CAT QUANTITATIVE APTITUDE
GEOMETRY PREPARATION

FORMULA SHEET • IMPORTANT TOPICS • MISTAKES TO AVOID

DOCUMENT MATRIX

Exam / Section	CAT — Quantitative Aptitude (Geometry Module)
Historical Weightage	3–5 questions per paper — approximately 12–15% of the QA section score
Core Influence	Interlocks with Algebra (quadratic constraints on coordinate paths) and Trigonometry; drives most spatial-optimization questions
Guide Length / Best Used As	8 Sections — Formula Sheet, Key Concepts, and Revision Checklist

HISTORICAL TREND ANALYSIS — LAST 4 CAT CYCLES

CAT Cycle	Triangles	Circles	Polygons	Coord. Geometry	3D Mensuration	Total Qs
CAT 2022	1	1	0-1	1	1	4
CAT 2023	1	1	1	1	0-1	4-5
CAT 2024	0-1	1	1	1-2	1	5
CAT 2025	1	0-1	1	1	1	4-5
CAT 2026 (Forecast)	1	1	1 (hybrid w/ Coord.)	1-2 (locus-heavy)	1 (frustum/cylinder combos)	5

TREND READING
The last two cycles show Coordinate Geometry increasingly fused with Locus and Quadratic constraints rather than appearing as a standalone plot-the-points question — treat Section 4C as high-priority.

1 | INTRODUCTION & PURPOSE

CAT Geometry rarely tests a single formula in isolation. Most questions combine two or three shapes — for example, a circle inscribed in a triangle sitting inside a square — and ask you to find one relationship connecting them. The real skill being tested is not formula recall but the ability to spot an invariant property (a fixed ratio, a constant angle, a preserved symmetry) inside a cluttered diagram. Memorising formulas without practicing this kind of pattern recognition is the main reason candidates run out of time on these questions.

2 | CORE CONCEPTS

Term & Definition Matrix

Term	Definition	Where It Bites
Orthocenter	Point of concurrency of the three altitudes of a triangle. In a right triangle it sits exactly at the right-angle vertex; in an obtuse triangle it lies outside the triangle.	Questions describing an altitude-based construction almost always want the orthocenter's distance from a vertex, not the centroid's.
Inradius & Circumradius	Inradius r = Area / s (s = semi-perimeter); Circumradius R = (a·b·c) / (4·Area). Together they satisfy Euler's relation $OI^2 = R^2 - 2Rr$.	Examiners exploit R and r together to force you to compute Area twice using two different formulas and equate them.
Cyclic Quadrilateral	A quadrilateral whose four vertices lie on one circle. Opposite angles are supplementary, and Ptolemy's theorem applies: product of diagonals = sum of products of opposite sides.	The 'cyclic' label is often buried inside a phrase like 'all four points lie on a circle' rather than stated directly.
Secant & Tangent Power	Power of a point P w.r.t. a circle: for a tangent PT and secant PAB, $PA \cdot PB = PT^2$. This value is identical for every line drawn through P.	Questions give two different secants from the same external point purely to let you equate two Power-of-a-Point expressions.
Centroidal Ratio	The centroid divides every median in the ratio 2:1 from the vertex. It is also the average of the three vertex coordinates.	In coordinate-geometry hybrids, 'the centroid is given at (x,y)' is a disguised way of supplying one linear equation in the unknown vertex.

Key Insight — Scaling Laws

KEY INSIGHT

If every linear dimension of a shape is scaled by a factor k, perimeter scales by k, Area scales by k^2, and Volume scales by k^3. This single fact resolves a large share of 'similar triangles' and '3D solid ratio' questions without a single extra construction — convert the given ratio to a linear scale factor first, then raise it to the correct power for whatever quantity (length, area, volume) the question actually asks for.

Key Question to Ask

KEY QUESTION

"What auxiliary line can I draw, or what symmetry can I exploit, to connect the known radius or side to the unknown segment?" — a dropped perpendicular from the center to a chord, a line joining two given centers, or a diagonal of a cyclic quadrilateral usually provides the missing connection.

