GUJCET 2026 – Mathematics

Questions with Full Solutions and Answers

Q1.

$$\int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx = \_\_\_\_\_\_\_\_\_\_ +C$$

(A) $-\dfrac{e^x}{1+x^2}$ (B) $\dfrac{e^x}{1+x^2}$ (C) $\dfrac{e^x}{(1+x^2)^2}$ (D) $\dfrac{e^x}{1+x}$

Solution: Use the standard result $\int e^x[f(x)+f'(x)]\,dx = e^x f(x)+C$. Take $f(x)=\dfrac{1}{1+x^2}$, so $f'(x)=\dfrac{-2x}{(1+x^2)^2}$.

Differentiate the candidate answer to check: $\dfrac{d}{dx}\left(\dfrac{e^x}{1+x^2}\right) = \dfrac{e^x(1+x^2)-e^x(2x)}{(1+x^2)^2} = \dfrac{e^x(1-x)^2}{(1+x^2)^2} = e^x\left(\dfrac{1-x}{1+x^2}\right)^2$.

This matches the integrand exactly, so

Answer: (B) $\dfrac{e^x}{1+x^2}$

Q2.

$$\int \frac{e^{2025+x}-e^{2025-x}}{e^{2026+x}+e^{2026-x}}\,dx = \_\_\_\_\_\_\_\_\_\_ +C$$

(A) $\log|e^x+e^{-x}|$ (B) $e\log|e^x+e^{-x}|$ (C) $\dfrac{1}{e}\log|e^x+e^{-x}|$ (D) $-\dfrac{1}{e}\log|e^x+e^{-x}|$

Solution: Factor: numerator $= e^{2025}(e^x-e^{-x})$, denominator $= e^{2026}(e^x+e^{-x})$.

So the integrand $= \dfrac{1}{e}\cdot\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$.

Let $u=e^x+e^{-x}$, so $du=(e^x-e^{-x})\,dx$. The integral becomes $\dfrac{1}{e}\displaystyle\int\frac{du}{u} = \frac{1}{e}\log|u|+C$.

Answer: (C) $\dfrac{1}{e}\log|e^x+e^{-x}|$

Q3.

Area lying in the first quadrant and bounded by ellipse $4x^2+9y^2=144$ is __________.

(A) 12π (B) 24π (C) 8π (D) 6π

Solution: Rewrite as $\dfrac{x^2}{36}+\dfrac{y^2}{16}=1$, so $a=6,\ b=4$.

Area of full ellipse $=\pi ab=24\pi$. The first-quadrant area is one quarter of the full ellipse: $\dfrac{24\pi}{4}=6\pi$.

Answer: (D) 6π

Q4.

The area bounded by the curve $y=x|x|$, the X-axis, and the ordinates $x=-1$ and $x=1$ is __________.

(A) $\dfrac{1}{3}$ (B) 0 (C) $\dfrac{2}{3}$ (D) $\dfrac{4}{3}$

Solution: For $x\geq 0$, $y=x^2$; for $x<0$, $y=-x^2$ (below the axis). Since the curve is symmetric (odd function), the total *area* (not signed area) is

$$2\int_0^1 x^2\,dx=2\left[\frac{x^3}{3}\right]_0^1=\frac{2}{3}.$$

Answer: (C) $\dfrac{2}{3}$

Q5.

The order and the degree of the differential equation

$$\sqrt{1+\left(\frac{d^2y}{dx^2}\right)^2}=\sqrt[3]{x+\left(\frac{dy}{dx}\right)^3}$$

is respectively ________ and ________.

(A) $3,2$ (B) $2,3$ (C) $1,6$ (D) $2,6$

Solution: The highest derivative present is $\dfrac{d^2y}{dx^2}$, so the **order** is 2.

To find the degree, remove the radicals (LCM of 2 and 3 is 6) by raising both sides to the 6th power:

$$\left[1+\left(\frac{d^2y}{dx^2}\right)^2\right]^3=\left[x+\left(\frac{dy}{dx}\right)^3\right]^2.$$

Now the highest-order derivative $\dfrac{d^2y}{dx^2}$ appears raised to the power $2\times 3=6$, so the **degree** is 6.

