CAT QUANTITATIVE APTITUDE | LOGARITHMS

COMPLETE STUDENT REFERENCE & TOPIC GUIDE

STRATEGIC BLUEPRINT ROADMAP

- **Section 1 & 2:** Master logarithmic notations, base constraints, and weightage metrics.
- **Section 3 & 4:** Deep dive into core operational rules, product scaling, and hidden multi-base configurations.
- **Section 5:** Review the CAT Trap Master List to protect against boundary errors and invalid roots.
- **Section 6 & 7:** Internalize speed math indicators, digit counts, and target revision summaries before practicing mocks.

Comprehensive Preparation Suite for Logarithmic Concepts & Master Shortcuts

Core Properties • 5 Foundational Sub-topics • 20 Core Traps • Speed Math Techniques • Quick Revision Cards

Target: 99.99 Percentile in QA

1. Introduction & Historical Trends

Logarithms represent an exceptionally high-yield, compact segment of the CAT Quantitative Aptitude section. While arithmetic tests logical endurance and wordy formatting, Logarithms evaluate your structural grasp of operational properties, base transformations, and boundary domains. Beyond direct questions, logarithmic equations are frequently embedded inside complex multi-variable functions, algebraic series, and sequence progressions, influencing up to 15% of the total QA landscape.

Historical Question Trends

CAT YEAR	DIRECT LOGARITHM QUESTIONS	TOTAL MARKS	CORE SUB-TOPICS TESTED
CAT 2021	2 Questions	6 Marks	Hidden Quadratic Substitutions, Base Change Rule
CAT 2022	1 Question	3 Marks	Logarithmic Inequalities & Domain Constraints
CAT 2023	2 Questions	6 Marks	Telescoping Series Loops, Character Mapping
CAT 2024	1 Question	3 Marks	Variable Multi-base Systems
CAT 2025 (Expected)	2 Questions	6 Marks	Domain Restrictions, Inequality Intercepts

2. Core Concepts & Definitions

TERM	MATHEMATICAL STRUCTURAL MEANING	DEFINITION & CONCRETE OPERATIONAL EXAMPLE
Logarithm	Inverse of Exponentiation	If $a^y = x$, then $\log_a x = y$. Example: Since $2^3 = 8$, then $\log_2 8 = 3$.
Argument	Input value of the log	The value x inside $\log_a x$. Must be strictly positive ($x > 0$).
Base	The scaling foundation	The index value a inside $\log_a x$. Must satisfy: $a > 0$ and $a\ eq\ 1$.
Common Log	Base-10 system	Written simply as $\log x$ where the underlying base is implicitly understood as 10.
Natural Log	Base-e system ($e\quad pprox\ 2.718$)	Written as $\ln x$, extensively leveraged in structural calculus and continuous rate equations.
Characteristic	Integer part of log value	If $\log_{10} 250 = 2.3979$, the characteristic is 2 (indicates the order of magnitude).
Mantissa	Fractional part of log value	If $\log_{10} 250 = 2.3979$, the mantissa is 0.3979 (always kept non-negative).

THE LOGARITHMIC FOUNDATIONAL AXIOM

In advanced CAT problems, the mechanical property application is secondary to the domain conditions. The single biggest reason for wrong selections is solving an equation flawlessly but accepting a root that violates the argument constraint ($Argument > 0$) or the base constraints ($Base > 0,\ eq\ 1$).

3. Formula Sheet & Fundamental Properties

3A. Core Operational Rules

- **Product Rule:** \log_a (xy) = \log_a x + \log_a y
- **Quotient Rule:** \log_a \left(rac{x}{y} ight) = \log_a x - \log_a y
- **Power Rule:** \log_a (x^k) = k \cdot \log_a x
- **Base Power Rule:** \log_{a^m} x = rac{1}{m} \cdot \log_a x
- **Base Change Theorem:** \log_a b = rac{\log_c b}{\log_c a} = rac{\ln b}{\ln a}
- **Reciprocal Theorem:** \log_a b = rac{1}{\log_b a}
- **Power Change Rule:** a^{\log_c b} = b^{\log_c a} (Crucial transformation shortcut)
- **Identity Collapses:** \log_a a = 1 \quad;\quad \log_a 1 = 0 \quad;\quad a^{\log_a x} = x

4. Topic-Wise Concept Summaries & Solved Examples

4A. Standard Algebra Property Applications

CAT frequently tests rapid identification of intersecting properties where equations are compressed to extract unknown values.