3 | FORMULA SHEET

3A | Basic Formulas

Concept	Formula
Area (Heron's)	Area = $\sqrt{[s(s-a)(s-b)(s-c)]}$, where $s = (a+b+c)/2$
Area (Inradius-based)	Area = $r \times s$
Area (Sine-based)	Area = $(1/2)\cdot a\cdot b\cdot \sin(C)$
Chord Property	Perpendicular from the center to a chord bisects the chord
Tangent–Secant Theorem	$PA \cdot PB = PT^2$ (Power of a Point from external point P)
Angle in Semicircle	An angle subtended by a diameter on the circle is always 90°

3B | Advanced / Sub-Topic Formulas

Theorem	Formula / Statement
Apollonius' Theorem	For median AD to side BC: $AB^2 + AC^2 = 2AD^2 + 2BD^2$
Ptolemy's Theorem	For cyclic quadrilateral ABCD: $AC \cdot BD = AB \cdot CD + AD \cdot BC$
Stewart's Theorem	For cevian AD to BC with BD = m, DC = n, AD = d: $b^2m + c^2n - d^2a = a\cdot m\cdot n$ $(a=m+n)$
Section Formula (Internal)	Point dividing (x_1,y_1) & (x_2,y_2) in ratio m:n = $((mx_2+nx_1)/(m+n), (my_2+ny_1)/(m+n))$
Section Formula (External)	Same as above with n replaced by –n
Area via Coordinates	Area = $(1/2)\lvert x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\rvert$

3C | Special / Boundary Cases

Case	Governing Rule
Max area polygon in a fixed circle	Among all polygons with n sides inscribed in a given circle, the regular n-gon has the maximum area
Max area triangle with fixed perimeter	The equilateral triangle maximizes area for a given perimeter
Regular Hexagon	Area = $(3\sqrt{3}/2)\cdot a^2$; longest diagonal = 2a; hexagon splits into 6 equilateral triangles of side a
Rectangle in a semicircle	Max-area inscribed rectangle has width = radius × $\sqrt{2}$ for the side sitting on the diameter

Deriving Apollonius' Theorem If You Forget It

QUICK DERIVATION

Drop a perpendicular from vertex A to BC, hitting it at foot F. Apply the Pythagorean theorem separately in triangles ABF and ACF, then substitute BD = DC = a/2 (D is the midpoint). Adding the two Pythagorean equations cancels the AF^2 term and regenerates $AB^2 + AC^2 = 2AD^2 + 2BD^2$ directly.

4 | TOPIC-WISE CONCEPT SUMMARIES & SOLVED EXAMPLES

4A | Properties of Triangles (Similarity, Congruence, Cevian Theorems)

Core Concepts

- AA similarity is sufficient to lock every angle and every side ratio — you never need a third condition once two angles match.
- A cevian (median, angle bisector, or altitude) splits the opposite side in a ratio governed by a different theorem each time: median → equal halves, angle bisector → ratio of adjacent sides, altitude → depends on the two right triangles formed.
- Congruence (SSS, SAS, ASA, RHS) locks every element, whereas similarity only locks ratios — watch for questions that give you a similarity condition but then ask for an absolute length, which requires one more piece of scale information.

> **CAT TIP**
> *Never trust your own rough sketch to decide whether two lines are parallel, whether a triangle is acute or obtuse, or whether a point lies inside or outside a circle — confirm every such claim algebraically before using it in the next step.*

SOLVED EXAMPLE

Q. In triangle ABC, D lies on BC such that AD is the median. If AB = 7, AC = 9, and AD = 6, find BC.

Apollonius' Theorem → $AB^2 + AC^2 = 2AD^2 + 2BD^2$

$\rightarrow 49 + 81 = 2(36) + 2BD^2$

$\rightarrow 130 = 72 + 2BD^2 \rightarrow BD^2 = 29$

$\rightarrow BD = \sqrt{29}$, **so** $BC = 2BD = 2\sqrt{29}$

Answer: $BC = 2\sqrt{29}$

4B | Circle Theorems, Tangents, and Intersecting Chords

Core Concepts

- Two tangents drawn from the same external point to a circle are always equal in length — this single fact quietly resolves most 'tangent quadrilateral' perimeter questions.
- When two chords intersect inside a circle, the products of their segments are equal ($PA \cdot PB = PC \cdot PD$); when they intersect outside, use the Power-of-a-Point tangent-secant form instead.
- Angle-chasing in circles reduces to two rules used repeatedly: the angle subtended at the center is twice the angle at the circumference, and angles in the same segment are equal.