Answer: (D) Order $=2$, Degree $=6$

Q6.

The number of arbitrary constants in the particular solution of a differential equation of third order are __________.

(A) 2 (B) 3 (C) 1 (D) 0

Solution: A *particular* solution is obtained by assigning specific numerical values to all the arbitrary constants present in the general solution. Hence it contains no arbitrary constants at all, regardless of the order of the equation.

Answer: (D) 0

Q7.

The general solution of the differential equation $\dfrac{dy}{dx} = e^{x+y}$ is __________.

(A) $e^x + e^y = C$ (B) $e^x + e^{-y} = C$ (C) $e^{-x} + e^y = C$ (D) $e^{-x} + e^{-y} = C$

Solution: $\dfrac{dy}{dx} = e^x \cdot e^y \implies e^{-y}\,dy = e^x\,dx.$

Integrating: $-e^{-y} = e^x + C_1 \implies e^x + e^{-y} = C.$

Answer: (B) $e^x + e^{-y} = C$

Q8.

If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2,\ |\vec{b}| = 3$ and $\vec{a}\cdot\vec{b} = 4$, then $|\vec{a} - \vec{b}| =$ __________.

(A) 5 (B) $\sqrt{5}$ (C) 13 (D) $\sqrt{17}$

Solution: $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\,\vec{a}\cdot\vec{b} = 4 + 9 - 8 = 5.$

So $|\vec{a} - \vec{b}| = \sqrt{5}.$

Answer: (B) $\sqrt{5}$

Q9.

The area of the triangle with vertices $A(1,1,2)$, $B(2,3,5)$ and $C(1,5,5)$ is __________.

(A) $\sqrt{61}$ (B) $\sqrt{43}$ (C) $\dfrac{\sqrt{43}}{2}$ (D) $\dfrac{\sqrt{61}}{2}$

Solution: $\vec{AB} = (1,2,3),\ \vec{AC} = (0,4,3).$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = (2\cdot 3 - 3\cdot 4)\vec{i} - (1\cdot 3 - 3\cdot 0)\vec{j} + (1\cdot 4 - 2\cdot 0)\vec{k} = (-6,-3,4).$$

$|\vec{AB} \times \vec{AC}| = \sqrt{36 + 9 + 16} = \sqrt{61}$.

Area $= \dfrac{1}{2}\sqrt{61}$.

Answer: (D) $\dfrac{\sqrt{61}}{2}$

Q10.

The value of $\hat{i} \cdot (\hat{k} \times \hat{j}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{j} \times \hat{i})$ is __________.

(A) -1 (B) 0 (C) 1 (D) 3

Solution: Using $\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i},\ \hat{k} \times \hat{i} = \hat{j}$:

$\hat{k} \times \hat{j} = -\hat{i} \Rightarrow \hat{i} \cdot (-\hat{i}) = -1$

$\hat{k} \times \hat{i} = \hat{j} \Rightarrow \hat{j} \cdot \hat{j} = 1$

$\hat{j} \times \hat{i} = -\hat{k} \Rightarrow \hat{k} \cdot (-\hat{k}) = -1$

Sum $= -1 + 1 - 1 = -1$.

Answer: (A) -1

Q11.

The angle between the pair of lines given by $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ is __________.

(A) $\cos^{-1}\left(-\dfrac{19}{21}\right)$ (B) $\cos^{-1}\left(\dfrac{19}{21}\right)$ (C) $\sin^{-1}\left(\dfrac{19}{21}\right)$ (D) $\cos^{-1}\left(\dfrac{\sqrt{19}}{21}\right)$

Solution: Direction vectors: $\vec{d_1} = (1, 2, 2)$, $\vec{d_2} = (3, 2, 6)$.

$$\cos\theta = \frac{|\vec{d_1} \cdot \vec{d_2}|}{|\vec{d_1}||\vec{d_2}|} = \frac{|3 + 4 + 12|}{\sqrt{1 + 4 + 4} \cdot \sqrt{9 + 4 + 36}} = \frac{19}{3 \times 7} = \frac{19}{21}.$$

Answer: (B) $\cos^{-1}\left(\dfrac{19}{21}\right)$

Q12.