Example: If \log_3 x + \log_9 x + \log_{27} x = 11, find the value of x.
Solution: Convert all bases to uniform base 3 via the Base Power Rule:
\log_3 x + rac{1}{2}\log_3 x + rac{1}{3}\log_3 x = 11
Take \log_3 x common: \log_3 x \left(1 + rac{1}{2} + rac{1}{3} ight) = 11 \implies \log_3 x \left(rac{6+3+2}{6} ight) = 11
\log_3 x \cdot \left(rac{11}{6} ight) = 11 \implies \log_3 x = 6 \implies x = 3^6 = 729.
Answer: 729

4B. Base Change and Multi-Base Operations

When multiple distinct bases exist in an expression, immediate base equalization via the Base Change rule is mandatory.

Example: Evaluate the expression: \log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdots \log_{63} 64.
Solution: Apply the Base Change rule to convert each term into standard natural log fractions:
rac{\ln 3}{\ln 2} \cdot rac{\ln 4}{\ln 3} \cdot rac{\ln 5}{\ln 4} \cdots rac{\ln 64}{\ln 63}
Notice the systematic telescoping cancellation of cross numerators and denominators. Only the initial denominator and final numerator survive:
ext{Result} = rac{\ln 64}{\ln 2} = \log_2 64 = 6.
Answer: 6

4C. Hidden Quadratic Setups (Placeholder Substitutions)

Intimidating variable structures are often secretly basic quadratic systems hidden underneath placeholder parameters.

Example: Solve for real numbers: (\log_2 x)^2 - \log_2 (x^3) - 4 = 0.
Solution: Simplify using the power rule: (\log_2 x)^2 - 3\log_2 x - 4 = 0.
Substitute placeholder t = \log_2 x: t^2 - 3t - 4 = 0 \implies (t-4)(t+1) = 0.
Thus, t = 4 or t = -1.
Case 1: \log_2 x = 4 \implies x = 2^4 = 16.
Case 2: \log_2 x = -1 \implies x = 2^{-1} = 0.5.
Both values yield positive arguments (16 > 0 and 0.5 > 0), so both solutions are fully valid.
Answer: 16, 0.5

4D. Logarithmic Inequalities & Domain Conditions

When solving log inequalities, the structural direction of the inequality sign shifts completely based on whether the base is greater than 1 or between 0 and 1.

Rule: If $\log_a x > \log_a y$:
- If $a > 1$, then $x > y$ (Inequality holds direction)
- If $0 < a < 1$, then $x < y$ (Inequality flips direction)

Example: Solve for x: $\log_{0.5} (2x - 4) \ge -1$.
Solution: First, handle core domain limits: $2x - 4 > 0 \implies x > 2$.
Next, drop the log. Because base is 0.5 (less than 1), flip the inequality sign:
$2x - 4 \le (0.5)^{-1} \implies 2x - 4 \le 2 \implies 2x \le 6 \implies x \le 3$.
Intersecting our constraints ($x > 2$ and $x \le 3$) provides the structural domain range: $(2, 3]$.
Answer: $2 < x \le 3$ or $(2, 3]$

4E. Logarithmic Series and Progressions

Logarithmic transformations are often used to convert multi-term geometric products into classic arithmetic summations.