> **CAT TIP**
> *A scratchpad circle that 'looks tangent' to a line is not proof of tangency — only a stated perpendicularity to the radius, or an explicitly given single point of contact, confirms it.*

SOLVED EXAMPLE

Q. Two chords AB and CD of a circle intersect at point P inside the circle. If AP = 6, PB = 8, and CP = 4, find PD.

Intersecting Chords Theorem → AP · PB = CP · PD

→ 6 × 8 = 4 × PD

→ 48 = 4 · PD → PD = 12

Answer: PD = 12

4C | Coordinate Geometry & 3D Mensuration Optimization

Core Concepts

- A locus condition (e.g. 'equidistant from two points') almost always collapses into the equation of a straight line or circle — identify which one before setting up coordinates.
- In 3D solids formed by combining a cylinder, cone, or hemisphere, the shared radius is the invariant that links every sub-formula; write the shared variable once and express every other quantity in terms of it.
- Optimization questions (maximum volume for fixed surface area, minimum surface area for fixed volume) are solved by expressing the target quantity as a single-variable function and testing the boundary or symmetric case first, since CAT rarely requires calculus.

CAT TIP

Do not assume an axis of symmetry or a right angle at the origin just because your scratch plot looks that way — re-derive it from the given coordinates using the distance or slope formula.

SOLVED EXAMPLE

Q. A solid right circular cylinder and a cone share the same base radius r and the same height h. If the volume of the cylinder is 3 times the volume of the cone, verify the ratio and find the ratio of their curved surface areas given $h = r\sqrt{3}$ (cone) is not assumed — use volumes only.

Volume of cylinder = $\pi r^2 h$; Volume of cone = $(1/3)\pi r^2 h$

→ Ratio (cylinder : cone) = $\pi r^2 h : (1/3)\pi r^2 h = 3 : 1$ (always true for equal r, h — confirms the given ratio without extra data)

→ This confirms the shared-radius, shared-height invariant discussed above resolves the question without needing the slant height at all

Answer: The 3:1 volume ratio holds for any r, h with equal base and height; no further data is needed.

5 | CAT TRAP IDENTIFIER

The Trap Matrix

Trap	How It Operates	Defense
The Diagram Illusion Trap	The figure visually suggests two lines are parallel or perpendicular purely because of how it was drawn, with no algebraic statement to that effect.	Verify parallelism via equal slopes or corresponding angles, and perpendicularity via the product of slopes = -1, before using either in a calculation.
The Missing Case Trap	The question's conditions are satisfied by two distinct configurations (commonly an acute and an obtuse triangle, or a point on either arc of a circle), but only one is tested.	After finding one valid configuration, check whether flipping a point to the other side of a line or arc still satisfies every given condition.
The Boundary Overlap Trap	An inscribed or overlapping shape is assumed to touch the boundary at a 'nice' symmetric point, when the given constraints actually place the point elsewhere.	Re-derive the touching point using the actual given ratio or distance, rather than defaulting to the midpoint or the visually obvious spot.

Pre-Attempt Mental Checklist

- Have I confirmed every 'looks parallel / looks perpendicular / looks tangent' claim algebraically?
- Could the described triangle be obtuse as well as acute, and does that change which point is the orthocenter or where the circumcenter lies?
- Is there a second valid position for the point / intersection / vertex that also satisfies all given conditions?
- Have I mixed up two different centers (centroid vs. incenter vs. circumcenter vs. orthocenter) anywhere in my working?
- If I assumed a special case (equilateral, right-angled, isosceles) to simplify, does the question's wording actually permit that assumption?
- Does my final answer match the units and the scale (linear vs. area vs. volume) that the question is actually asking for?

6 | SPEED TECHNIQUES & SHORTCUTS

Table A — Pythagorean Triplets & Extreme Variations

Base Triplet	x2 Scaled	x3 Scaled	Notes
3-4-5	6-8-10	9-12-15	Most frequent; recognize instantly in any right-triangle setup
5-12-13	10-24-26	15-36-39	Common in coordinate-distance questions with 'nice' integer answers
8-15-17	16-30-34	24-45-51	Appears in ladder/wall and tower-shadow style problems
7-24-25	14-48-50	21-72-75	Watch for disguised versions inside larger compound figures
20-21-29	40-42-58	60-63-87	Less common but appears in 'find the missing rational side' traps
9-40-41	18-80-82	27-120-123	Extreme/narrow triplet; useful for elimination in MCQ options

Table B — Interior / Exterior Angles of Regular Polygons

Polygon (n sides)	Interior Angle	Exterior Angle	Sum of Interior Angles
Pentagon (5)	108°	72°	540°
Hexagon (6)	120°	60°	720°
Heptagon (7)	≈128.57°	≈51.43°	900°
Octagon (8)	135°	45°	1080°
Decagon (10)	144°	36°	1440°
n-gon (general)	((n–2)×180°)/n	360°/n	(n–2)×180°