If the lines $\dfrac{1 - x}{3} = \dfrac{7y - 14}{2p} = \dfrac{3 - z}{-2}$ and $\dfrac{7 - 7x}{3p} = \dfrac{y - 5}{1} = \dfrac{6 - z}{5}$ are perpendicular, then the value of p is __________.

(A) $\dfrac{35}{11}$ (B) $\dfrac{11}{70}$ (C) $\dfrac{70}{11}$ (D) $-\dfrac{70}{11}$

Solution: Rewrite both lines in standard form $\dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}$.

Line 1: $\dfrac{1-x}{3}=\dfrac{x-1}{-3}$, $\dfrac{7y-14}{2p}=\dfrac{y-2}{2p/7}$, $\dfrac{3-z}{-2}=\dfrac{z-3}{2}$. So $(a_1,b_1,c_1)=\left(-3,\ \dfrac{2p}{7},\ 2\right)$.

Line 2: $\dfrac{7-7x}{3p}=\dfrac{x-1}{-3p/7}$, and $b_2=1$, $\dfrac{6-z}{5}=\dfrac{z-6}{-5}$. So $(a_2,b_2,c_2)=\left(-\dfrac{3p}{7},\ 1,\ -5\right)$.

Perpendicularity: $a_1a_2+b_1b_2+c_1c_2=0$:

$$(-3)\left(-\frac{3p}{7}\right)+\left(\frac{2p}{7}\right)(1)+(2)(-5)=0 \implies \frac{9p}{7}+\frac{2p}{7}-10=0 \implies \frac{11p}{7}=10.$$

$$p=\frac{70}{11}.$$

Answer: (C) $\dfrac{70}{11}$

Q13.

The vector equation of the line passing through the point $(1,2,-4)$ and perpendicular to the two lines $\dfrac{x-8}{3}=\dfrac{y+19}{-16}=\dfrac{z-10}{7}$ and $\dfrac{x-15}{3}=\dfrac{y-29}{8}=\dfrac{z-5}{-5}$ is __________.

(A) $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}-6\hat{k})$ (B) $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}-3\hat{j}+6\hat{k})$ (C) $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ (D) $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}-3\hat{j}-6\hat{k})$

Solution: The required direction is perpendicular to both given lines, so it is their cross product. Let $\vec{u}=(3,-16,7)$, $\vec{v}=(3,8,-5)$.

$$\vec{u}\times\vec{v}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\3&-16&7\\3&8&-5\end{vmatrix}=\big((-16)(-5)-7(8)\big)\vec{i}-\big(3(-5)-7(3)\big)\vec{j}+\big(3(8)-(-16)(3)\big)\vec{k}$$
$$=(80-56)\vec{i}-(-15-21)\vec{j}+(24+48)\vec{k}=(24,36,72)=12(2,3,6).$$

So the direction ratios are $(2,3,6)$, giving

$$\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k}).$$

Answer: (C) $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$

Q14.

The coordinates of the corner points of the bounded feasible region are $(0,10)$, $(5,5)$, $(15,15)$, $(0,20)$. The minimum of the objective function $z=3x+9y$ is __________.

(A) 90 (B) 180 (C) 30 (D) 60

Solution: Evaluate z at each corner point: $z(0,10) = 90, \quad z(5,5) = 60, \quad z(15,15) = 180, \quad z(0,20) = 180$.

The minimum value is 60.

Answer: (D) 60

Q15.

For a linear programming problem, the objective function is $z = px + qy$, $p, q > 0$. If at the corner points $(0,10)$ and $(5,5)$ the values of z are 90 and 60 respectively, then the relation between p and q is __________.

(A) $q = 3p$ (B) $p = 3q$ (C) $q = 2p$ (D) $p = 2q$

Solution: At $(0,10)$: $10q = 90 \Rightarrow q = 9$.