Example: If a, b, c are in GP, prove that $\log x, \log y, \log z$ elements are in AP given standard scaling.
More practically, consider standard CAT questions: Find value of $rac{1}{\log_2 100!} + rac{1}{\log_3 100!} + \dots + rac{1}{\log_{100} 100!}$.
Solution: Apply the reciprocal theorem to invert each term: $\log_{100!} 2 + \log_{100!} 3 + \dots + \log_{100!} 100$.
Apply the product rule across matching bases: $\log_{100!} (2 \cdot 3 \cdot 4 \cdots 100) = \log_{100!} (100!) = 1$.
Answer: 1

5. CAT Trap Identifier: Top Core Pitfalls

Trap 1: The Invalid Negative Argument Blindspot

Flawed Approach: Solving structural variable combinations blindly via placeholder quadratics and assuming all resulting numbers are valid.
Correct Approach: Always check your output parameters back against the primary equation. If any value results in an input argument less than or equal to zero, discard it immediately.
CAT Paradigm: Solving $\log_2(x-2) + \log_2(x-4) = 3$ yields $x=6$ and $x=0$. Substituting $x=0$ yields $\log_2(-2)$, a total domain failure. Discard 0.

Trap 2: Fractional Base Inequality Inversion Failure

Flawed Approach: Dropping logarithm prefixes from inequalities involving base fractions without changing the inequality's direction.
Correct Approach: If base a falls in the interval $0 < a < 1$, the function decays. Dropping logs requires a complete inversion of the inequality sign.
CAT Paradigm: Assuming $\log_{0.2} x > \log_{0.2} 5 \implies x > 5$ is totally wrong. The mathematically true conversion is $x < 5$ (subject to $x > 0$).

Trap 3: The Base = 1 Boundary Disregard

Flawed Approach: Retaining variable solutions that inadvertently force log bases to equal exactly 1.
Correct Approach: A log base must never equal 1 (since $1^y = 1$ cannot model arbitrary values). Ensure variable filters discard this boundary state.
CAT Paradigm: Solving an equation where the base is $(x-2)$ yields answers $x=5, x=3$. If $x=3$, base becomes 1, rendering the entire equation invalid.

Trap 4: Structural Distributive Fantasy Expansion

Flawed Approach: Expansively claiming that \log(x+y) = \log x + \log y or that rac{\log x}{\log y} = \log x - \log y.
Correct Approach: Remember properties strictly: \log(xy) = \log x + \log y, and rac{\log x}{\log y} = \log_y x. Do not invent custom distribution rules.
CAT Paradigm: Simplification mistakes on wordy expressions lead directly to wrong answers deliberately placed in standard multiple-choice configurations.

6. Speed Techniques & Mental Math Shortcuts

- **Number of Digits Discovery Shortcut:** To discover the total digit count of a high-value exponential form a^n, compute its log base 10 value: V = \log_{10} (a^n) = n \cdot \log_{10} a. The total digit count is simply equal to:

  ext{Number of Digits} = \lfloor V floor + 1 \quad (ext{Characteristic} + 1)

- **Leading Zeros for Small Decimals:** For a small fraction or decimal F^n, if \log_{10} (F^n) = -C \cdot M, then the total number of consecutive zeros immediately following the decimal point before hitting a non-zero digit is exactly equal to C - 1 (where C is the positive integer component).
- **Ratio-Based Log Estimation:** Memorize basic base-10 values to cross-check boundary ranges on options without accurate computation tools:

  \log_{10} 2 pprox 0.3010 \quad;\quad \log_{10} 3 pprox 0.4771 \quad;\quad \log_{10} 5 pprox 0.6990 \quad;\quad \log_{10} 7 pprox 0.8451

7. Quick Revision Cards

REVISION CARD 1: DOMAIN MAPPING RULES

For structural expressions \log_B A Regel:
1. Argument Constraint: A > 0
2. Base Constraints: B > 0 and B eq 1
Core Trap: Always resolve domain bounds *before* applying expansions like power drops or base changes.

REVISION CARD 2: KEY PROPERTY TRANSFORMATIONS

- \log_a b \cdot \log_b a = 1
- \log_{a^m} b^n = rac{n}{m} \log_a b
- x^{\log_y z} = z^{\log_y x}
- \log_b a = x \implies a = b^x

REVISION CARD 3: TARGET EXECUTION STRATEGY

- Direct log questions must be solved in under 45–60 seconds using property compressions.
- Complex inequalities with variable bases must be solved in under 90 seconds by splitting inputs into > 1 and 0 < x < 1 check intervals.
- Target accuracy metrics for this specific chapter: 95%+.