Strategic Directives — When to Deploy Which Method

Situation	Preferred Method
Points are given as explicit (x, y) coordinates, or a locus condition is described	Deploy Coordinate Geometry — set up equations directly rather than searching for a synthetic construction
Pure angle and side relationships within a single static figure, no coordinates given	Deploy pure Euclidean geometry — congruence, similarity, and circle theorems resolve it faster than forcing coordinates
A triangle's angles are given and a length ratio is asked for, or vice-versa	Deploy Trigonometric scaling (Sine Rule / Cosine Rule) — converts angle information into length ratios directly

Approximation & Scaling Framework

ELIMINATION SHORTCUT

When a question describes 'a general triangle' or 'a general quadrilateral' with no constraint that rules out a special case, assume the most convenient special case — equilateral, right-angled, or square — solve quickly with that assumption, and use the resulting number to eliminate options; the ratio-based nature of most CAT geometry questions means the special-case answer usually matches the general one.

7 | COMMON MISTAKES TO AVOID

MISTAKE 1 | Confusing Centroid and Incenter Ratios

WRONG Applying the 2:1 centroid ratio to an angle bisector, assuming every cevian splits in the same proportion.

CORRECT Only the median splits in a fixed 2:1 ratio (vertex to centroid). An angle bisector splits the opposite side in the ratio of the two adjacent sides (Angle Bisector Theorem), which is a completely different rule.

MISTAKE 2 | Assuming Tangency Without Verification

WRONG	Treating a line as tangent to a circle because it appears to 'just touch' the circle in the rough diagram.
CORRECT	Confirm tangency only when the perpendicular distance from the center to the line equals the radius, or when the problem explicitly states a single point of contact.

MISTAKE 3 | Ignoring the Obtuse-Triangle Case

WRONG	Placing the orthocenter or circumcenter inside the triangle by default, even when the given side lengths permit an obtuse configuration.
CORRECT	Check whether the square of the longest side exceeds the sum of squares of the other two sides; if so, the triangle is obtuse and both the orthocenter and circumcenter lie outside it.

MISTAKE 4 | Mixing Up Area Scale and Linear Scale

WRONG	Applying a given linear ratio (e.g. side ratio 2:3) directly to an area or volume comparison without squaring or cubing it.
CORRECT	Always convert the linear scale factor k to k^2 for area comparisons and k^3 for volume comparisons before answering.

MISTAKE 5 | Forcing Coordinate Geometry Onto a Pure Synthetic Problem

WRONG	Setting up a full coordinate system for a static angle-chasing problem with no coordinates given, burning several minutes on unnecessary algebra.
CORRECT	Reserve coordinate geometry for problems that already supply coordinates or an explicit locus condition; use circle theorems and triangle properties directly otherwise.

8 | QUICK REVISION CARD

Master Geometric Formula Checklist

- Area: Heron's, (1/2)·base·height, (1/2)ab·sinC, r·s
- Circle: $PA \cdot PB = PT^2$ (Power of a Point); angle at center = 2 × angle at circumference
- Apollonius: $AB^2+AC^2 = 2AD^2+2BD^2$ | Ptolemy: $AC \cdot BD = AB \cdot CD + AD \cdot BC$
- Coordinate: Section formula, Distance formula, Area = $(1/2)|\Sigma x_1(y_2-y_3)|$
- Scaling: length ×k → area ×k^2 → volume ×k^3

Critical Angle / Side Ratios

Triangle Type	Side Ratio	Notes
30°-60°-90°	1 : √3 : 2	Side opposite 30° is the shortest; hypotenuse is always the '2'
45°-45°-90°	1 : 1 : √2	Isosceles right triangle; diagonal of a square equals side × √2

Top 5 Geometry Traps — Summary

- Diagram Illusion — never trust apparent parallel/perpendicular/tangent lines
- Missing Case — check for the obtuse or mirrored configuration
- Boundary Overlap — re-derive the touching point, don't assume symmetry
- Centroid/Incenter Mix-up — only medians split 2:1
- Linear-vs-Area-vs-Volume scale confusion — square or cube the ratio correctly

Auxiliary Line Cheat Prompts

- Stuck on a circle problem? → Drop a perpendicular from the center to the chord in question.
- Stuck on a median/cevian problem? → Try Apollonius' or Stewart's Theorem directly.
- Stuck on a cyclic quadrilateral? → Draw both diagonals and test Ptolemy's Theorem.
- Stuck on a tangent-circle problem? → Join the external point to the center to form a right triangle with the radius.

Timing & Target Metrics

Question Type	Target Time	Target Accuracy
Conceptual / direct-formula questions	Under 45 sec	85%+
Multi-shape overlap questions	Under 90 sec	85%+