At $(5,5)$: $5p + 5q = 60 \Rightarrow p + q = 12 \Rightarrow p = 12 - 9 = 3$.

So $q = 9 = 3(3) = 3p$.

Answer: (A) $q = 3p$

Q16.

Let A and B be two events such that $P(A) = \dfrac{5}{11}$, $P(B) = \dfrac{2}{11}$ and $P(A \cup B) = \dfrac{3}{11}$, then $P(A' \mid B') =$ __________.

(A) $\dfrac{1}{2}$ (B) $\dfrac{8}{9}$ (C) $\dfrac{3}{5}$ (D) $\dfrac{2}{9}$

Solution: Using $P(A' \mid B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{P\big((A \cup B)'\big)}{P(B')} = \dfrac{1 - P(A \cup B)}{1 - P(B)}$:

$$P(A' \mid B') = \frac{1 - \frac{3}{11}}{1 - \frac{2}{11}} = \frac{\frac{8}{11}}{\frac{9}{11}} = \frac{8}{9}.$$

Answer: (B) $\dfrac{8}{9}$

Q17.

If A and B are any two events such that $P(A) + P(B) - P(A \cap B) = P(A)$ then __________.

(A) $P(A \mid B) = 1$ (B) $P(B \mid A) = 1$ (C) $P(B \mid A) = 0$ (D) $P(A \mid B) = 0$

Solution: From the given condition: $P(B) - P(A \cap B) = 0 \Rightarrow P(A \cap B) = P(B)$.

Then $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(B)}{P(B)} = 1$.

Answer: (A) $P(A \mid B) = 1$

Q18.

Three cards are drawn successively, without replacement, from a pack of 52 well-shuffled cards. The probability that the first two cards are kings and the third card drawn is an ace is ________.

(A) $\frac{3}{5525}$ (B) $\frac{1}{135200}$ (C) $\frac{2}{5525}$ (D) $\frac{3}{135200}$

Solution:

$$P = \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} = \frac{48}{132600} = \frac{2}{5525}.$$

Answer: (C) $\frac{2}{5525}$

Q19.

Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2,\ b < 6\}$ then ________.

(A) $(8, 3) \in R$ (B) $(6, 8) \in R$ (C) $(8, 7) \in R$ (D) $(2, 4) \in R$

Solution: Check $(2, 4)$: $a = 2, b = 4$. Is $a = b - 2$? $4 - 2 = 2 = a$ ✓. Is $b < 6$? $4 < 6$ ✓. Valid.

The other options fail either the equation or the inequality $b < 6$.

Answer: (D) $(2, 4) \in R$

Q20.

Let $f : N \to N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & n \text{ odd} \\ \frac{n}{2}, & n \text{ even} \end{cases}$ for all $n \in N$, then f is ________.

(A) One-one but not onto (B) One-one and onto (C) Many-one and onto (D) Neither one-one nor onto

Solution: $f(1) = \frac{1+1}{2} = 1$ and $f(2) = \frac{2}{2} = 1$, so $f(1) = f(2) \Rightarrow$ not one-one (many-one).

For any $m \in N$, taking $n = 2m$ (even) gives $f(n) = m$, so every $m \in N$ is attained $\Rightarrow$ onto.

Answer: (C) Many-one and onto

Q21.

If $\cos^{-1} x = y$ then ________.

(A) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (B) $0 \le y \le \pi$ (C) $0 < y < \pi$ (D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Solution: By definition, the principal value branch (range) of $\cos^{-1} x$ is $[0, \pi]$.

Answer: (B) $0 \le y \le \pi$

Q22.

$\sin^{-1}\left(\sin \frac{3\pi}{5}\right) =$ ________

(A) $\frac{2\pi}{5}$ (B) $\frac{\pi}{5}$ (C) $\frac{3\pi}{5}$ (D) $\frac{4\pi}{5}$

Solution: $\frac{3\pi}{5}$ lies outside the principal range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ of $\sin^{-1}$.

