + r$ (common remainder problems)

CRITICAL RATIOS / VALUES TO REMEMBER

- Perfect squares always have an **odd** number of total factors.
- $\phi(p) = p - 1$ for any prime p.
- Co-prime factor pairs of N: 2^{k-1}, where k = distinct prime count.
- 1 is neither prime nor composite — always exclude it from prime counts.

TOP 5 CORE TOPIC TRAPS (COMPRESSED)

1. Forgetting the "+1" on every exponent when counting factors.
2. Using Euler's Theorem without checking co-primality first.
3. Inverting the HCF/LCM fraction rules.
4. Leaving a remainder negative instead of normalizing it.
5. Confusing "number of factors" with "number of distinct prime factors."

PRE-SOLVE CHECKLIST (QUICK-FIRE TRIGGERS)

- Have I prime-factorized fully?
- Is the divisor prime, composite, or splittable into co-primes?
- Is Euler's Theorem actually valid here?
- Are all remainders in positive form?
- Does the question want factors, prime factors, or co-prime factor pairs?

Shortcut / Timing Targets

QUESTION TYPE	TARGET TIME	ACCURACY TARGET
Direct factor/divisibility questions	Under 45–60 sec	90%+
Remainder/cyclicity problems	Under 60–75 sec	90%+
Complex/multi-step or DI-embedded sets	Under 75–90 sec	85%+