Since $\sin(\pi - \theta) = \sin\theta$: $\sin\frac{3\pi}{5} = \sin\left(\pi - \frac{3\pi}{5}\right) = \sin\frac{2\pi}{5}$, and $\frac{2\pi}{5}$ lies within the principal range.

So $\sin^{-1}\left(\sin\frac{3\pi}{5}\right) = \frac{2\pi}{5}$.

Answer: (A) $\frac{2\pi}{5}$

Q23.

$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] =$ ________.

(A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) $-\frac{3\pi}{4}$

Solution: $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$, so $2 \times \frac{\pi}{6} = \frac{\pi}{3}$.

$\cos\frac{\pi}{3} = \frac{1}{2}$, so $2 \times \frac{1}{2} = 1$.

$\tan^{-1}(1) = \frac{\pi}{4}$.

Answer: (B) $\frac{\pi}{4}$

Q24.

If $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is such that $A^2 = I$, then ________.

(A) $1 - a^2 + bc = 0$ (B) $1 + a^2 + bc = 0$ (C) $1 - a^2 - bc = 0$ (D) $1 + a^2 - bc = 0$

Solution:

$$A^2 = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}\begin{bmatrix} a & b \\ c & -a \end{bmatrix} = \begin{bmatrix} a^2+bc & 0 \\ 0 & a^2+bc \end{bmatrix} = (a^2+bc)\,I.$$

Setting $A^2 = I$ gives $a^2 + bc = 1$, i.e., $1 - a^2 - bc = 0$.

Answer: (C) $1 - a^2 - bc = 0$

Q25.

If A and B are skew-symmetric matrices of the same order, then $AB - BA$ is a __________.
(A) Symmetric matrix (B) Skew-symmetric matrix (C) Zero matrix (D) Identity matrix
Solution: Since $A^T = -A$ and $B^T = -B$:

$$(AB - BA)^T = B^T A^T - A^T B^T = (-B)(-A) - (-A)(-B) = BA - AB = -(AB - BA).$$

So $AB - BA$ is skew-symmetric.

Answer: (B) Skew-symmetric matrix

Q26.

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, then $A^2 + I =$ __________.
(A) $A - I$ (B) $A - 2I$ (C) $A + I$ (D) $I - A$
Solution:

$$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}.$$

$$A^2 + I = \begin{bmatrix} 2 & -2 \\ 4 & -3 \end{bmatrix}, \qquad A - I = \begin{bmatrix} 2 & -2 \\ 4 & -3 \end{bmatrix}.$$

These are equal.

Answer: (A) $A - I$

Q27.

If area of a triangle is 35 sq. units with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$, then k is __________.
(A) -2 (B) 12 (C) $-12, -2$ (D) $12, -2$
Solution:

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|2(4-4) + 5(4 - (-6)) + k(-6-4)\right| = \frac{1}{2}|50 - 10k|.$$

Setting this equal to 35: $|50 - 10k| = 70$.
Case 1: $50 - 10k = 70 \Rightarrow k = -2$. Case 2: $50 - 10k = -70 \Rightarrow k = 12$.

Answer: (D) $12, -2$

Q28.

If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$, then $A^2 + B^2 =$ __________.

Solution: For diagonal matrices, squares are found by squaring each diagonal entry:

$$A^2 = \text{diag}(4, 9, 16), \qquad B^2 = \text{diag}(1, 4, 9).$$

$$A^2 + B^2 = \text{diag}(5, 13, 25) = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 25 \end{bmatrix}.$$

Answer: $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 25 \end{bmatrix}$

Q29.

If the inverse matrix of $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} a & \frac{3}{11} \\ \frac{1}{11} & b \end{bmatrix}$, then $a + b =$ __________.

(A) $-\dfrac{2}{11}$ (B) $\dfrac{2}{11}$ (C) $\dfrac{6}{11}$ (D) $-\dfrac{6}{11}$

Solution: $\det A = 2(-4) - 3(1) = -11$.

$$A^{-1} = \frac{1}{-11} \begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{1}{11} & -\frac{2}{11} \end{bmatrix}.$$

So $a = \dfrac{4}{11}$, $b = -\dfrac{2}{11}$, giving $a + b = \dfrac{2}{11}$.

Answer: (B) $\dfrac{2}{11}$

Q30.

If the function f is continuous at point $x = \pi$ and $f(x) = \begin{cases} kx + 1, & x \leq \pi \\ \cos x, & x > \pi \end{cases}$, then the value of k is __________.

(A) $\dfrac{1}{\pi}$ (B) $\dfrac{2}{\pi}$ (C) $-\dfrac{2}{\pi}$ (D) 0

Solution: For continuity at $x = \pi$: $k\pi + 1 = \cos\pi = -1 \Rightarrow k\pi = -2 \Rightarrow k = -\dfrac{2}{\pi}$.

Answer: (C) $-\dfrac{2}{\pi}$

Q31.

If $x = at^2$, $y = 2at$ then $\dfrac{d^2y}{dx^2} =$ ____________.

(A) $-\dfrac{a}{xy}$ (B) $\dfrac{a}{xy}$ (C) $\dfrac{ax}{y}$ (D) $-\dfrac{ax}{y}$

Solution: $\dfrac{dy}{dt} = 2a$, $\dfrac{dx}{dt} = 2at$, so $\dfrac{dy}{dx} = \dfrac{1}{t}$.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{t}\right) = \frac{-1/t^2}{2at} = -\frac{1}{2at^3}.$$

Since $t = \dfrac{y}{2a}$ and $t^2 = \dfrac{x}{a}$: $t^3 = \dfrac{y}{2a} \cdot \dfrac{x}{a} = \dfrac{xy}{2a^2}$.

$$\frac{d^2y}{dx^2} = -\frac{1}{2a \cdot \frac{xy}{2a^2}} = -\frac{a}{xy}.$$

Answer: (A) $-\dfrac{a}{xy}$

Q32.

If $y = \log_{2026}(\log_{2025} x)$ then $\dfrac{dy}{dx} =$ ____________.

(A) $\dfrac{1}{2025\,x\log x}$ (B) $\dfrac{1}{x\log x\log 2025}$ (C) $\dfrac{1}{x\log x\log 2026}$ (D) $\dfrac{1}{2026\,x\log x}$

Solution: Write $y = \dfrac{\ln(\log_{2025} x)}{\ln 2026} = \dfrac{\ln\left(\frac{\ln x}{\ln 2025}\right)}{\ln 2026}$.

$$\frac{dy}{dx} = \frac{1}{\ln 2026} \cdot \frac{1}{\frac{\ln x}{\ln 2025}} \cdot \frac{1}{\ln 2025} \cdot \frac{1}{x} = \frac{1}{x\ln x\,\ln 2026}.$$

Answer: (C) $\dfrac{1}{x\log x\log 2026}$

Q33.

If $e^y(x+1) = 1$ then $\dfrac{d^2y}{dx^2} - \left(\dfrac{dy}{dx}\right)^2 =$ ____________.

(A) $-\dfrac{1}{x+1}$ (B) e^y (C) $\dfrac{1}{x+1}$ (D) 0

Solution: $e^y = \dfrac{1}{x+1} \Rightarrow y = -\ln(x+1)$.

$$\frac{dy}{dx} = -\frac{1}{x+1}, \qquad \frac{d^2y}{dx^2} = \frac{1}{(x+1)^2}.$$

$$\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = \frac{1}{(x+1)^2} - \frac{1}{(x+1)^2} = 0.$$

Answer: (D) 0

Q34.

The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$ is __________.

(A) 96 (B) 116 (C) 90 (D) 126

Solution: Marginal revenue $= R'(x) = 6x + 36$.

At $x = 15$: $R'(15) = 90 + 36 = 126$.

Answer: (D) 126

Q35.

The maximum value of the function $f(x) = -|x+1| + 3,\ x \in R$ is __________.

(A) -2 (B) 2 (C) 3 (D) 4

Solution: Since $|x+1| \geq 0$ always, $f(x)$ is maximized when $|x+1| = 0$, i.e. $x = -1$.

Maximum value $= -0 + 3 = 3$.

Answer: (C) 3

Q36.

The interval in which $y = x^2e^{-x}$ is decreasing is __________.

(A) $(-2, 0)$ (B) $(-\infty, \infty)$ (C) $(2, \infty)$ (D) $(0, 2)$

Solution: $y' = 2xe^{-x} - x^2e^{-x} = e^{-x}x(2-x)$.

Since $e^{-x} > 0$ always, the sign of y' matches the sign of $x(2-x)$, which is negative when $x > 2$ (and also when $x < 0$). Among the given options, the decreasing interval is:

Answer: (C) $(2, \infty)$

Q37.

$$\int \sec^2 x \cdot \text{cosec}^2 x \cdot dx = \_\_\_\_\_\_\_\_\_\_ +C$$

(A) $\tan x - \cot x$ (B) $\tan x + \cot x$ (C) $\tan x \cdot \cot x$ (D) $\tan x - \cot 2x$

Solution: $\sec^2 x \cdot \text{cosec}^2 x = \dfrac{1}{\sin^2 x \cos^2 x} = \dfrac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} = \sec^2 x + \text{cosec}^2 x.$

$$\int (\sec^2 x + \text{cosec}^2 x)\, dx = \tan x - \cot x + C.$$

Answer: (A) $\tan x - \cot x$

Q38.

$$\int \frac{dx}{\sqrt{9x - 4x^2}} = \_\_\_\_\_\_\_\_\_\_ +C$$

(A) $\dfrac{1}{2} \sin^{-1}\left(\dfrac{8x-9}{9}\right)$ (B) $\dfrac{1}{9} \sin^{-1}\left(\dfrac{9x-8}{8}\right)$ (C) $\dfrac{1}{3} \sin^{-1}\left(\dfrac{9x-8}{8}\right)$ (D) $\dfrac{1}{2} \sin^{-1}\left(\dfrac{9x-8}{9}\right)$

Solution: Complete the square: $9x - 4x^2 = \dfrac{81}{16} - 4\left(x - \dfrac{9}{8}\right)^2 = 4\left[\left(\dfrac{9}{8}\right)^2 - \left(x - \dfrac{9}{8}\right)^2\right].$

$$\int \frac{dx}{2\sqrt{\left(\frac{9}{8}\right)^2 - \left(x - \frac{9}{8}\right)^2}} = \frac{1}{2} \sin^{-1}\left(\frac{x - \frac{9}{8}}{\frac{9}{8}}\right) + C = \frac{1}{2} \sin^{-1}\left(\frac{8x-9}{9}\right) + C.$$

Answer: (A) $\dfrac{1}{2} \sin^{-1}\left(\dfrac{8x-9}{9}\right)$

Q39.

$$\int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}\right) dx = \_\_\_\_\_\_\_\_\_\_$$

(A) 1 (B) 0 (C) -1 (D) 2

Solution: Using $\cos x = \cos^2 \dfrac{x}{2} - \sin^2 \dfrac{x}{2}$, the integrand equals $-\cos x$.

$$\int_0^{\pi} (-\cos x)\, dx = -\big[\sin x\big]_0^{\pi} = -(0 - 0) = 0.$$

Answer: (B) 0

Q40.

$\displaystyle\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}} =$ __________

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) 0 (D) 1

Solution: Let $I = \displaystyle\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$. Using $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ with $a+b = \frac{\pi}{2}$, and $\cot\left(\frac{\pi}{2}-x\right) = \tan x$:

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}.$$

Adding the two expressions for I:

$$2I = \int_{\pi/6}^{\pi/3} \left[\frac{1}{1+\sqrt{\cot x}} + \frac{1}{1+\sqrt{\tan x}}\right] dx.$$

Since $\sqrt{\cot x} = \frac{1}{\sqrt{\tan x}}$, the first term becomes $\frac{\sqrt{\tan x}}{\sqrt{\tan x}+1}$, so the bracket simplifies to 1.

$$2I = \int_{\pi/6}^{\pi/3} 1\,dx = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \implies I = \frac{\pi}{12}.$$

Answer: (A) $\frac{\pi}{12}$